



INTENSE ELECTRON BEAMS WORKSHOP

CORNELL UNIVERSITY, JUNE 17-19, 2015

Nuclear and nucleon structure effects in low-energy parity-violating electron scattering

The carbon 12 case and
a detour to neutrino coherent scattering

O. Moreno and T. W. Donnelly

Center for Theoretical Physics and
Laboratory for Nuclear Science,
Massachusetts Institute of Technology





SUMMARY

- Motivation and goals
- Introduction
- Analysis of nuclear and nucleon structure effects
 - Coulomb distortion
 - Nuclear isospin mixing
 - Nucleon strangeness content
 - Meson exchange currents
 - Inelastic transitions
- Conclusions
- References



MOTIVATION

- Current interest in low-energy, high-luminosity polarized electron beams for high precision parity-violating experiments: MESA@Mainz, FEL@JLab, C β @Cornell.



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- They are aimed at the evaluation of the weak mixing angle, the size and momentum-dependence of higher-order electroweak radiative corrections, ...
- To this end, knowledge is required of the size and uncertainty of the nuclear and nucleon structure effects involved.



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- Computation of nuclear and nucleon structure effects on the parity-violating observables in electron scattering. Focus on carbon 12 ($J^\Pi = 0^+$, $N=Z$) as a target.



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- Estimation of the theoretical uncertainties related to the previous calculations.
- Determination of the kinematic conditions for acceptable theoretical and statistical uncertainties.



INTRODUCTION: PARITY-VIOLATING ASYMMETRY

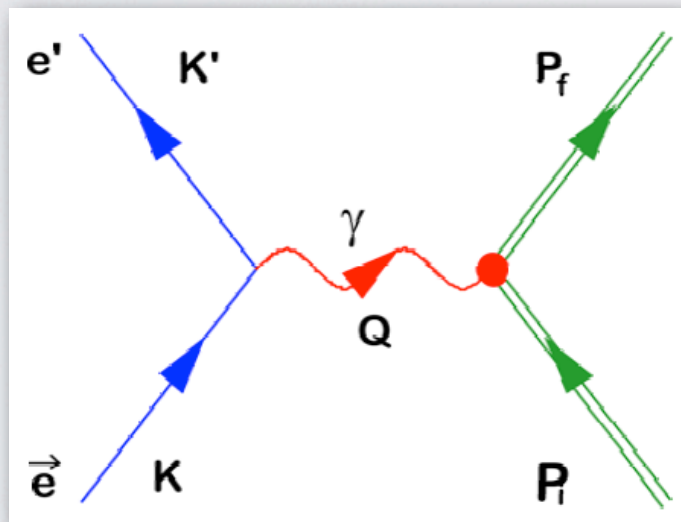
Definition of parity-violating (helicity) asymmetry:

$$A = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

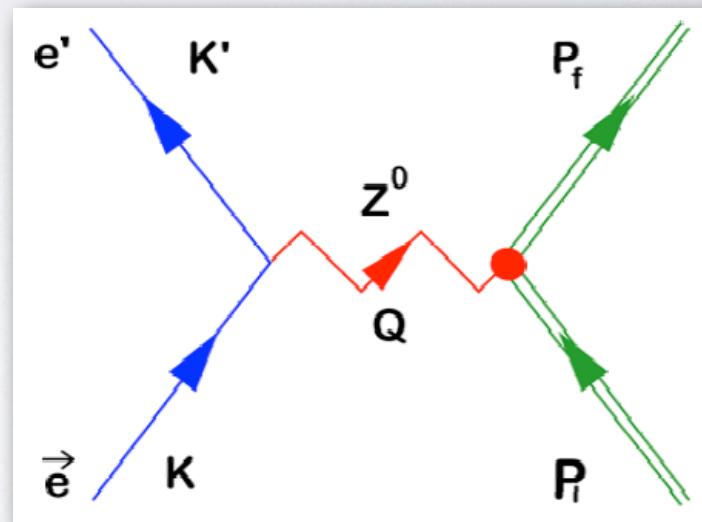
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Within plane wave Born approximation:

$$A = \frac{G_F}{2\sqrt{2}\pi\alpha} |Q^2| \frac{a_A^e(v_L \hat{W}_L + v_T \hat{W}_T) + a_V^e v_{T'} \hat{W}_{T'}}{v_L W_L + v_T W_T}$$

\downarrow
 $1.8 \cdot 10^{-4} \text{ GeV}^{-2}$



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For **isospin-0**, **spin-0** targets ('reference asymmetry'):

$$\mathcal{A} = \mathcal{A}^{\text{ref}} \equiv -\frac{G_F}{\sqrt{2}\pi\alpha} |Q^2| a_A^e \sin^2 \theta_W \sim 10^{-6}$$



INTRODUCTION: PARITY-VIOLATING ASYMMETRY

Assumptions:

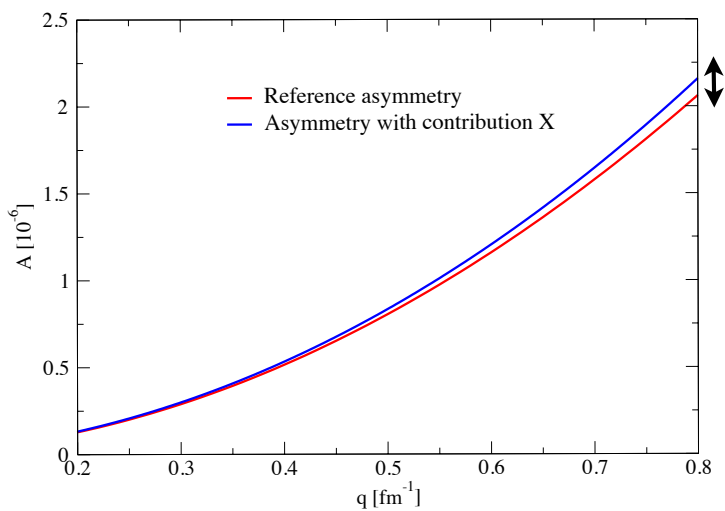
- Only one photon and one Z^0 are exchanged.
- Electron wave functions are not distorted by the target charge.
- Target state is spin-0, isospin-0 (non isospin mixing).

$$\mathcal{A} = \mathcal{A}^{\text{ref}} \equiv -\frac{G_F}{\sqrt{2}\pi\alpha} |Q^2| a_A^e \sin^2 \theta_W$$



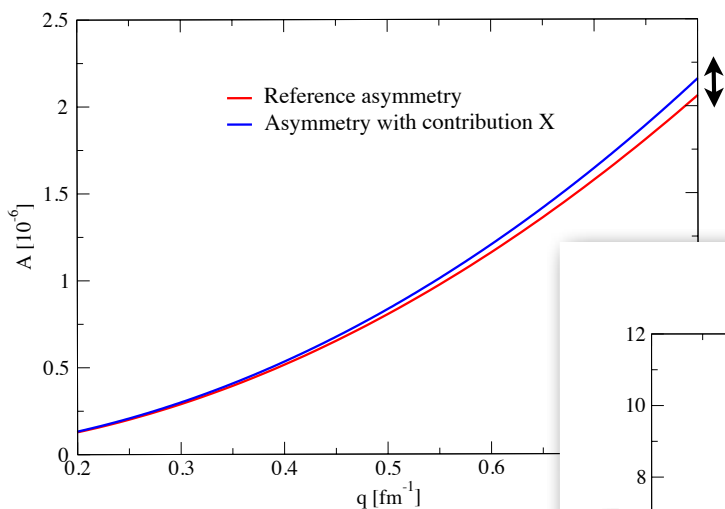
INTRODUCTION: THEORETICAL UNCERTAINTY

Absolute effect of feature X

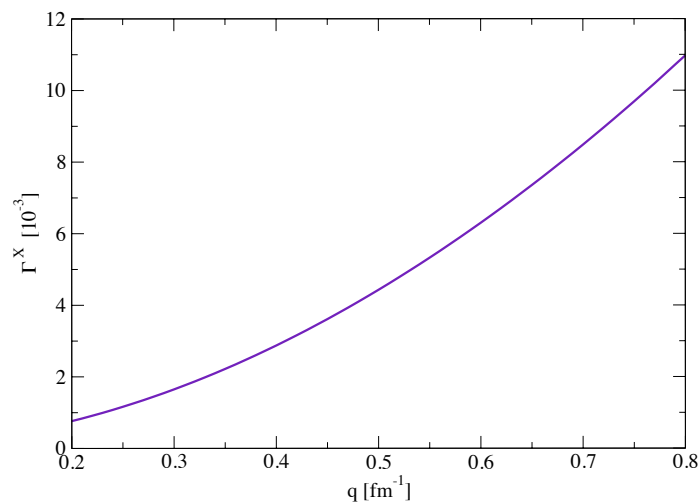


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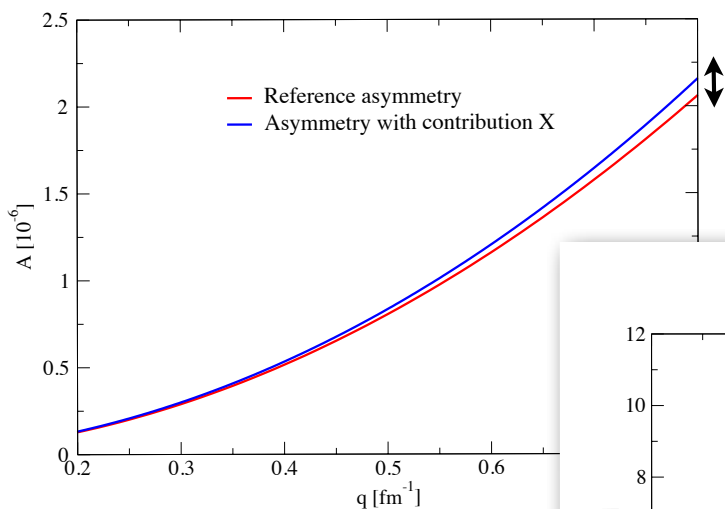
Relative effect of feature X



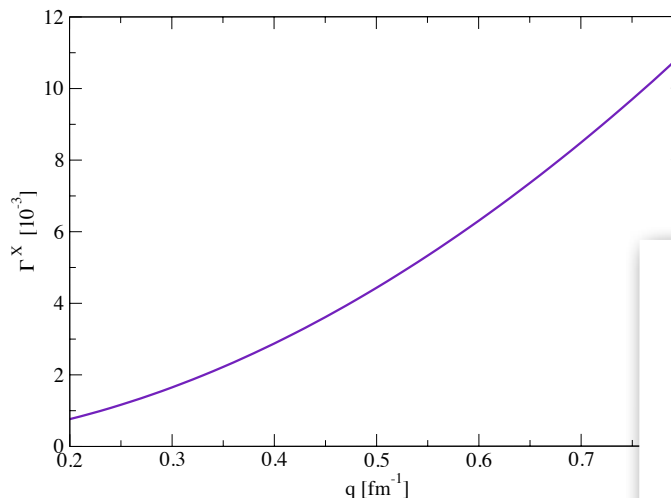
$$\Gamma^X = \frac{A^X - A^{\text{ref}}}{A^{\text{ref}}}$$

INTRODUCTION: THEORETICAL UNCERTAINTY

Absolute effect of feature X

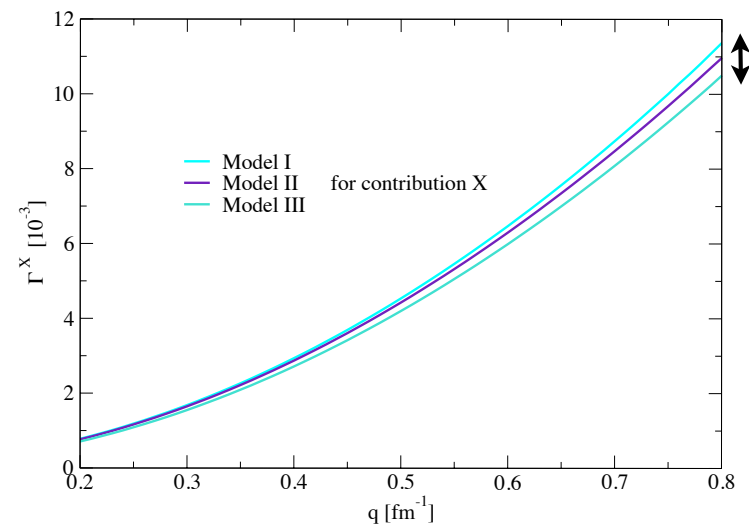


Relative effect of feature X



$$\Gamma^X = \frac{A^X - A^{\text{ref}}}{A^{\text{ref}}}$$

Relative uncertainty of feature X



$$\Delta\Gamma^X = \Gamma^{X_a} - \Gamma^{X_b} = \frac{A^{X_a} - A^{X_b}}{A^0} = \frac{\Delta A^X}{A^0}$$



ANALYSIS OF CONTRIBUTIONS

- Distortion of the electron wave function due to the Coulomb field created by the nuclear charge distribution: from plane-wave (PW) to distorted-wave (DW) calculations.



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- Meson exchange currents among the nucleons affecting differently the isoscalar and the isovector nuclear responses.
- Inelastic transitions to excited nuclear states differing significantly from the ground state (e.g. different nominal isospin).



COULOMB DISTORTION EFFECT

Theoretical uncertainties in the PV asymmetry due to a reasonable variability in the nuclear charge distributions

We use a three-parameter Fermi distribution:

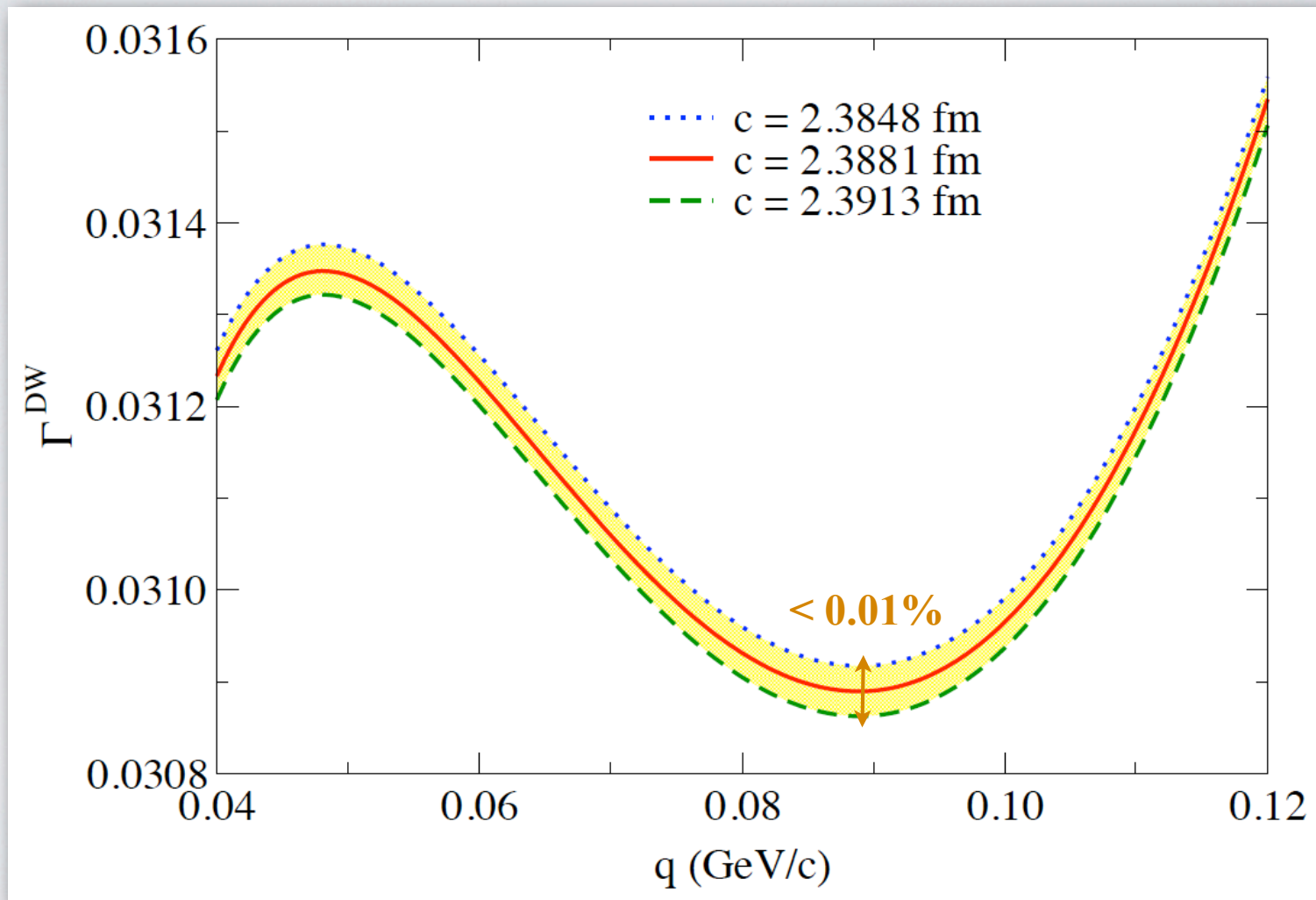
$$\rho(r) = \rho_0 \frac{1 + \frac{wr^2}{c^2}}{1 + e^{\frac{r-c}{d}}}$$

and modify the parameters keeping the rms charge radius within the experimental range. They generate different Coulomb fields which distort the electron wave functions differently.



COULOMB DISTORTION EFFECT

Theoretical uncertainties in the PV asymmetry due to a reasonable variability in the nuclear charge distributions





NUCLEAR ISOSPIN MIXING EFFECT

Theoretical uncertainties in the PV asymmetry from a reasonable variability in the neutron vs. the proton distributions

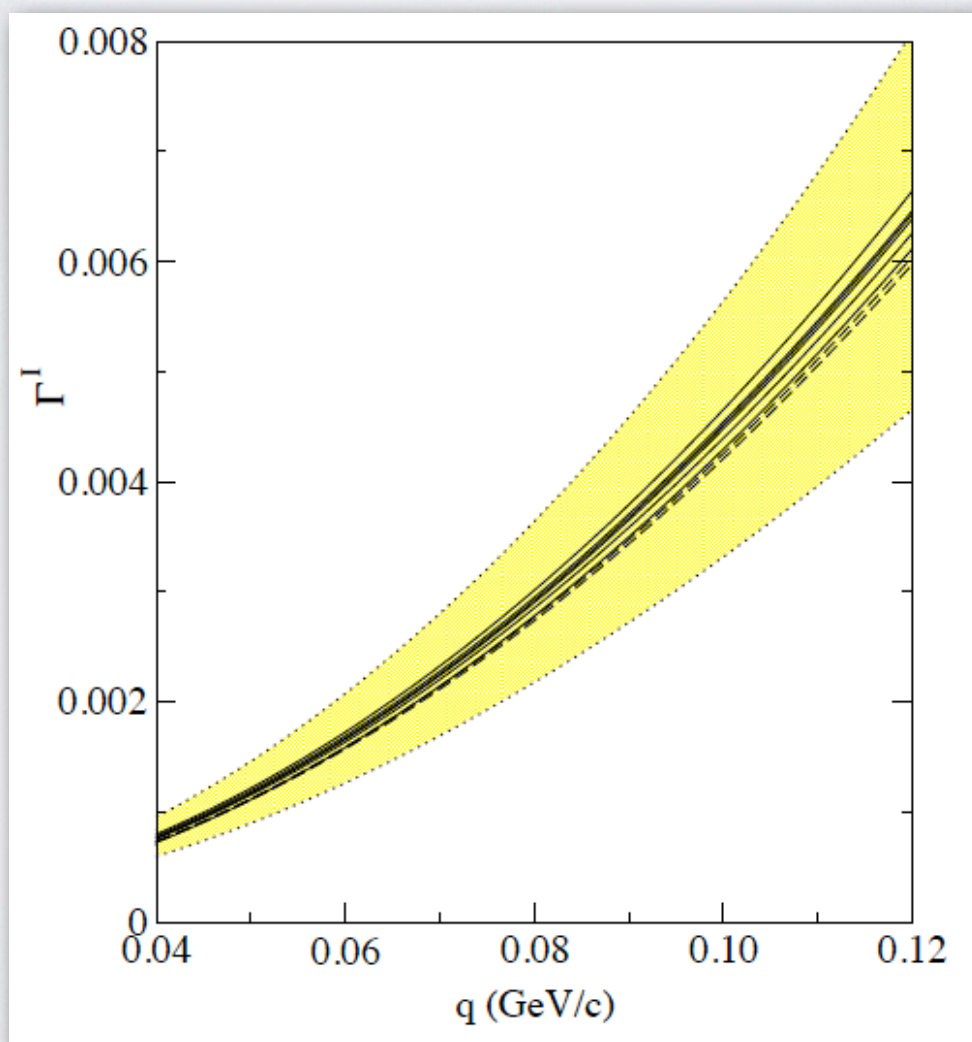
We perform an axially-deformed Hartree-Fock mean field calculation with BCS pairing correlations, using Skyrme nucleon-nucleon interactions:

$$\begin{aligned} V_{12}^{Sk} = & t_0(1+x_0P_\sigma)\delta(\vec{r}_1-\vec{r}_2) + \frac{1}{2}t_1(1+x_1P_\sigma)[\delta(\vec{r}_1-\vec{r}_2)k^2 + k'^2\delta(\vec{r}_1-\vec{r}_2)] \\ & + t_2(1-x_2P_\sigma)\vec{k}'\delta(\vec{r}_1-\vec{r}_2)\vec{k} + iW_0(\vec{\sigma}_1+\vec{\sigma}_2)\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k} \\ & + \frac{1}{6}t_3(1+x_3P_\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right), \end{aligned}$$

NUCLEAR ISOSPIN MIXING EFFECT

Theoretical uncertainties in the PV asymmetry from a reasonable variability in the neutron vs. the proton distributions

DIFFERENT NUCLEON-NUCLEON FORCES: HF MEAN FIELD



NUCLEAR STRANGENESS CONTENT EFFECT

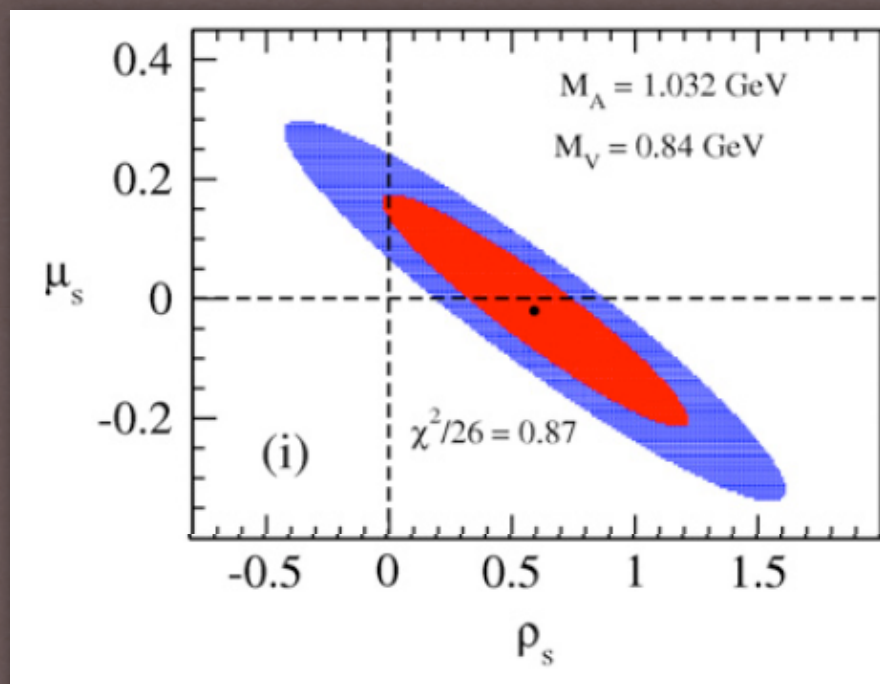
Theoretical uncertainties in the PV asymmetry from nucleon **strangeness** content uncertainties

Experimental ranges of electric (ρ_s) and magnetic (μ_s) strangeness parameters

$$G_E^{(s)} = \frac{|Q^2|}{4m_N^2} \frac{\rho_s}{(1 + |Q^2|/M_V^2)^P}$$

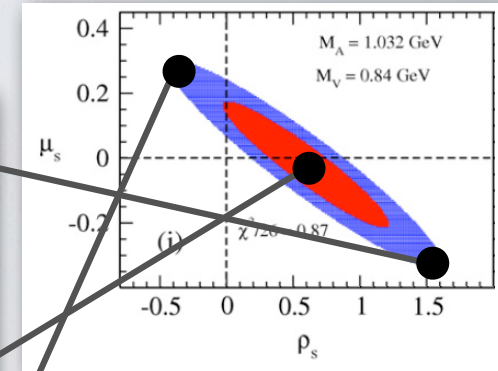
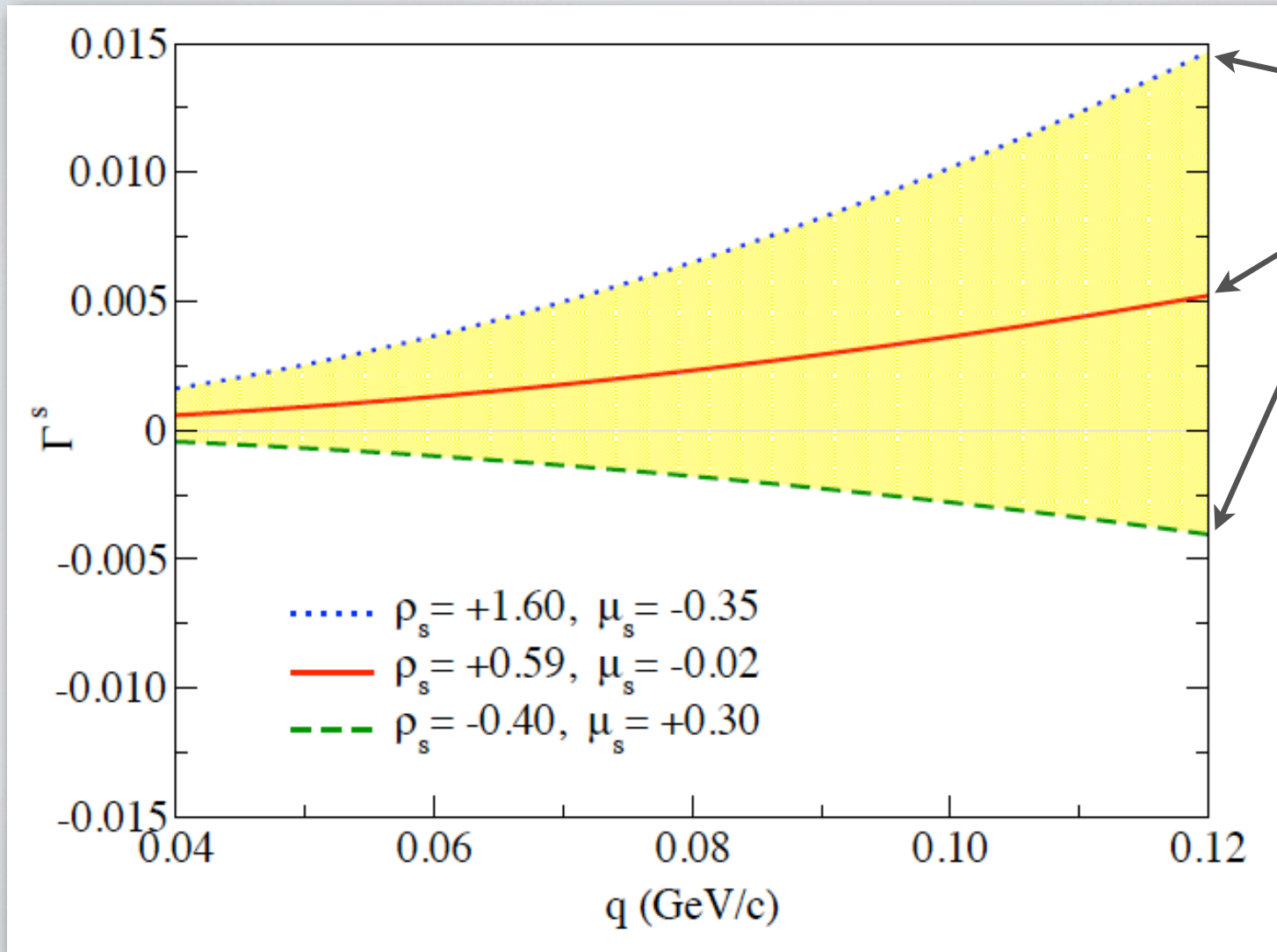
$$G_M^{(s)} = \frac{\mu_s}{(1 + |Q^2|/\tilde{M}_V^2)^P}$$

$$\chi^2 = \sum_{j=1}^{28} \left(\frac{\mathcal{A}_j^{exp} - \mathcal{A}_j^{th}}{\Delta \mathcal{A}_j^{exp}} \right)^2$$



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MESON EXCHANGE CURRENT EFFECT

- In absence of nuclear isospin mixing and of nucleon strangeness content: no contribution of MEC.



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- With nuclear isospin mixing: the isoscalar and the isovector nuclear responses are affected differently, resulting in an overall $\sim 10\%$ change of the isospin mixing deviation.



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- In absence of nuclear isospin mixing and of nucleon strangeness content: no contribution of MEC.
- With nuclear isospin mixing: the isoscalar and the isovector nuclear responses are affected differently, resulting in an overall $\sim 10\%$ change of the isospin mixing deviation.
- With nuclear isospin mixing and nucleon strangeness content: any MEC uncertainty is clearly exceeded by the strangeness content uncertainties.

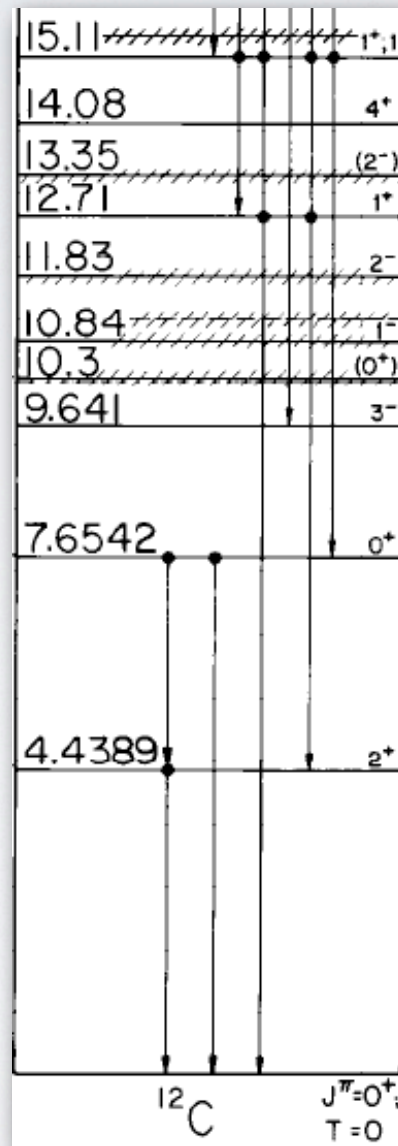
INELASTIC TRANSITIONS EFFECT

Theoretical uncertainties in the PV asymmetry due to possible transitions to excited states in the target

Total asymmetry:

$$A = \sum_{i=0}^n f_i A_i$$

$$f_i = \frac{\sigma_i^{EM}}{\sum_{k=0}^n \sigma_k^{EM}}$$



T = 0 levels:
 Same asymmetry
 as for ground state
 ($\Gamma^{\text{inel.}} = 0$)
 (in absence of strangeness
 and at tree-level)

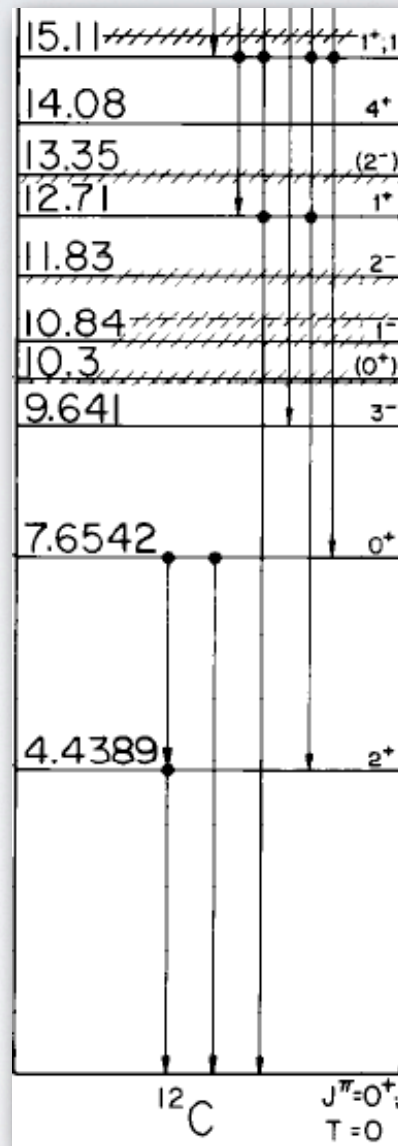
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T = 1 levels:
 Different asymmetry
 (isovector contribution)

T = 0 levels:
 Same asymmetry
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 (in absence of strangeness
 and at tree-level)



CONCLUSIONS

SUMMARY OF SIZES AND UNCERTAINTIES

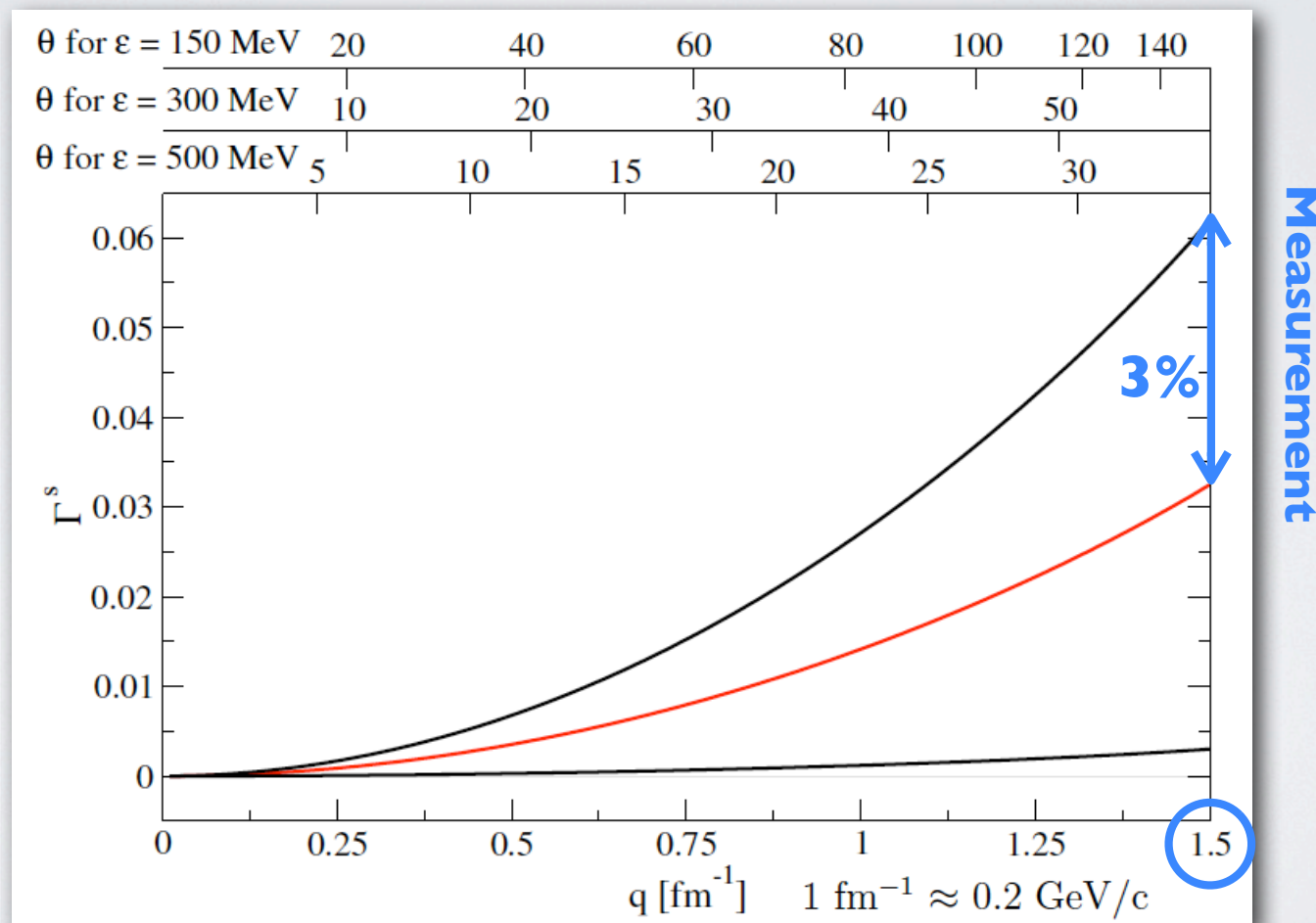
150 MeV incident energy, 25°-45° scattering angular range

Contribution to PV asymmetry	Relative size	Relative uncertainty
Coulomb distortion of projectile wave function	3%	0.01%
Nuclear isospin mixing (electromagnetic origin)	0.4%	0.05%
Nucleon strangeness content (mainly electric)	0 - 1 %	1%
Meson exchange currents	< 0.1 %	< 0.1 %
Inelastic contributions	< 0.1 %	--

CONCLUSIONS

STRATEGY OUTLOOK

Focus on the momentum transfer range where the strangeness content uncertainty is large:

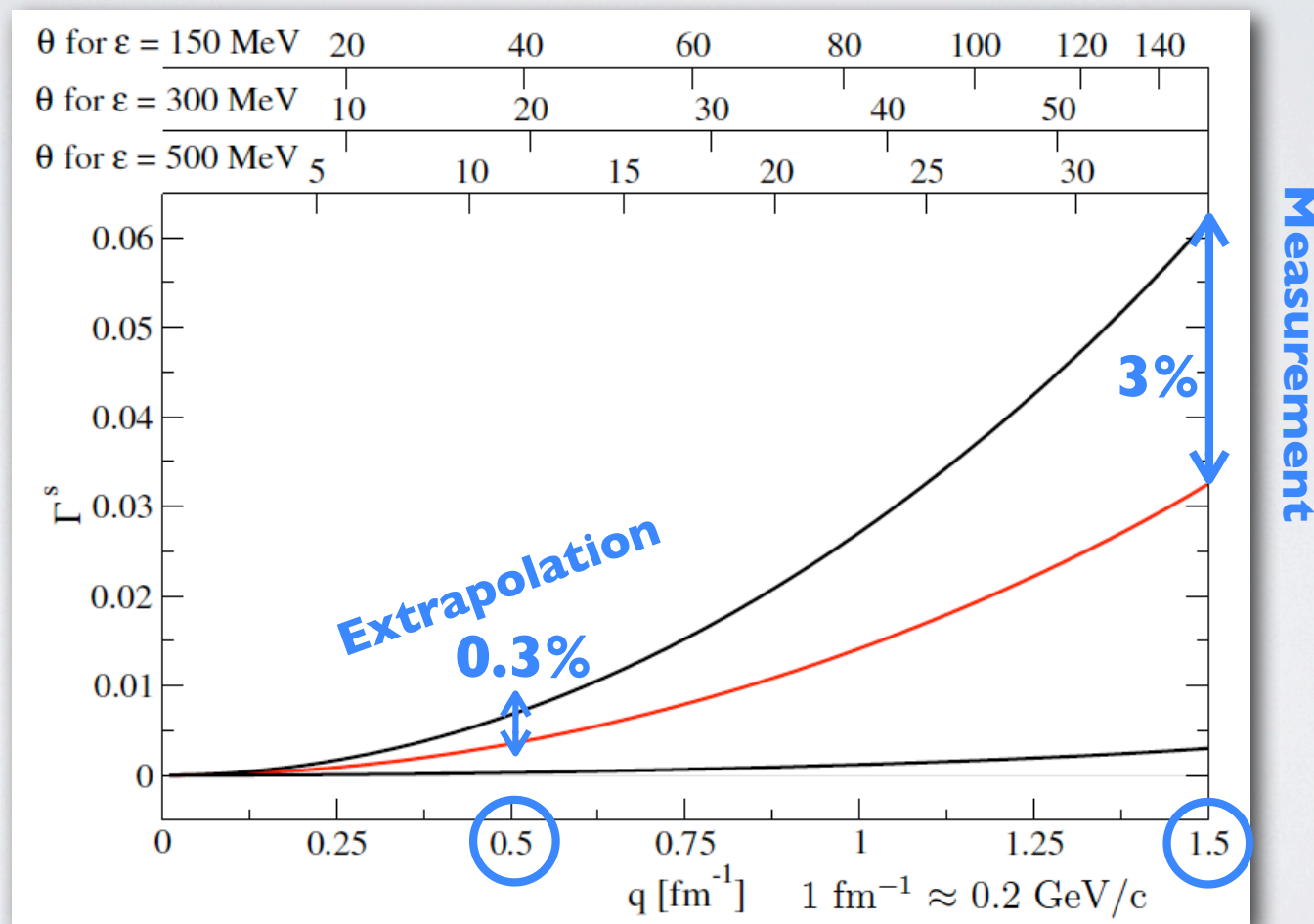




CONCLUSIONS

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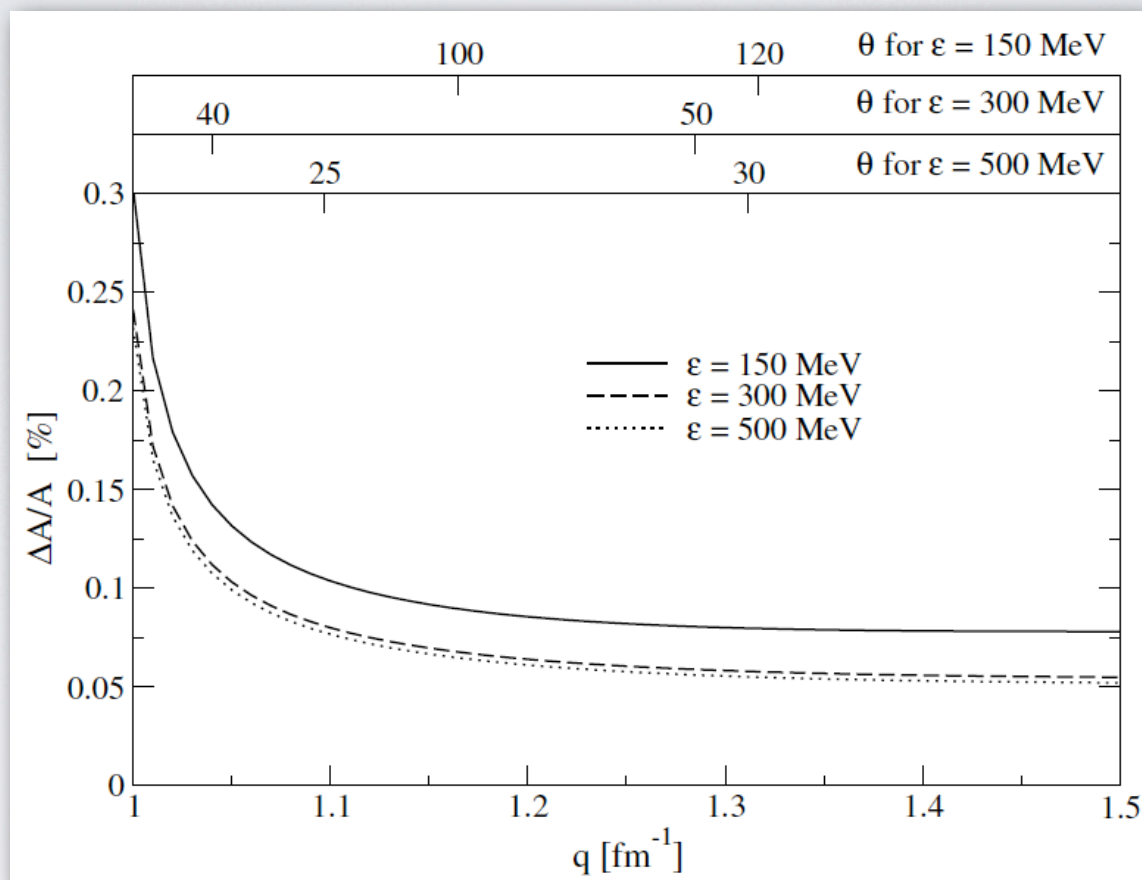
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CONCLUSIONS

STRATEGY OUTLOOK

Statistical uncertainty in this momentum transfer region



$\Delta\phi = 2\pi$
 $L = 5 \cdot 10^{38} \text{ cm}^{-2}\text{s}^{-1}$
 $T = 10^7 \text{ s}$
 $P_e = 100 \%$
 $\theta_i = 25^\circ$
 $\epsilon = 150 \text{ MeV}$

$$\frac{\Delta A}{A} = \frac{1}{[\Delta\phi L T]^{1/2}} \frac{\left[\int_{\theta_i}^{\theta_f} d\theta \frac{d\sigma}{d\Omega}(\theta) \sin\theta \right]^{1/2}}{\int_{\theta_i}^{\theta_f} d\theta A(\theta) \frac{d\sigma}{d\Omega}(\theta) \sin\theta}$$



REFERENCES

- Asymmetry in PVES from nuclei and definition of reference (Standard Model) value:
G. Feinberg, *Phys. Rev. D* 12 (1975) 3575 & J. D. Walecka, *Nucl. Phys. A* 285 (1977) 349.
- Exploration of isospin mixing in PVES from nuclei and proposal to use PV electron scattering to measure the neutron distribution in the nucleus (PREX, CREX, etc.):
T. W. Donnelly, J. Dubach and I. Sick, *Nucl. Phys. A* 503 (1989) 589.
- Extension of the previous study using improved nuclear models of light $N=Z$ nuclei, including carbon 12:
O. Moreno, P. Sarriguren, E. Moya de Guerra, J. M. Udías, T. W. Donnelly and I. Sick, *Nucl. Phys. A* 828 (2009) 306.
- Analysis of nuclear uncertainties in PVES from carbon 12:
O. Moreno and T. W. Donnelly, *Phys. Rev. C* 89 (2014) 015501.
- World data analysis of strangeness content parameters extracted from PVES experiments:
R. González-Jiménez, J. A. Caballero, T. W. Donnelly, *Phys. Rep.* 524 (2013) 1.
- Theoretical uncertainties in PVES from nucleons and nuclei:
O. Moreno T. W. Donnelly, R. González-Jiménez, J. A. Caballero, *J. Phys. G* 42 (2015) 034006.



COHERENT NEUTRINO SCATTERING

as an alternative method to determine electroweak constants
at low momentum transfer



COHERENT NEUTRINO SCATTERING

as an alternative method to determine electroweak constants
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and other applications:

- disentangling higher-order corrections,
- finding possible differences in electron and neutrino weak neutral currents (BSM),
- determining the axial structure of the target, ...



COHERENT NEUTRINO SCATTERING

- WNC process, neutrino in and neutrino out \Rightarrow detection of nuclear recoil.
- The target nucleus remains in its ground state (elastic scattering).
- Valid when $q \approx 1/R \approx 160 A^{-1/3}$ MeV ($\Rightarrow q \approx 70$ MeV for ^{12}C).
- The cross-section is roughly proportional to N^2 .
- It is the only elastic contribution for even-even nuclear targets, and usually dominant for other targets.

$$\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)}^{ref} = \frac{G_F^2}{2\pi^2} [(a_A^\nu)^2 + (a_V^\nu)^2] \varepsilon_\nu'^2 \cos^2(\theta_\nu/2) f_{rec}^{-1} A^2 \underline{\sin^4 \theta_W}$$



COHERENT NEUTRINO SCATTERING

- Relationship between coherent electron-nucleus and coherent neutrino-nucleus cross-sections in PWBA:

$$\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)} = \left[\frac{(a_V^\nu)^2 + (a_A^\nu)^2}{2(a_A^e)^2}\right] \mathcal{A}_{(e,e)}^2 \left(\frac{d\sigma}{d\Omega}\right)_{(e,e)}$$

Deviations from this prediction:

- Coulomb distortion.
- Effect of higher order corrections.
- Different coupling of Z^0 to neutrinos and charged leptons.



COHERENT NEUTRINO SCATTERING

- Relationship between relative uncertainties:

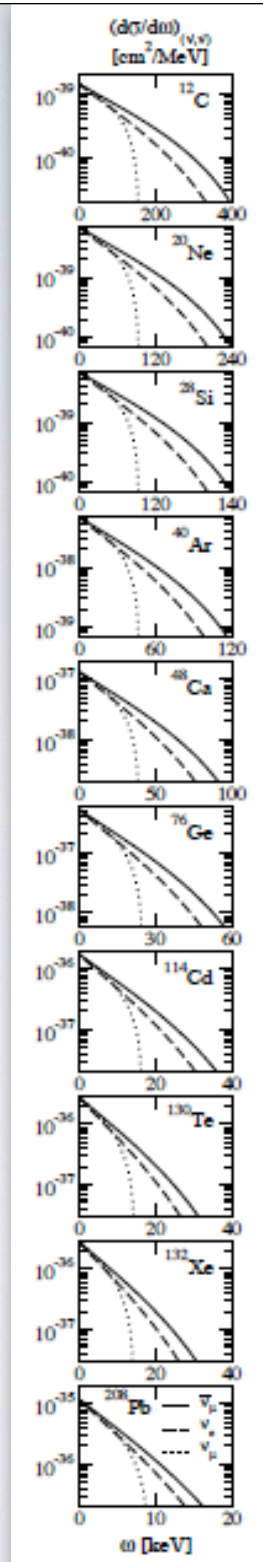
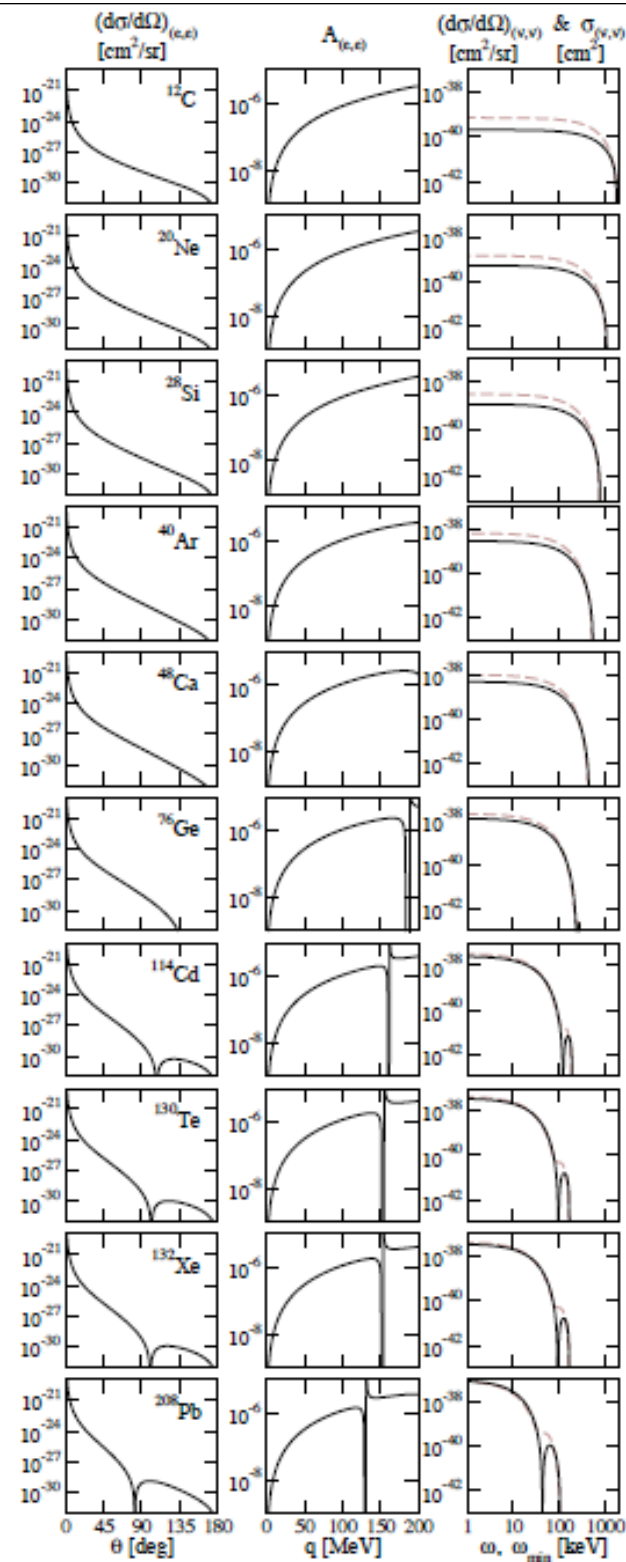
$$\mathcal{E}\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)} \approx 2 \mathcal{E}_{\mathcal{A}(e,e)}$$

to which the PV experiment statistical contribution is:

$$\mathcal{E}^{\text{stat.}}\left(\frac{d\sigma}{d\Omega}\right)_{(\nu,\nu)} \approx 2 \mathcal{X}_{PV}^{-\frac{1}{2}} \mathcal{F}_{PV}^{-\frac{1}{2}}$$



COHERENT NEUTRINO SCATTERING





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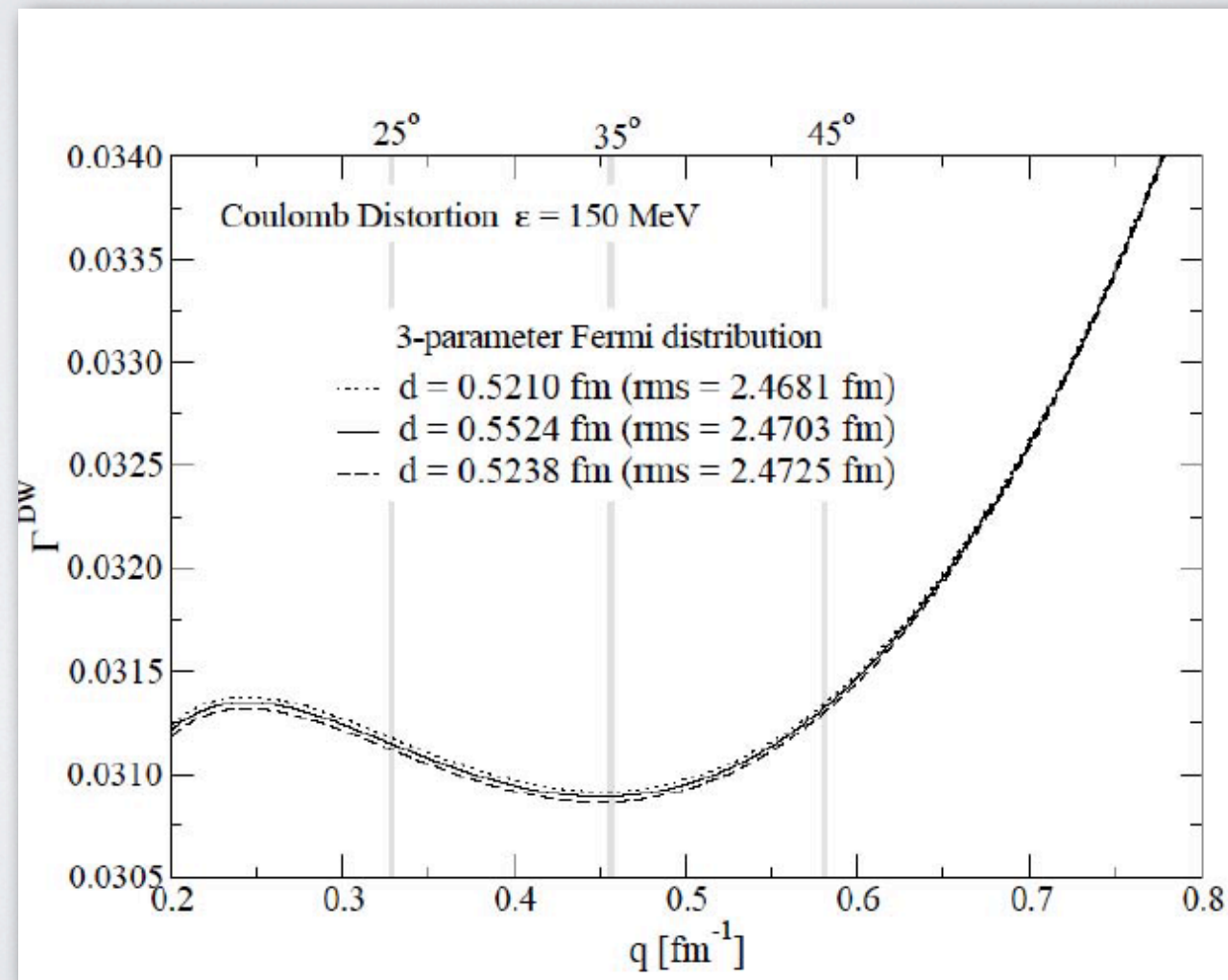
Extra material



COULOMB DISTORTION EFFECT

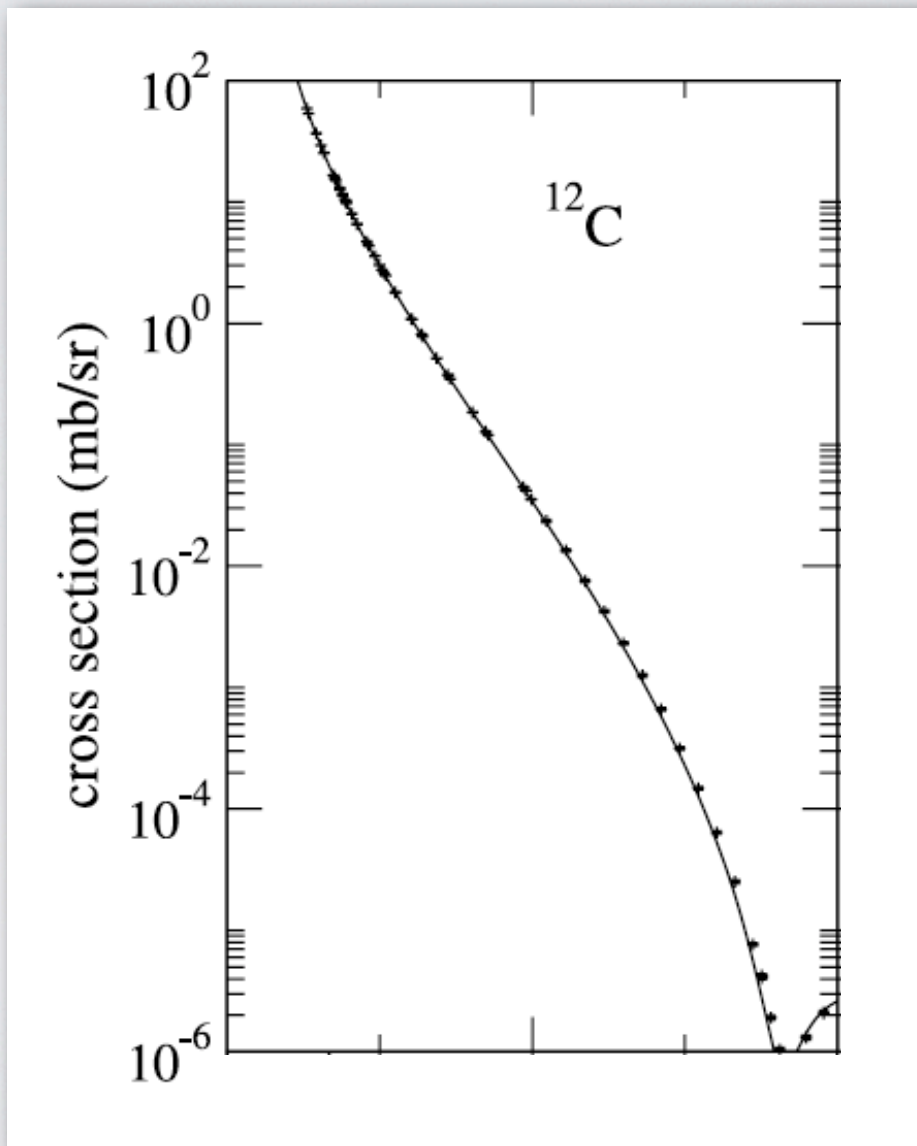
Theoretical uncertainties in the PV asymmetry due to a reasonable variability in the nuclear charge distributions

Deviation due to Coulomb distortion for nuclear charge distributions differing in the diffuseness parameter of the Fermi distribution





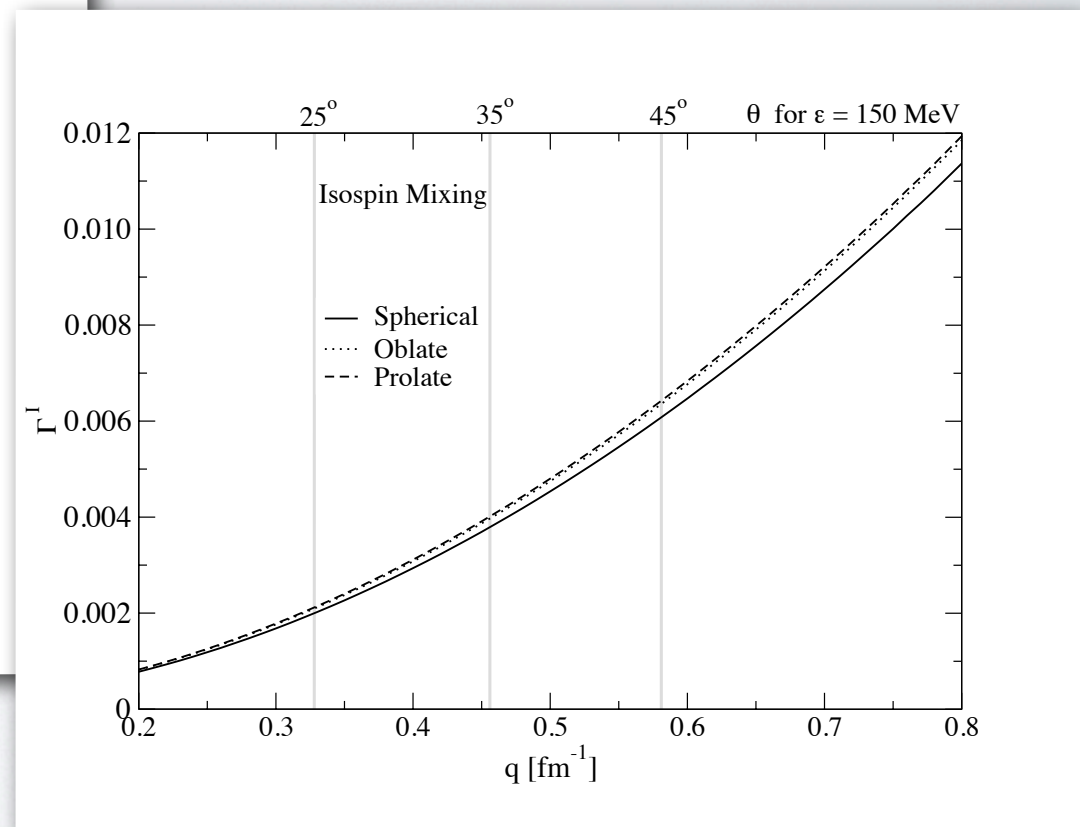
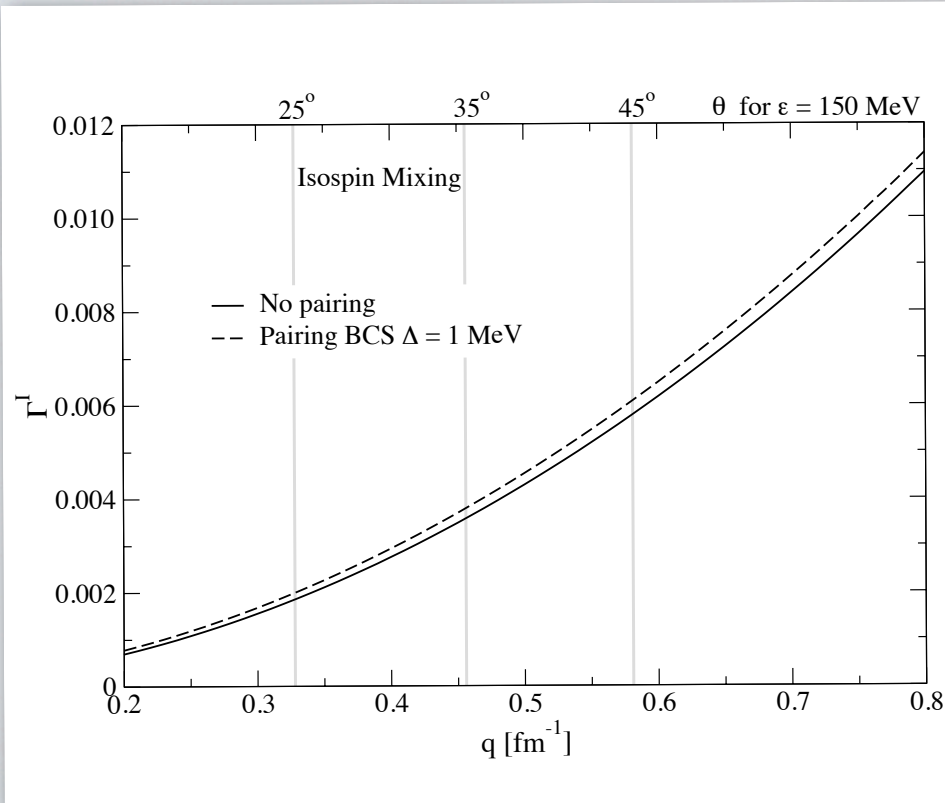
Experimental and theoretical (HF+BCS, DWBA) cross-section (1 GeV electrons)



NUCLEAR ISOSPIN MIXING EFFECT

Theoretical uncertainties in the PV asymmetry from a reasonable variability in the neutron vs. the proton distributions

OTHER MODIFICATIONS OF THE MEAN FIELD (PAIRING, SHAPE)



NUCLEAR STRANGENESS CONTENT EFFECT

Theoretical uncertainties in the PV asymmetry from nucleon **strangeness** content uncertainties

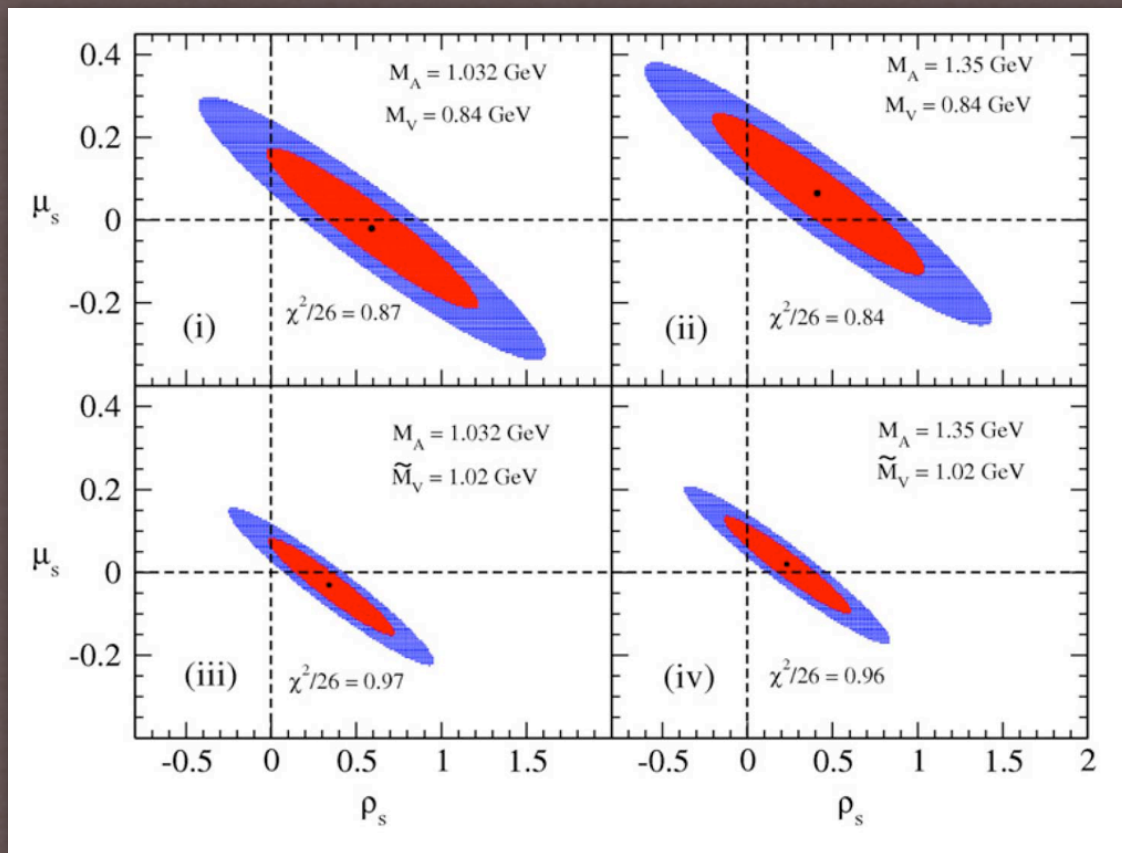
Experimental ranges of electric (Q_s) and magnetic (μ_s) strangeness parameters

Dipole (P = 2)
 [M_V = 0.84 GeV] →

$$G_E^{(s)} = \frac{|Q^2|}{4m_N^2} \frac{\rho_s}{(1 + |Q^2|/M_V^2)^P}$$

$$G_M^{(s)} = \frac{\mu_s}{(1 + |Q^2|/M_V^2)^P}$$

Monopole (P = 1)
 [M_V = 1.02 GeV] →



$$G_A = \frac{G_A(0)}{(1 + |Q^2|/M_A^2)^2}$$

M_A = 1.032 GeV ↑

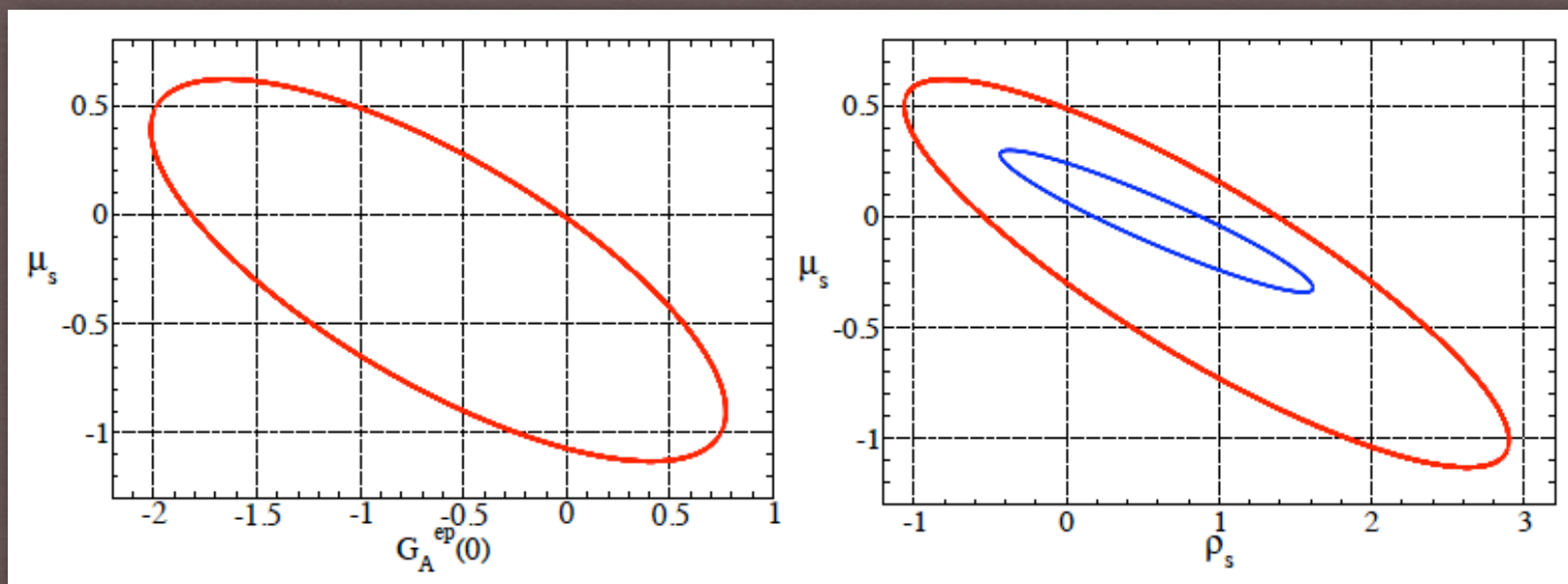
M_A = 1.35 GeV ↑

NUCLEAR STRANGENESS CONTENT EFFECT

Theoretical uncertainties in the PV asymmetry from
nucleon **strangeness** content uncertainties

Experimental ranges of electric (Q_s) and magnetic (μ_s) strangeness parameters

$$\xi_V^p, \xi_V^n, G_A^{ep}(0), \rho_s, \mu_s$$

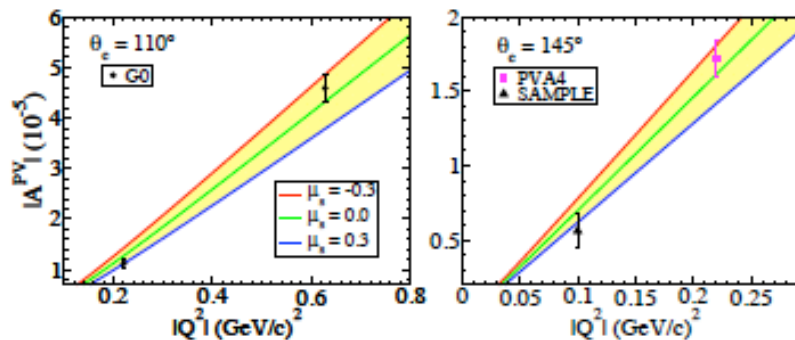


R. González-Jiménez, J. A. Caballero, T. W. Donnelly, Phys. Rep. 524 (2013) 1

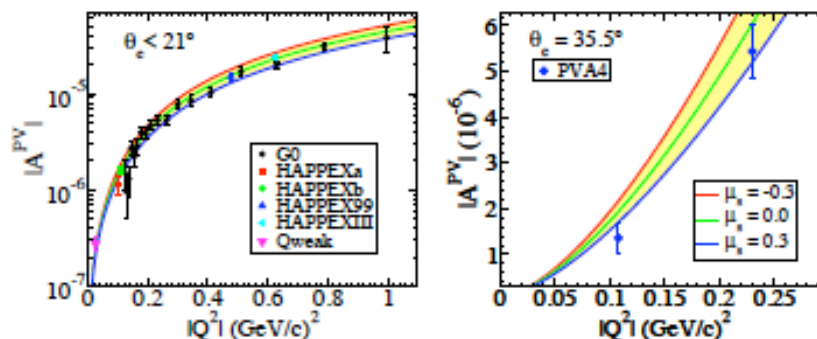
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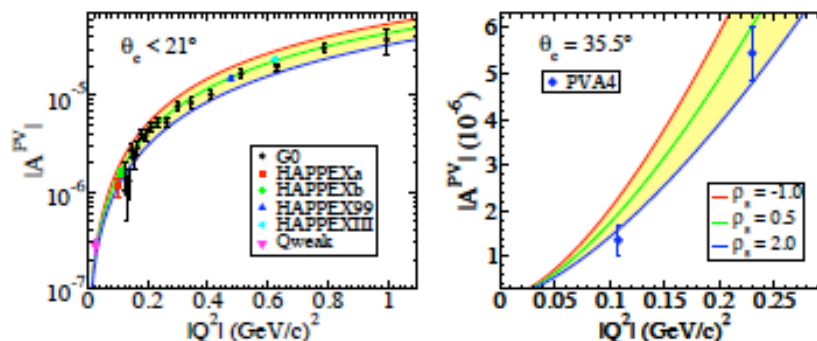
Backward angles, magnetic strangeness dependence



Forward angles, magnetic strangeness dependence



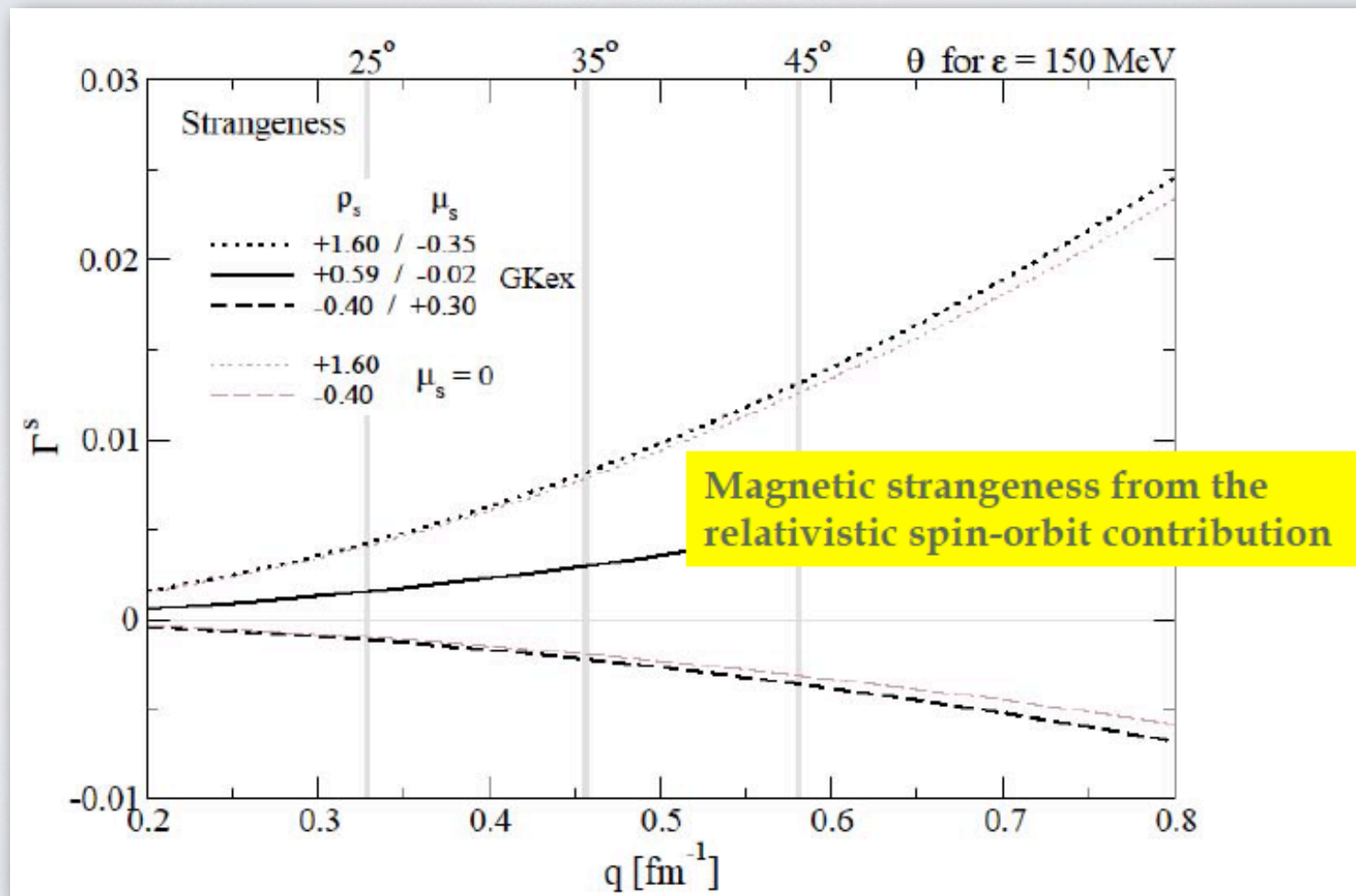
Forward angles, electric strangeness dependence



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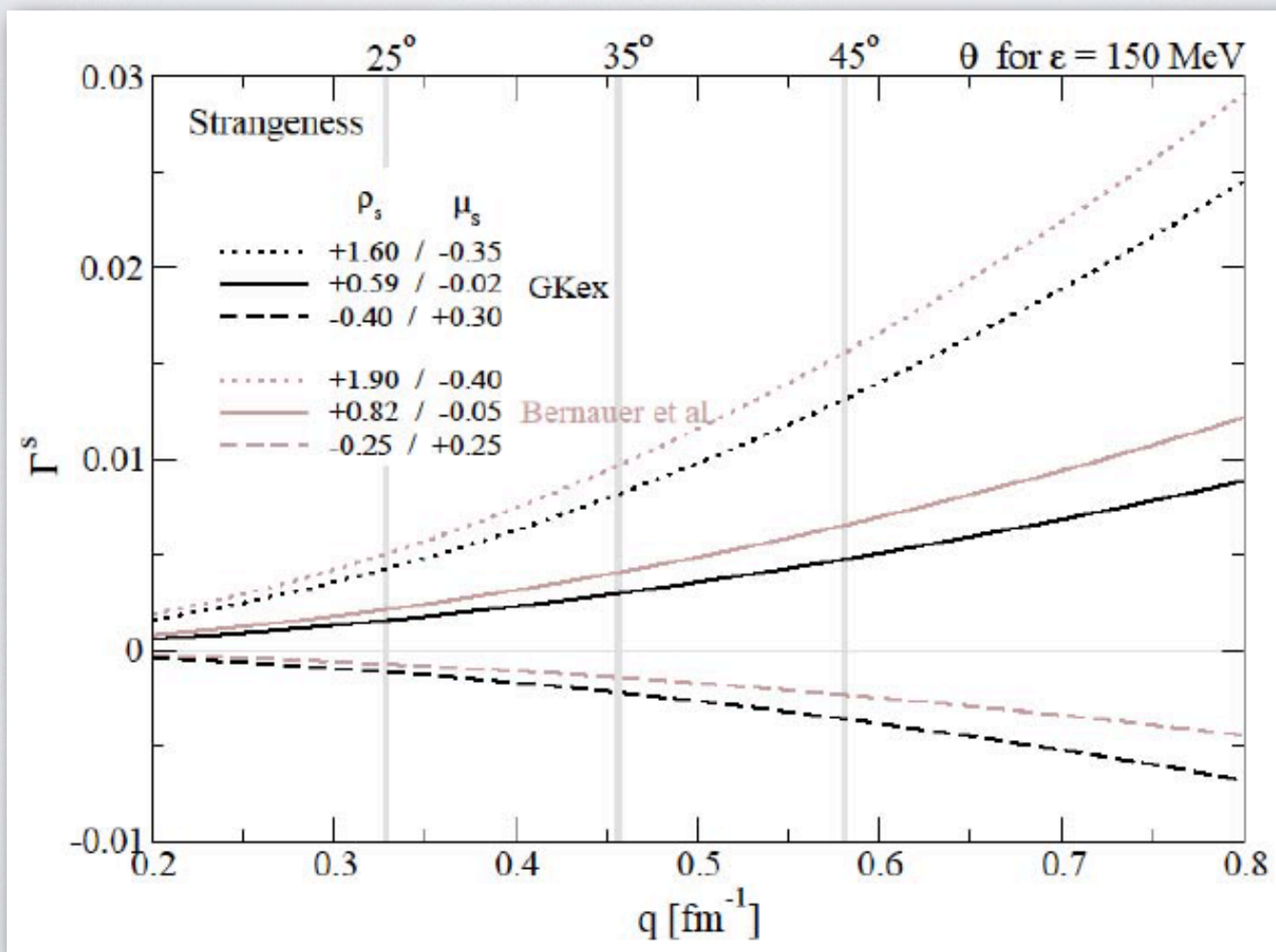
Contribution of magnetic strangeness content (through spin-orbit correction in operator)



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Theoretical uncertainties in the PV asymmetry from nucleon **strangeness** content uncertainties

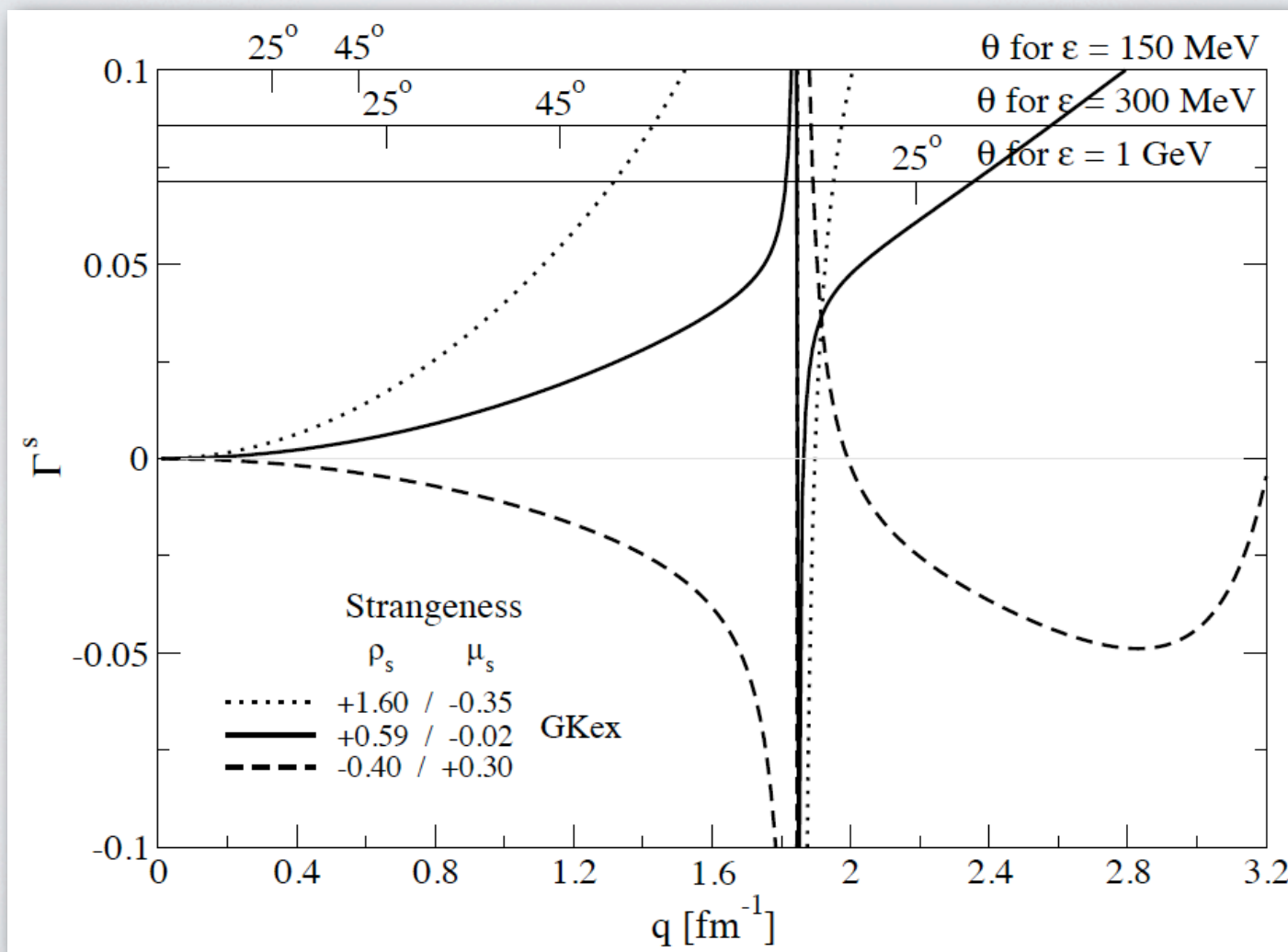
Strangeness contribution for two different extractions of experimental ranges of strangeness content parameters



NUCLEAR STRANGENESS CONTENT EFFECT

Theoretical uncertainties in the PV asymmetry from nucleon **strangeness** content uncertainties

Strangeness contribution within a large range of momentum transfer





MESON EXCHANGE CURRENT EFFECT

- In absence of nuclear isospin mixing and of nucleon strangeness content: no contribution of MEC.
- With nuclear isospin mixing: the isoscalar and the isovector nuclear responses are affected differently, resulting in an overall $\sim 10\%$ change of the isospin mixing deviation.
- With nuclear isospin mixing and nucleon strangeness content: any MEC uncertainty is clearly exceeded by the strangeness content uncertainties.
- As for the strangeness content of exchanged mesons (two-body strangeness): effects of this exotic MEC expected to be smaller than conventional MEC effects at low momentum transfer.

[see M. J. Musolf, R. Schiavilla and T. W. Donnelly, Phys. Rev. C50 (1994) 2173]