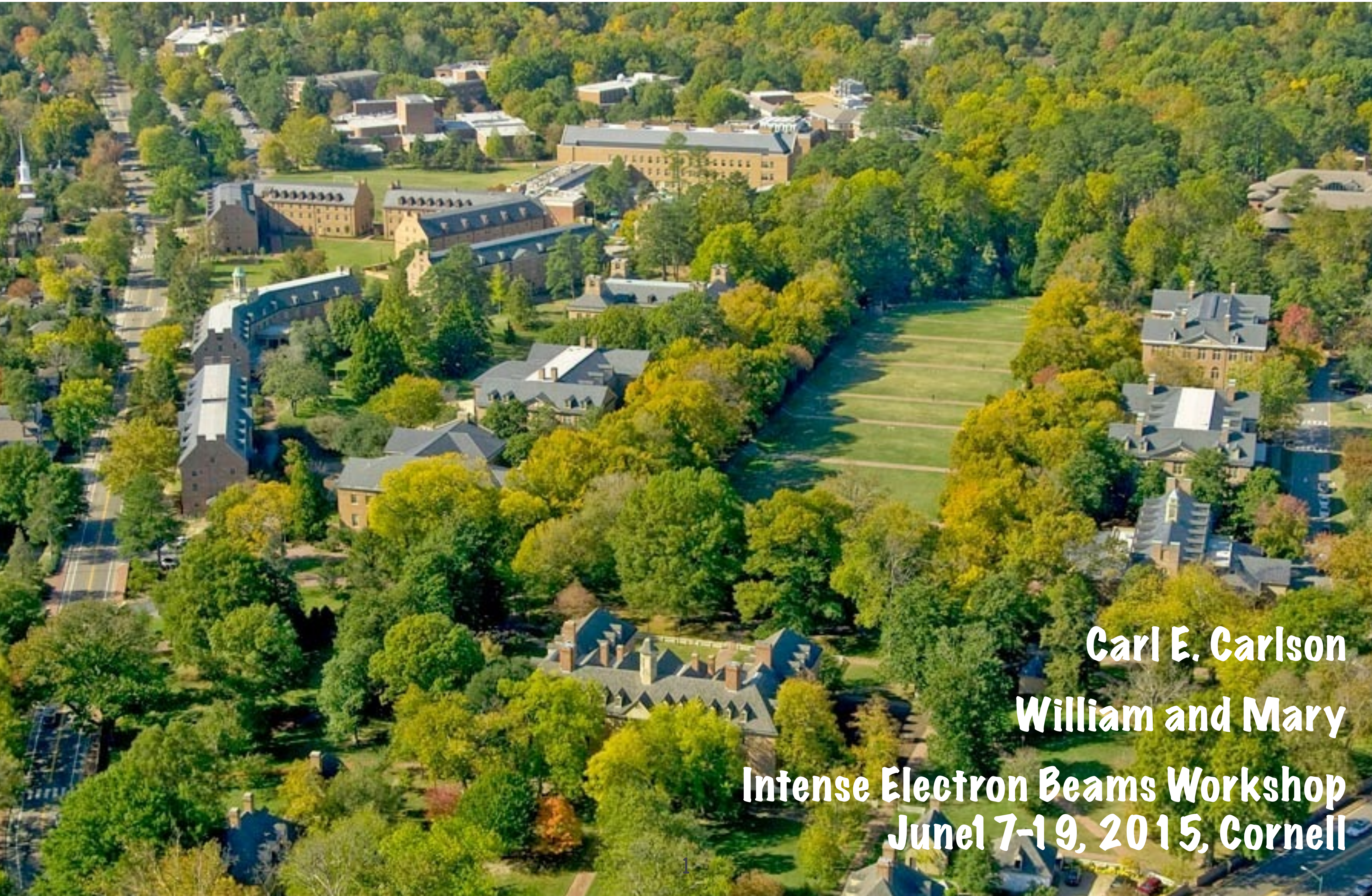
Calculations of γZ corrections-Box diagrams



Carl E. Carlson

William and Mary

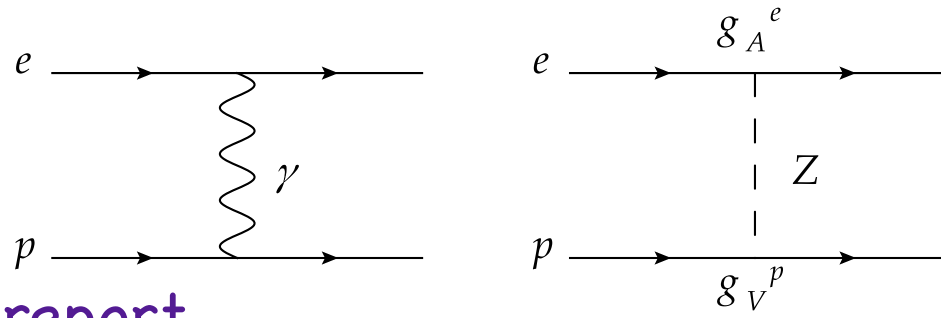
Intense Electron Beams Workshop
June17-19, 2015, Cornell

Topics

- PV in ep scattering and QWeak
- A startling (at least in 2009) calculation
- It may be settled
- But we would like to be sure
- How PVDIS can help

relevant for today

- Parity violating (PV) electron scattering
- Usually, polarized electron, unpolarized target
- Parity violation exists in SM, from (small at low energy) Z-exchange



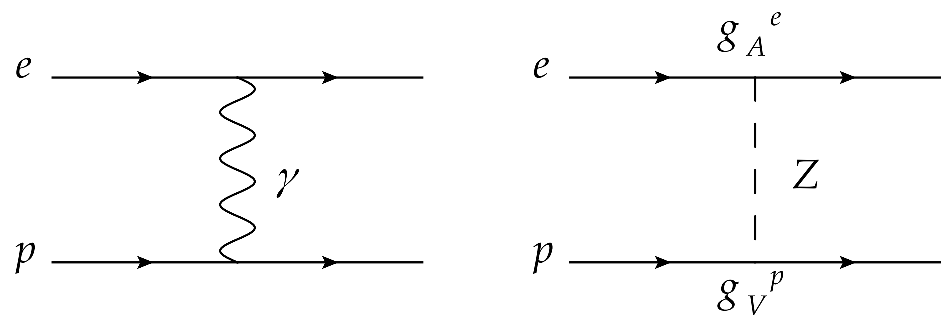
- Usually report

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

(R,L = helicity of electron)

QWeak---from elastic ep scatt.

- At LO, asymmetry comes from interference between photon exchange and Z-boson exchange,



- For Q^2->0,

$$A_{PV} = -\frac{G_F}{4\pi\alpha\sqrt{2}} Q^2 Q_W^P$$

- LO only,

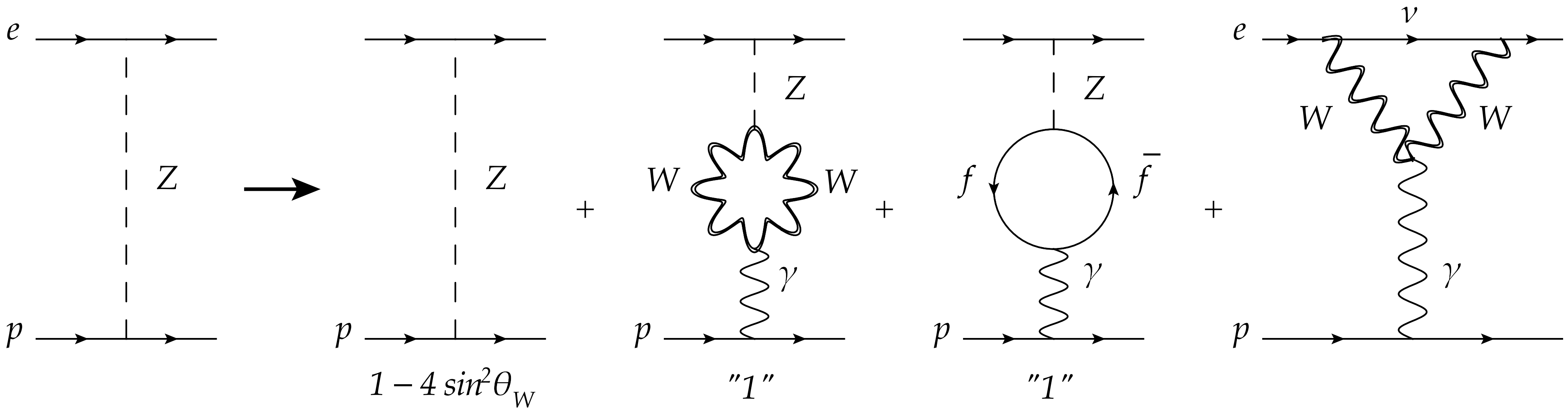
$$Q_W^{p,LO} = 1 - 4\sin^2\theta_W$$

For later, JLab QWeak runs at E_{elec}=1.165 GeV, Q^2 = 0.026 GeV^2

Mainz (P2 at MESA) plans for E_{elec} = 150 MeV

QWeak

- Interesting because of HO corrections, e.g.,



- Changes balance between "1" and "4sin² θ_W".

$$1 - 4\sin^2\theta_W \rightarrow 1 - 4\,\kappa(Q^2)\sin^2\theta_W \equiv 1 - 4\,\sin^2\theta_W(Q^2)$$

- Thus, $\sin^2\theta_W$ "runs" or "evolves" with Q^2.

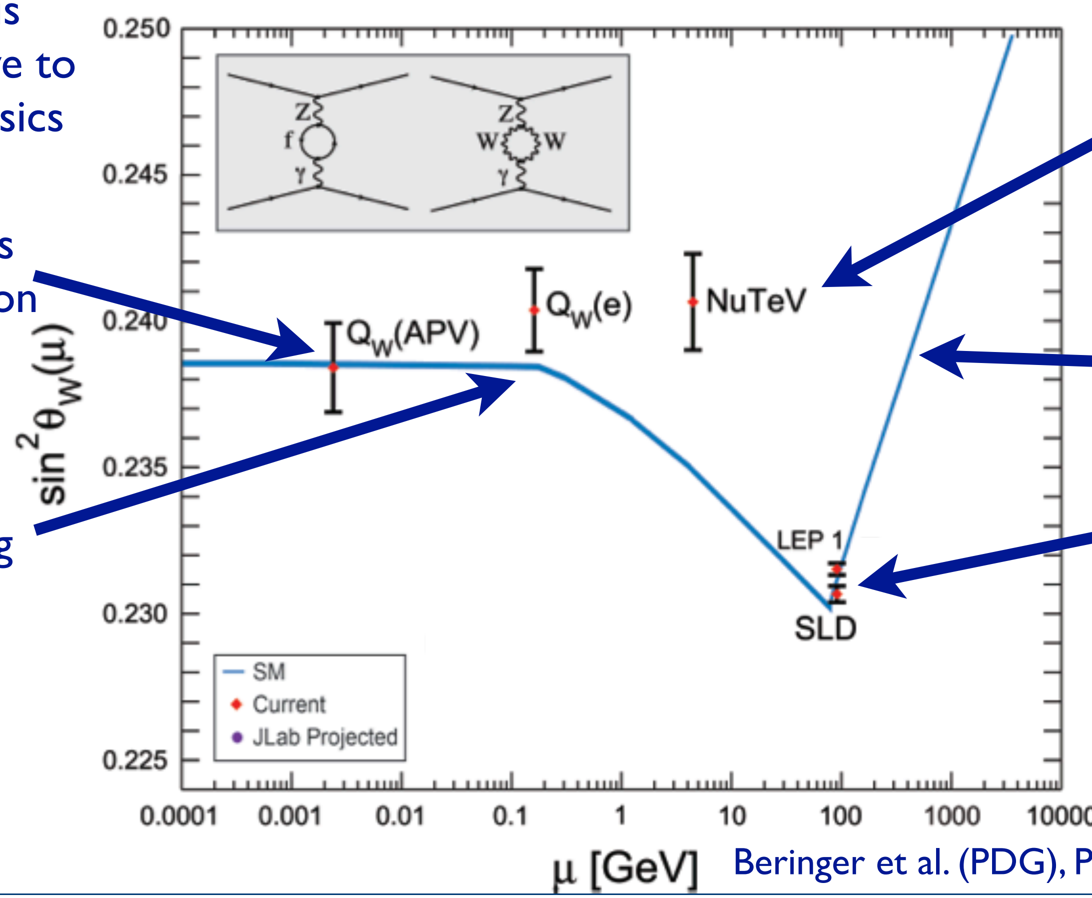
- If SM complete---particle content and interactions known---evolution can be precisely calculated.

SM $\sin^2\theta_W$ evolution

Each experiment is differently sensitive to potential new physics

$6S \rightarrow 7S$ ^{133}Cs atomic transition

Parity violating moller scattering



neutrino deep-inelastic scattering cross-sections (controversial hadronic corrections not included)

Standard Model electroweak fit with uncertainty

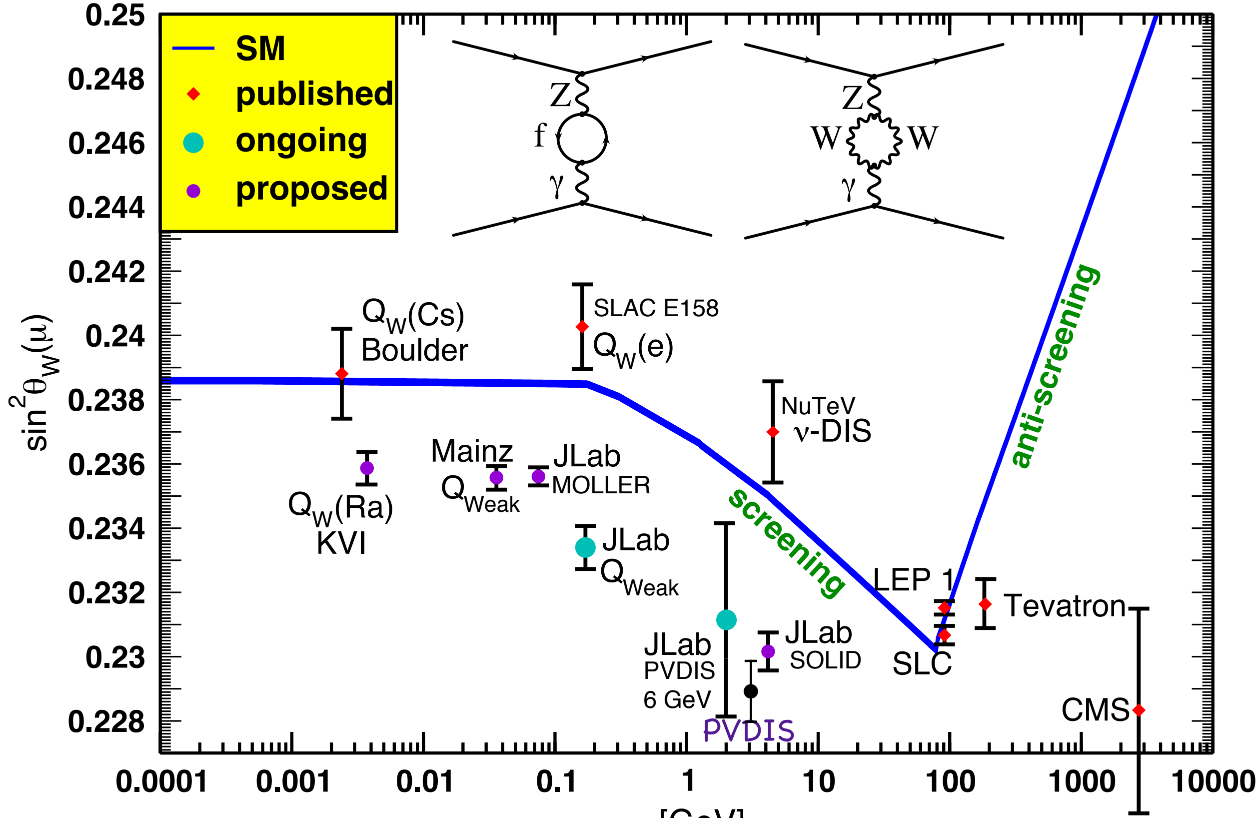
Colliders

Beringer et al. (PDG), Phys. Rev. D86, 010001 (2012)

- If SM correct, result from QWeak will lie on curve.
- If not ....
- Precision needed!

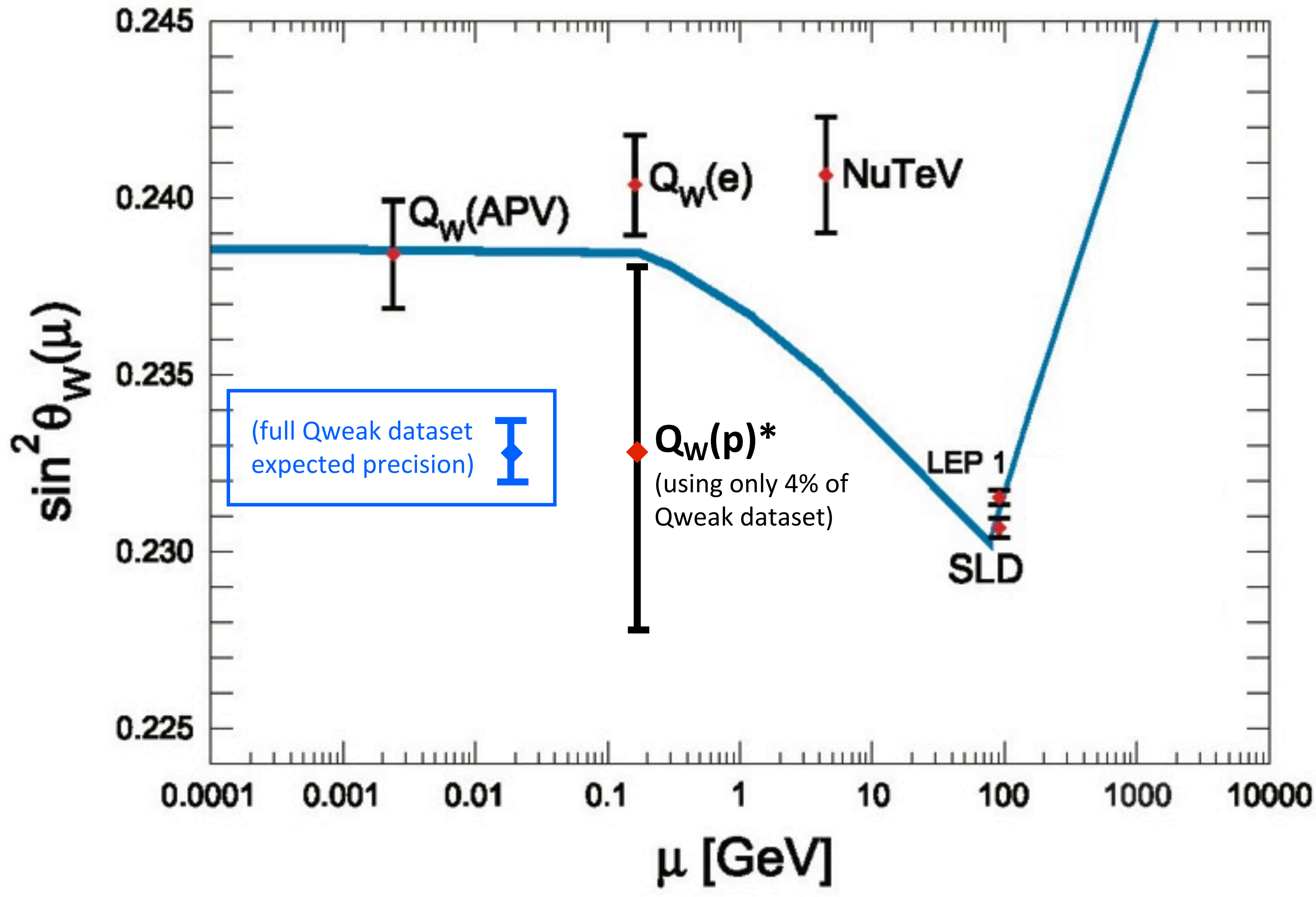
and still more data will come

- From PDG,
  or from Erler,
  1208.6262,
- with future hopes



Report: QWeak has data

4% of total data



from Mark Dalton, APS/DNP meeting, Fall 2012
Publ.: PRL 111 (2013) 14, 141803

But there are other corrections

Correction to ρ

$1 - 4\sin^2\theta_W(0)$

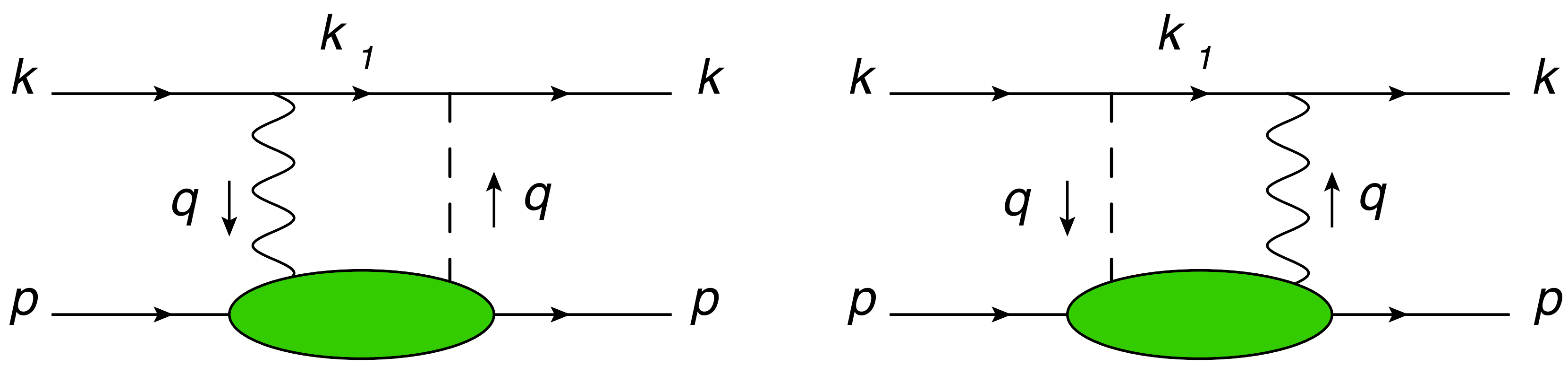
Troublesome box

$$Q_W^p = (1 + \Delta\rho + \Delta_e)\left(Q_W^{p,LO} + \Delta_e'\right) + \Box_{WW} + \Box_{ZZ} + \mathrm{Re}\,\Box_{\gamma Z}$$

Corrections to the Z-boson and photon vertices

Well understood box corrections

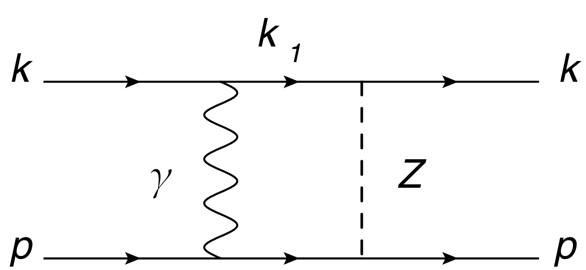
γ-Z Box



- (Dashed line for Z.)
- Only one heavy propagator. Low momenta dominate loop.
- Both vector and axial Z-proton couplings contribute. Abbreviated $\Box_{\gamma Z}^{V}$ and $\Box_{\gamma Z}^{A}$.

Now starts a story

- Big note: $\Box_{\gamma Z}^{V}(E)$ is odd in E; $\Box_{\gamma Z}^{A}$ is even in E (electron beam en.) (Crossing symmetry argument… .)
- Old days (< 2009), calculated basic box at threshold E=0. Thought actual E low enough to use this result.



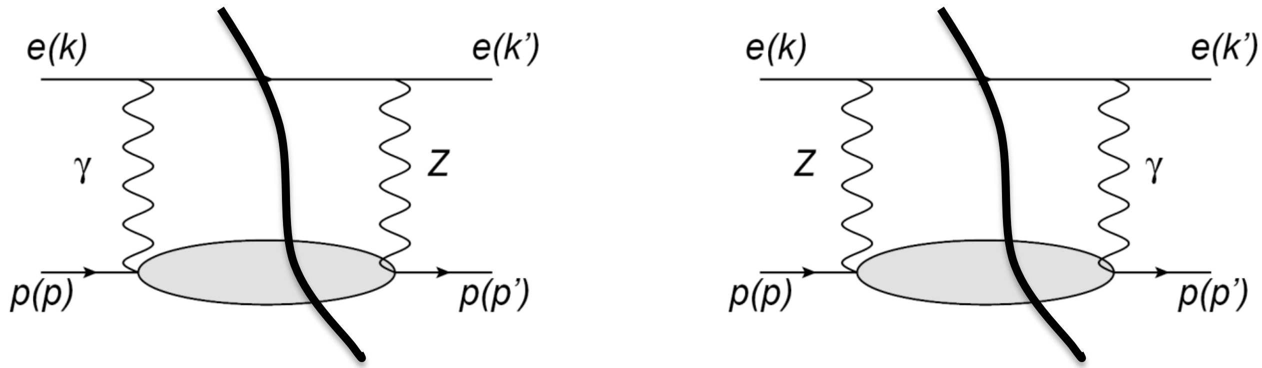
(+ reverse and crosses)

- Still old days: Dumped $\Box_{\gamma Z}^{V}$.

- Defacto just $\Box_{\gamma Z}^{A}$. (Will hardly talk about it today.)

γ-Z Box

- Gorchtein and Horowitz (PRL 102, 091806 (2009)) had insight to calculate the amplitude dispersively



- DR $\rightarrow$ calculate whole amplitude form imaginary part.
- Imaginary part comes when intermediate states on shell.
- Like inelastic amplitude squared, i.e., for DIS. Squares given and measured as structure functions F_i .
- Only problem: $F_i^{\gamma\gamma}$ measured, not the interference term $F_i^{\gamma Z}$.

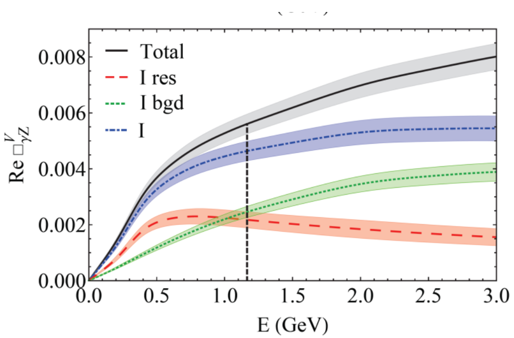
Maybe a problem

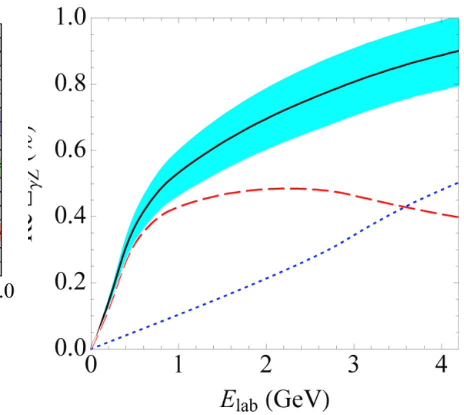
- Gorchtein-Horowitz first estimate of $\Box_{\gamma Z}^{V}$ (the thing that was supposed to be zero) was twice the size of the projected experimental uncertainty of the Q_{Weak} experiment.

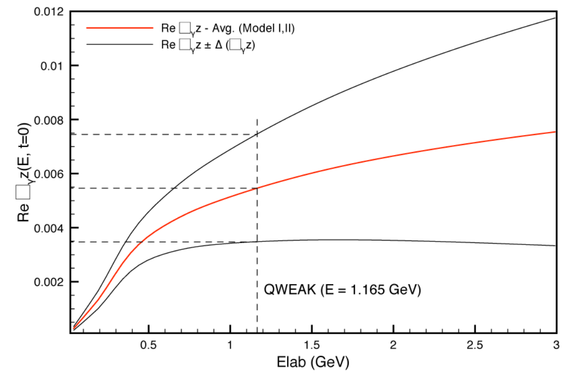
- People got busy.

Vector box plots today

Hall *et al.*	Carlson and Rislow	Gorchtein *et al.*
PRD 88, 013011 (2013)	PRD 83, 113007 (2011)	PRC 84, 015502 (2011)







$$\mathrm{Re}\square^V_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

$$(5.6 \pm 0.36) \times 10^{-3} \quad (5.7 \pm 0.9) \times 10^{-3} \quad (5.4 \pm 2.0) \times 10^{-3}$$

- Central values close
- Differences come from the treatment of the structure functions
- BTW, we combined errors directly, Hall et al. in quadrature. Could repeat:

$$\mathrm{Re}\,\square^V_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

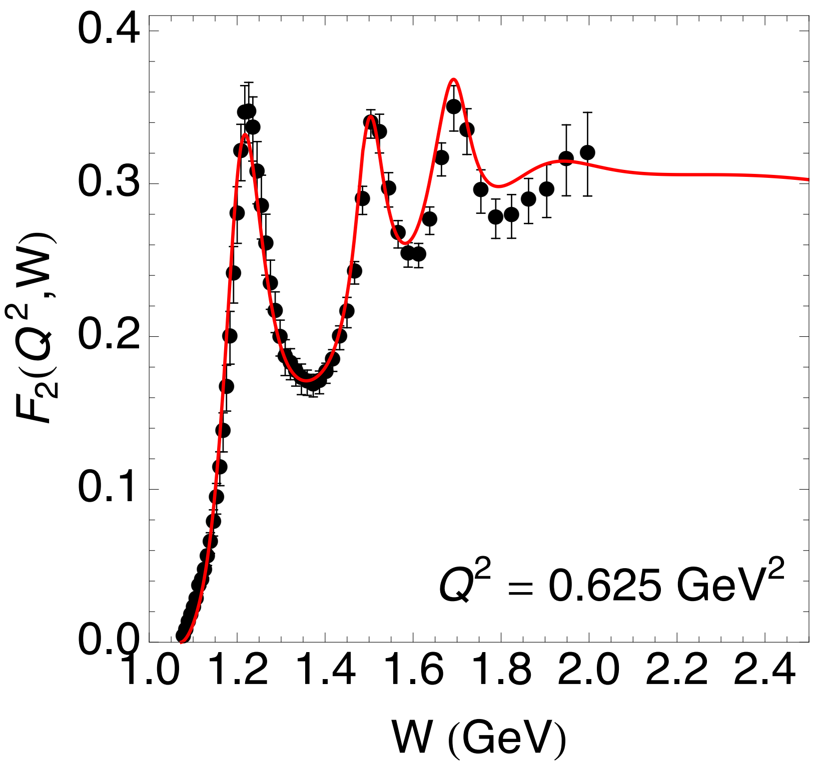
$$(5.6 \pm 0.36) \times 10^{-3} \quad (5.7 \pm 0.52) \times 10^{-3} \quad (5.4 \pm 2.0) \times 10^{-3}$$

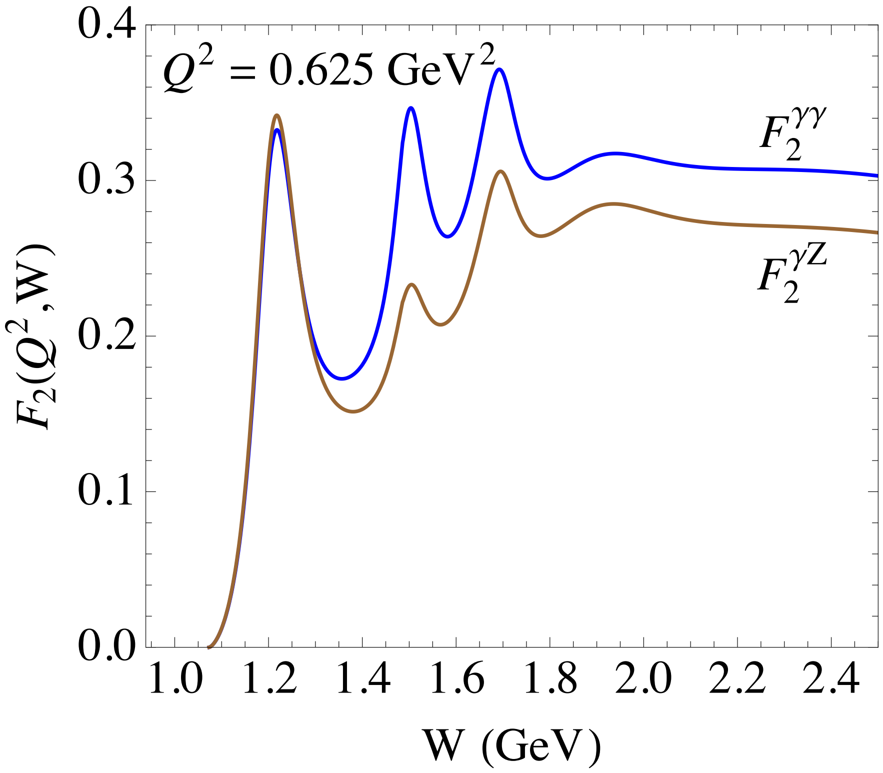
Why not be happy?

- Where from came results?
- Resonance contributions: basically from fit of Bosted and Christy for $F_i^{\gamma\gamma}$ modified using
  - NR quark model (Rislow and me)
  - Isospin rotations and neutron data (GHRM, Hall et al.), getting p/n ratio from PDG, finessing Q^2 dependence
  - As above, getting resonant amplitudes and Q^2 dependence from MAID fits (Rislow and me, later attempt)

Data plots and functions

- The Bosted-Christy fits are good. Sample:
- 2nd plot shows difference $F_i^{\gamma\gamma}$ to $F_i^{\gamma Z}$





Note on isospin rotations

- Basic relation

$$2\langle R^+|J_\mu^{Z_V}|p\rangle = (1-4\sin^2\theta_W)\langle R^+|J_\mu^\gamma|p\rangle - \langle R^0|J_\mu^\gamma|n\rangle - \langle R^+|\bar{s}\gamma_\mu s|p\rangle$$

- Neglect contribution of strange quark (A4, G0, HAPPEX)
- Need two things: Proton electromagnetic matrix elements
  - GHRM get them from identifiable resonance terms in Christy-Bosted fit
  - (as we did also)
- and then need neutron matrix elements. GHRM obtain matrix elements at Q^2 = 0 from PDG, form n/p ratios, and then use above relation. Omitted Q^2 dependence in n/p ratios.
- Can also get resonance electroproduction amplitudes from MAID.
- Above is for resonances. Background, both under (in) resonance region and above resonance region still to be discussed.

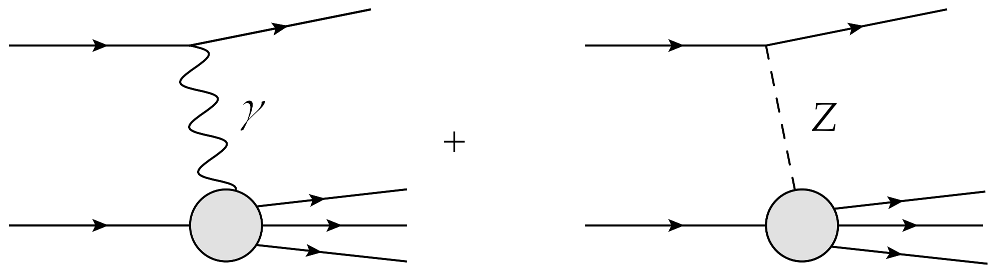
Note on non-resonant contributions

- The difficult region is low Q^2 and high W
- We took Christy-Bosted background, got guidance from scaling region to argue that for the γZ version was between 2/3 and 3/3 of the $\gamma\gamma$ values.
- GHRM took two $\gamma\gamma$ fits to HERA and ZEUS data (much higher energies) and extrapolated to the support region for the present case. Difference between the two extrapolations gave the bulk of their uncertainty.

Think of something!

- Although results similar, they come after doing some integrals, and there are regions where the integrands are fairly different.
- The interference structure functions $F_i^{\gamma Z}$ actually are measurable. Use Parity Violating Deep Inelastic Scattering (PVDIS).

PVDIS, esp. in res. reg.



- PVDIS asymmetry directly depends on $F_i^{\gamma Z}$

$$A_{PVDIS} = g_A^e \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{xy^2 F_1^{\gamma Z} + \left(1 - y - \frac{x^2 y^2 M^2}{Q^2}\right) F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}\left(y - \frac{y^2}{2}\right) x F_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + \left(1 - y - \frac{x^2 y^2 M^2}{Q^2}\right) F_2^{\gamma\gamma}}$$

- $x = Q^2/2m_p\nu$; $y = \nu/E$; $g_A^e = -½$; $g_V^e = -½+2\sin^2\theta_W$
- with unlimited data can obtain all $F_i^{\gamma Z}(\nu,Q^2)$
- with some data, can check other models
- for $\square_{\gamma Z}^V$, resonance region dominates integrals

for context—scaling region

- write $F_i^{\gamma Z}$ in terms of quark distribution functions,

$$A_{PVDIS} = \frac{3G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{2C_{1u}(u_A + \bar{u}_A) - C_{1d}(d_A + \bar{d}_A + s_A + \bar{s}_A) + Y\left(2C_{2u}(u_A - \bar{u}_A) - C_{2d}(d_A - \bar{d}_A)\right)}{4(u_A + \bar{u}_A) + d_A + \bar{d}_A + s_A + \bar{s}_A}$$

$$Y(y) = \frac{1-(1-y)^2}{1+(1-y)^2} \quad , \quad C_{1q} = 2g_A^e g_V^q \quad , \quad C_{2q} = 2g_V^e g_A^q$$

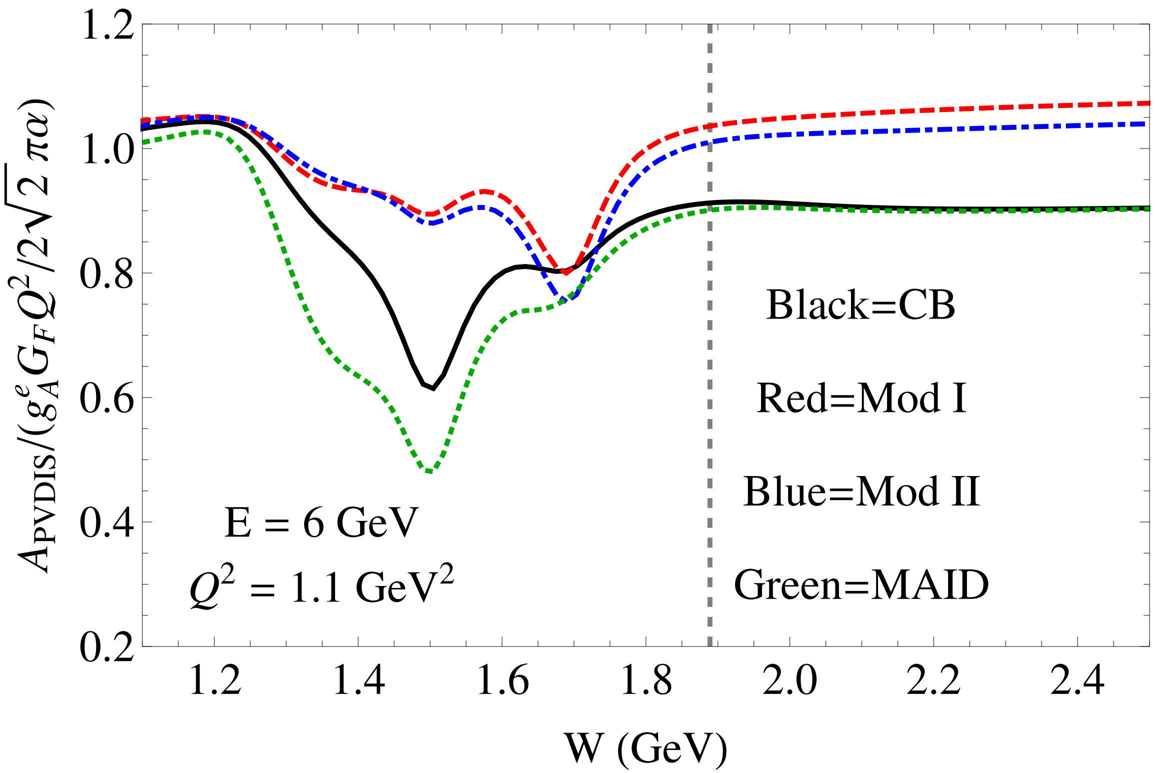
- Scaling region is $x \to 1$, $y \to 1$, $Y \to 1$, antiquark and strange distributions $\to 0$, and for deuteron, $u_A = d_A$,

$$A_{PVDIS} = \frac{3G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{2C_{1u} - C_{1d} + 2C_{2u} - C_{2d}}{5}$$

- The C_1's are better known, can test BSM for C_2's.

PVDIS in res. reg.

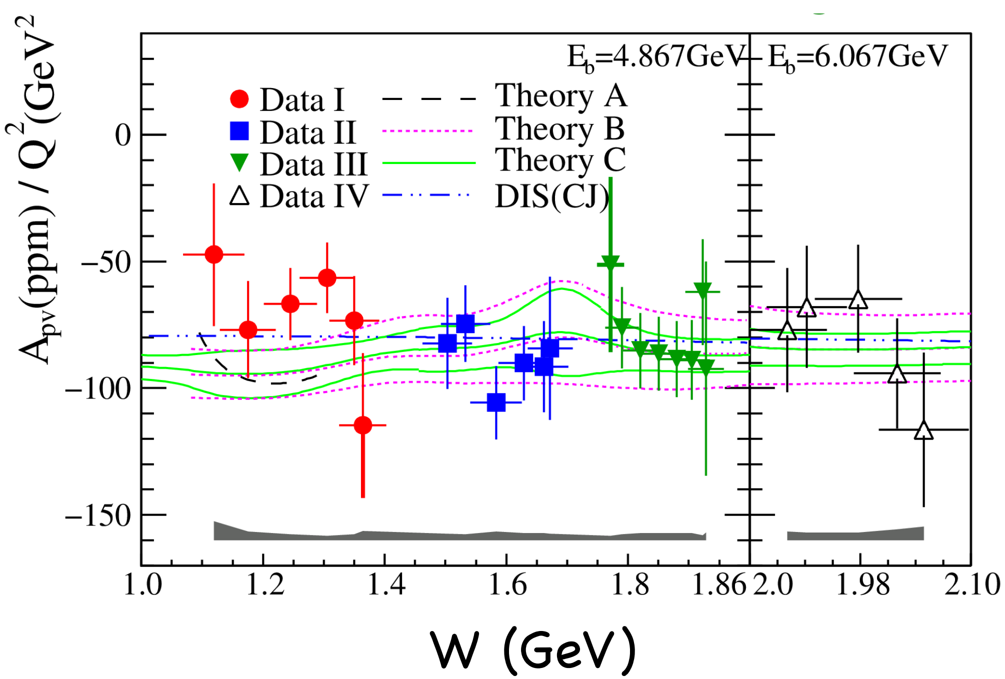
- For sparser data case, here are **predictions** from existing models,

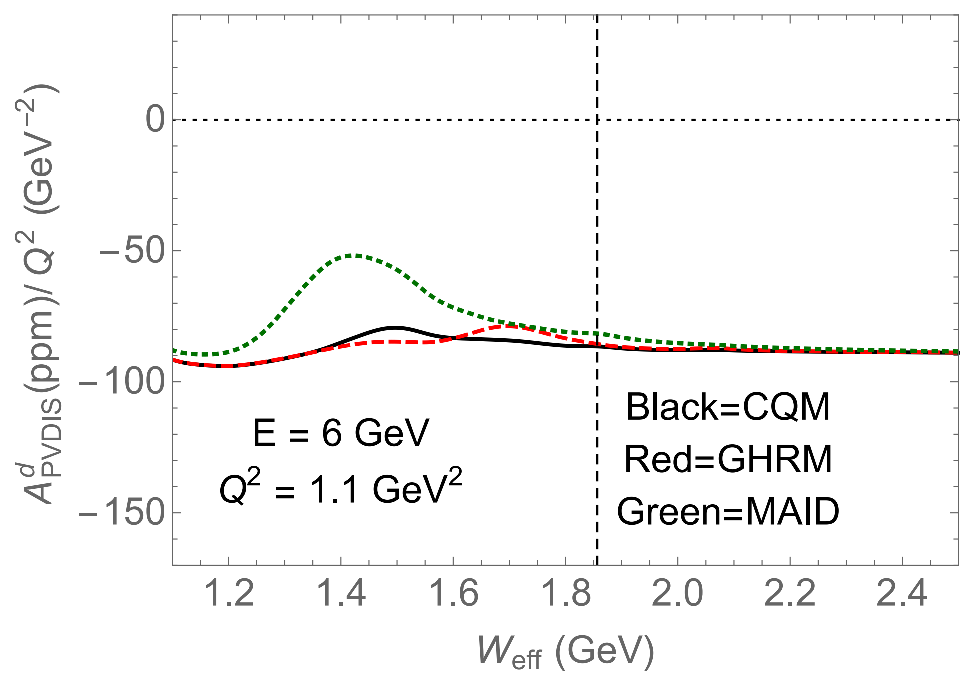


- **this is proton target**
- CB = CQM modified Christy-Bosted $F_{1,2}^{\gamma\gamma}$ fit
- Model I, II = GHRM based results
- MAID from isospin rotated MAID p & n EM fits
- Vertical dashed line = 6 GeV PVDIS expt. point
- JLab expt has some public data in scaling region

deuteron predictions and data

- for the deuteron, there is PVDIS data in the resonance region: Wang et al., PRL 111, 082501 (2013)
- Calc: Rislow and me, PRD 85, 073002 (2012),
  Matsui et al. (2005); Gorchtein et al. (2011);
  Hall et al (2013).





general statements regarding data

- also want data on proton
- more precise
- useful: lower Q^2 (few tenths GeV^2) and high W. This is where the background disagreements lie.

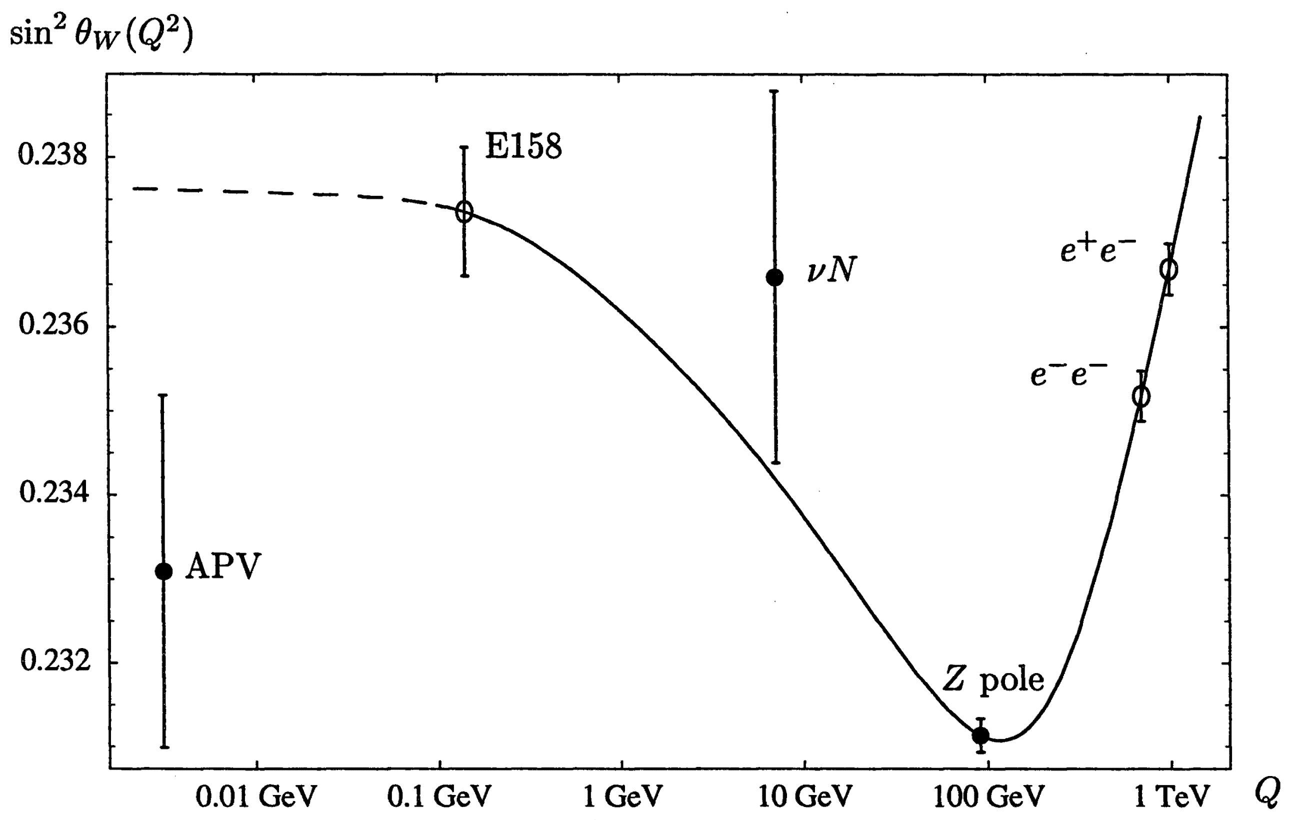
Summary

- The world is saved—maybe—regarding the γZ corr. to Q_{Weak}.
- I.e., $\square_{\gamma Z}^{V}$ now calculated.
- About (8.1±1.4)% of Q_W^p at E_{elec}=1.165 GeV. Proportional to E_{elec}.
- Not discussed here: $\square_{\gamma Z}^{A}$ also now calculated w/o guesswork certain log terms
- About (6.3±0.6%) of Q_W^p at E_{elec} threshold. Small dependence on E_{elec}. Might still like to improve.
- For goal of 1% or better measurement of QWeak (Mesa), energy is about 1/6 of JLab experiment, and corrections and error in $\square_{\gamma Z}^{V}$ scale with energy.
- PVDIS can help shrink uncertainty limits.

Beyond the end

Cusps and kinks

- A smoother view, albeit from year 2000



Czarnecki & Marciano

Comments on $\Box_{\gamma Z}{}^{A}$

- For some of integral, $F_3{}^{\gamma Z}$ is in resonance region. No e.m. analog (parity violating). Get by
  - fits to neutrino resonance region data (Lalakulich et al., '06)
    - but there is ≈ no data
  - or by quark modeled modifications of e.m. case.
- Published results (BMT) are with first. Rislow and I have done the second. Not wildly different overall for $\Box_{\gamma Z}{}^{A}$ although noticeably different for resonance part alone. Adds to uncertainty.