# Elastic Lepton-Proton Scattering and Higher-Order QED Effects

Andrei Afanasev

The George Washington University, Washington, DC, USA



## Plan of talk

#### **Radiative corrections for charged lepton scattering**

. Model-independent and model-dependent; soft and hard photons

#### **Two-photon exchange effects**

- . Soft-photon exchange approximation and IR regularization
- . Novel effects in muon scattering
- . Single-spin asymmetries from two-photon exchange

#### Summary





## Basic Approaches to QED Corrections

- L.W. Mo, Y.S. Tsai, Rev. Mod. Phys. 41, 205 (1969); Y.S. Tsai, Preprint SLAC-PUB-848 (1971).
  - . Considered both elastic and inelastic inclusive cases. No polarization.
- D.Yu. Bardin, N.M. Shumeiko, Nucl. Phys. B127, 242 (1977).
  - Covariant approach to the IR problem. Later extended to inclusive, semiexclusive and exclusive reactions with polarization.
- E.A. Kuraev, V.S. Fadin, Yad.Fiz. 41, 7333 (1985); E.A. Kuraev, N.P.Merenkov, V.S. Fadin, Yad. Fiz. 47, 1593 (1988).
  - Developed a method of <u>electron structure functions</u> based on Drell-Yan representation; currently widely used at e<sup>+</sup>e<sup>-</sup> colliders
  - Applied for polarized electron-proton scattering by AA et al, JETP 98, 403 (2004).



# Complete radiative correction in $O(\alpha_{em})$



Radiative Corrections:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and VCS (g,h)
- Corrections (e-h) depend on the nucleon structure

•Meister&Yennie; Mo&Tsai

•Further work by Bardin&Shumeiko;

Maximon&Tjon; AA, Akushevich, Merenkov;

•Guichon&Vanderhaeghen' 03: Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3% ...

#### Main issue: Corrections dependent on nucleon structure

Model calculations:

- •Blunden, Melnitchouk, Tjon, Phys.Rev.Lett.91:142304,2003
- •Chen, AA, Brodsky, Carlson, Vanderhaeghen, Phys.Rev.Lett.93:122301,2004



# Bremsstrahlung for Relativistic vs Nonrelativistic Lepton Scattering

- Accelerated charge always radiates, but the magnitude of the effect depends on kinematics
- . See Bjorken&Drell (Vol.1, Ch.8):
  - . For large  $Q^2 >> m_e^2$  the rad.correction is enhanced by a large logarithm,  $\log(Q^2/m_e^2) \sim 15$  for GeV<sup>2</sup> momentum transfers
  - . For small  $Q^2 \ll m_e^2$ , rad.correction suppressed by  $Q^2/m_e^2$
  - . For intermediate  $Q^2 \sim m_e^2$ , neither enhancement nor suppression, rad correction of the order  $2\alpha/\pi$
- Implications for COMPASS @CERN: rad. corrections reduce for  $log(Q^2/m_{\mu}^2) \sim 3$  by about a factor of 5 compared to electrons (*good news!*) and become comparable in magnitude to two-photon effects (*bad news!*)



# Separating soft 2-photon exchange

- Tsai; Maximon & Tjon ( $k \rightarrow 0$ ); similar to Coulomb corrections at low Q<sup>2</sup>
- . Grammer & Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- . Shown is the resulting (soft) QED correction to cross section
- . <u>Already included in experimental data analysis for elastic ep</u>
  - Also done for pion electroproduction in AA, Aleksejevs, Barkanova, Phys.Rev. D88 (2013) 5, 053008 (inclusion of lepton masses is straightforward)





Lepton mass is not essential for TPE calculation in ultra-relativistic case; Two-photon effect below 1% for lower energies and  $Q^2 < 0.1 GeV^2$ 

# Calculations using Generalized Parton Distributions



Model schematics:

• Hard eq-interaction

•GPDs describe quark emission/ absorption

•Soft/hard separation

•Use Grammer-Yennie prescription

Hard interaction with a quark

AA, Brodsky, Carlson, Chen, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004; Phys.Rev.D**72**:013008,2005



### Updated Ge/Gm plot

AA, Brodsky, Carlson, Chen, Vanderhaeghen, Phys.Rev.Lett.93:122301, 2004; Phys.Rev.D72:013008, 2005 Review: Carlson, Vanderhaeghen, Ann.Rev.Nucl.Part.Sci. 57 (2007) 171-204



- Significant part of the discrepancy is removed by the TPE mechanism
- Verification coming from
  - VEPP: PRL 114 (2015) 6, 062005
  - CLAS 114 (2015) 6, 062003
  - OLYMPUS (coming 2015)



#### Hard Bremsstrahlung

. Need to include radiative lepton tensor in a complete form: AA et al, **Phys.Rev. D64 (2001) 113009; PLB 514, 269 (2001)**: terms ~ k emitted photon momentum) usually neglected in rad.correction calculations, but can lead to ~1% effect for Rosenbluth slope at high  $Q^2$ 

$$L^{r}_{\mu\nu} = -\frac{1}{2}Tr(\hat{k}_{2} + m)\Gamma_{\mu\alpha}(1 + \gamma_{5}\hat{\xi}_{e})(\hat{k}_{1} + m)\overline{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k \cdot k_{1}} - \frac{k_{2\alpha}}{k \cdot k_{2}}\right)\gamma_{\mu} - \frac{\gamma_{\mu}\hat{k}\gamma_{\alpha}}{2k \cdot k_{1}} - \frac{\gamma_{\alpha}\hat{k}\gamma_{\mu}}{2k \cdot k_{2}}\right)$$

$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k \cdot k_{1}} - \frac{k_{2\alpha}}{k \cdot k_{2}}\right)\gamma_{\nu} - \frac{\gamma_{\nu}\hat{k}\gamma_{\alpha}}{2k \cdot k_{1}} - \frac{\gamma_{\nu}\hat{k}\gamma_{\alpha}}{2k \cdot k_{2}}\right)$$
additional terms, about 1% effect  
THE GEORGE  
WASHINGTON  
UNIVERSITY  
Andrei Afanasev, Intense Electron Beams Workshop, Cornell University, 6/17/2015

WASHINGTON, DC

### Coulomb and Two-Photon Corrections

- . Coulomb correction calculations are well justified at lower energies and Q2
- Hard two-photon exchange (TPE) contributions cannot be calculated with the same level of precision as the other contributions.
- . Two-photon exchange is independent on the lepton mass in an ultrarelativistic case.
- . <u>Issue:</u> For energies ~ mass TPE amplitude is described by 6 independent generalized form factors; but experimental data on TPE are for ultrarelativistic electrons, hence independent info on 3 other form factors will be missing.
- Theoretical models show the trend that TPE has a smaller effect at lower  $Q^2$ . The reason is that "hard" TPE amplitudes do not have a  $1/Q^2$  Coulomb singularity, as opposed to the Born amplitude.



#### Lepton Mass Effects

- Standard approximations keep the lepton mass in the logarithms but neglect it in power terms. May be justified in the ultrarelativistic case and  $Q^2 >> (lepton mass)^2$
- . Most of analysis codes use exact mass dependence for hard brem, but use above approximations for the "soft" part of brem correction
- Revised approach is required that will NOT result in new theoretical uncertainties
- . New rad.correction codes no longer use peaking approximation (justified for relatively small lepton masses)
- Formalism and Monte-Carlo generators can be adapted for this analysis (ELRADGEN; MASCARAD, etc;

more on <a href="https://www.jlab.org/RC">www.jlab.org/RC</a>); HAPRAD for SIDIS of muons



#### **ELRADGEN Results for 100MeV-beams**

MUSE: Proposed experiment at PSI to measure proton charge radius in elastic scattering of muons, arXiv:1303.2160

Ilyichev (Minsk) and AA: updated ELRADGEN Monte Carlo (Afanasev et al., Czech. J. Phys. 53 (2003) B449; Akushevich et al., Comput. Phys. Commun. 183 (2012) 1448) to include (a) mass effects and (b) two-photon effects (c) hard brem included



Left: Radiative correction for elastic electron-proton scattering as a function of lab scattering angle in MUSE kinematics. Dashed lines show the effect of a kinematic cut. Right: Same result but for the scattering of muons.



#### C-odd Effects in ELRADGEN

- Order- $\alpha$  corrections due to (a) two-photon exchange and (b) lepton-hadron brem interference for opposite-sign leptons are also opposite in sign
  - ELRADGEN included TPE (soft photons only) and brem interference), predicted charge asymmetry in JLAB CLAS kinematics (electrons)





Andrei Afanasev, Intense Electron Beams Workshop, Cornell University, 6/17/2015

# Helicity amplitudes for $\mu p$ elastic scattering

- Total of 6 amplitudes:
  - . 3 helicity-conserving, 3 helicity flip
  - . Helicity-flip amplitudes neglected in ultra-relativistic  $E\mu >> m\mu$
  - Exception: single-spin beam asymmetries caused by interference of helicity-conserving and helicity-flip
- For muon scattering at ~100 MeV ultra-relativistic approximation no longer applies
- Model-independent analysis of two-photon exchange requires to fit amplitudes



#### Elastic contribution to TPE

TPE for elastic mu-p scattering calculated by Tomalak&Vanderhaeghen, PRD 90 (2014) 013006; included only elastic intermediate state described by form factors



FIG. 4: TPE correction to the unpolarized elastic  $\mu^- p$  cross section for three different muon beam momenta. The total correction is shown by the black solid curves, the contribution from the F1F1 structure of photon-proton-proton vertices is shown by the red dashed curves, the contribution from the F1F2 structure by the green dashed-dotted curves, and the contribution from the F2F2 structure by the blue dotted curves.

WASHINGTON UNIVERSITY WASHINGTON, DC

THE GEORGE

#### Helicity-Flip in TPE; estimate of inelastic contribution

- . New dynamics from scalars ( $\sigma$ , f-mesons). No pseudo-scalar contribution for unpolarized particles
- . Scalar t-channel exchange contributes to TPE (no longer setting m<sub>lepton</sub>



. No information on  $F_{\sigma\mu\mu}$  is available. Need model estimates. From sigma-pole contribution to nucleon polarizability, we estimate for Q<sup>2</sup>=0.01 GeV<sup>2</sup>  $\delta_{\sigma}^{2\gamma}$  is about 10<sup>-4</sup>, lepton helicity-flip is important, scales as  $\sqrt{\tau}$ ,  $\tau = Q^2 / 4M_N^2$ Can be studied directly in the ratio of  $\mu$ + and  $\mu$ - cross sections UNIVERSITY Andrei Afanasey. Intense Electron Beams Workshop, Cornell University, 6/17/2015

WASHINGTON, DC

### Conclusions

#### MUSE:

. The effort on the radiative corrections aims at proper accounting of the radiative effects, that appear to show significant difference between electron and muon scattering

- Radiative corrections shown to be <1% for muons; included in MUSE analysis
- . Two-photon effects can be studied directly in the ratio of  $\mu$ + and  $\mu$ cross sections



### Single-Spin Asymmetries in Elastic Scattering

#### Parity-conserving

. Observed spin-momentum correlation of the type:

$$\vec{s} \cdot \vec{k}_1 \times \vec{k}_2$$

where  $k_{1,2}$  are initial and final electron momenta, *s* is a polarization vector of a target OR beam

• For elastic scattering asymmetries are due to *absorptive part* of 2-photon exchange amplitude

Parity-Violating

$$\vec{s} \cdot \vec{k_1}$$



#### Normal Beam Asymmetry in Moller Scattering

Pure QED process,  $e^++e^- \rightarrow e^-+e^-$ 

- . Barut, Fronsdal , Phys.Rev.120:1871 (1960): Calculated the asymmetry in first non-vanishing order in QED  $O(\alpha)$
- Dixon, Schreiber, Phys.Rev.D69:113001,2004, Erratumibid.D71:059903,2005: Calculated O(α) correction to the asymmetry







#### Single-Spin Target Asymmetry

$$\vec{s}_T \cdot \vec{k}_1 \times \vec{k}_2$$

De Rujula, Kaplan, De Rafael, Nucl.Phys. B53, 545 (1973): Transverse polarization effect is due to the absorptive part of the non-forward Compton amplitude for off-shell photons scattering from nucleons See also AA, Akushevich, Merenkov, hep-ph/0208260





Figure 2. Integration region over  $Q_1^2$  and  $Q_2^2$  in Eq.(2) for elastic  $(W^2 = M^2)$  and inelastic contributions. The latter (left) is given for  $Q^2=4$  GeV<sup>2</sup> and two values of  $W^2$ , which is an integration variable in this case. The elastic case is shown on the right as a function of external  $Q^2$ . The electron beam energy is  $E_b = 5$  GeV.

Andrei Afanasev, Intense Electron Beams Workshop, Cornell University, 6/17/2015

#### WASHINGTON, DC

#### Quark+Nucleon Contributions to Target Asymmetry

- Single-spin asymmetry or polarization normal to the scattering plane •
- Handbag mechanism prediction for single-spin asymmetry of elastic eN-scattering on a polarized nucleon target (AA, Brodsky, Carlson, Chen, Vanderhaeghen)

$$A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}} \frac{1}{\sigma_R} \left[ G_E \operatorname{Im}(A) - \sqrt{\frac{1+\varepsilon}{2\varepsilon}} G_M \operatorname{Im}(B) \right]$$

*Only minor role of quark mass* 



Andrei Afanasev, Intense Electron Beams Workshop, Cornell University, 6/17/2015

WASHINGTON, DC

#### Single-spin Asymmetries at JLAB

- Polarized target (He3) JLAB E-05-015 (arXiv:1502.02636)
- . Recoil polarimetry (proton)





## Single-Spin Asymmetry in Elastic Scattering Early Calculations

*Spin-orbit interaction of electron moving in a Coulomb field* 

*Need in spin-flip and spin-nonflip+phase difference* 

- N.F. Mott, Proc. Roy. Soc. London, Set. A **135**, 429 (1932);
- Interference of one-photon and twophoton exchange Feynman diagrams in electron-muon scattering: Barut, Fronsdal, Phys.Rev.120, 1871 (1960)
- *Extended to quark-quark scattering SSA in pQCD*: Kane, Pumplin, Repko, Phys.Rev.Lett. 41, 1689 (1978)

$$A_n \propto \frac{\alpha \cdot m_e \cdot \theta^3}{E}$$
, for  $\theta \ll 1$   
(small – angle scattering)

 $\Delta(\vartheta) = \pm 2Z\alpha \frac{v\sqrt{1-v^2}}{1-v^2\sin^2(\vartheta/2)} \frac{\sin^3(\vartheta/2)}{\cos(\vartheta/2)} \ln \frac{1}{\sin(\vartheta/2)}.$ 





# Proton Mott Asymmetry at Higher Energies

- Asymmetry due to absorptive part of two-photon exchange amplitude; shown is elastic intermediate state contribution
- Nonzero effect first observed by SAMPLE Collaboration (S.Wells et al., PRC63:064001,2001) for 200 MeV electrons

THE.GEORGE alculated by Diaconescu&Ramsey-Musolf (2004); used low-momentum WASHING TOTS ion, questionable in SAMPLE kinematics

•

•

# Phase Space Contributing to the absorptive part of $2\gamma$ -exchange amplitude

- 2-dimensional integration  $(Q_1^2, Q_2^2)$  for the elastic intermediate state
- 3-dimensional integration  $(Q_1^2, Q_2^2, W^2)$  for inelastic excitations



#### Andrei Afanasev, Intense Electron Beams Workshop, Cornell University, 6/17/2015

WASHINGTON, DC

UNIVERSITY

#### MAMI data on Mott Asymmetry





However, it doesn't make it into TPE for Rosenbluth

Andrei Afanasev, Intense Electron Beams Workshop, Cornell University, 6/17/2015

WASHINGTON, DC

#### Special property of Mott asymmetry

•Mott asymmetry above the nucleon resonance region

(a) does not decrease with beam energy

(b) is enhanced by large logs

(AA, Merenkov, PL B599 (2004)48; hep-ph/0407167v2 (erratum))
•Reason for the unexpected behavior: exchange of hard collinear quasireal photons and diffractive mechanism of nucleon Compton scattering

•For s>>-t and above the resonance region, the asymmetry is given by:

$$A_{n}^{e}(diffractive) = \sigma_{\gamma p} \frac{(-m_{e})\sqrt{Q^{2}}}{8\pi^{2}} \cdot \frac{F_{1} - \tau F_{2}}{F_{1}^{2} + \tau F_{2}^{2}} (\log(\frac{Q^{2}}{m_{e}^{2}}) - 2) \cdot Exp(-bQ^{2})$$

Compare with asymmetry caused by Coulomb distortion at small  $\theta =>$  may differ by orders of magnitude depending on scattering kinematics

$$A_n^e(Coulomb) \propto \alpha \frac{m_e}{\sqrt{s}} \theta^3 \rightarrow A_n^e(Diffractive) \propto \alpha m_e(\sqrt{s}) \theta \cdot R_{int}^2$$

THE GEORGE WASHINGTON UNIVERSITY WASHINGTON, DC

#### Input parameters

For small-angle (-t/s<<1) scattering of electrons with energies Ee, normal beam asymmetry is given by the energy-weighted integral

$$A_n \propto \frac{1}{E_e^2} \int_{v_{th}}^{E_e} dv \cdot v \sigma_{\gamma p}^{tot}(v; q_{1,2}^2 \approx 0)$$



<u>The integral is energy-weighed,</u> <u>higher energies enhanced</u>

 $\sigma_{\gamma p}$  from N. Bianchi at al., Phys.Rev.C54 (1996)1688 (resonance region) and Block&Halzen,

Phys.Rev. D70 (2004) 091901

-A<sub>n</sub> serves as an ideal tool to sum over a variety of intermediate states



#### Predictions vs experiment for Mott asymmetry

Use fit to experimental data on  $\sigma_{\gamma p}$  (dotted lines include only one-pion +nucleon intermediate states)



Andrei Afanasev, Intense Electron Beams Workshop, Cornell University, 6/17/2015

WASHINGTON, DC

# Predict no suppression for Mott asymmetry with energy at fixed $Q^2$



#### Comparison with E158 data



. SLAC E158:

An=-2.89±0.36(stat)±0.17(syst) ppm

(K. Kumar, private communication)

• Theory (AA, Merenkov): An=-3.2ppm

• Good agreement justifies application of this approach to the real part of two-boson exchange ( $\gamma Z$  box)



### Mott Asymmetry on Nuclei

Important systematic correction for parity-violation experiments (~-10ppm for HAPPEX on <sup>4</sup>He, ~-5ppm for PREX on Pb,), *see AA arXiv:0711.3065 [hep-ph]* ; also Gorchtein, Horowitz, Phys.Rev.C77:044606,2008

Coulomb distortion: only10<sup>-10</sup> effect (Cooper&Horowitz, Phys.Rev.C72:034602,2005)

![](_page_32_Figure_3.jpeg)

**Five orders of magnitude** enhancement in HAPPEX kinematics due to excitation of inelastic intermediate states in 2γ-exchange (AA, Merenkov; use Compton data from Erevan ) THE GEORGE WASHINGTON UNIVERSITY

# Transverse Beam Asymmetries on Nuclei (HAPPEX+PREX)

Abrahamyan et al, Phys.Rev.Lett. 109 (2012) 192501

- . Good agreement with theory for nucleon and light nuclei
- Puzzling disagreement for <sup>208</sup>Pb measurement; if confirmed, need to include additional electron interaction with highly excited intermediate nuclear state, magnetic terms, etc (= effects of higher order in  $\alpha_{em}$ ). Interesting nuclear effect! Experimentally, need additional measurements for intermediate-mass targets (e.g., Al, Ca, Fe)

![](_page_33_Figure_4.jpeg)

Target	Н	$^{4}\mathrm{He}$	<sup>12</sup> C	<sup>208</sup> Pb
$A_{\rm n}({\rm ppm})$	-6.80	-13.97	-6.49	0.28
$\sigma(A_{\rm n})({\rm ppm})$	$\pm 1.54$	$\pm 1.45$	$\pm 0.38$	$\pm 0.25$
$\sqrt{Q^2}$ (GeV)	0.31	0.28	0.099	0.094
A/Z	1.0	2.0	2.0	2.53
$\hat{A}_{n} (ppm/GeV)$	-21.9	-24.9	-32.8	+1.2
$\sigma(\hat{A}_{\rm n})({\rm ppm/GeV})$	$\pm 5.0$	$\pm 2.6$	$\pm 1.9$	$\pm 1.1$

### Inclusive Electroproduction of Pions

$$\vec{s}_e \cdot \vec{k}_e imes \vec{k}_{\pi}$$

- . Reaction  $p(e_{pol},\pi)X$ 
  - . Parity-conserving spin-momentum correlation
  - . Introduced in Donnelly, Raskin, Annals Phys. 169, 247 (1986)
    - . Can be shown to be a) due to  $R_{TL}$ , response function (=fifth structure function) and b) not to integrate to zero after integration over momenta of the scattered electron
  - This is NOT a two-photon exchange effect (but suppressed by an electron mass)
    - . Order-of magnitude estimate: An(ep-> $\pi X$ )~ A<sub>LT'</sub>(ep->e'  $\pi N$ )\*m<sub>e</sub>/E'/sin( $\theta_e$ )
      - Use MAMI data  $A_{LT'}(ep->e'\pi N)\sim7\%$ , from Bartsch et al Phys.Rev.Lett. 88:142001,2002 => An(ep->\pi X)~250ppm
  - Physics probe of (strong) final-state interactions in electroproduction reactions
    - . Why not simply measuring SF in A( $e_{pol}$ , $e\pi$ )X directly with
      - longitudinal polarization? Because transverse SSA gives access to very low Q<sup>2</sup>, may not available to spectrometers Andrei Afanasey, Intense Electron Beams Workshop, Cornell University, 6/17/2015

![](_page_34_Picture_12.jpeg)

WASHINGTON, DC

# Summary: SSA in Elastic ep- and eA-Scattering

- VCS amplitude in *beam asymmetry* is enhanced in different kinematic regions compared to *target asymmetry* or corrections to *Rosenbluth cross section*
- Physics probe of an absorptive part of a non-forward Compton amplitude
- . Important systematic effect for PREX,  $Q_{weak}$
- Mott asymmetry in small-angle ep-scattering above the pion threshold is controlled by quasi-real photoproduction cross section with photon energy approximately matching beam energy – similarity with Weizsacker-Williams Approximation – collinear photon exchange
- Due to excitation of inelastic intermediate states  $A_n$  is

(a) not suppressed with beam energy and

(b) does not grow with Z (proportional to instead A/Z) (c) At small angles  $\sim \theta$  (vs  $\theta^3$  for Coulomb distortion)

THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

. Confirmed experimentally for a wide range of beam energies

#### Outlook

- . Beam and target SSA for elastic electron scattering probe imaginary part of virtual Compton amplitude.
  - . Beam SSA: target helicity flip<sup>2</sup>+nonflip<sup>2</sup>
  - . Target SSA: Im[target helicity flip\*nonflip]
  - . Ideal " $4\pi$  detector" to probe electroproduction amplitudes for a variety of final states ( $\pi$ ,  $2\pi$ , etc)
- Beam SSA for nuclear targets in good agreement with theory except for a high-Z target 208Pb. Interesting nuclear physics effects beyond two-photon exchange
- . Beam SSA in Reaction  $A(e_{pol},\pi)X$  probes strong final-state interactions due to "fifth stucture function"

in A(e,e'  $\pi$ )X

![](_page_36_Picture_8.jpeg)

#### Physics Opportunities with High Intensity Beams

- . High intensities allow measurements with high statistical accuracy
- QED corrections limit interpretation of electron scattering measurements in terms of one-photon exchange quantities (eg, form factors)
- . Systematics from high-order QED can be studied by

(a) comparing electron and positron measurements (C-odd asymmetries) and

(b) studies of single-spin asymmetries (that are otherwise zero in first Born approximation)

![](_page_37_Picture_6.jpeg)