Overview of recent advances in calculations of two-photon exchange effects

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Intense Electron Beams Workshop

June 18, 2015

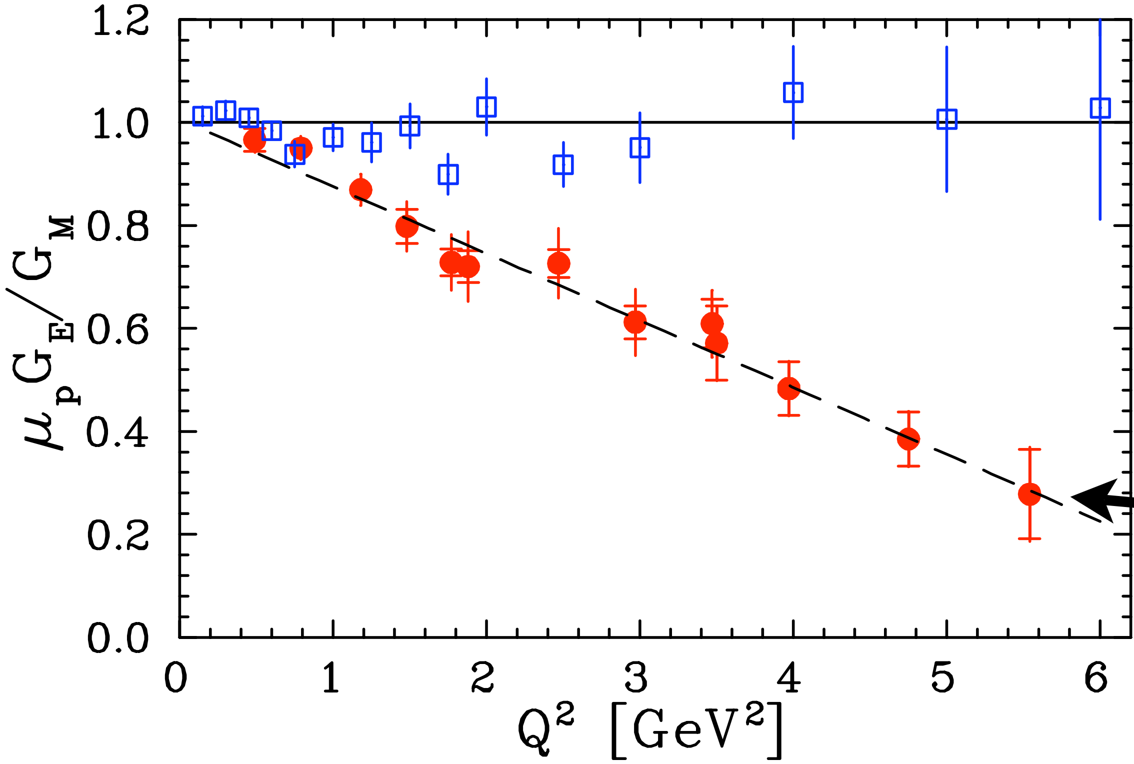
Outline

- Summary of key results (circa 2003-2008)

  Review: Arrington, PGB, Melnitchouk, Prog. Nucl. Part. Phys., (2011)

  – impact on form factor measurements
  – what is connection to 2nd Born approximation?
  – what happens at very low Q^2 ?
  – how do resonances and partonic description enter as Q^2 increases?
- Recent advances
  – improved hadronic model parameters (fit to data)
  – use of dispersion relations and connection to data
  – new experimental results

Proton G_E/G_M Ratio



Rosenbluth (<u>L</u>ongitudinal-<u>T</u>ransverse) Separation

<u>P</u>olarization <u>T</u>ransfer

<u>LT</u> method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

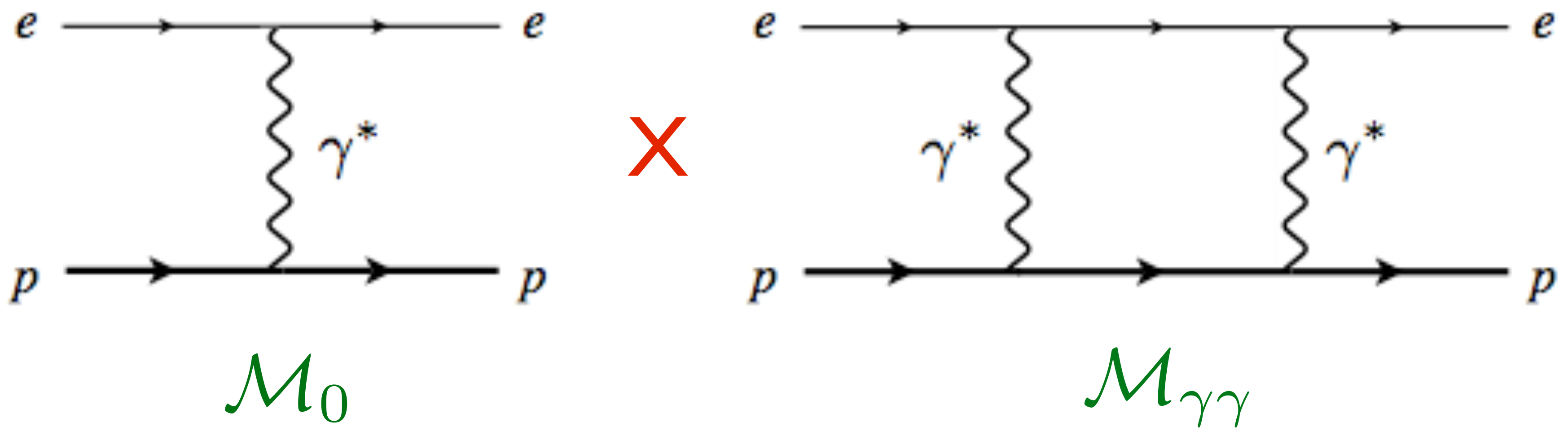
<u>PT</u> method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$

Two-photon exchange

- interference between Born and two-photon exchange amplitudes



- contribution to cross section:

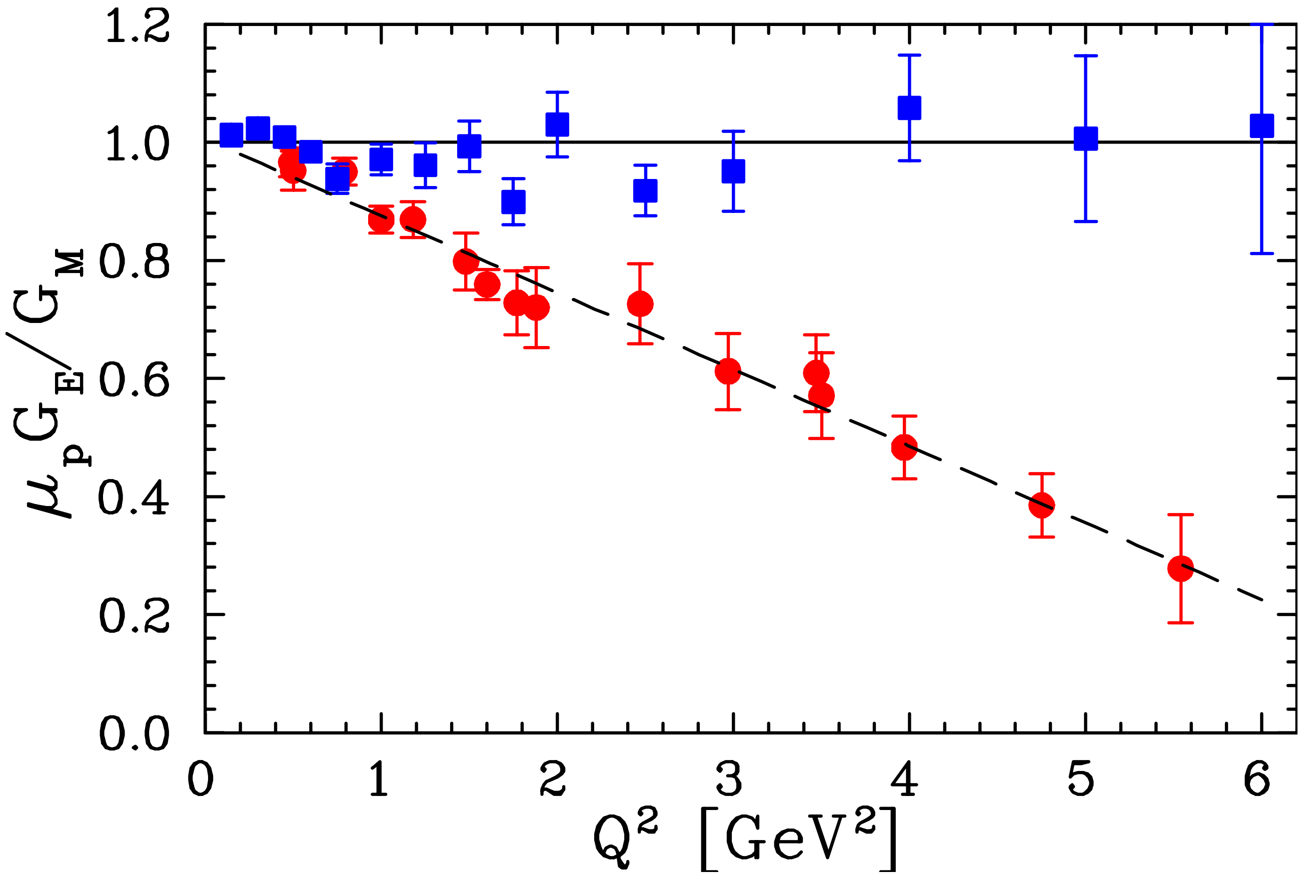
$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\, \mathcal{M}_{\gamma\gamma}\right\}}{|\mathcal{M}_0|^2}$$

- standard "soft photon approximation" (used in most data analyses)

$\longrightarrow$ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

$\longrightarrow$ neglect nucleon structure (no form factors) *Mo, Tsai (1969)*

Assuming OPE

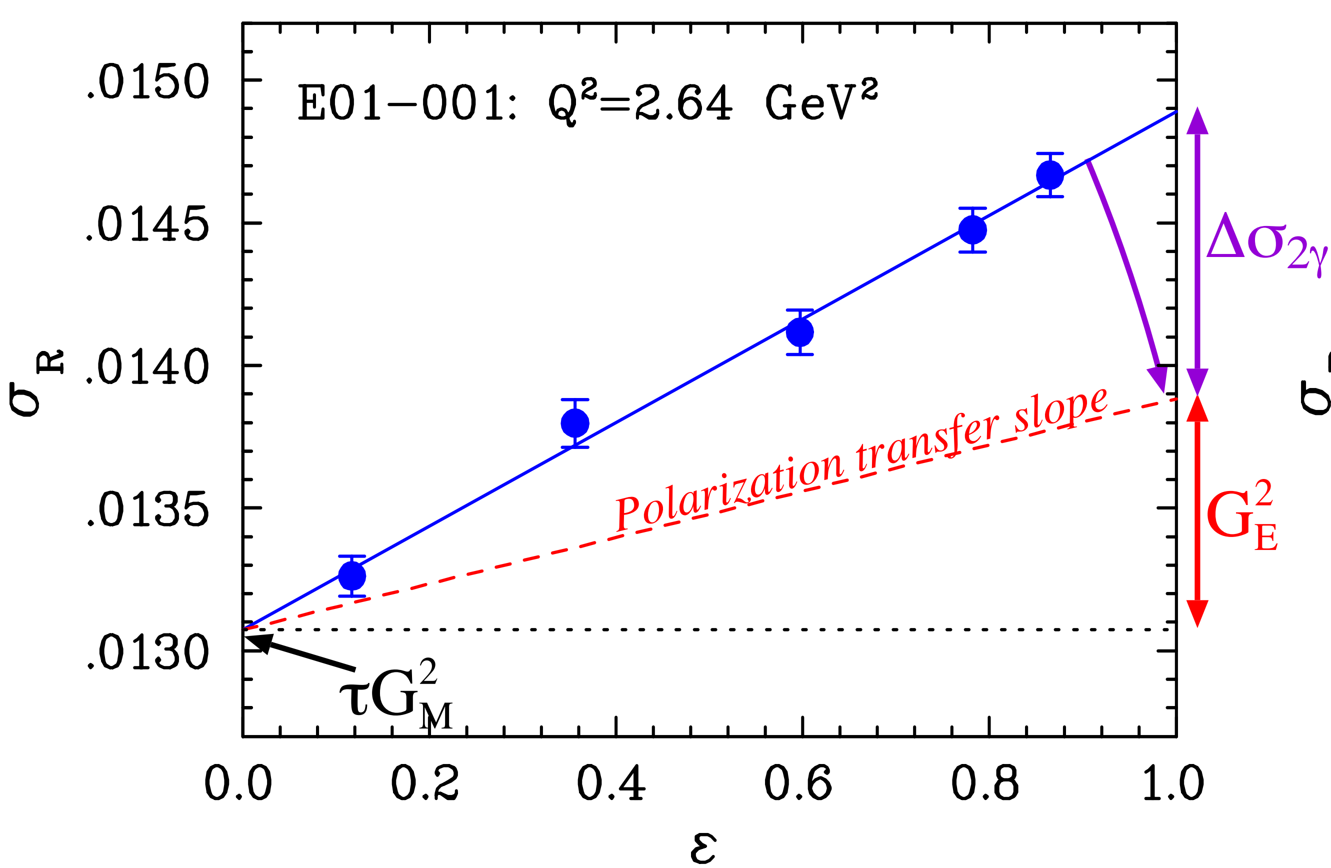


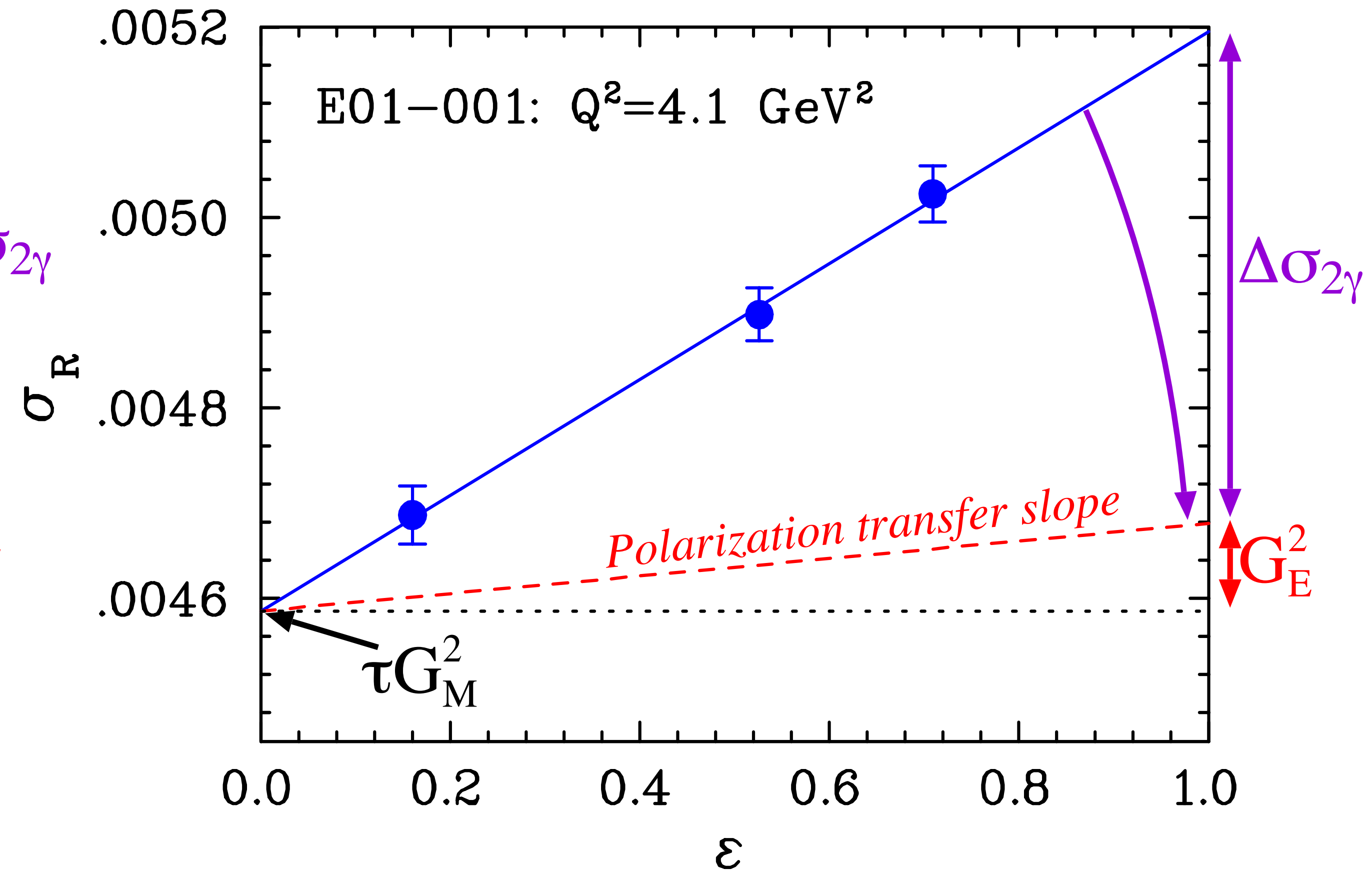
Rosenbluth

Pol. transfer

about 50% TPE + ??

about 80% TPE + ??



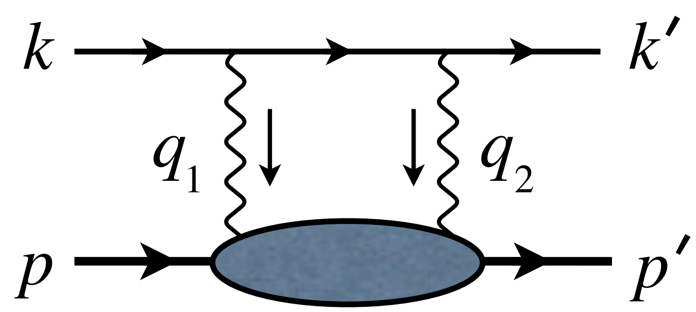


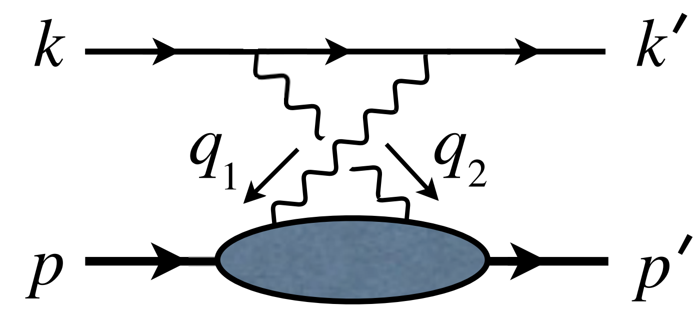
Various Approaches (circa 2003-2008)

Low to moderate Q^2:

hadronic: $\mathrm{N} + \Delta + \mathrm{N}^*$ etc.

- as Q^2 increases more and more parameters, less and less reliable

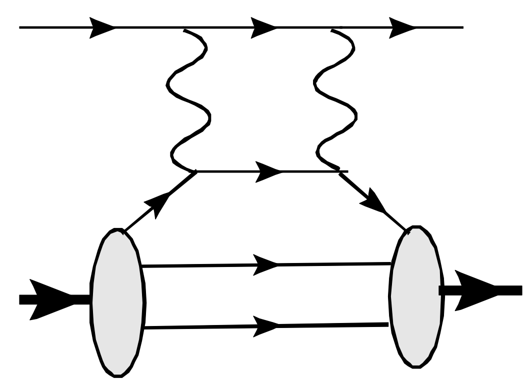




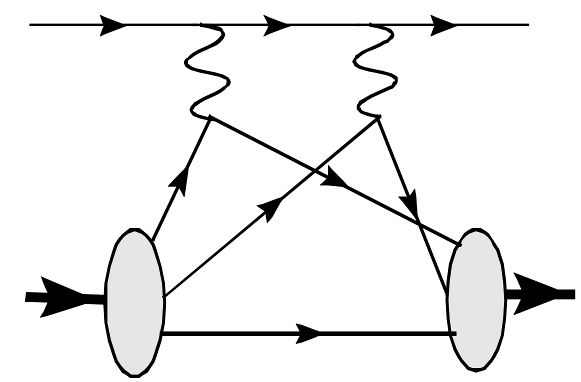
(PGB *et al.*, Phys. Rev. Lett **91**, 142304 (2003))

Moderate to high Q^2:

- GPD approach: assumption of hard photon interaction with 1 active quark
  - Embed in nucleon using Generalized Parton Distributions
  - Valid only in certain kinematic range ($|s,t,u| \gg M^2$)



"handbag"

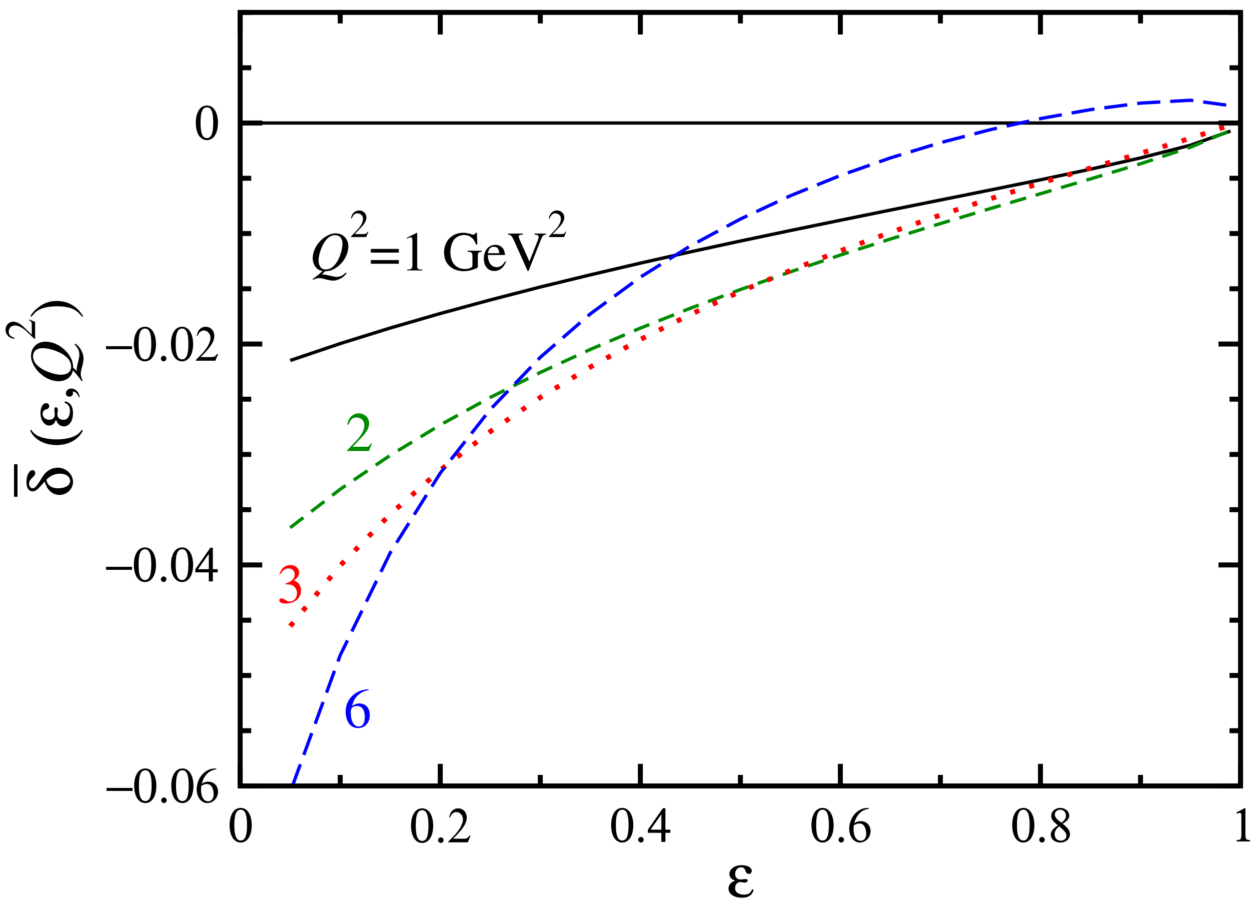


"cat's ears"

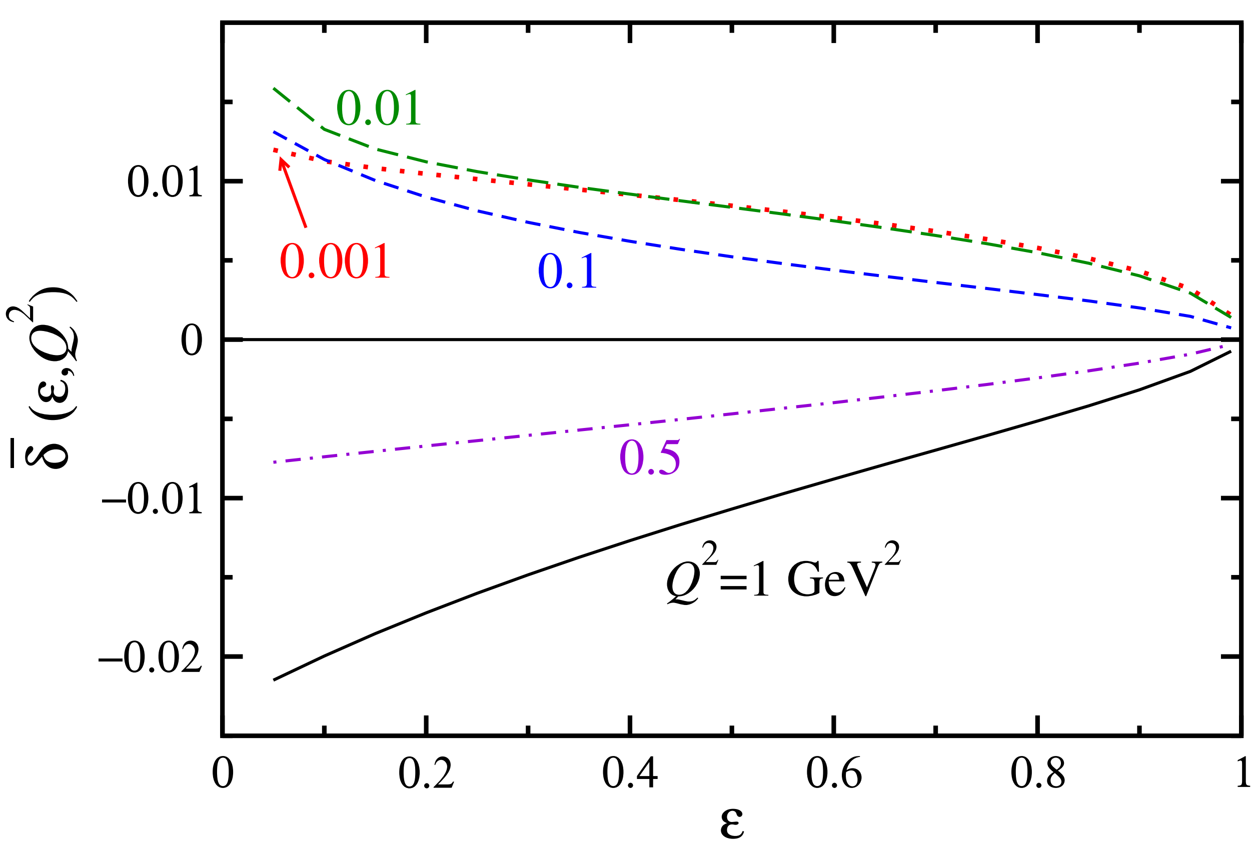
(Afanasev *et al.*, Phys. Rev. D **72**, 013008 (2005))

- pQCD: recent work indicates two active quarks dominate

Nucleon (elastic) intermediate state



- positive slope
- vanishes as $\varepsilon \longrightarrow 1$
- nonlinearity grows with increasing Q^2
- G_{M} dominates in loop integral



- changes sign at low Q^2
- agrees well with static limit for point particle (no form factors in loop and $Q^2 \longrightarrow 0$)
- G_{E} dominates in loop integral

Pointlike limit (e.g. $e^- \mu^+$)

$$\delta_{\gamma\gamma} = -\frac{2\alpha}{\pi} \ln \eta \ln \frac{Q^2}{\lambda^2} + \delta_{\rm hard}$$

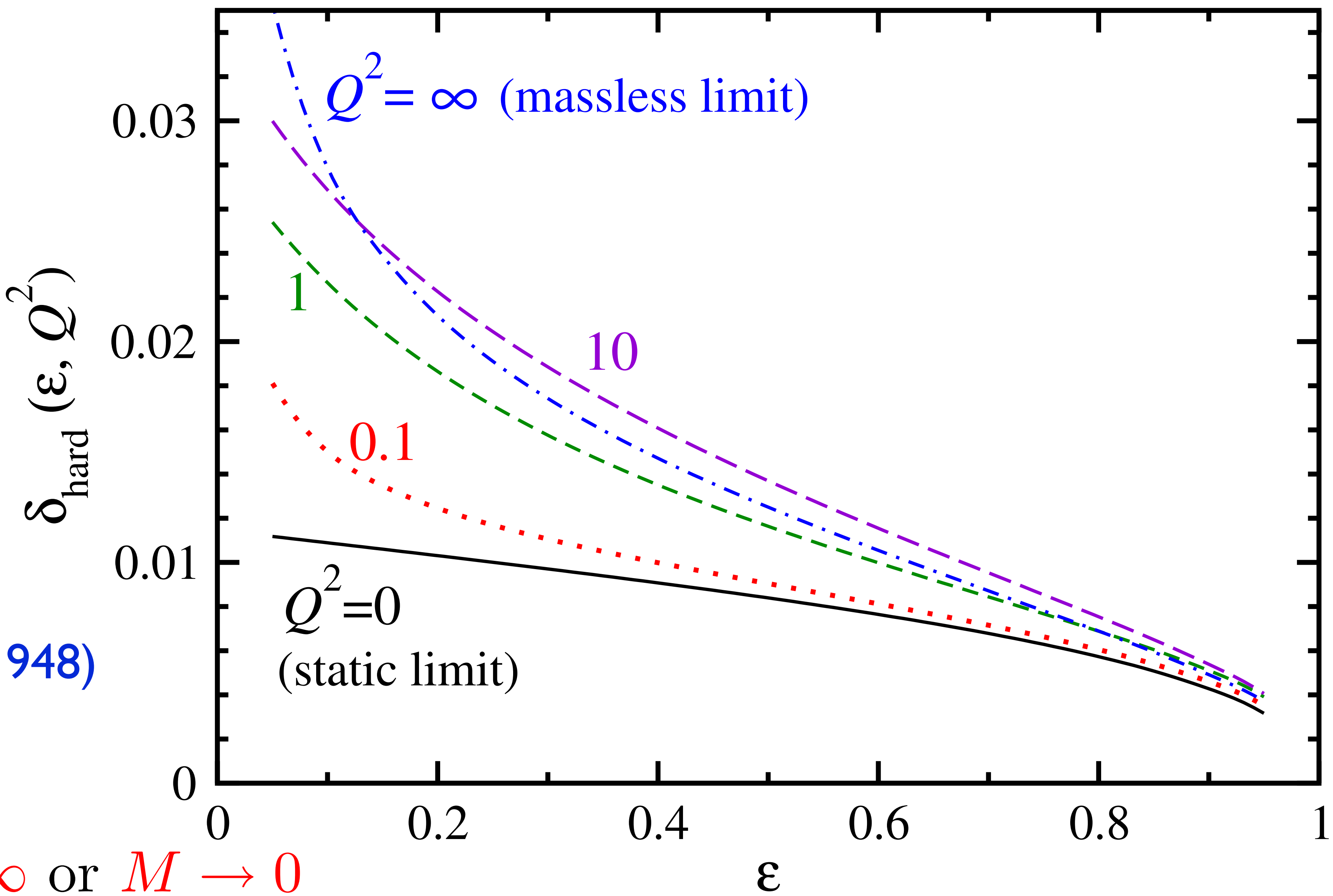
Static limit:

$$Q^2 \to 0 \text{ or } M \to \infty$$

$$\delta_{\rm hard} = \frac{\alpha\pi}{1+x}$$

$$x = \sqrt{(1+\epsilon)/(1-\epsilon)}$$

Agrees with McKinley & Feshbach (1948)
2nd Born result with $x=1/\sin(\theta/2)$



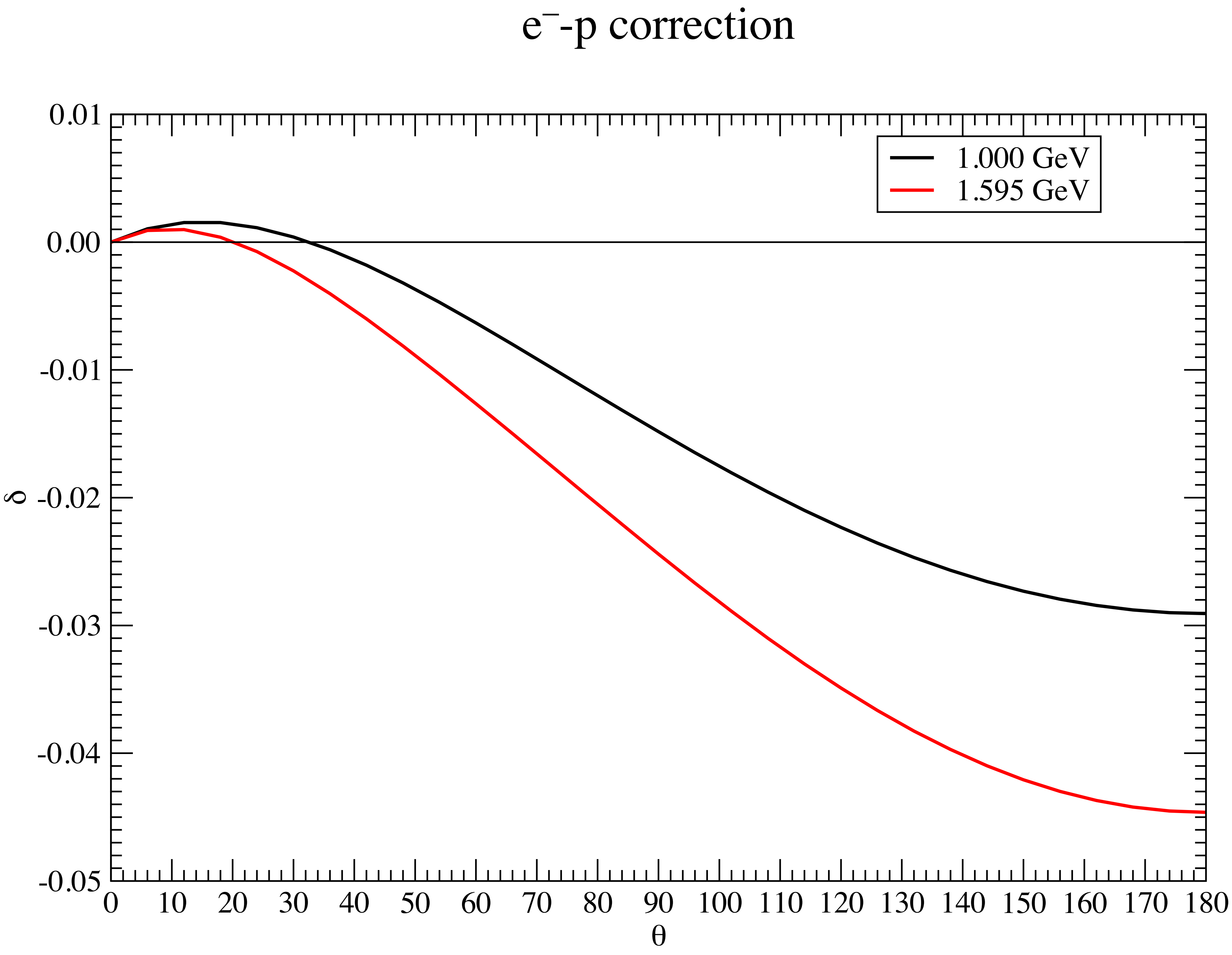
Massless limit: $Q^2 \to \infty$ or $M \to 0$

$$\delta_{\rm hard} = \frac{\alpha}{\pi(x^2+1)} \left\{ \ln\left(\frac{x+1}{x-1}\right) + x\left[\pi^2 + \ln^2\left(\frac{x+1}{2}\right) + \ln^2\left(\frac{x-1}{2}\right) - \ln\left(\frac{x^2-1}{4}\right)\right]\right\}$$

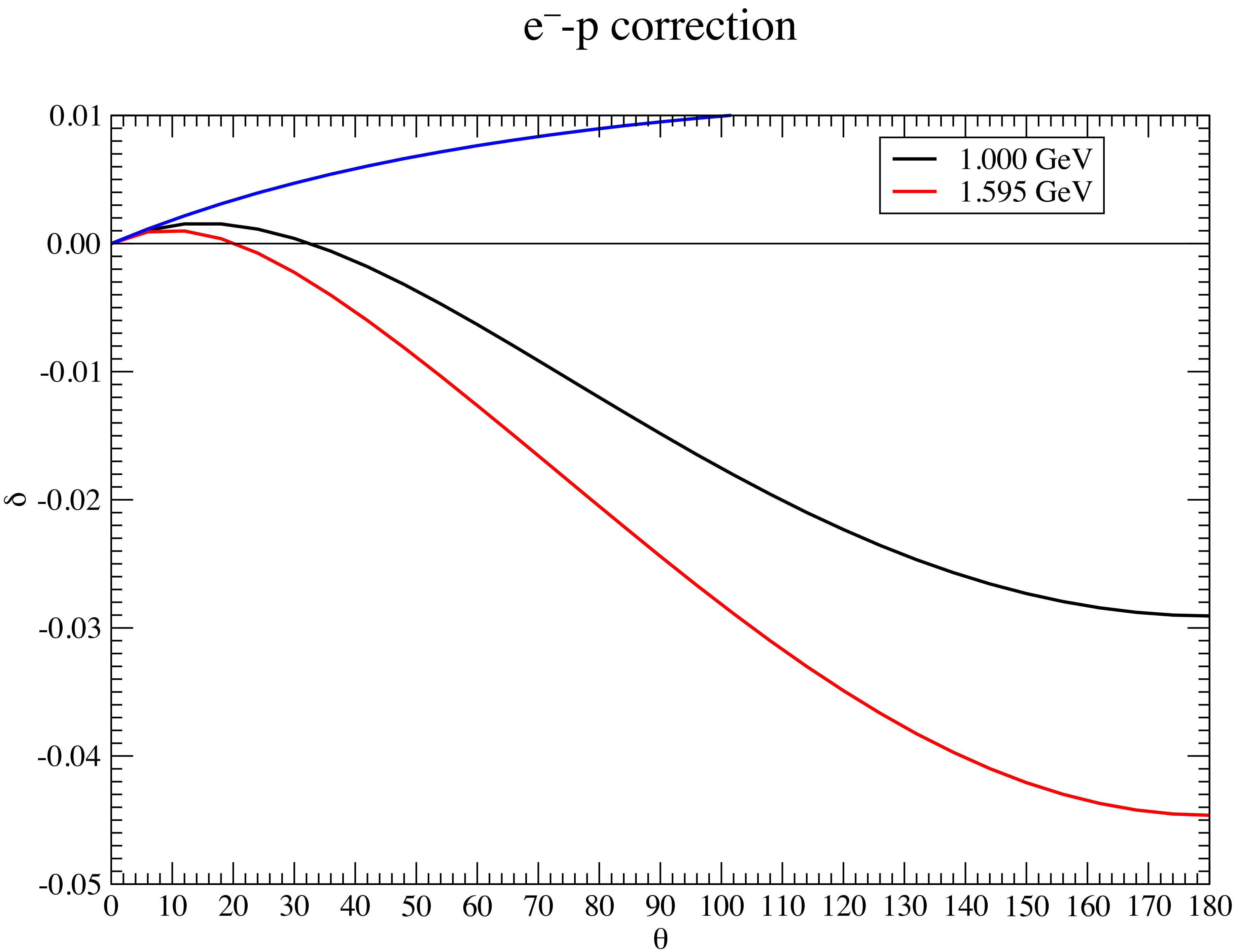
Agrees with Nieuwenhiuzen (1971) and Afanasev et al. (2005)

Suggests hard scattering from one active quark per se cannot be responsible for a reduction in cross section at backward angles.

Fixed E (Novosibirsk kinematics)



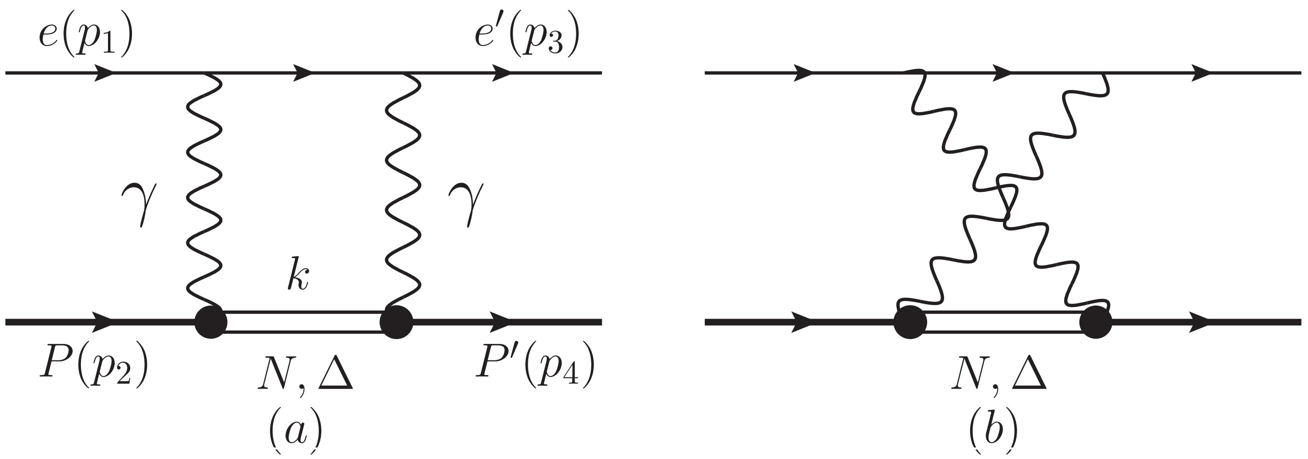
Fixed E (VEPP-3 Novosibirsk kinematics)



Agrees with 2nd Born expression at small angles

- At forward angles TPE dominated by Coulomb distortion, while at backward angles exchange of 2 hard photons contributes

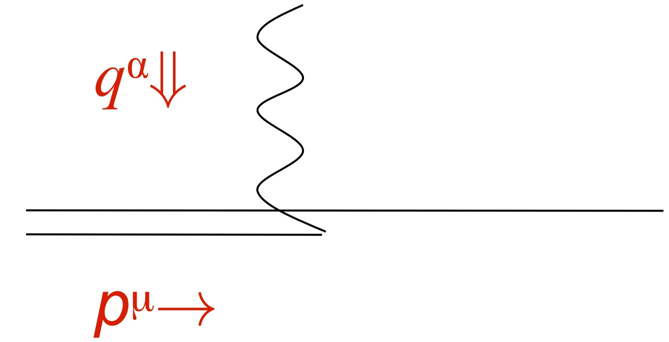
Delta intermediate states



- $\gamma N\Delta$ transition well-studied
- Dominant inelastic contribution
- More important as Q^2 increases

Resonance (Δ) contribution:

$\gamma(q^\alpha) + \Delta(p^\mu) \rightarrow \mathrm{N}$



$\gamma \mathrm{N} \Delta$ vertex

- Lorentz covariant form
- Spin ½ decoupled
- Obeys gauge symmetries

$$p_\mu \Gamma^{\alpha\mu}(p,q) = 0$$
$$q_\alpha \Gamma^{\alpha\mu}(p,q) = 0$$

$$\begin{aligned}\Gamma^{\alpha\mu}_{\gamma\Delta\rightarrow N}(p,q) = {} & \frac{ieF_\Delta(q^2)}{2M_\Delta^2}\{g_1(g^{\alpha\mu}\not{p}\not{q} - p^\alpha\gamma^\mu\not{q} - \gamma^\alpha\gamma^\mu p\cdot q + \gamma^\alpha\not{p}q^\mu) \\ & + g_2(p^\alpha q^\mu - g^{\alpha\mu}p\cdot q) \\ & + (g_3/M_\Delta)\left(q^2(p^\alpha\gamma^\mu - g^{\alpha\mu}\not{p}\right) + q^\alpha(q^\mu\not{p} - \gamma^\mu p\cdot q)\}\gamma_5 T_3\end{aligned}$$

3 coupling constants g_1, g_2, and g_3

At Δ pole:	g_1	Magnetic (dominant contribution)
	g_2-g_1	Electric
	g_3	Coulomb

Take dipole form factor $F_\Delta(q^2) = 1/(1-q^2/\Lambda^2)^2$

with $\Lambda = 0.75$ GeV (softer than nucleon form factors, with $\Lambda = 0.84$ GeV)

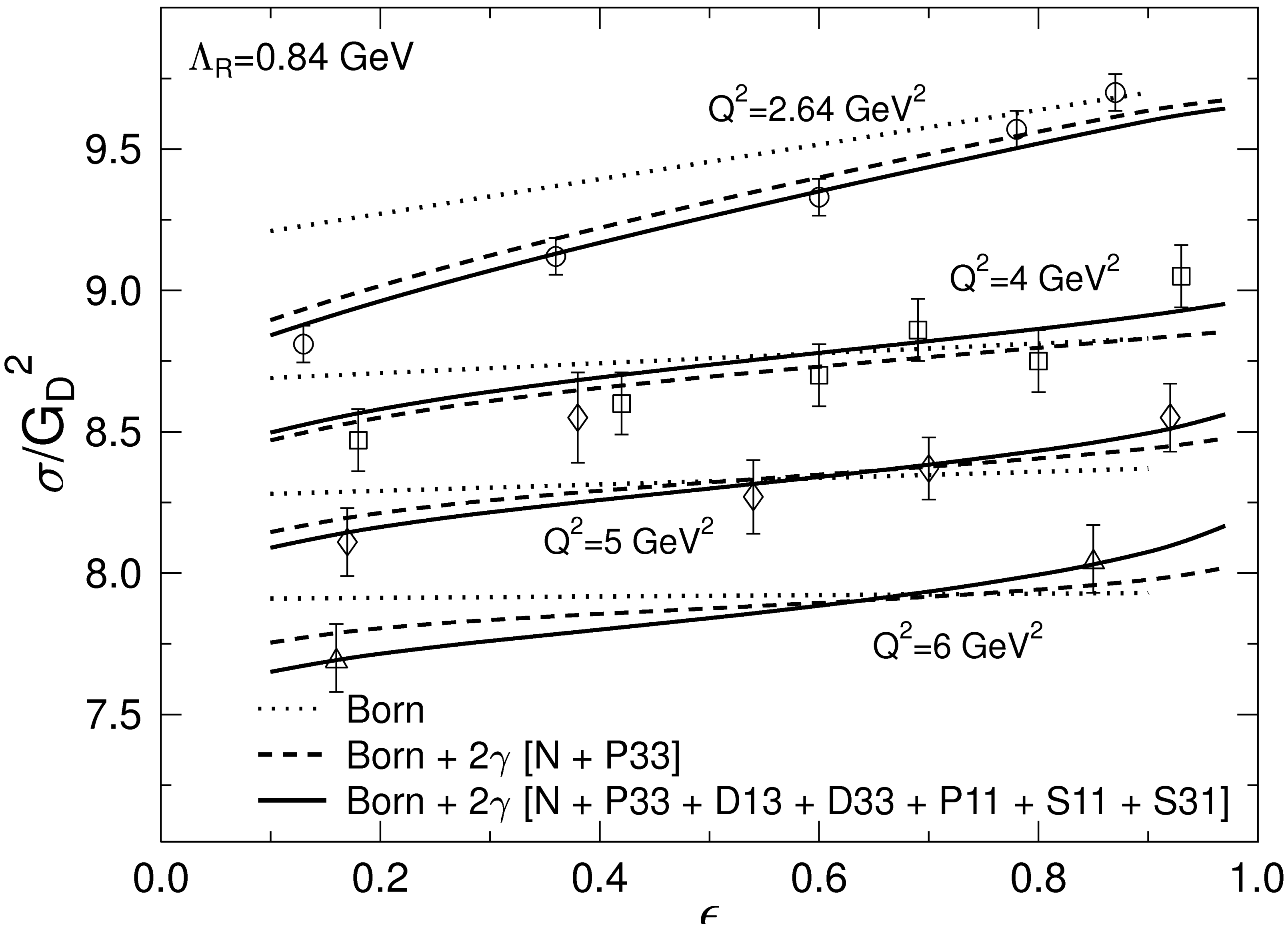
Zero width approximation (okay for Re part of δ)

Other resonances (Kondratyuk & PGB, PRC 2007)

- N (P11), Δ (P33) + D13, D33, P11, S11, S31
- Parameters from dressed K-matrix model

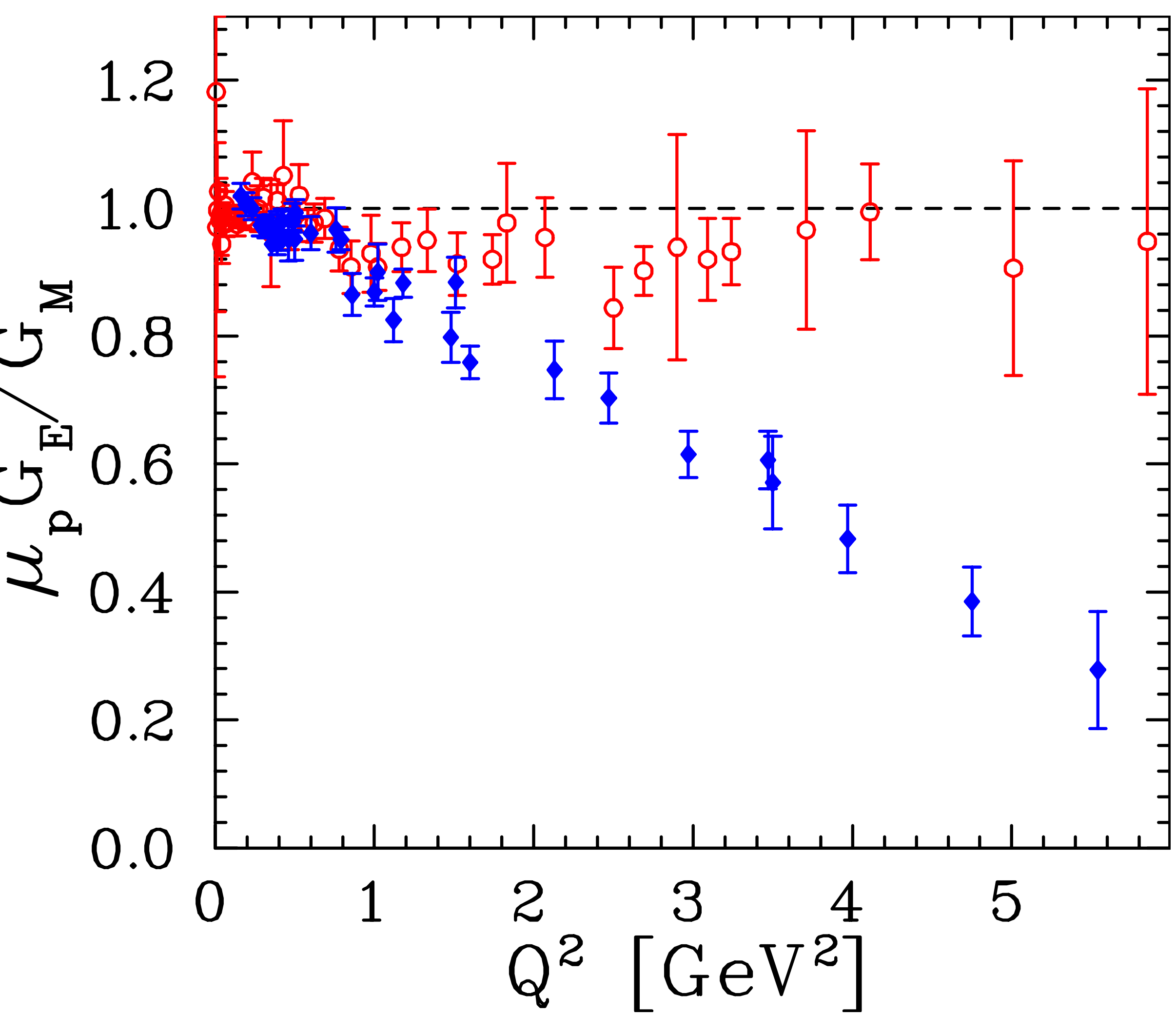
Results

- contribution of heavier resonances much smaller than N and Δ
- D13 next most important (consistent with second resonance shape of Compton scattering cross section)
- partial cancellation between spin 1/2 and spin 3/2
- leads to better agreement, especially at high Q^2

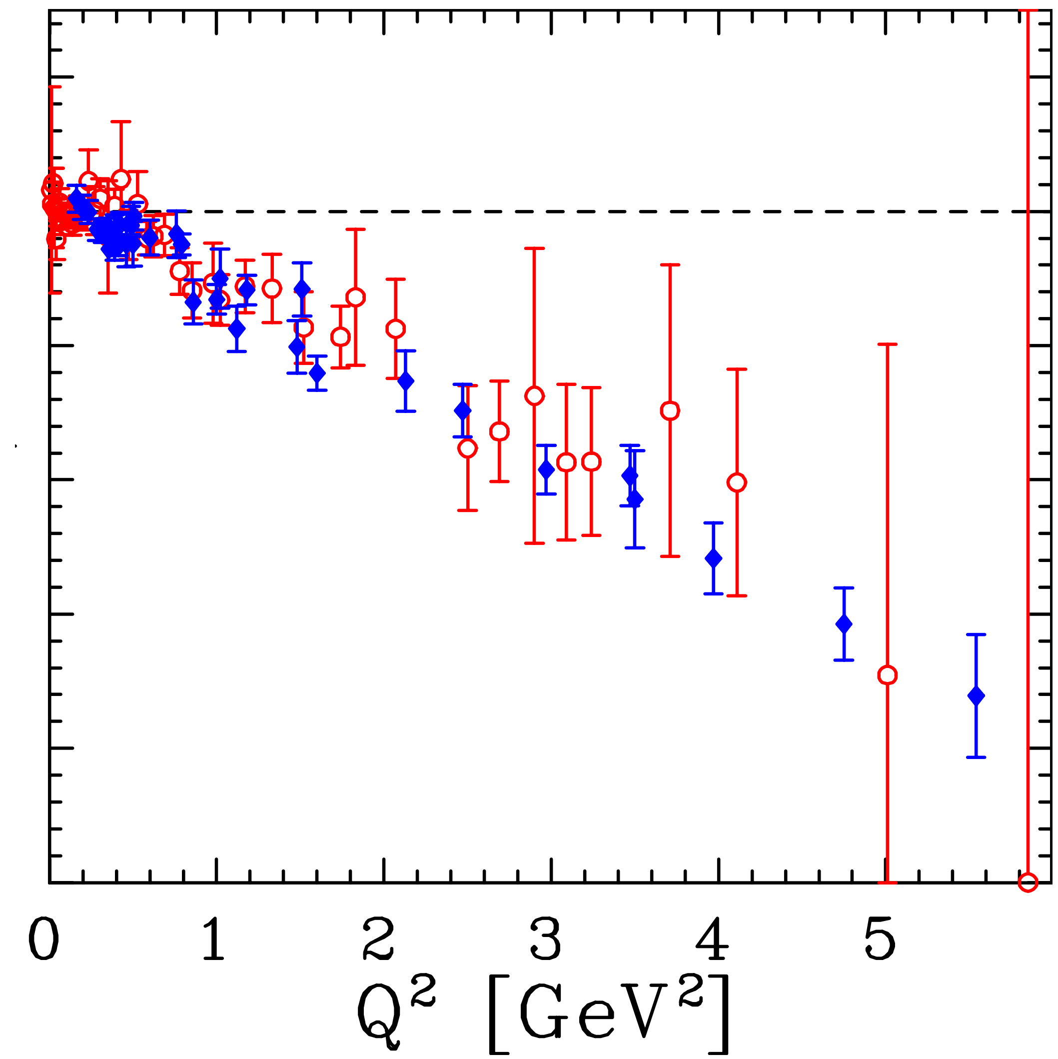


Fit to SuperRosenbluth (JLAB) data

Effect on ratio $\mu_p\, G_E/G_M$



Raw results



Corrected with TPE

Recent Advances

Experiment

- Qweak parity-violation experiment, and the γZ box diagram contribution
- Discrepancy between proton charge radius as measured in atomic H, muonic H, and electron scattering
- TPE effect on ratio of e^+p to e^-p cross sections

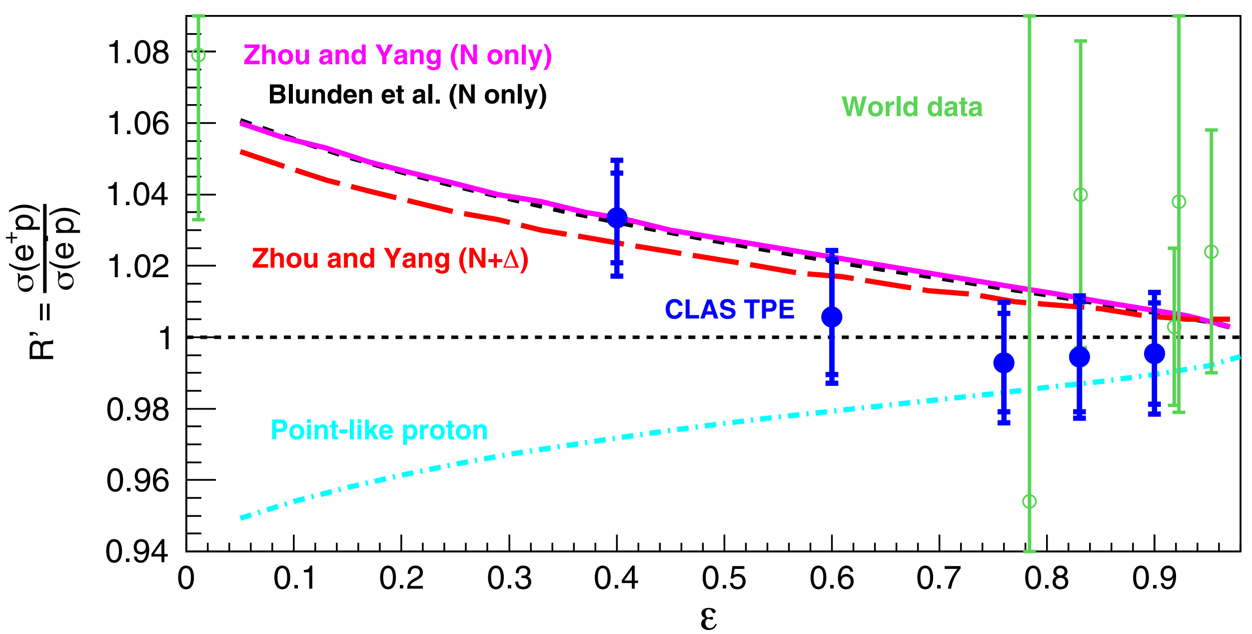
Theory

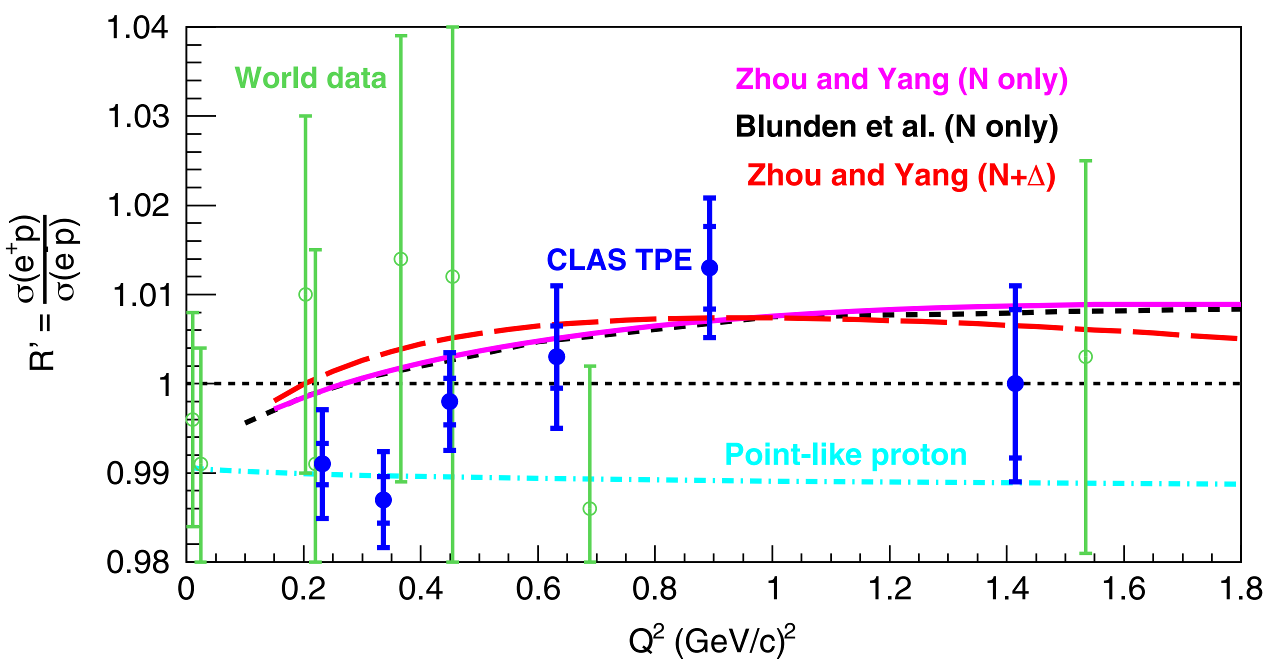
- Use improved $\gamma N\Delta$ form factors based on most recent data
- Use dispersion integrals to relate Real and Imaginary parts. Imaginary parts fixed by cross section data
  - Valid at forward angles: must use models to extrapolate
  - Incomplete: not all data is available (e.g. axial hadron coupling and isospin dependence in γZ diagrams
- Model-independent analysis of corrections in forward kinematics in dispersive formalism (sum rule based on total photoabsorption cross section)

TPE effect on ratio of e^+p to e^-p cross sections

CLAS collaboration (2015)

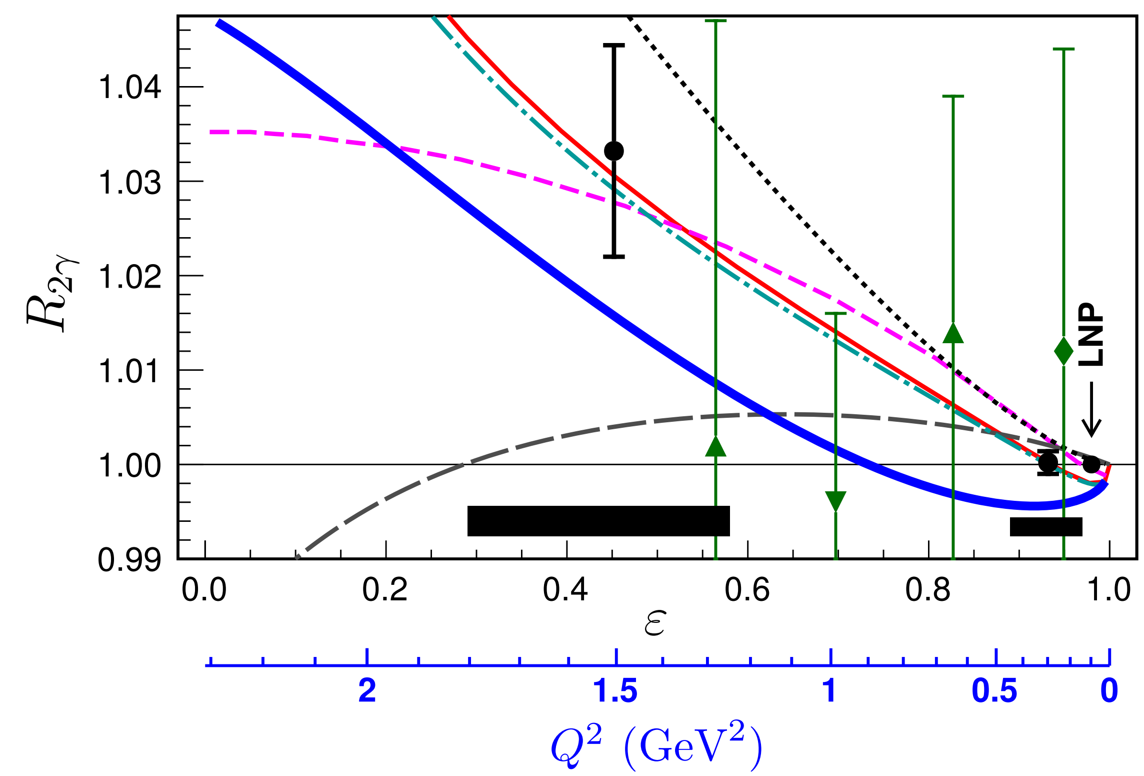
Q^2=1.45 GeV²

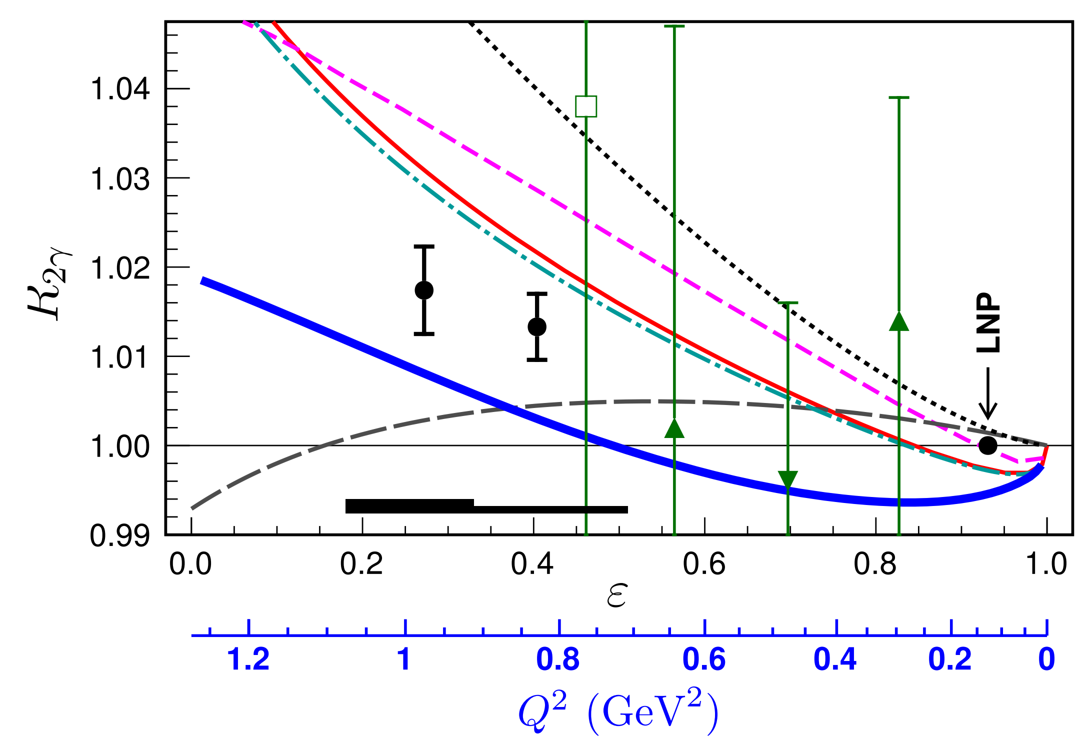




ε =0.88

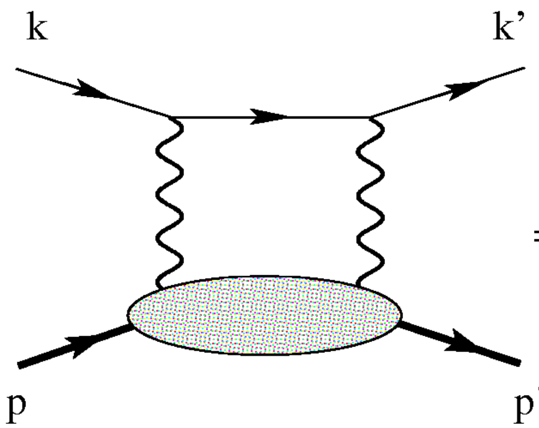
VEPP-3 Novosibirsk (2015)

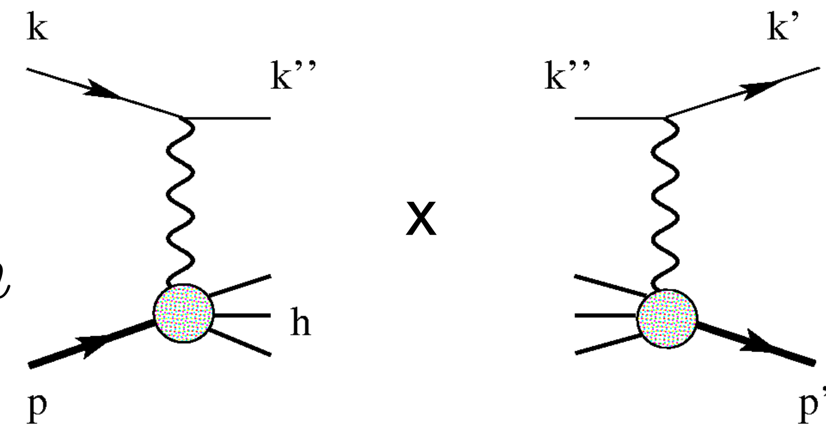




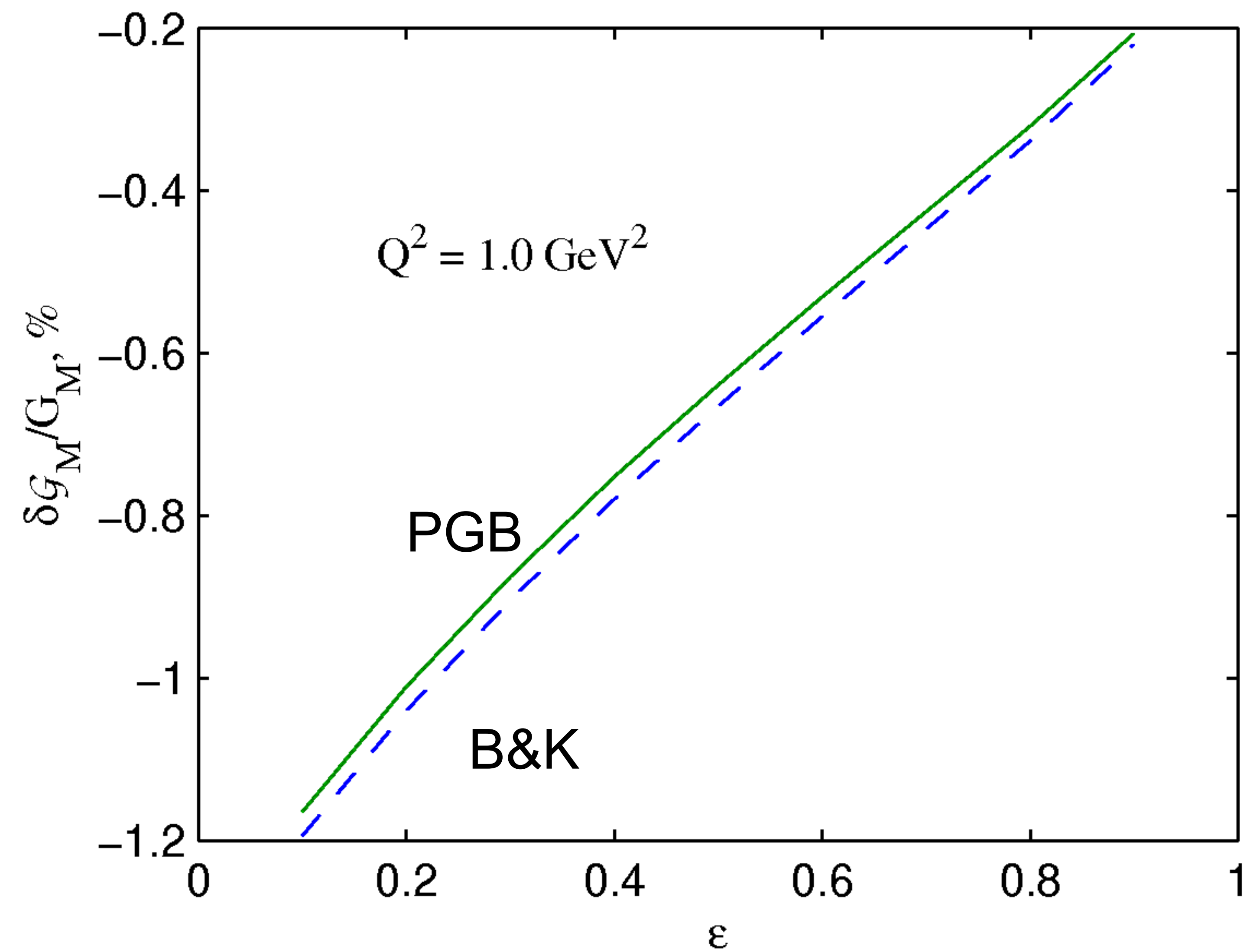
TPE using dispersion relations

(Borisyuk & Kobushkin, Phys. Rev. C **78,** 2008)

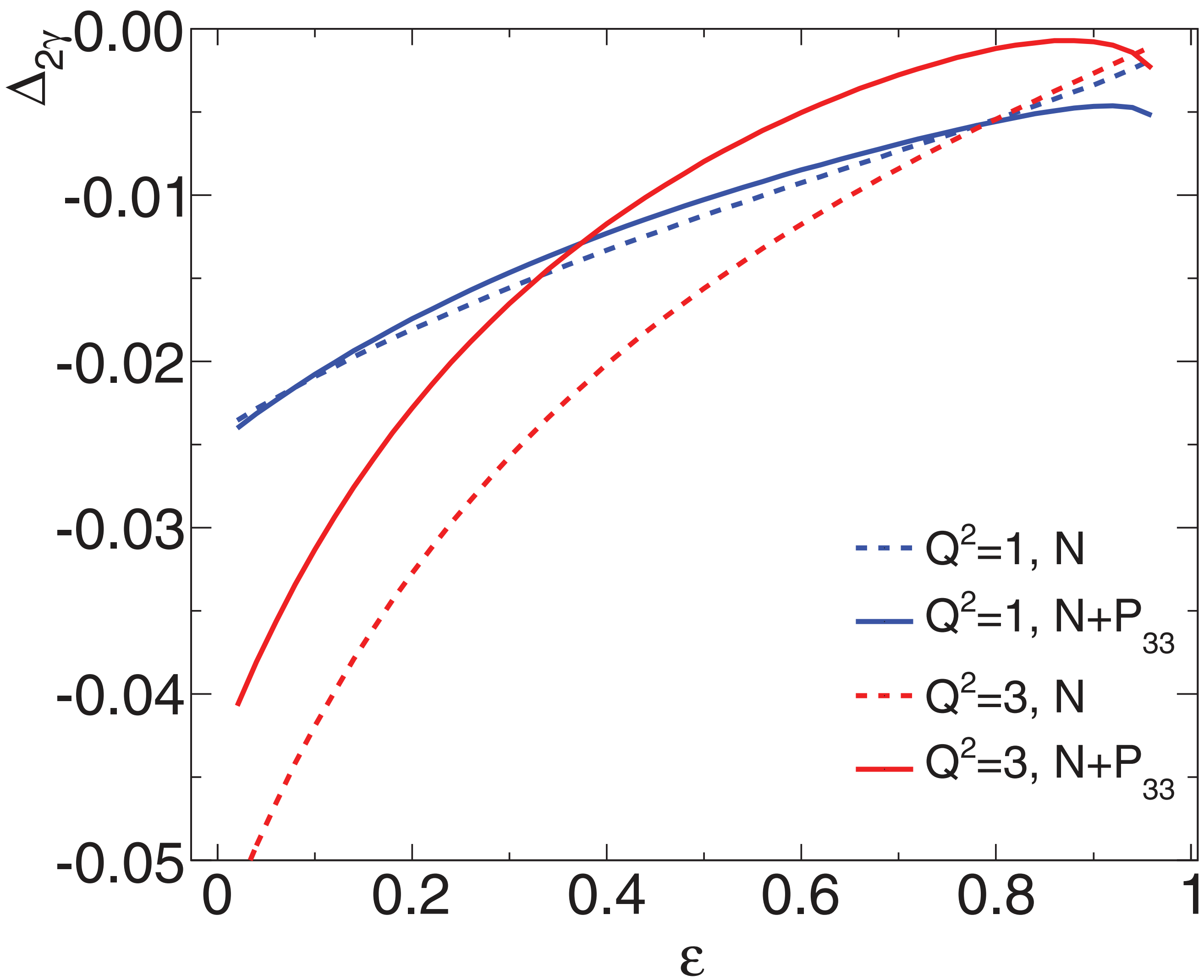
$2\,\mathrm{Im}$ 

$= \int dk'' \sum_h$ 

- Imaginary part determined by unitarity
- Only on-shell form factors
- Real part determined from dispersion relations
- For elastic (N) intermediate state, numerical differences between one loop (solid) and dispersion (dashed) analyses are tiny (all due to $(F_2 \times F_2)$ term in box vertices



See also recent work by Tomalak & Vanderhaeghen, Eur. Phys. J. A. (2015) **51**: 24

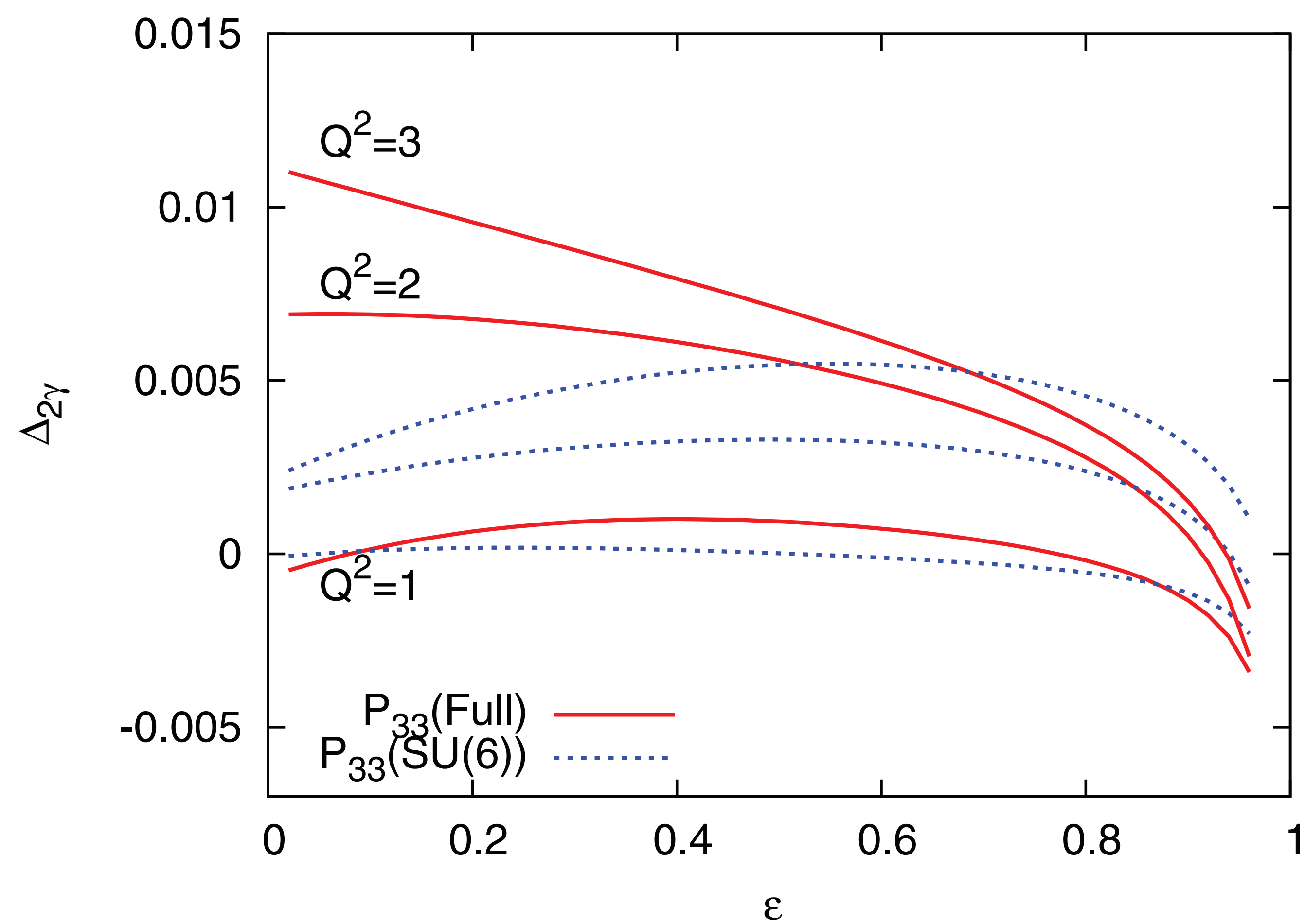


Graczyk, Phys. Rev. C **88**, 065205 (2013)

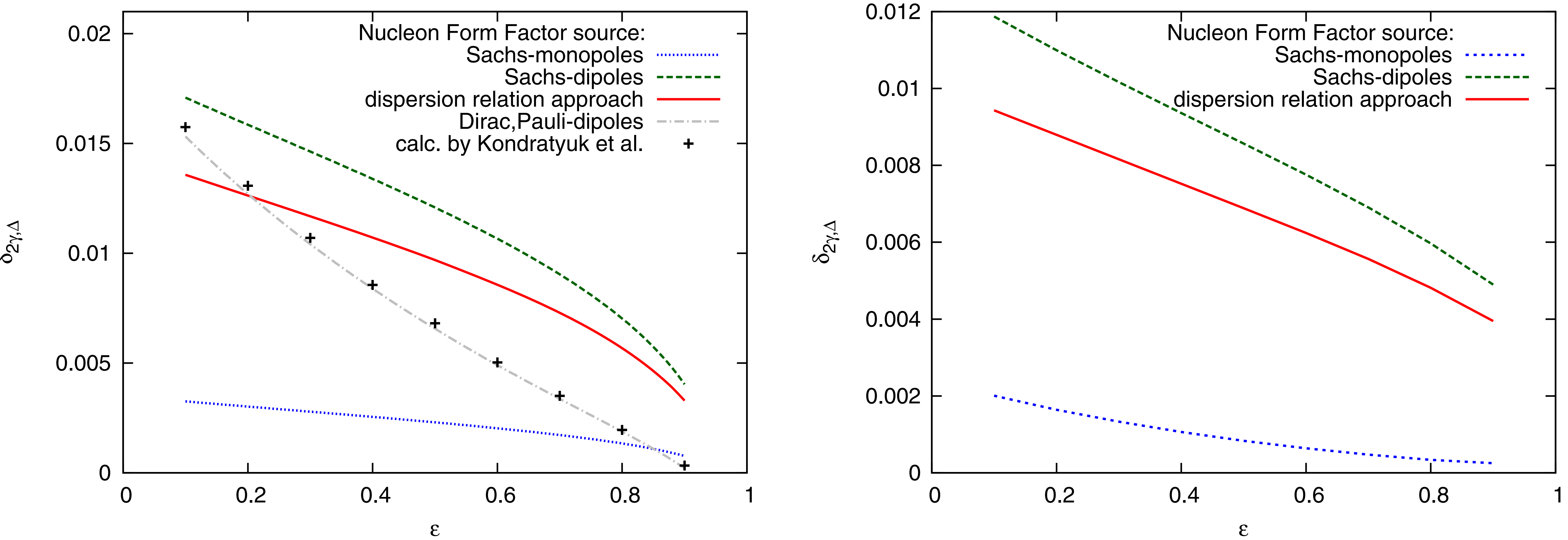
1. TPE extracted from data using Bayesian analysis
2. Use model fit that includes N and Δ

TABLE IV. Values of the proton radius $\sqrt{\langle r_E^2\rangle}$ obtained from the BNN and HM fits in femtometers.

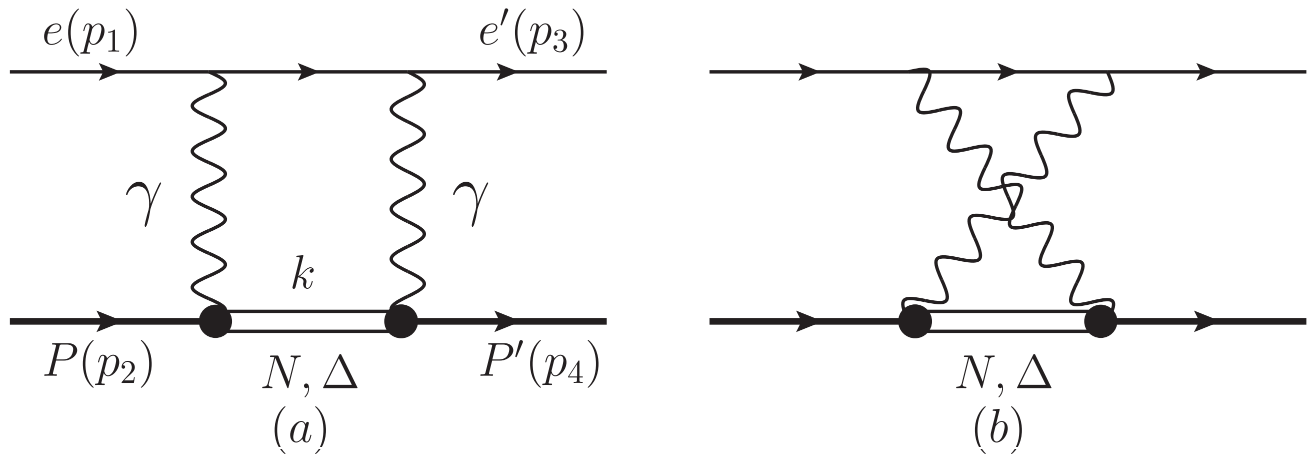
BNN	fit I	fit II
0.85 ± 0.01	0.898 ± 0.001	0.867 ± 0.002



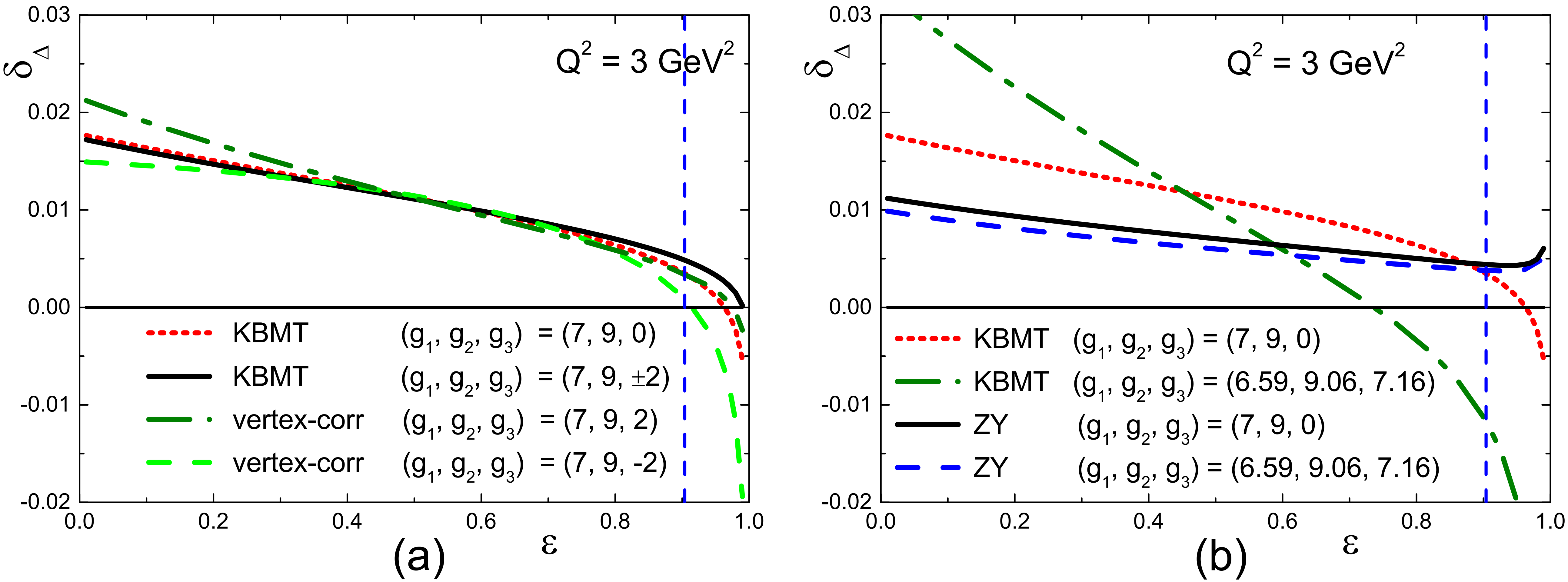
Lorenz *et al.*, Phys. Rev. D **91**, 014023 (2015)



- Used $\gamma N\Delta$ form factors fit to recent data
- Find smaller results than Kondratyuk & PGB
  - (consistent with softer form factor Λ=0.75 GeV than for nucleon)
- Claim substantial effect on the determination of the proton charge radius from scattering data



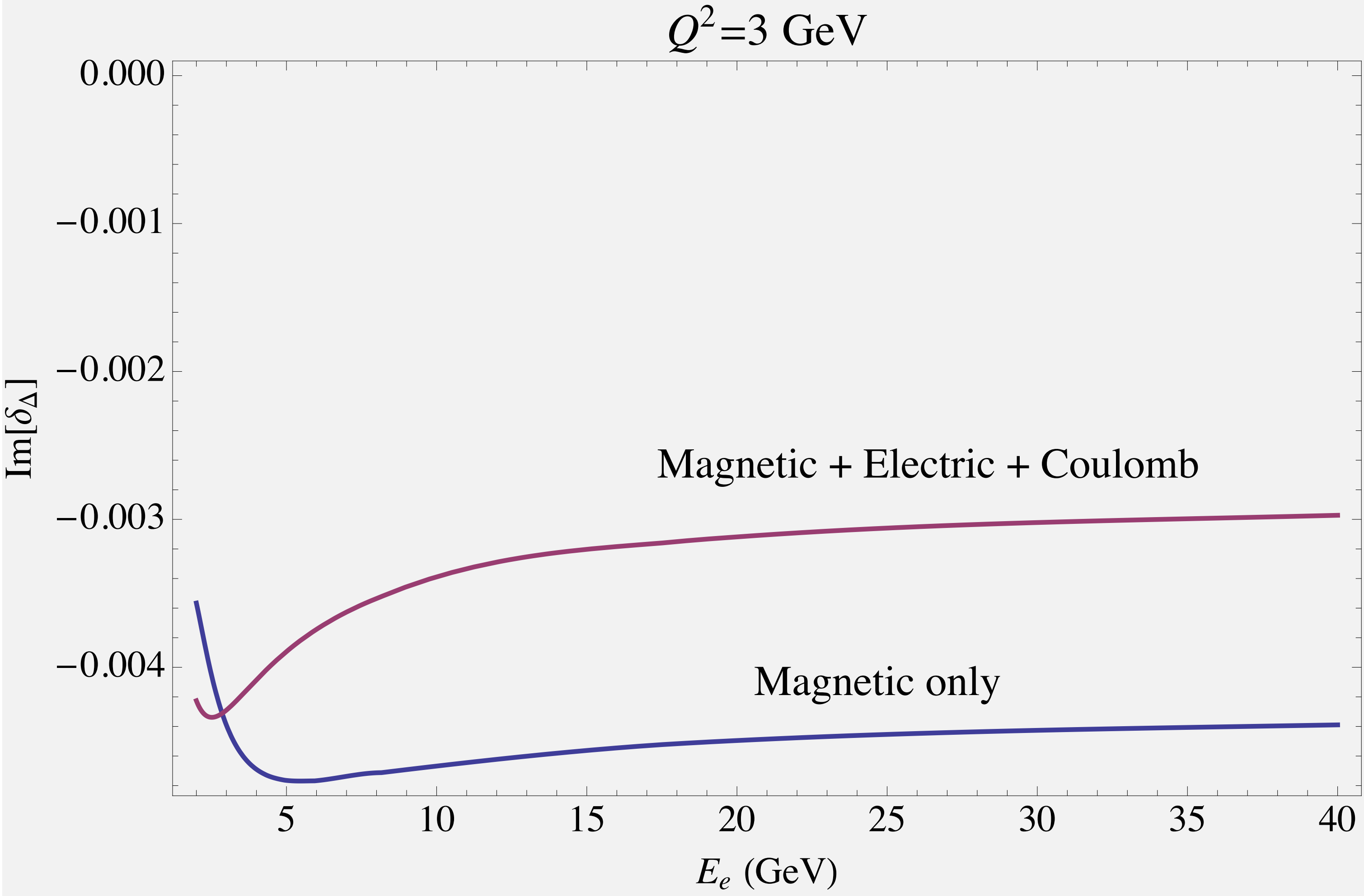
Zhou and Yang, arXiv: 1407.2711 (2015)

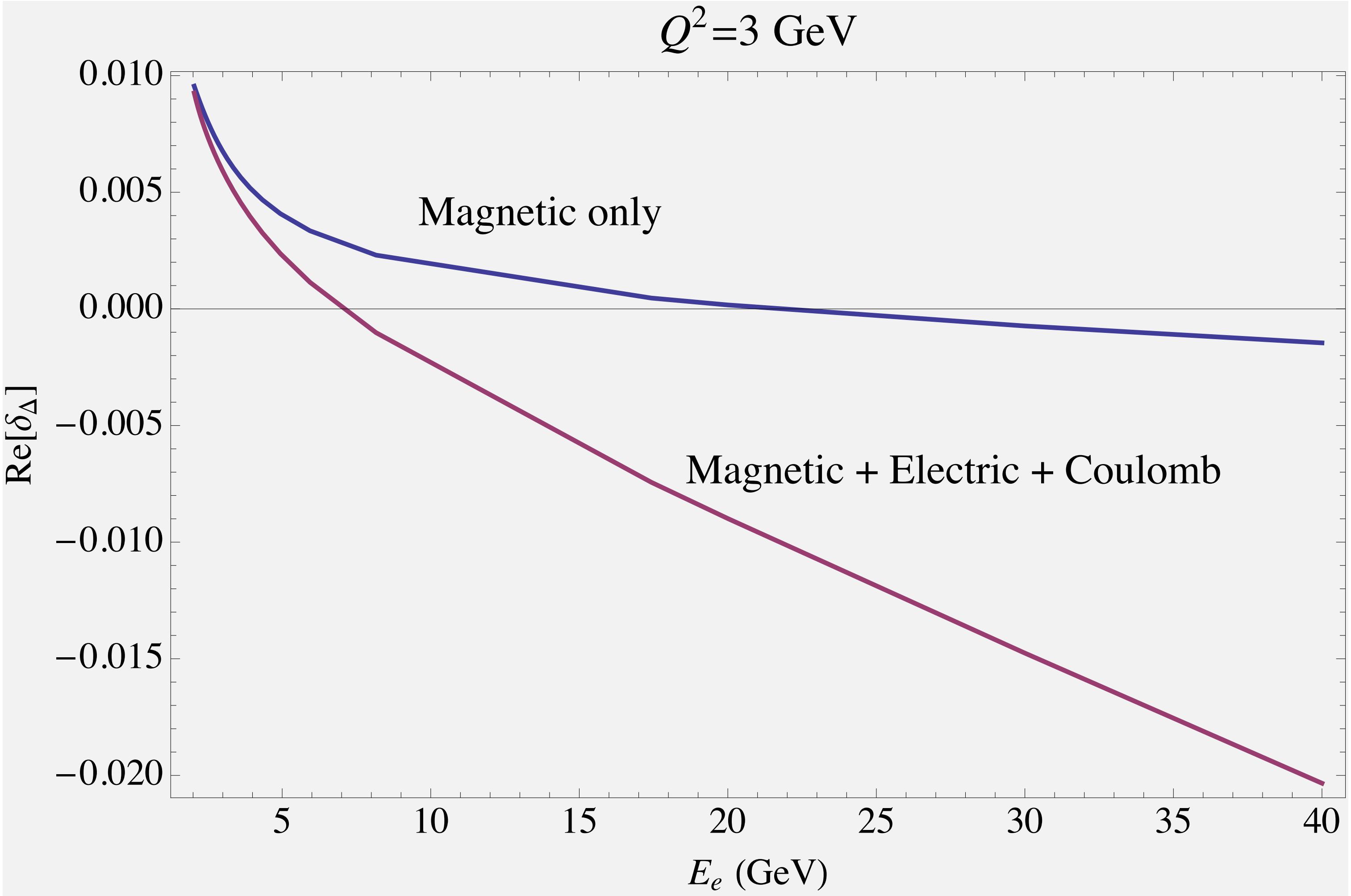


Include all 3 multipoles, with form factors fit to recent CLAS data

Plot vs. energy instead of ε

- Imaginary part well-behaved
- Dispersive integral also well-behaved
  (e.g. vanishes at $\varepsilon \to 0$)

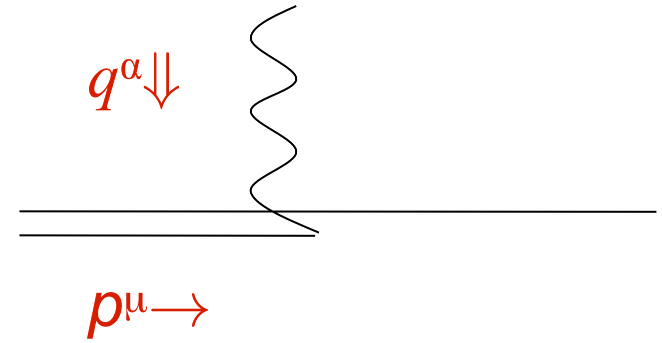




- Real part from loop calculation diverges linearly with energy (violation of Froissart bound)
- Problem due to momentum-dependent vertices, uncontrained by on-shell condition

Resonance (Δ) contribution:

$\gamma(q^{\alpha}) + \Delta(p^{\mu}) \rightarrow \mathrm{N}$



$\gamma \mathrm{N} \Delta$ vertex

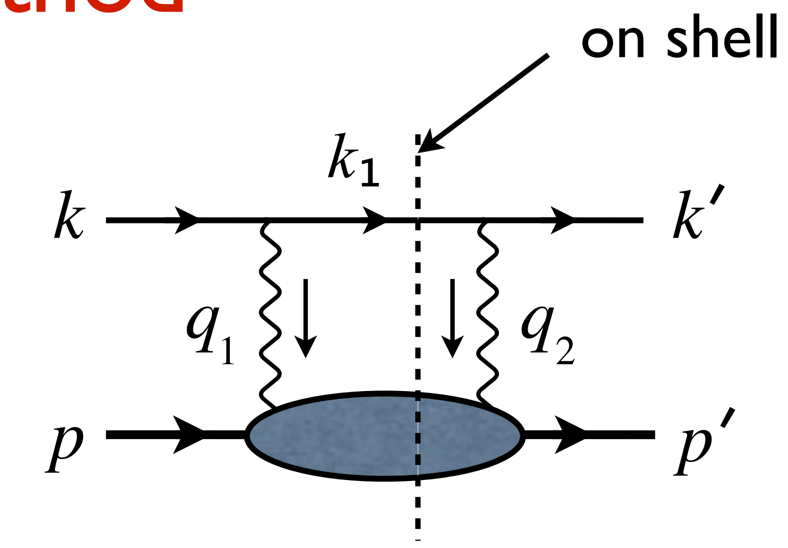
$$\Gamma^{\alpha\mu}_{\gamma\Delta\rightarrow N}(p,q) = \frac{ieF_{\Delta}(q^2)}{2M_{\Delta}^2}\{g_1(g^{\alpha\mu}\not{p}\not{q} - p^{\alpha}\gamma^{\mu}\not{q} - \gamma^{\alpha}\gamma^{\mu}p\cdot q + \gamma^{\alpha}\not{p}q^{\mu}) + g_2\,(p^{\alpha}q^{\mu} - g^{\alpha\mu}p\cdot q) + (g_3/M_{\Delta})\left(q^2(p^{\alpha}\gamma^{\mu} - g^{\alpha\mu}\not{p}\right) + q^{\alpha}\,(q^{\mu}\not{p} - \gamma^{\mu}p\cdot q)\}\gamma_5 T_3$$

3 coupling constants g_1, g_2, and g_3

At Δ pole:

	g_1	Magnetic (dominant contribution)
	g_2-g_1	Electric
	g_3	Coulomb

Dispersion method



$$S = 1 + i\mathcal{M}$$

$$S^\dagger = 1 - i\mathcal{M}^\dagger$$

$$SS^\dagger = 1$$

Unitarity → $-i\left(\mathcal{M} - \mathcal{M}^\dagger\right) = 2\Im m\, \mathcal{M} = \mathcal{M}^\dagger \mathcal{M}$

$$\Im m\, \langle f|\mathcal{M}|i\rangle = \frac{1}{2}\int d\rho\, \sum_n \langle f|\mathcal{M}^*|n\rangle\langle n|\mathcal{M}|i\rangle$$

$$d\rho = \frac{d^3 k_1}{(2\pi)^3 2E_{k_1}} \sim dW_n\, dQ_1^2\, dQ_2^2$$

→ dispersion relation

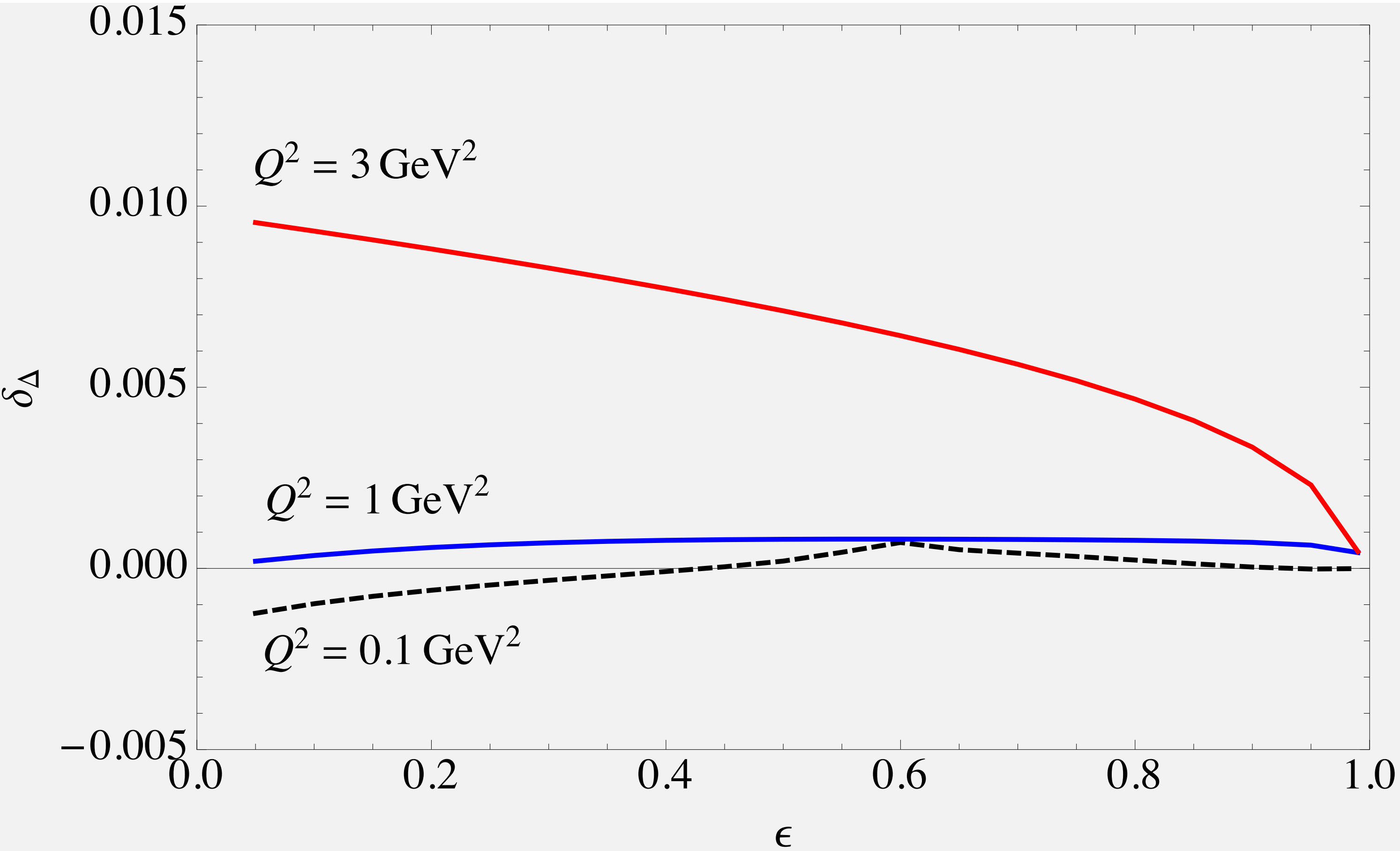
$$\Re e\delta(\nu') = \frac{2\nu'}{\pi}\int_{\nu_{\rm th}}^{\infty} d\nu \, \frac{1}{\nu^2 - \nu'^2} \Im m\delta(\nu); \qquad \nu = (s-u)/4$$

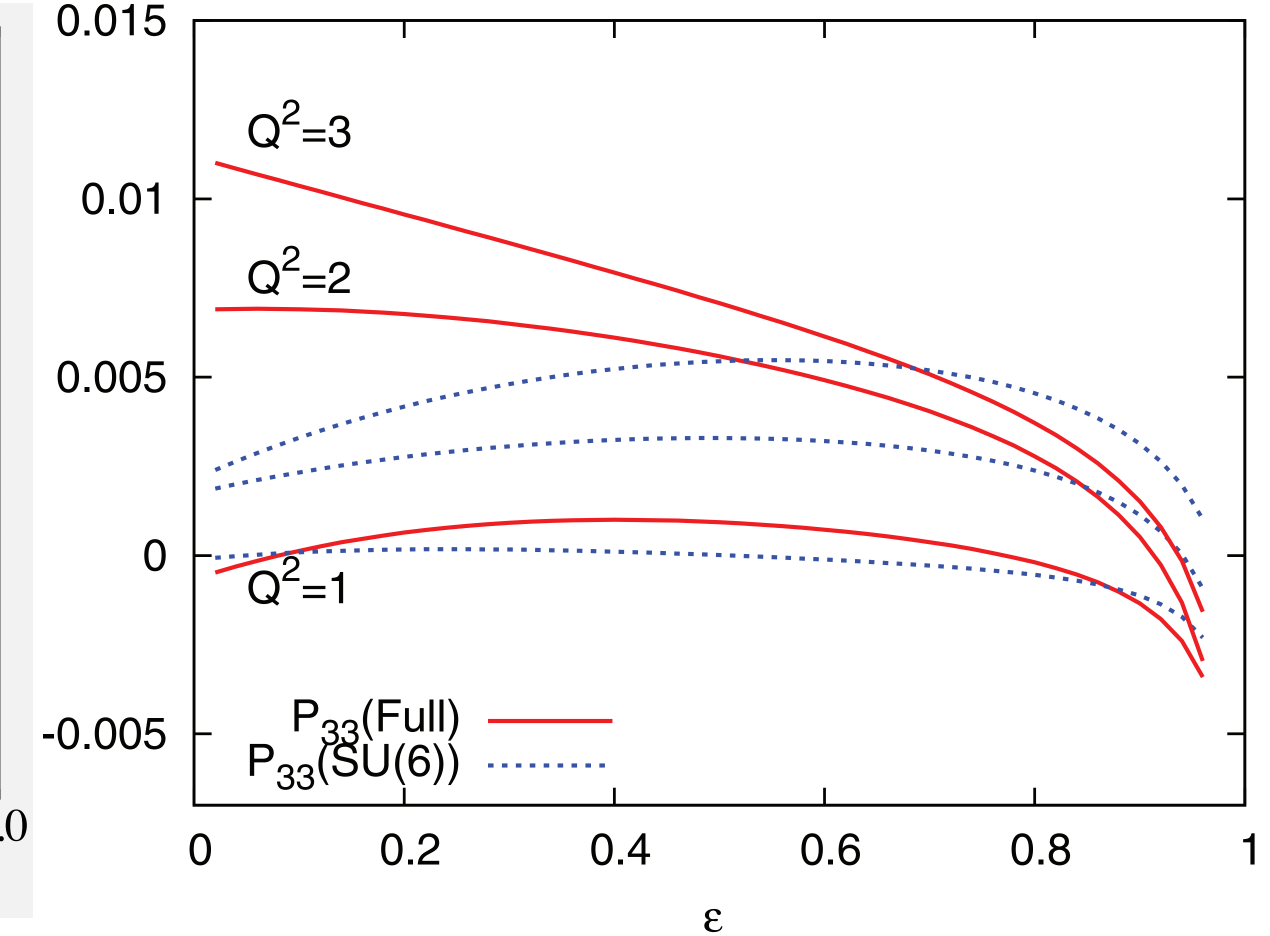
→ imaginary part given by

$$\Im m\delta(\nu) \sim \alpha \int dW \underbrace{\int dQ_1^2 \int dQ_2^2 \, \frac{1}{Q_1^2 Q_2^2} \left\{ L_{ijk} H^{ijk} \right\}}$$

- For dipole form factors, 2D integral can be done analytically; expressible in terms of elementary functions.
- Can also be done numerically for more general form factor parametrizations

$\nu_{\rm th}$ extends into the unphysical region (ε<0)



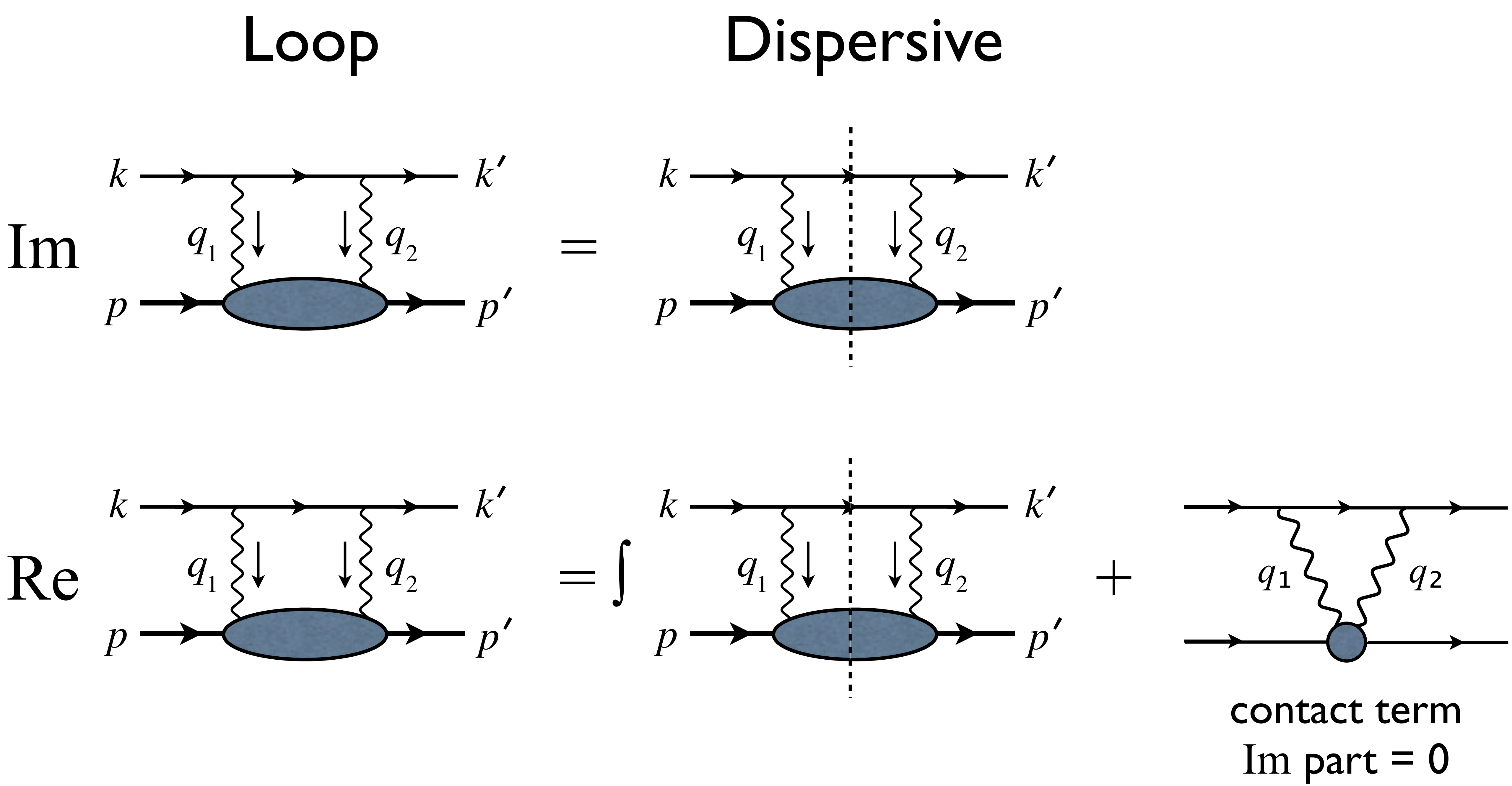


PGB: dispersive calculation

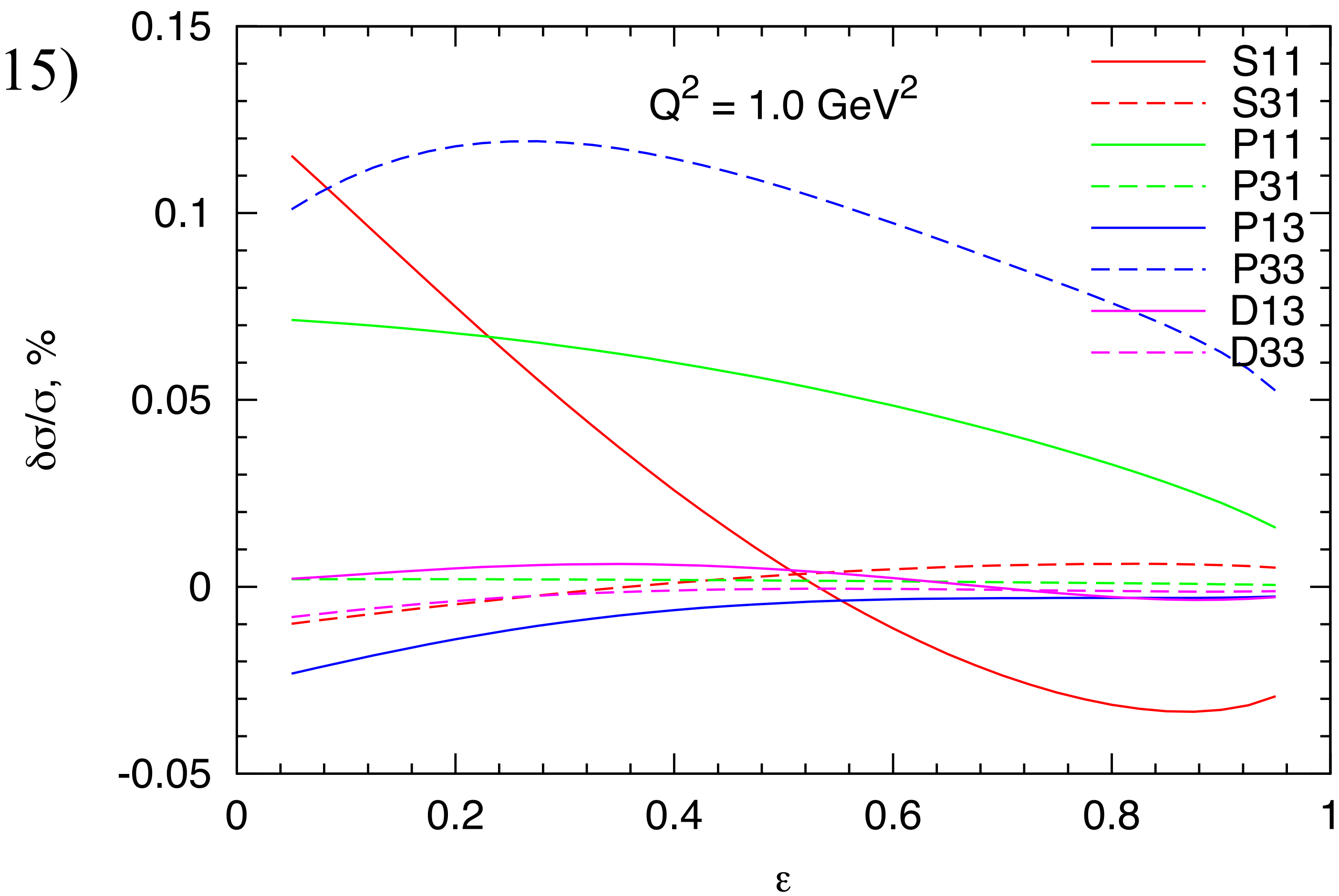
Graczyk, Phys. Rev. C **88**, 065205 (2013)
loop calculation

Both techniques agree reasonably well at low ε (small E), but only the dispersive method gives a vanishing contribution as $\varepsilon \to 1$.

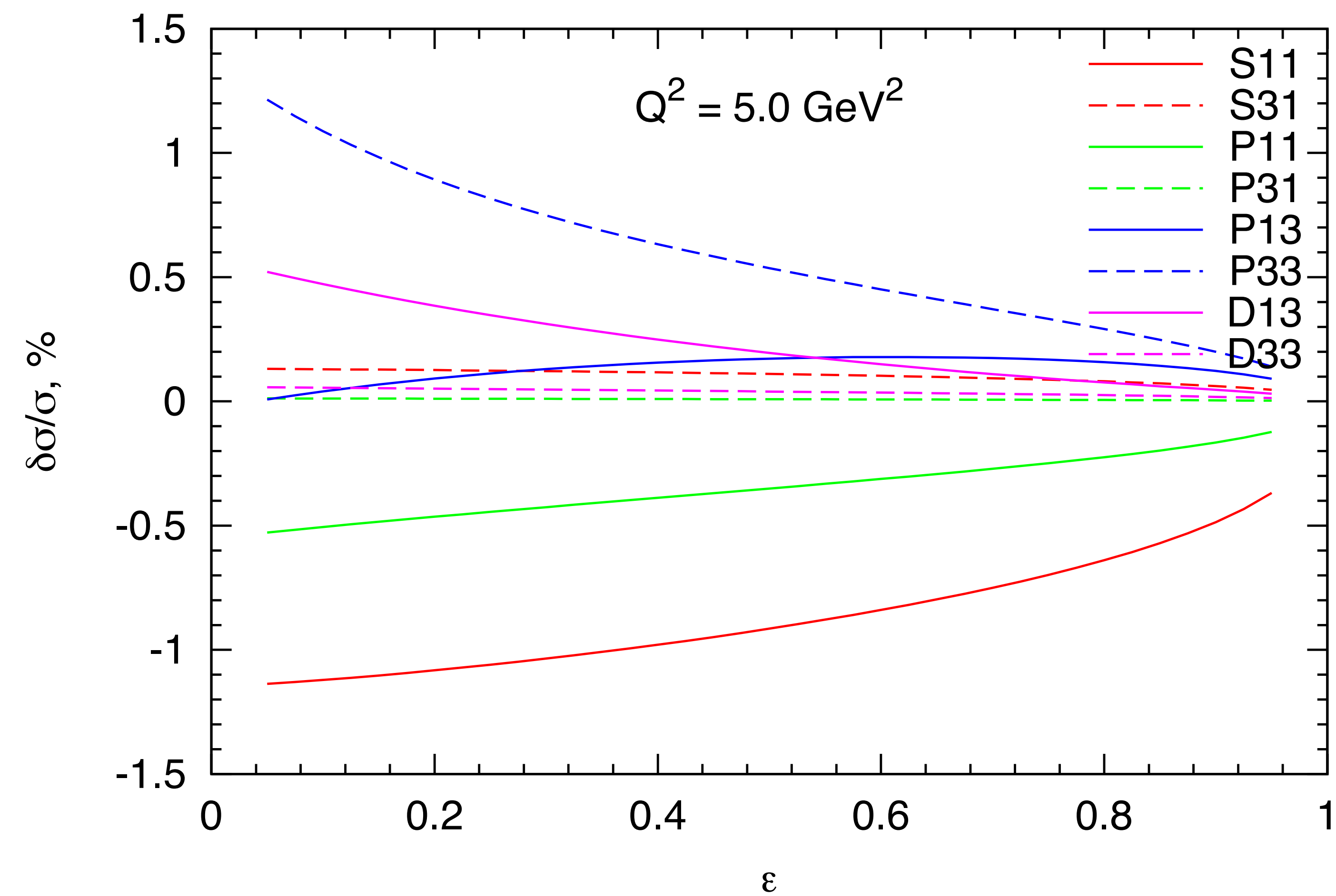
Why? Isn't this contrary to Cutkowsky rules?



Borisyuk & Kobushkin, arXiv:1506.02682 (2015)



- Include other spin 1/2 and 3/2 resonances using MAID helicity amplitudes
- Include a finite width
- Contributions tend to cancel, in qualitative agreement with Kondratyuk & Blunden result



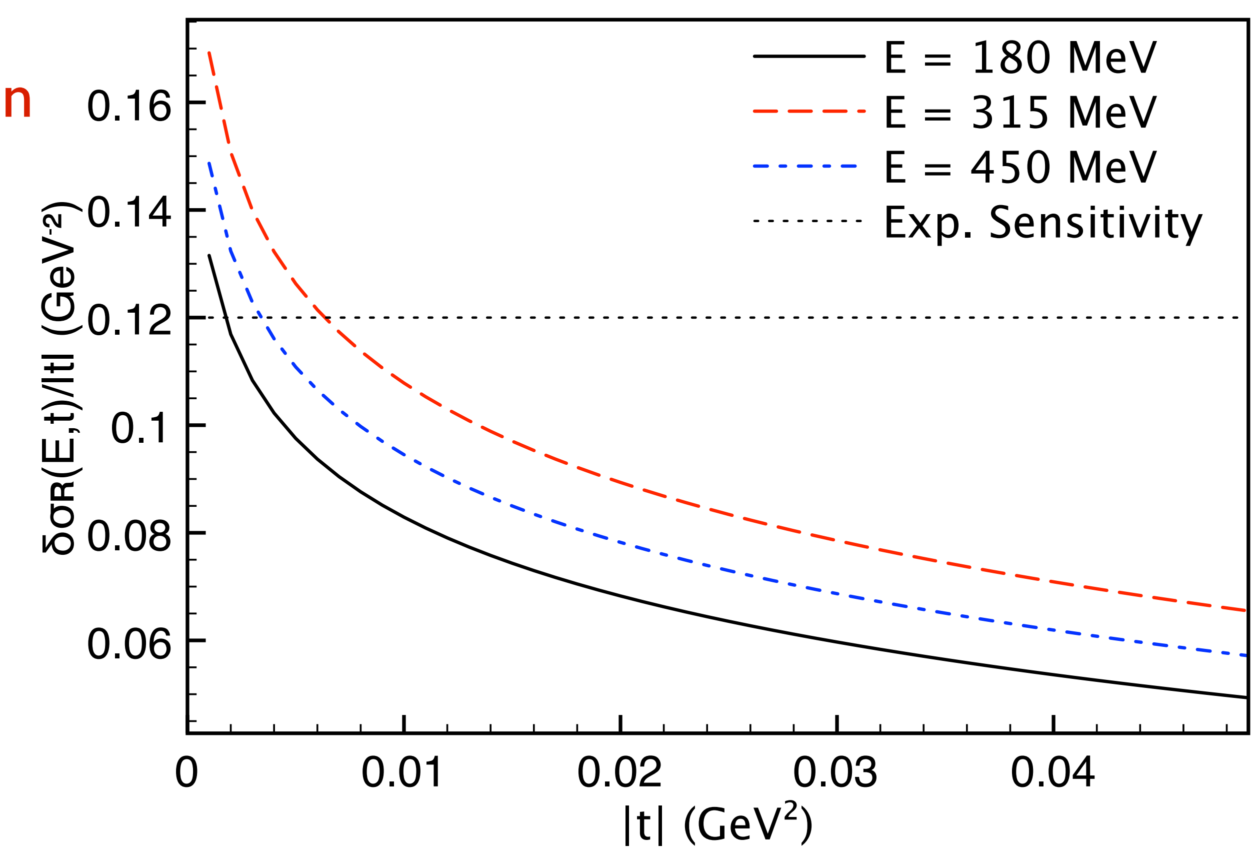
Gorchtein, arXiv: 1406.1612 (2014)

Model-independent analysis of corrections in forward kinematics (forward angles, low Q^2) using dispersive analysis

TPE amplitude $\Phi(E)$: See also Brown, Phys Rev D **1**, 1432 (1970)

$$\Im m \Phi(E) = \frac{Q^2}{4\pi^2} \int_{E_\pi}^{E} \frac{d\omega}{\omega} \sigma_T(\omega) \ln\left(\frac{4\omega_{\rm cm}^2}{Q^2}\right) \left[1 - \frac{\omega}{E} + \frac{\omega^2}{2E^2}\right]$$

Total photoabsorption cross section



Summary

- Lots of interesting new theoretical work motivated by new experimental results
- Dispersive method promising approach with connection to data in forward angle limit
  - A similar approach is essential for the γZ box in Qweak parity-violation kinematics