



JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ



THE LOW-ENERGY FRONTIER  
OF THE STANDARD MODEL

**IEB-Workshop, June 17-19 2015, Ithaca, NY, USA**

# **Monte Carlo simulations of a solenoid spectrometer for Project P2**

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JGU Mainz**



# Outline

- **Project P2 @ MESA:**

A new high precision determination of the electroweak mixing angle at low momentum transfer

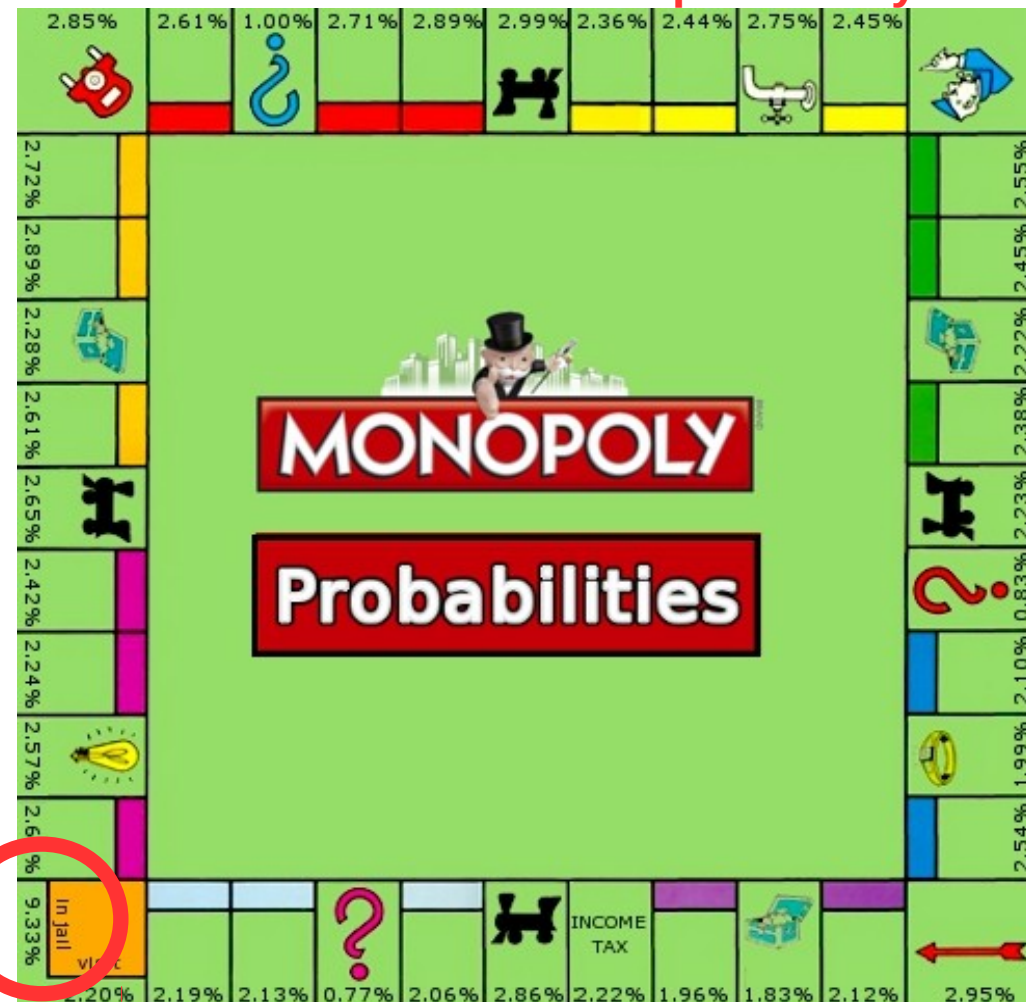
- **P2 main detector concept:**

Monte Carlo simulations of a solenoid spectrometer

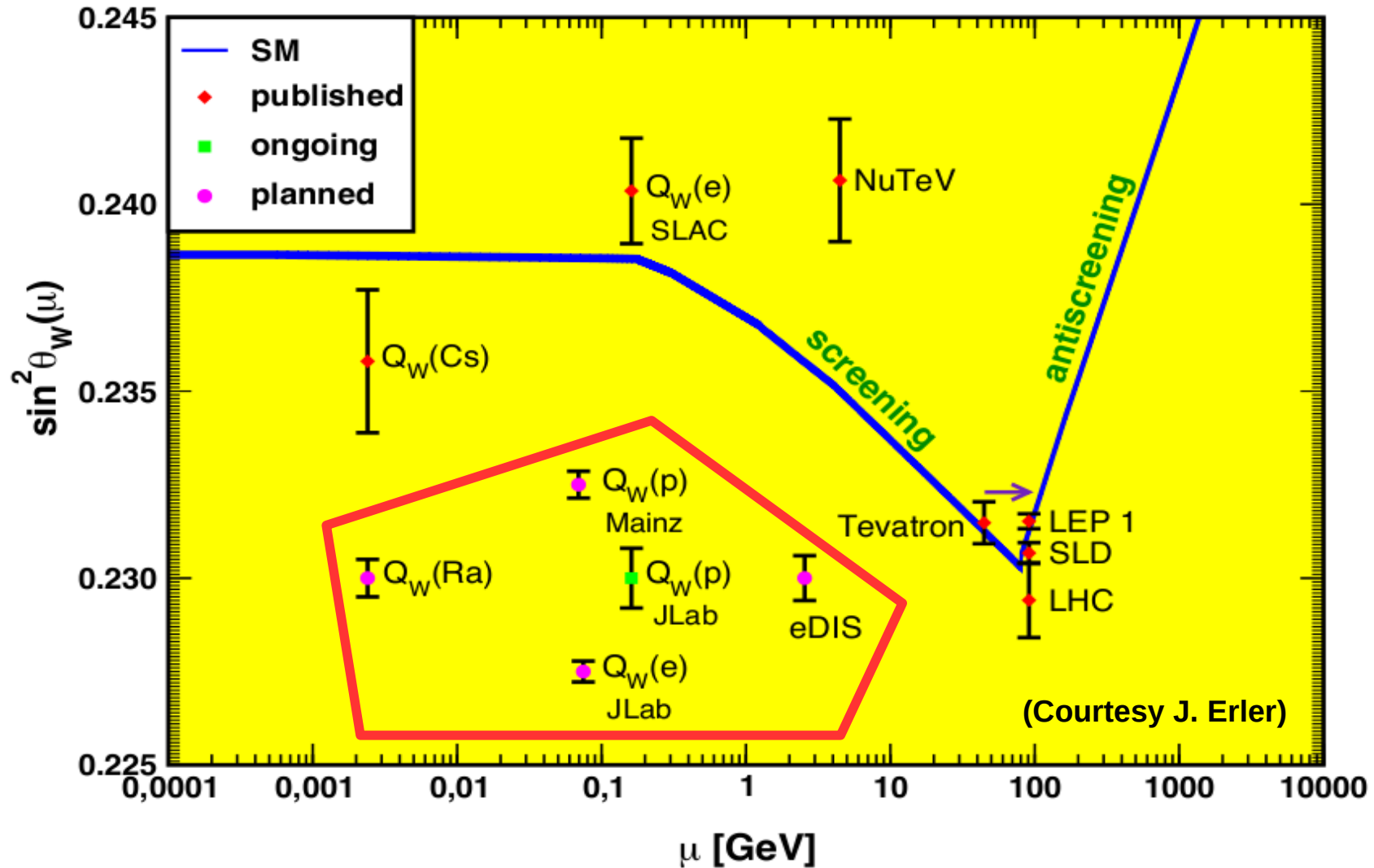
- Monte Carlo simulations regarding a precision measurement of the weak mixing angle at **higher beam energies and beam current**

Monte Carlo is all about probability...

Highest probability



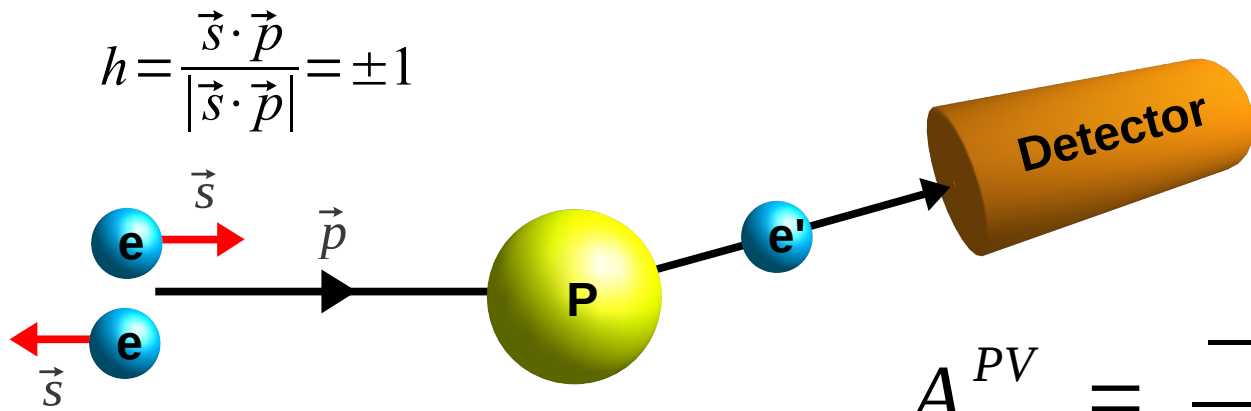
# The global situation



## Project P2 @ MESA:

- New high precision determination of the proton weak charge  $Q_W(p)$  at low  $Q^2 \sim 6 \cdot 10^{-3} \text{ GeV}^2/c^2$
- Precision goal:  $\Delta Q_W(p) = 1.9 \%$   
 $\Delta \sin^2 \theta_W = 0.15 \%$
- Measurement of  $Q_W(p)$  through parity violation in elastic e-p scattering

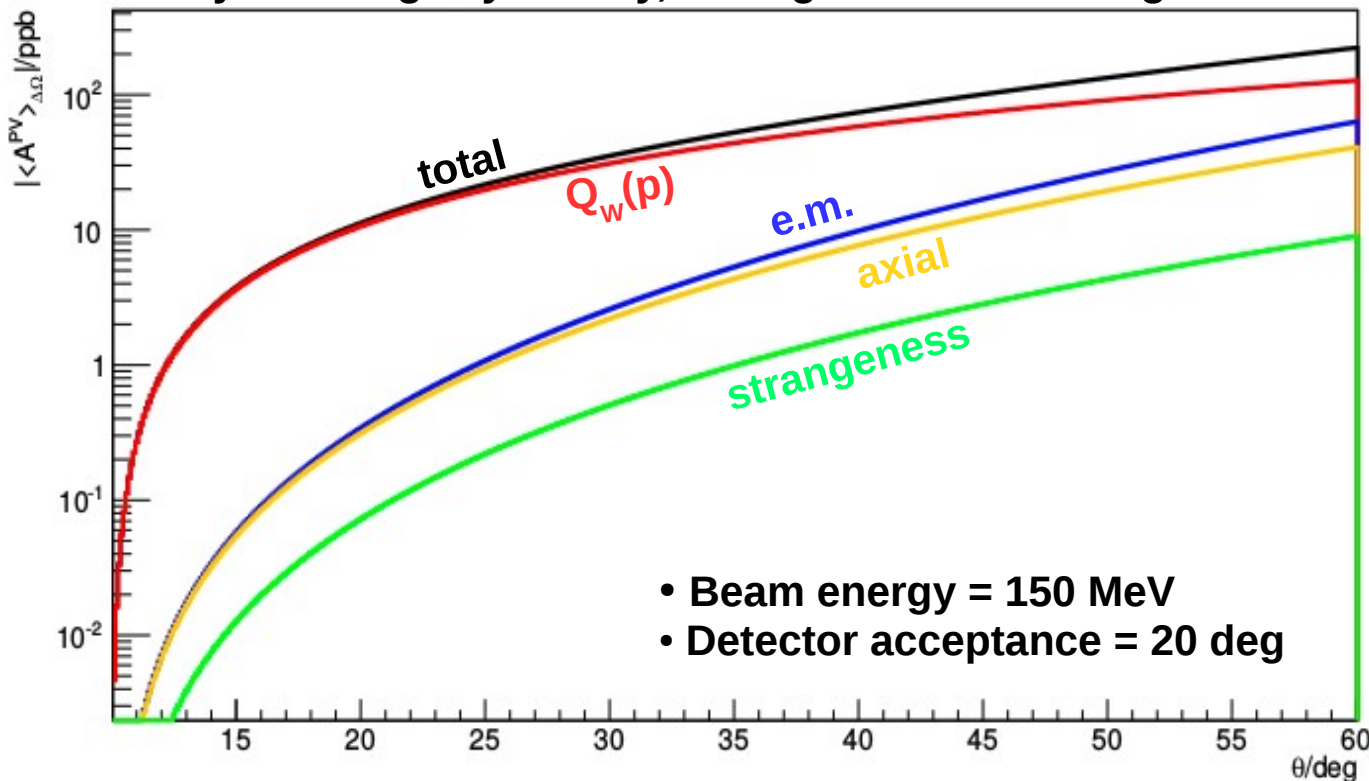
# Access to the weak mixing angle



Parity violating asymmetry in elastic e-p scattering:

$$A^{PV} = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha} [Q_W(p) - F(Q^2)]$$

Parity violating asymmetry, averaged over solid angle

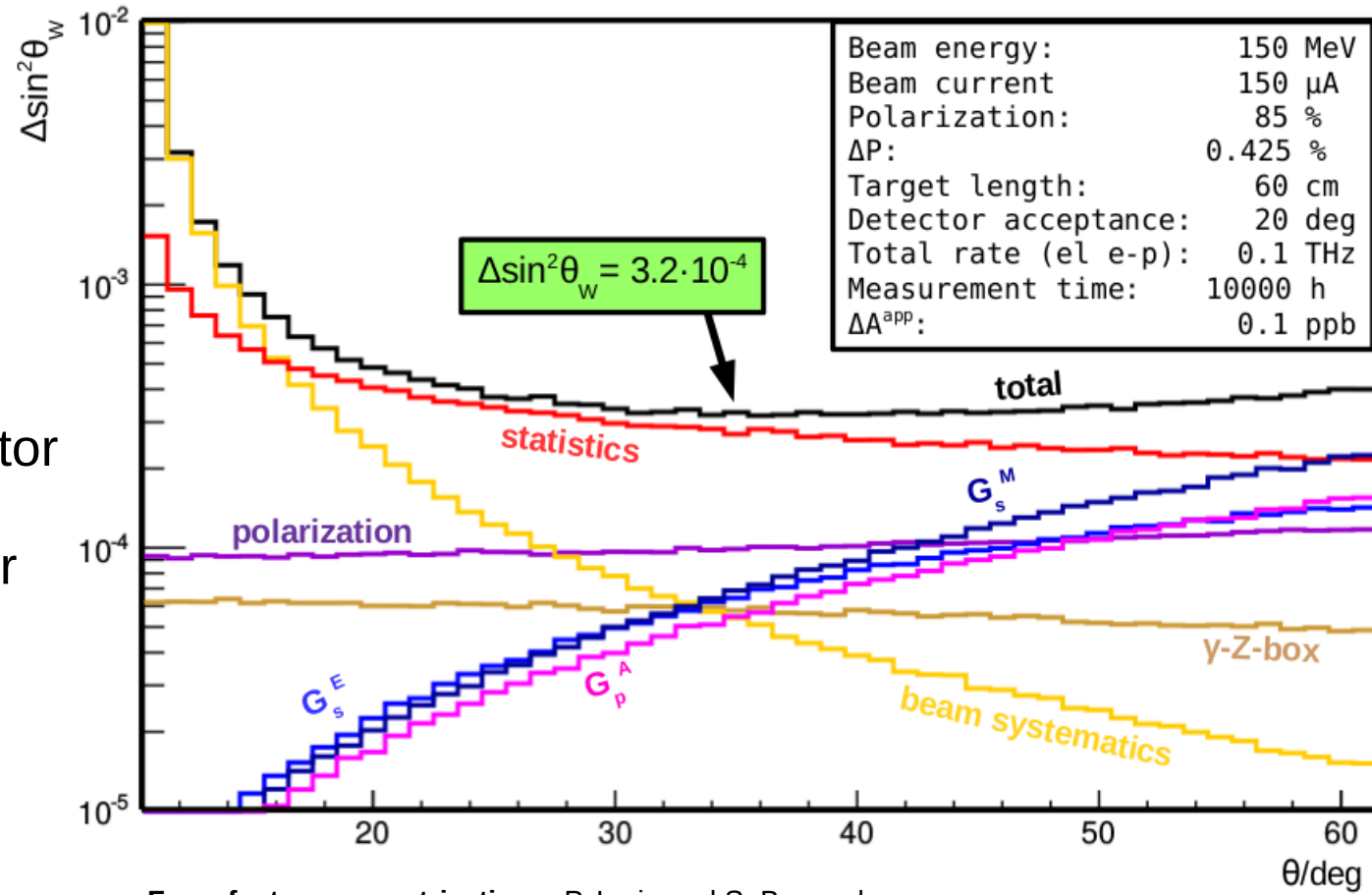


- $Q_W(p)$ : Proton weak charge,  $Q_W(p) = 1 - 4 \cdot \sin^2(\theta_W)$  (tree level)
- $F(Q^2)$ : Nucleon structure contribution, small at low  $Q^2$

$$A^{PV} \sim \sin^2 \theta_W$$

# Prediction of achievable precision and choice of kinematics

- Monte Carlo approach to error propagation calculation
- Assumption of back angle measurement of axial and strange magnetic form factor in P2
  - Reduction of form factor uncertainty by factor 4
- $A^{PV} = -39.80$  ppb
  - $\pm 0.54$  ppb (stat.)
  - $\pm 0.34$  ppb (other)



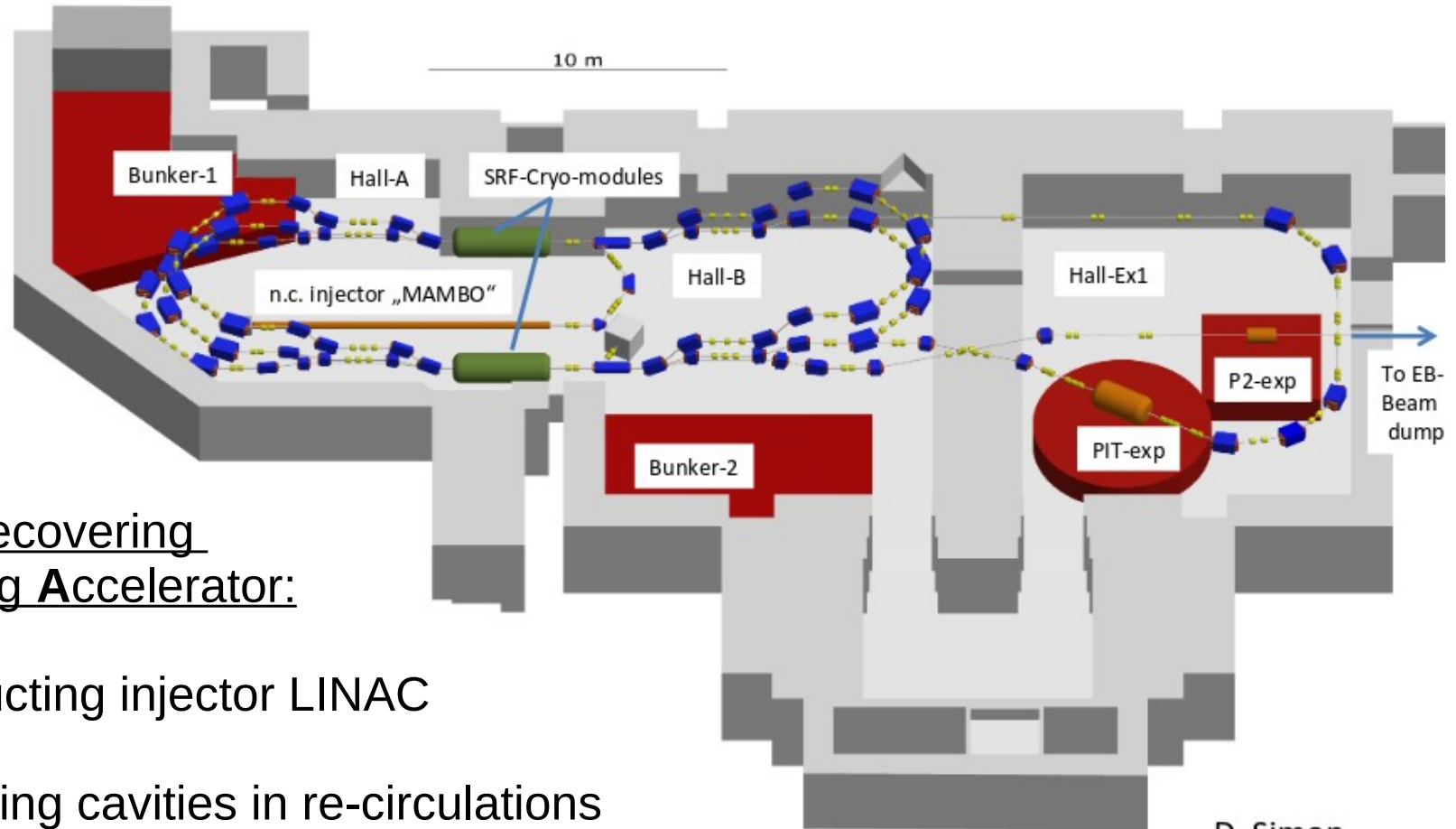
Form factor parametrizations: P. Larin and S, Baunack

y-Z-box according to: Gorchtein, Horowitz, Ramsey-Musolf 1102.3910 [nucl-th]

$$\Delta \sin^2(\theta_w) = 3.2 \cdot 10^{-4} (0.13 \%) \quad @$$

**Beam energy:** 150 MeV  
**Central scattering angle:** 35 deg  
**Detector acceptance:** 20 deg

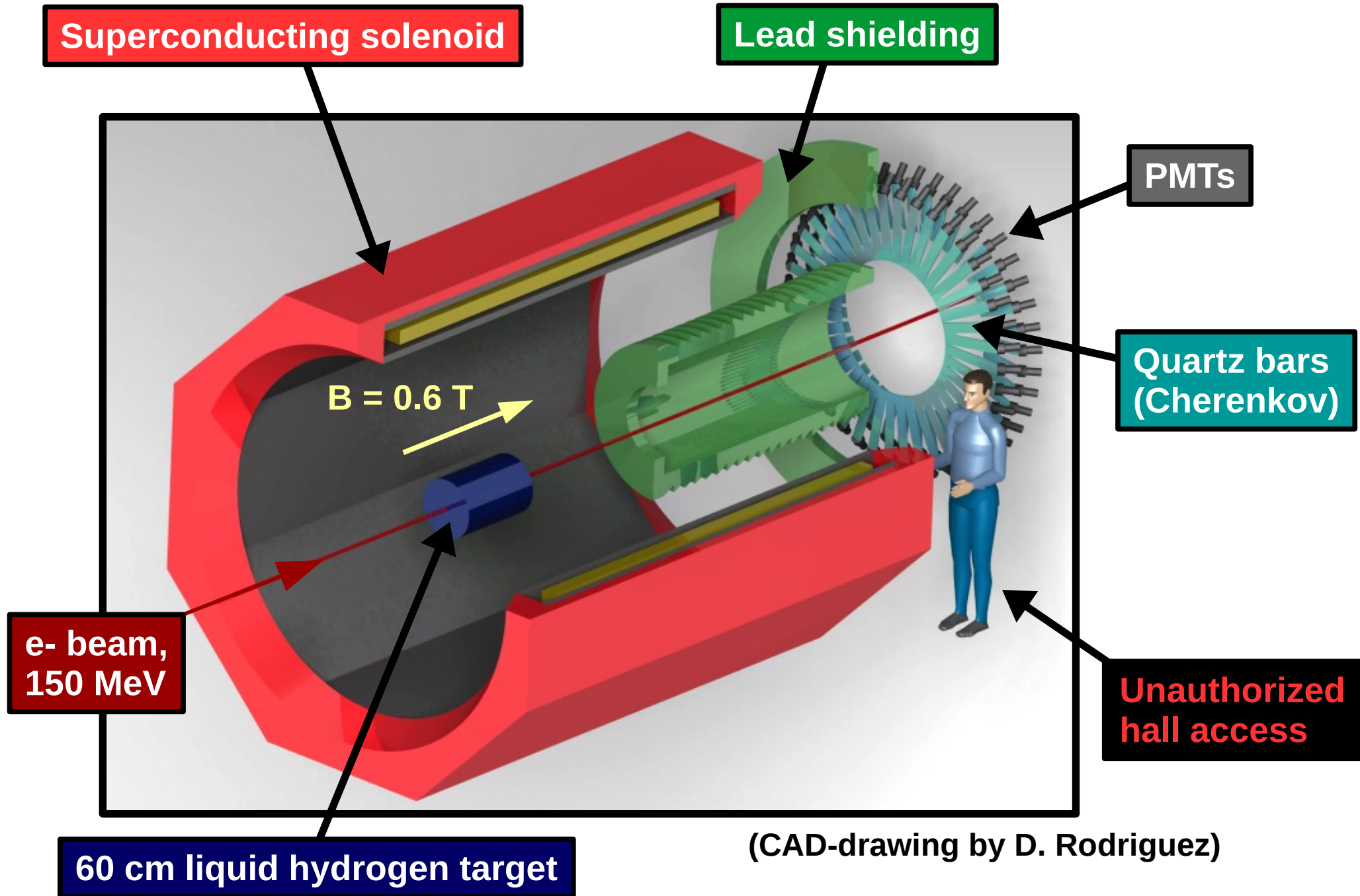
# The new M.E.S.A. facility in Mainz



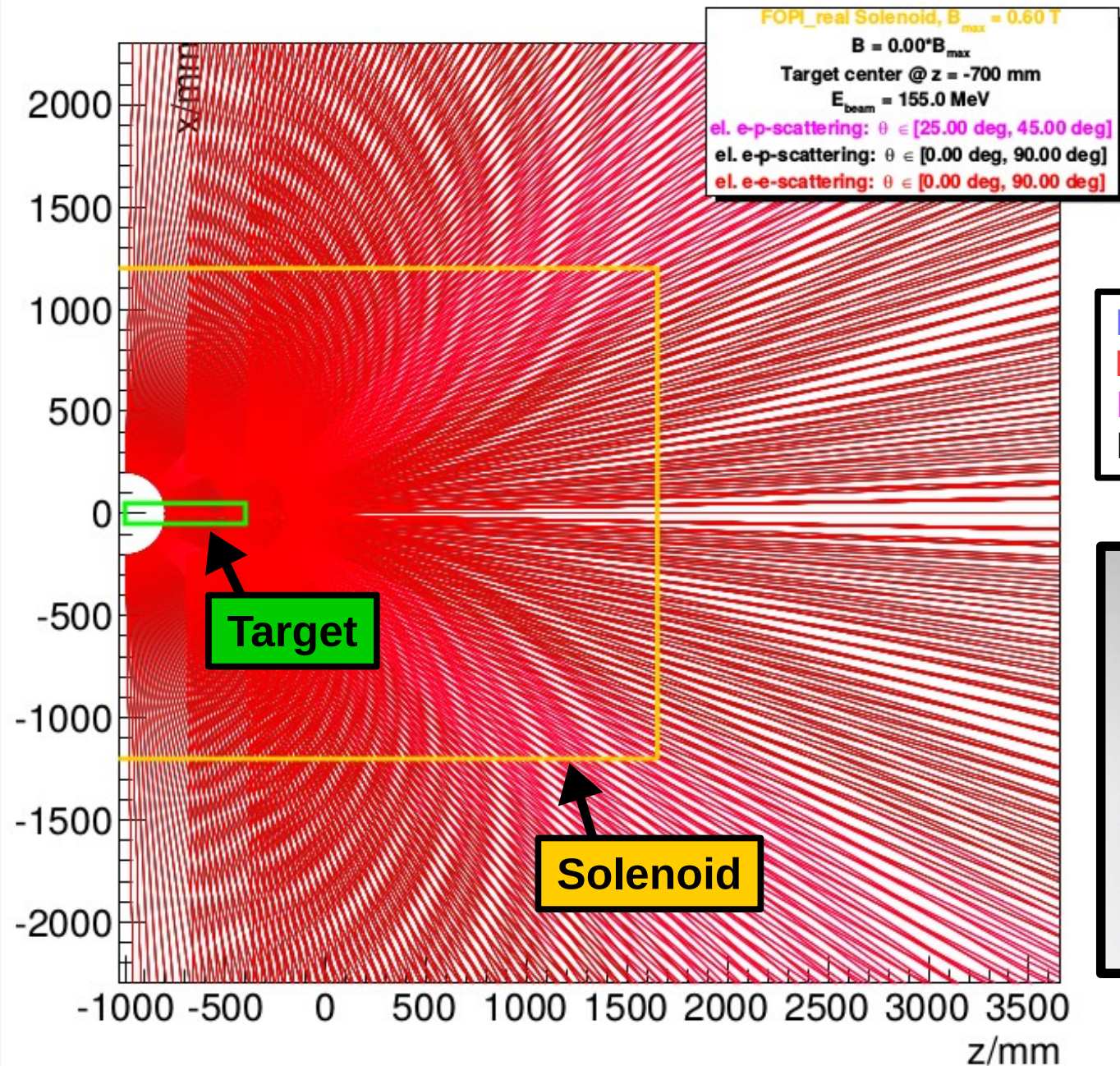
## Mainz Energy recovering Superconducting Accelerator:

- Normal-conducting injector LINAC
- Superconducting cavities in re-circulations
- Energy recovering mode: Unpolarized beam, 10 mA, 100 MeV, pseudo-internal gas-target,  $L \sim 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- External beam mode:  $P = 85\% \pm 0,5\%$ , 150  $\mu\text{A}$ , 155 MeV,  $L \sim 10^{39} \text{ cm}^{-2} \text{ s}^{-1}$ ,  $\langle \Delta A_{\text{app}} \rangle_{\Delta t} = 0.1 \text{ ppb}$

# Experimental setup under investigation

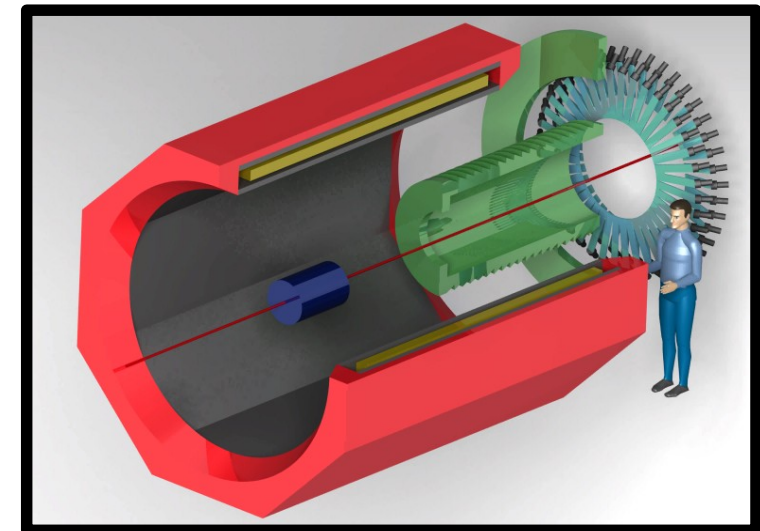


# Raytrace simulations in the magnetic field



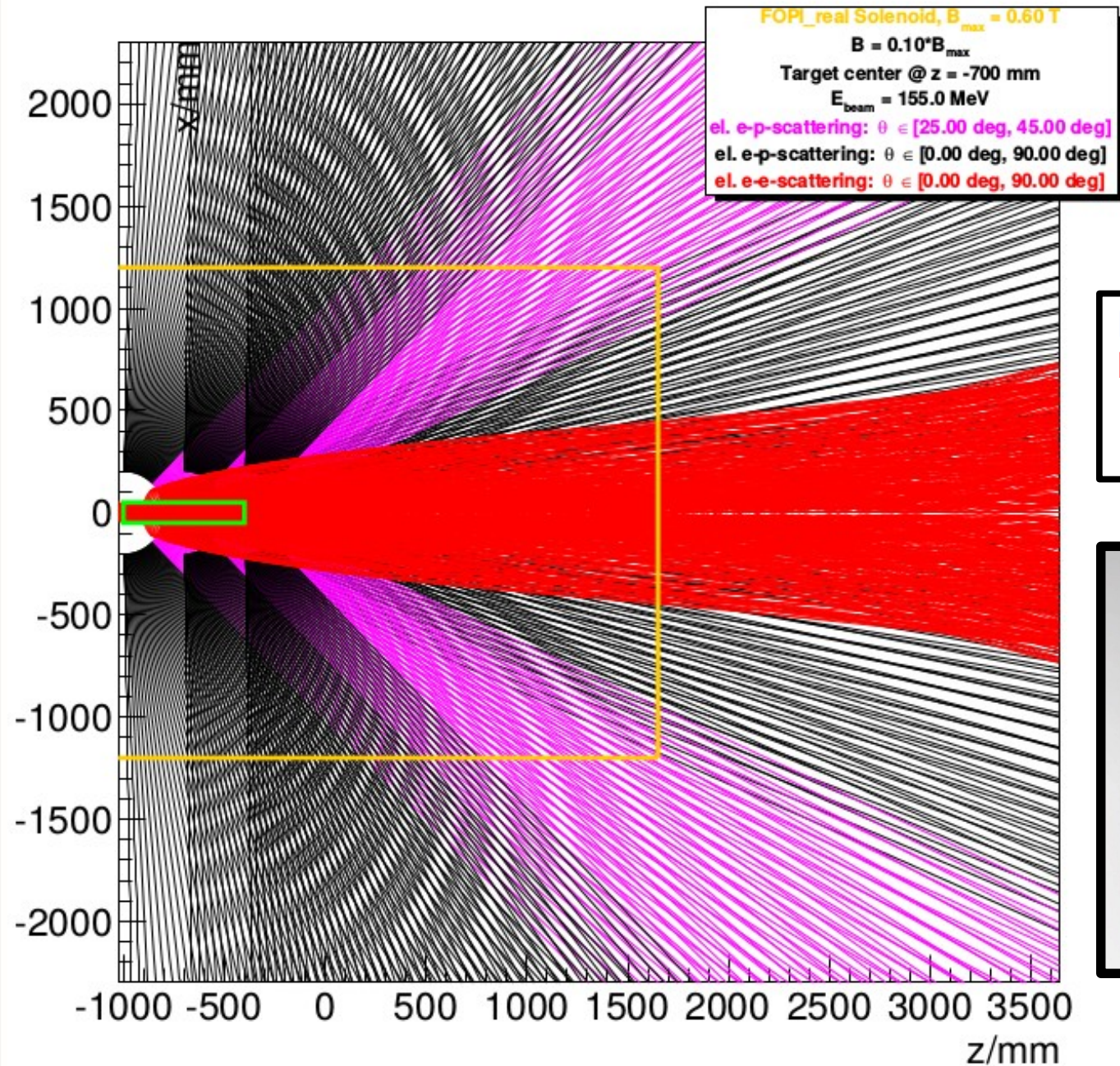
Magnetic field:  
**OFF**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$



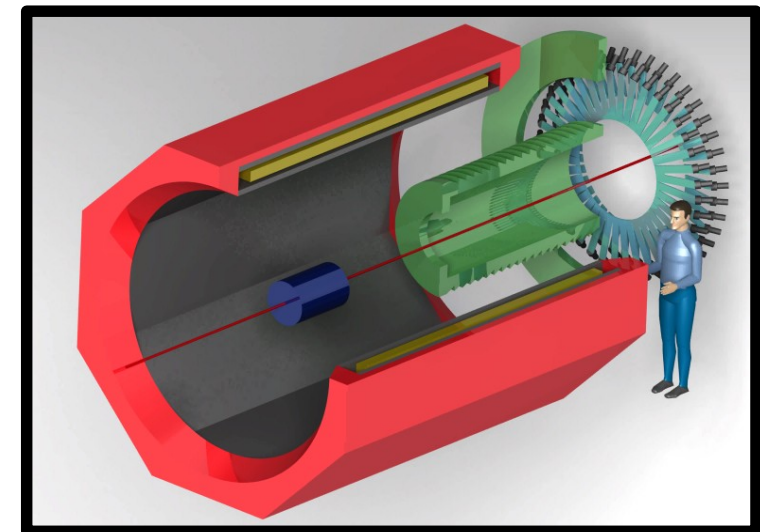


# Raytrace simulations in the magnetic field

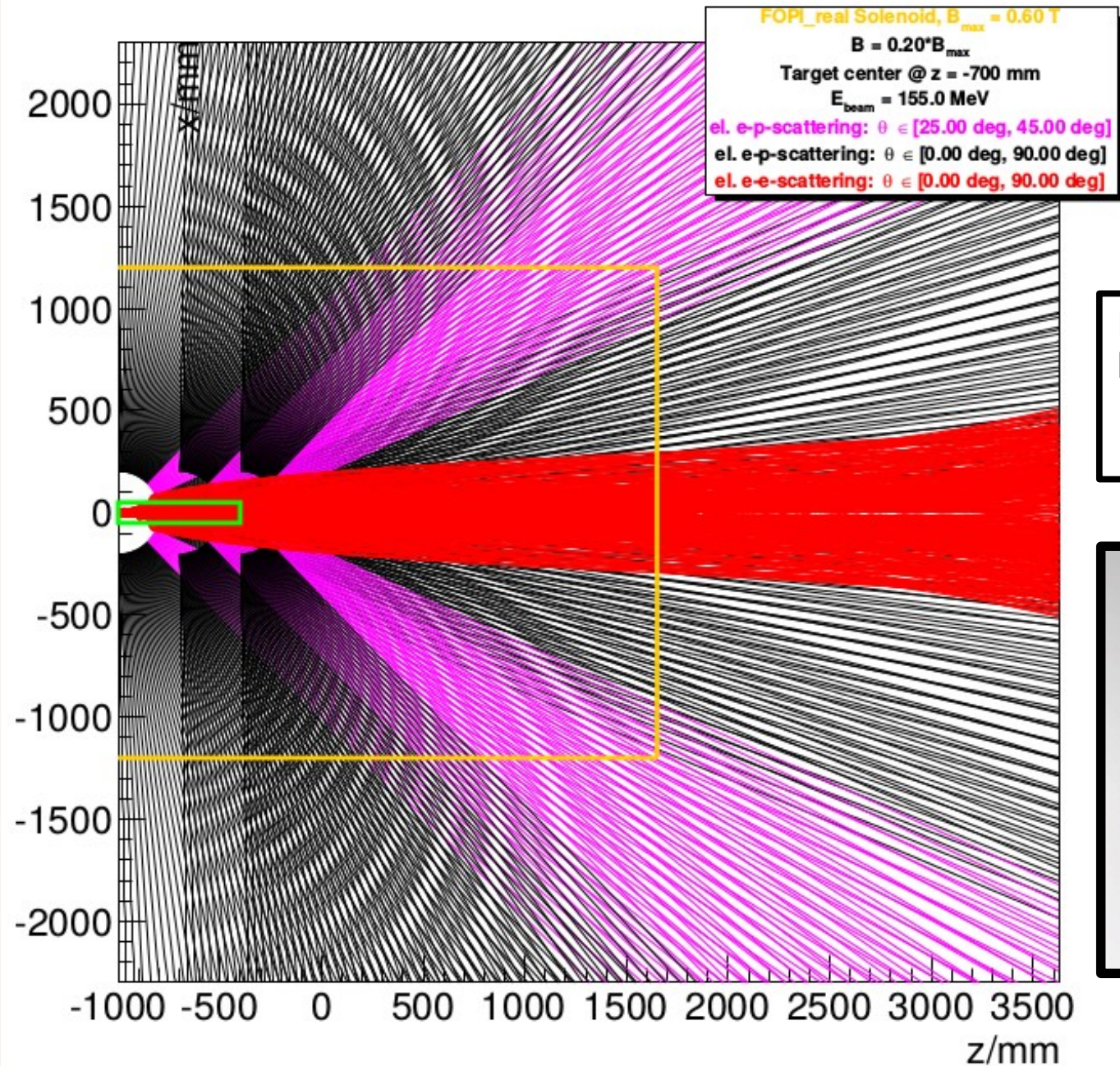


**Magnetic field:**  
**0.06 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

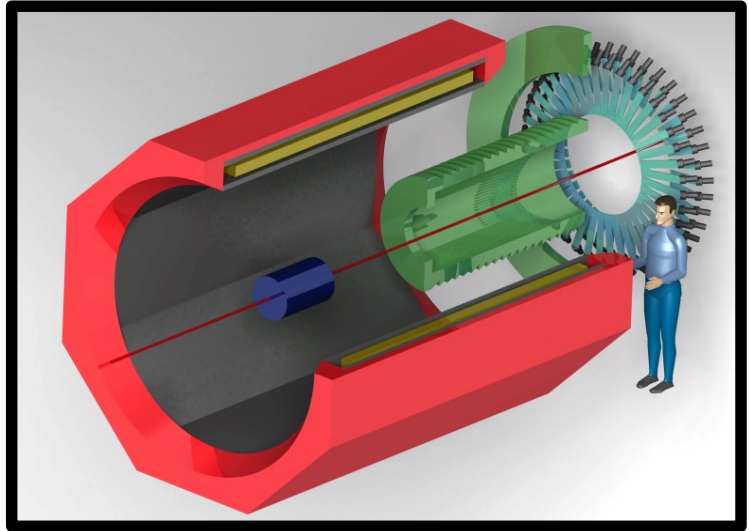


# Raytrace simulations in the magnetic field

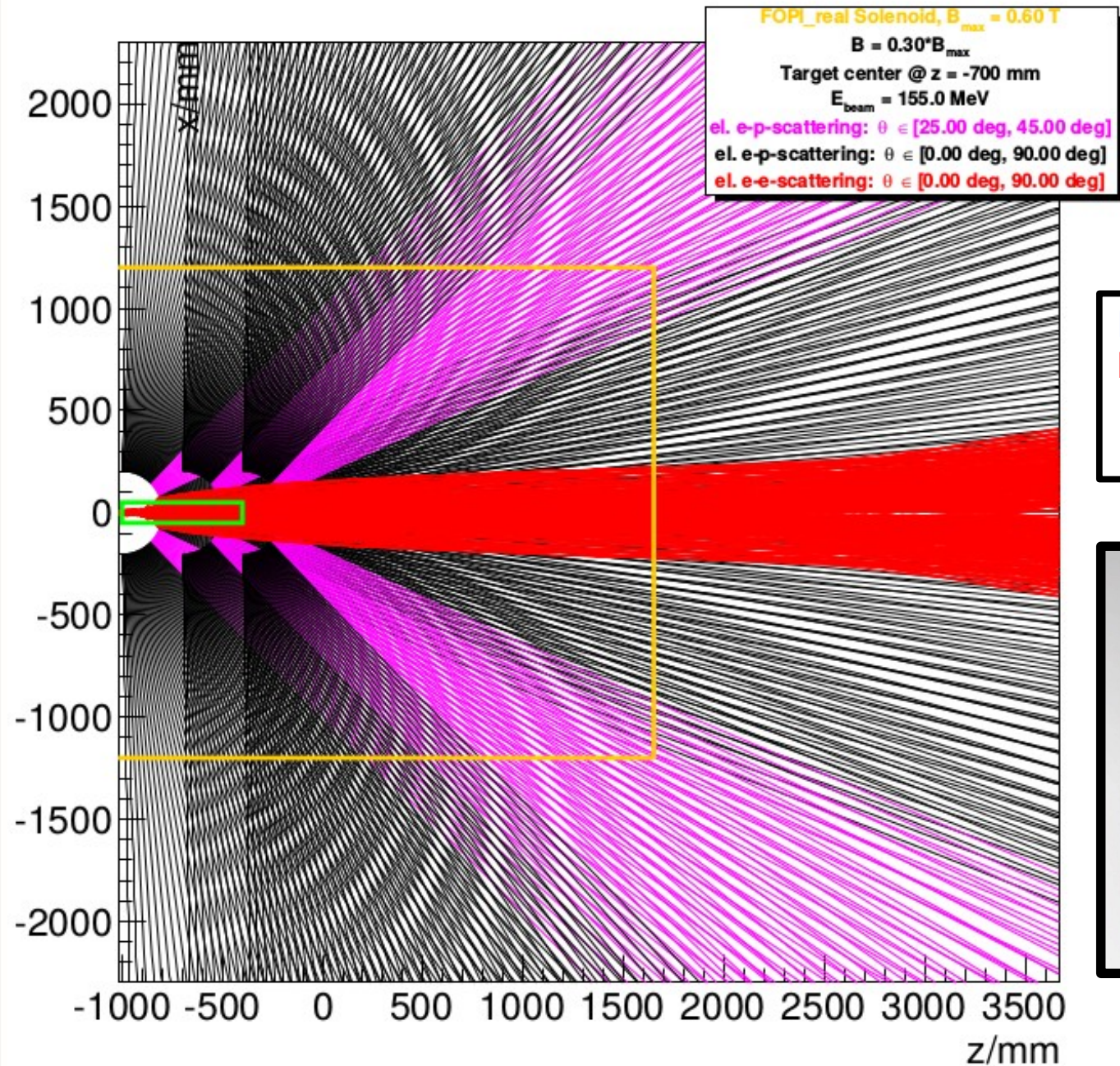


**Magnetic field:**  
**0.12 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

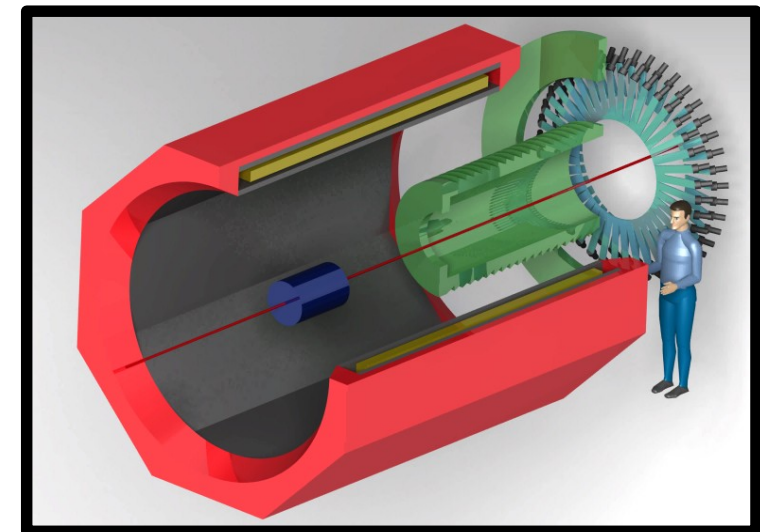


# Raytrace simulations in the magnetic field

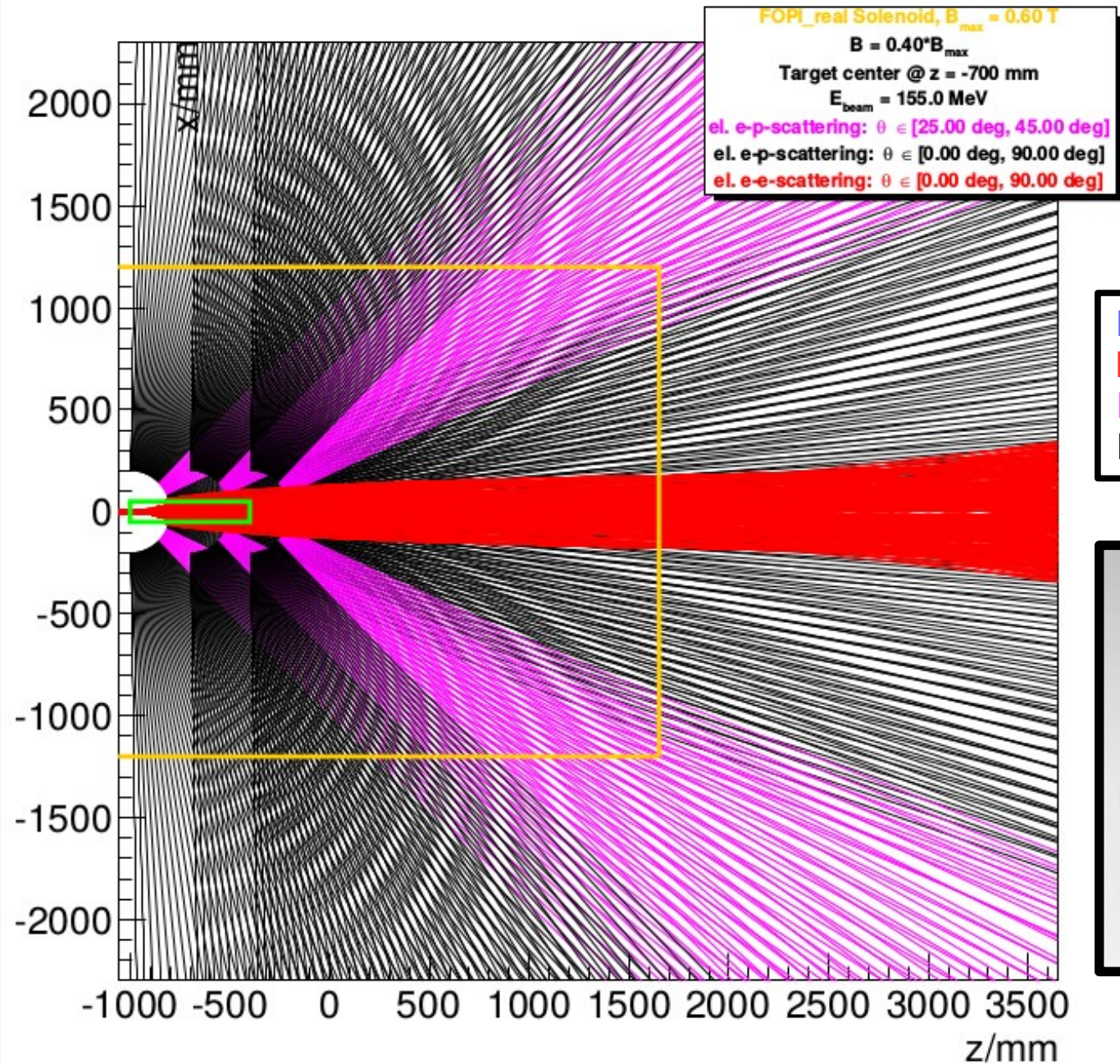


**Magnetic field:**  
**0.18 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

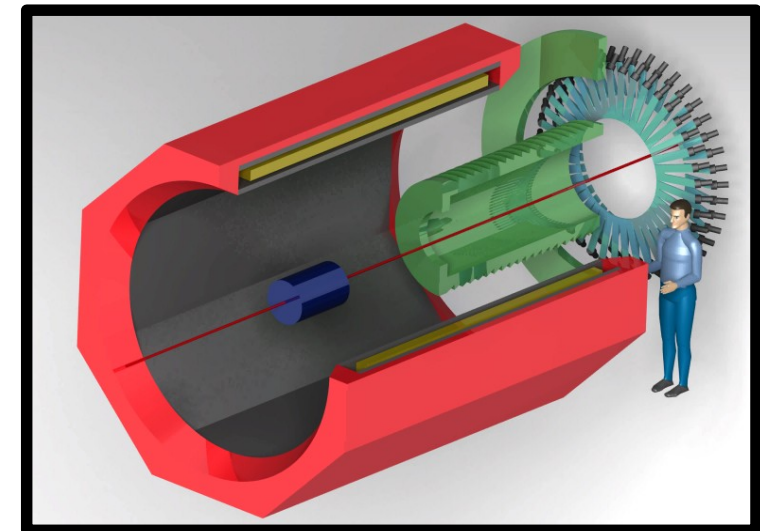


# Raytrace simulations in the magnetic field

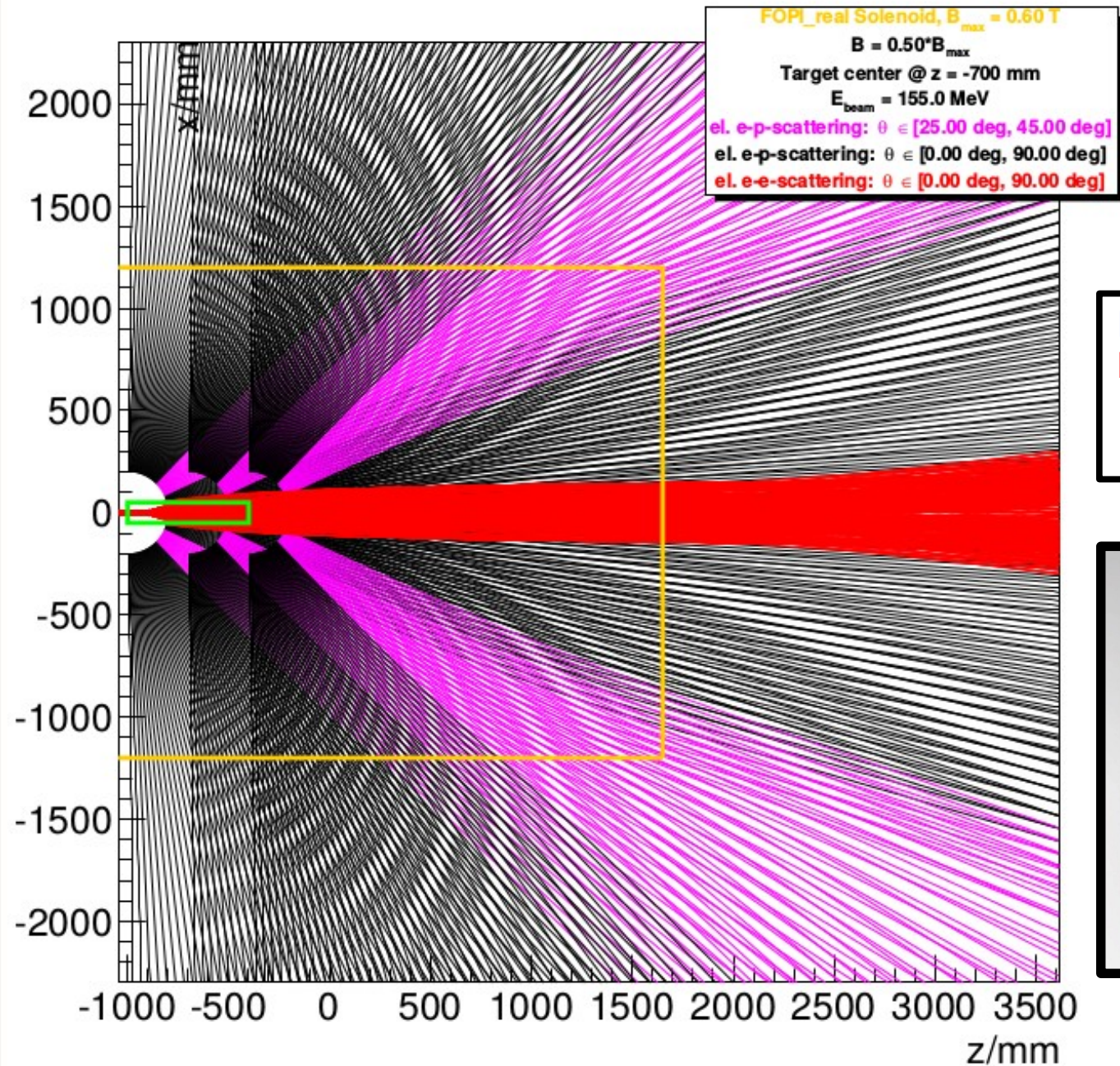


**Magnetic field:**  
**0.24 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

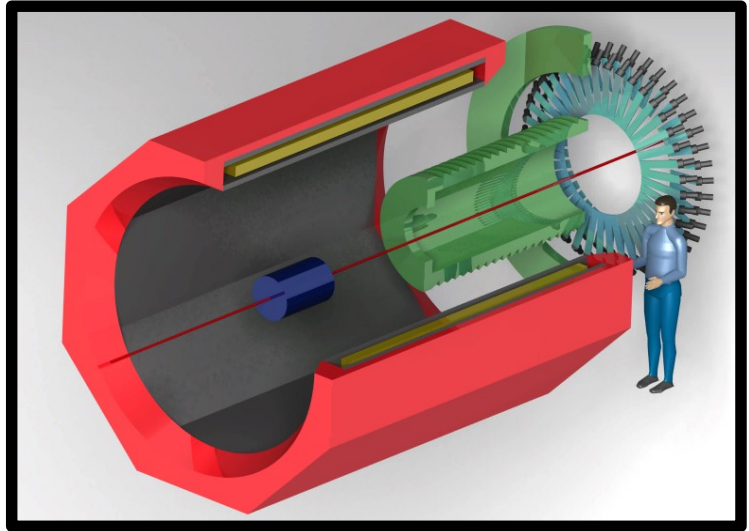


# Raytrace simulations in the magnetic field

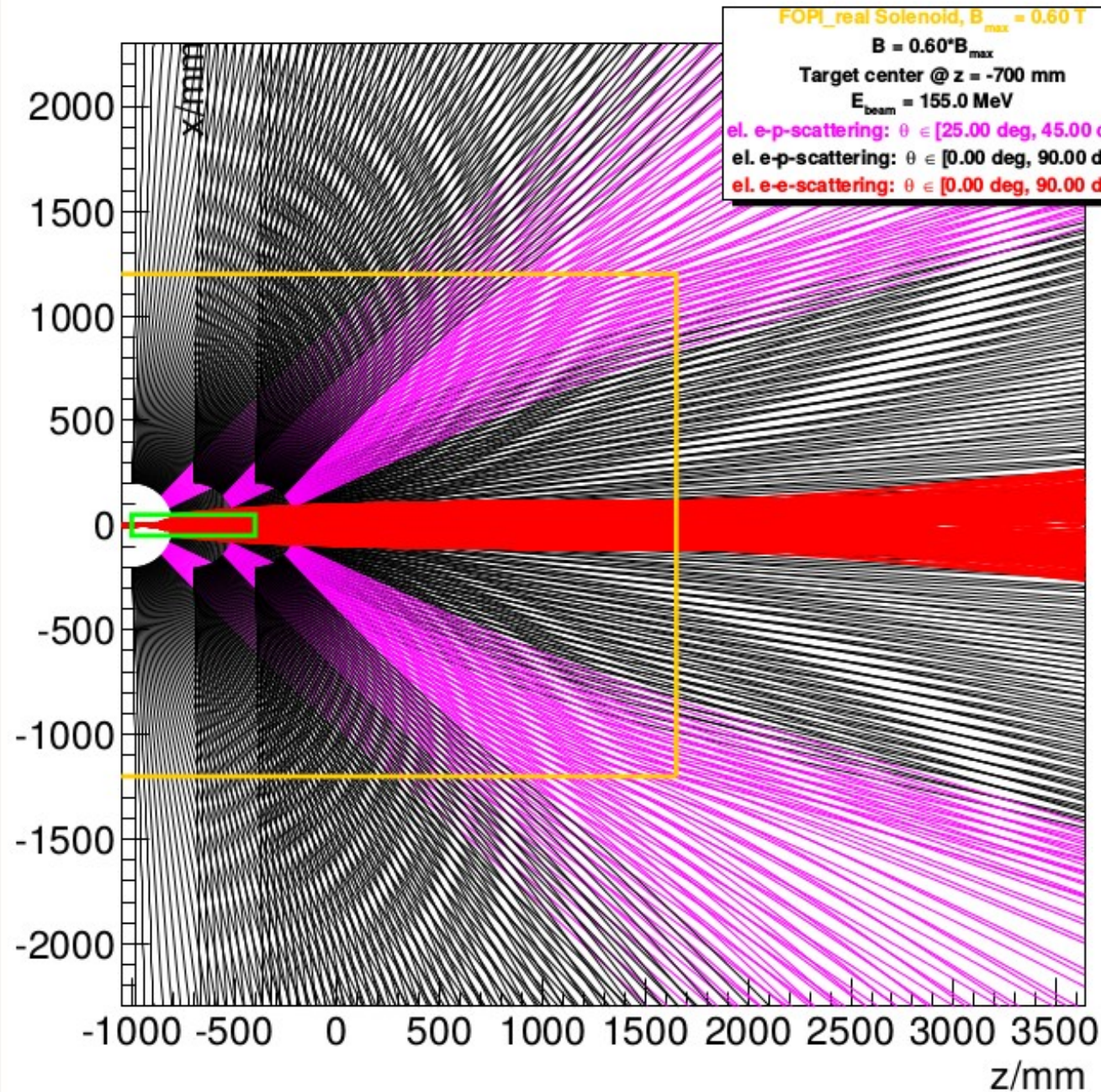


**Magnetic field:**  
**0.3 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

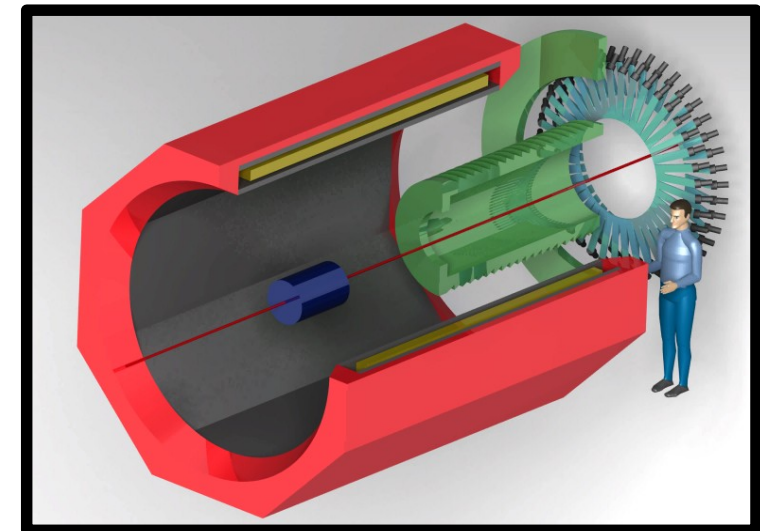


# Raytrace simulations in the magnetic field

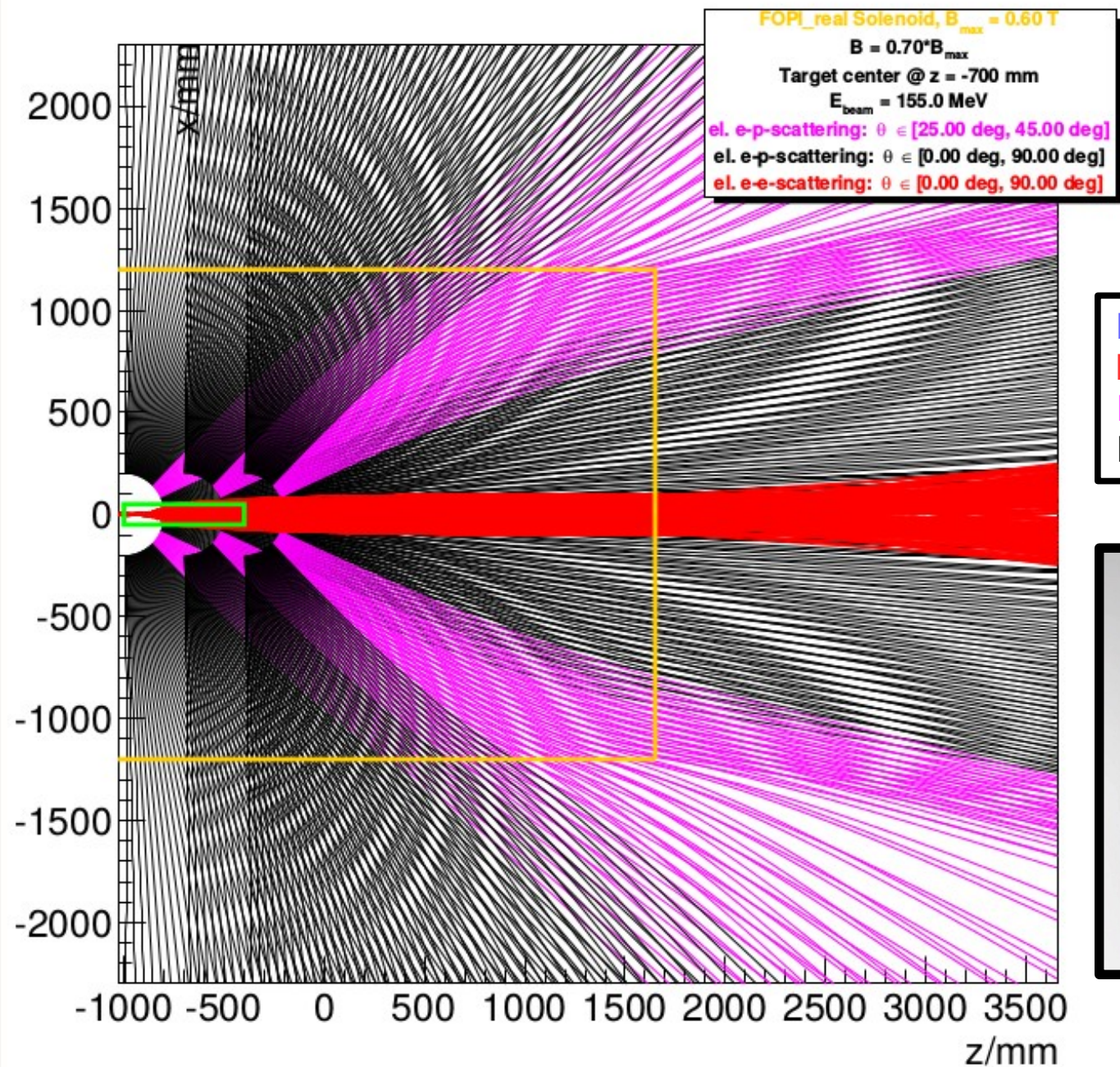


**Magnetic field:**  
**0.36 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

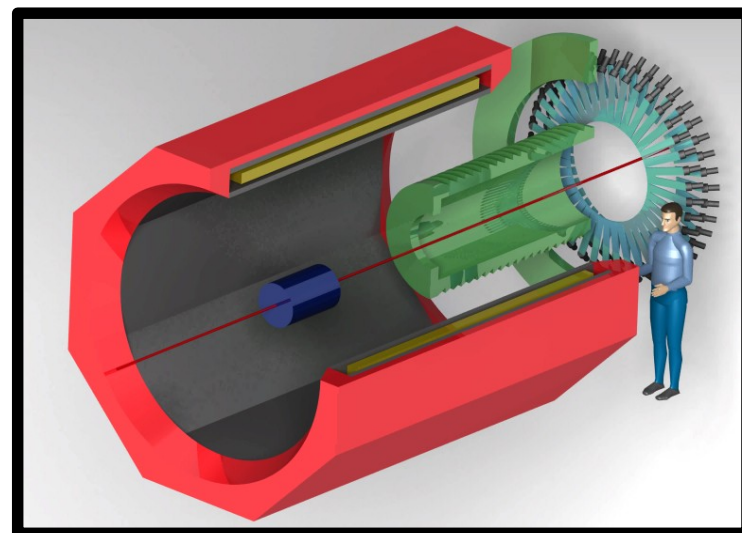


# Raytrace simulations in the magnetic field

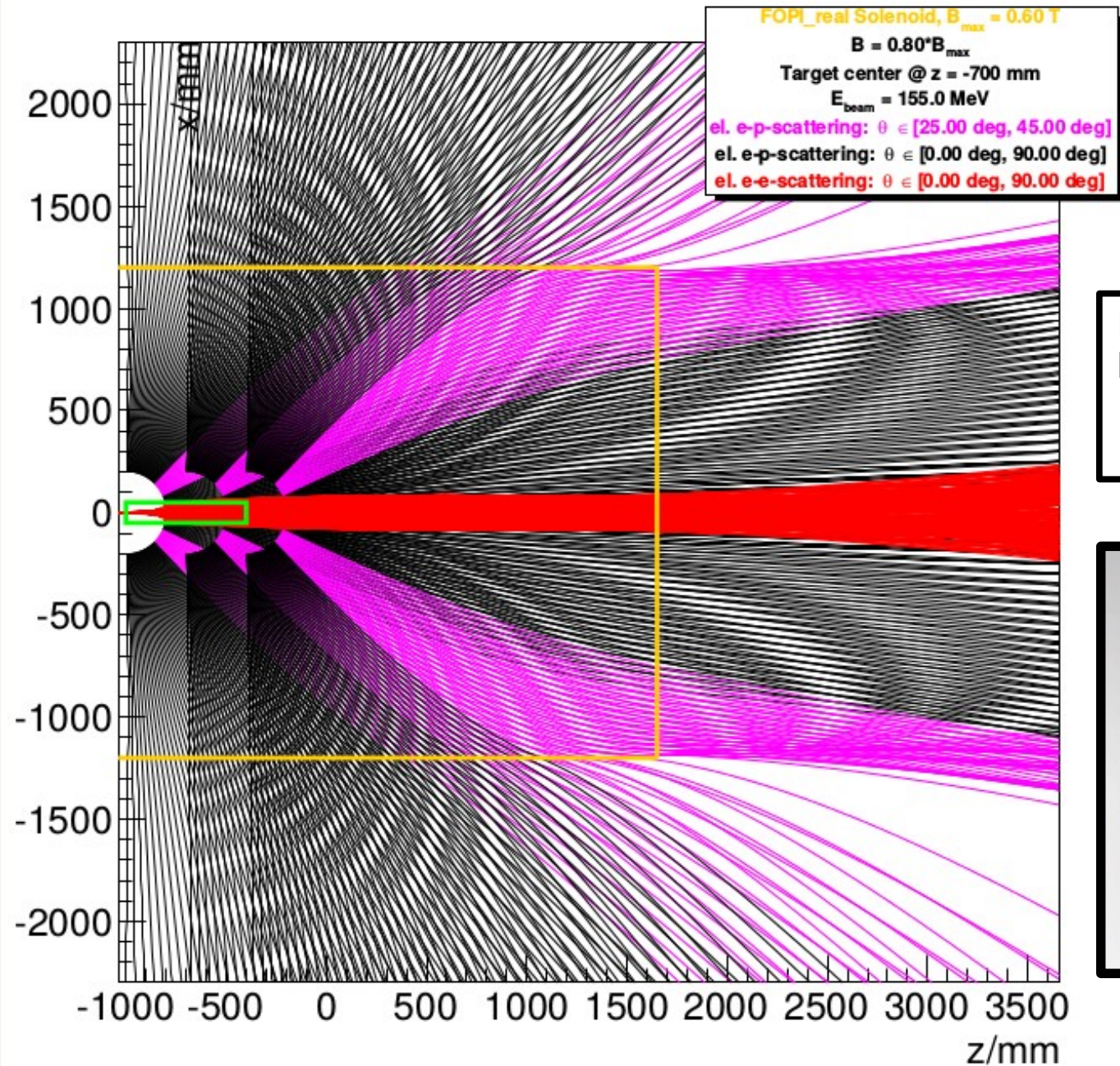


**Magnetic field:**  
**0.42 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

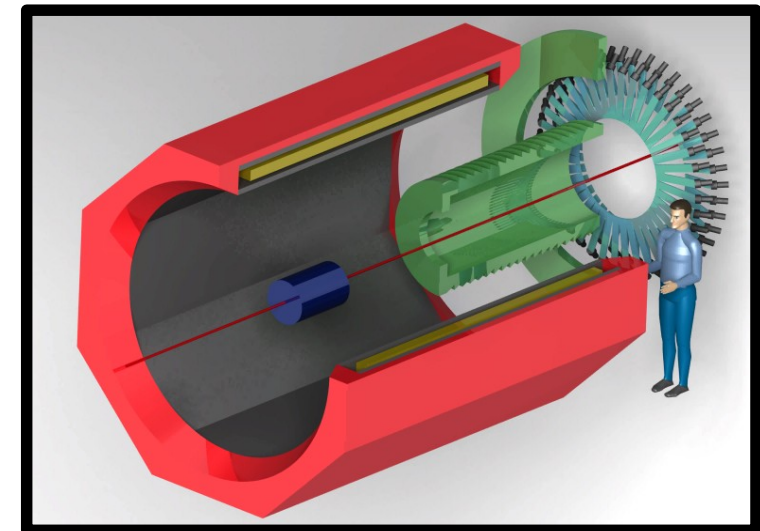


# Raytrace simulations in the magnetic field



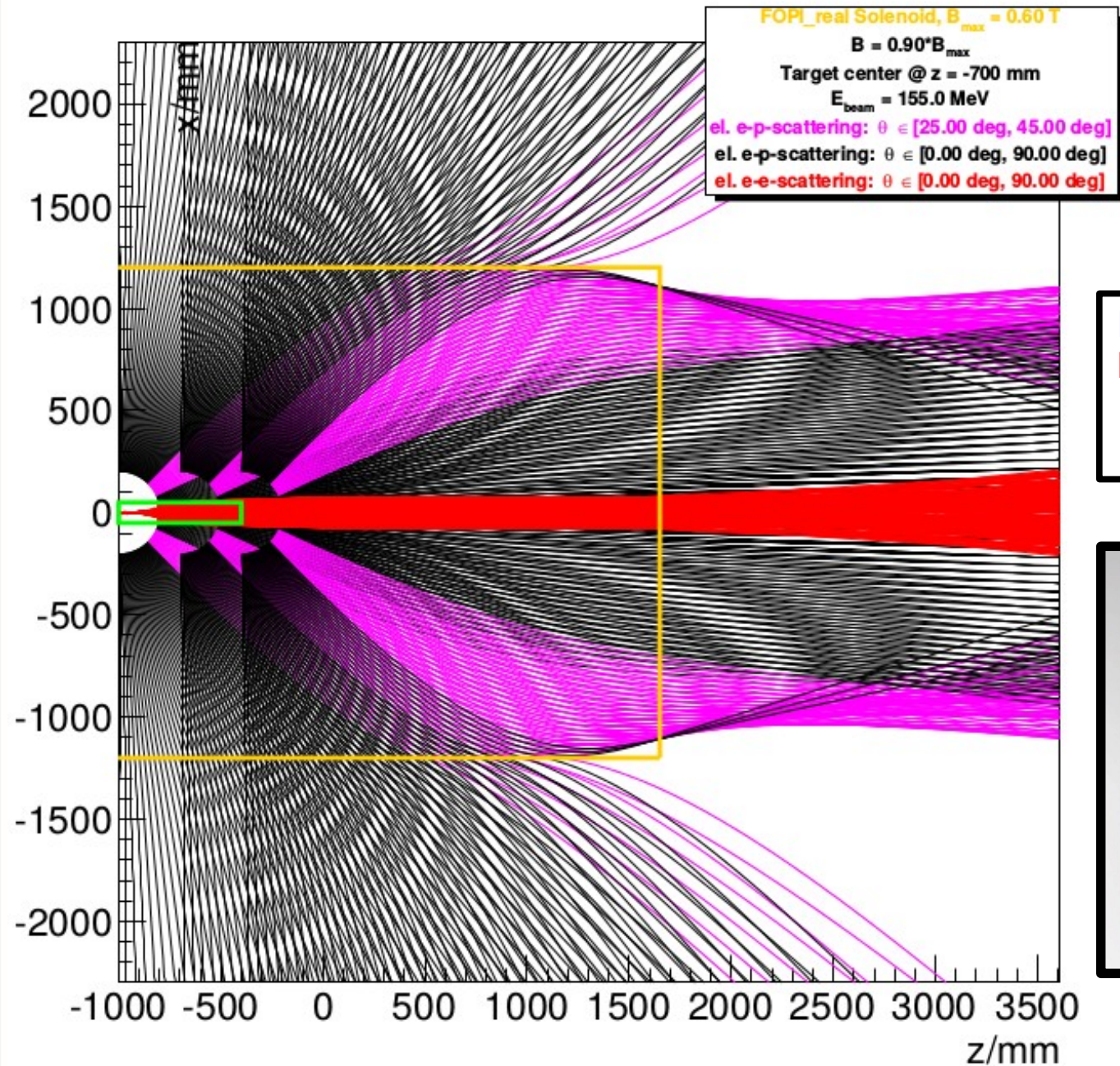
**Magnetic field:**  
**0.48 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$



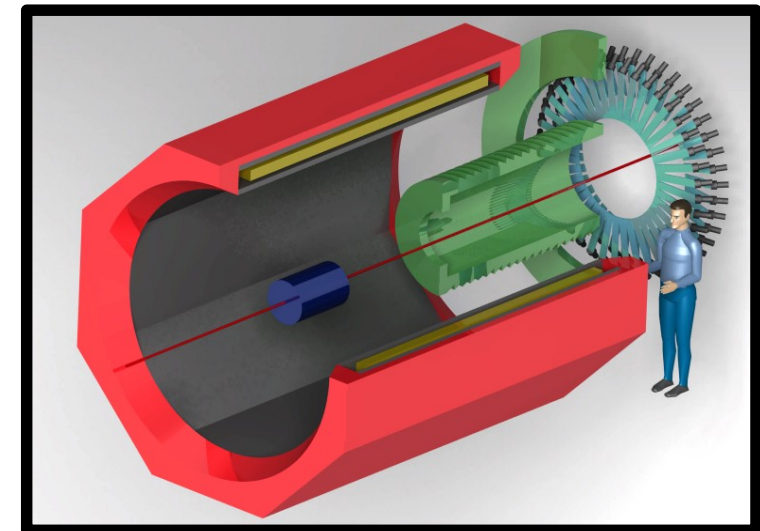


# Raytrace simulations in the magnetic field

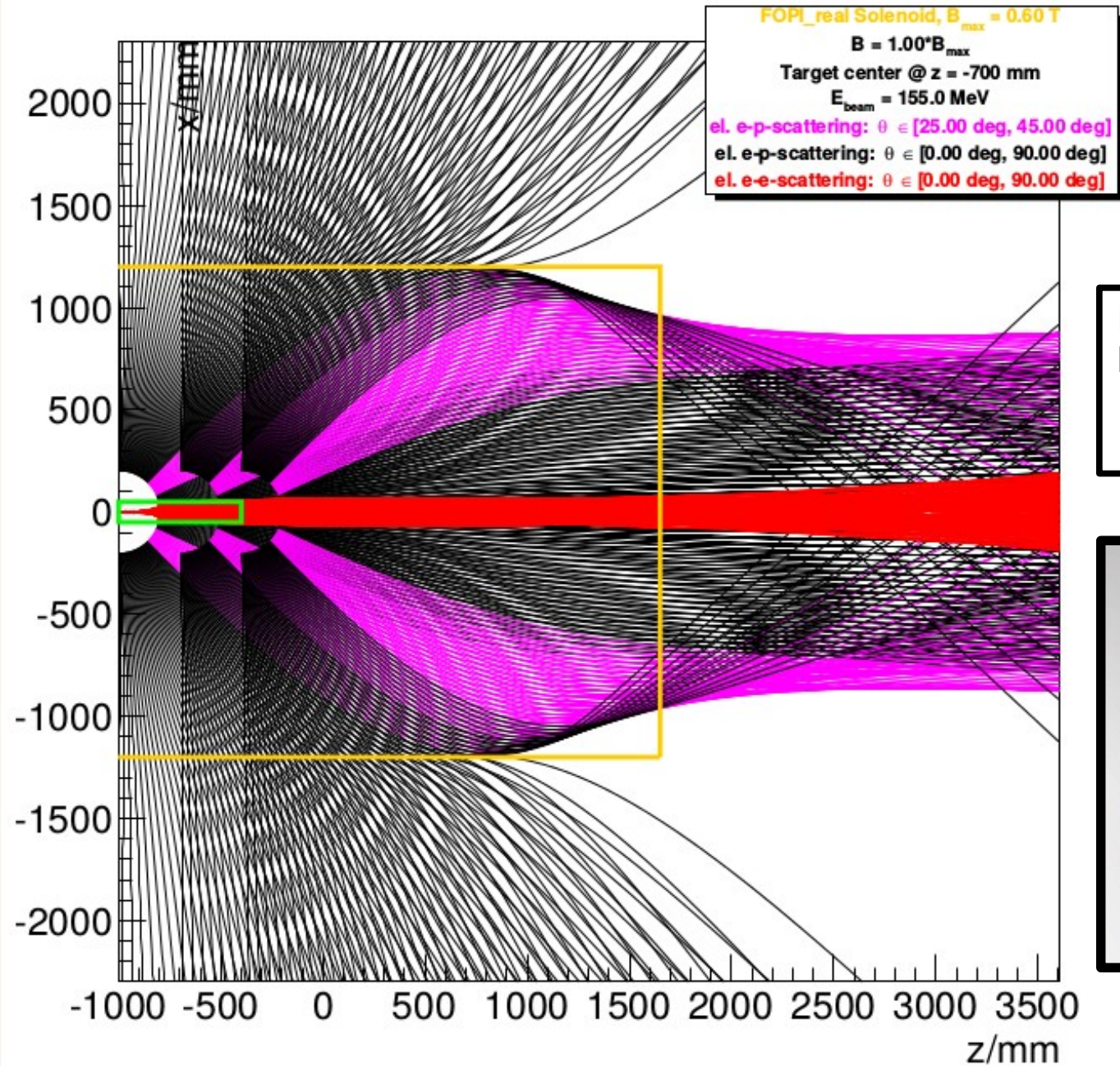


**Magnetic field:**  
**0.54 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

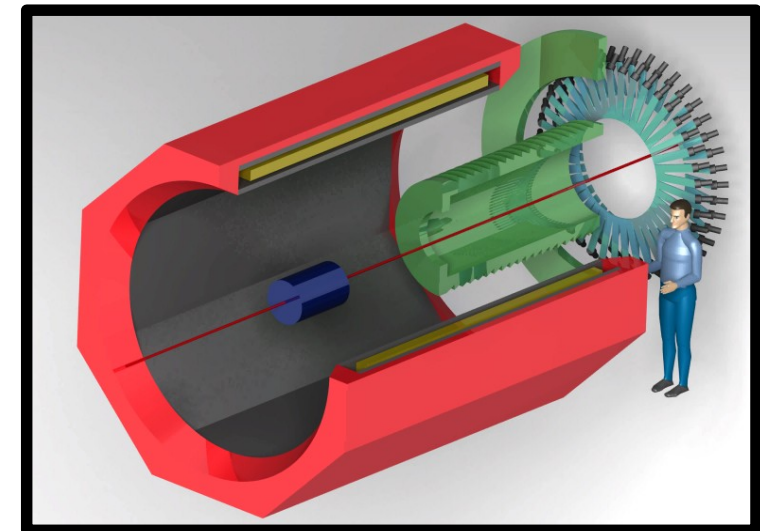


# Raytrace simulations in the magnetic field

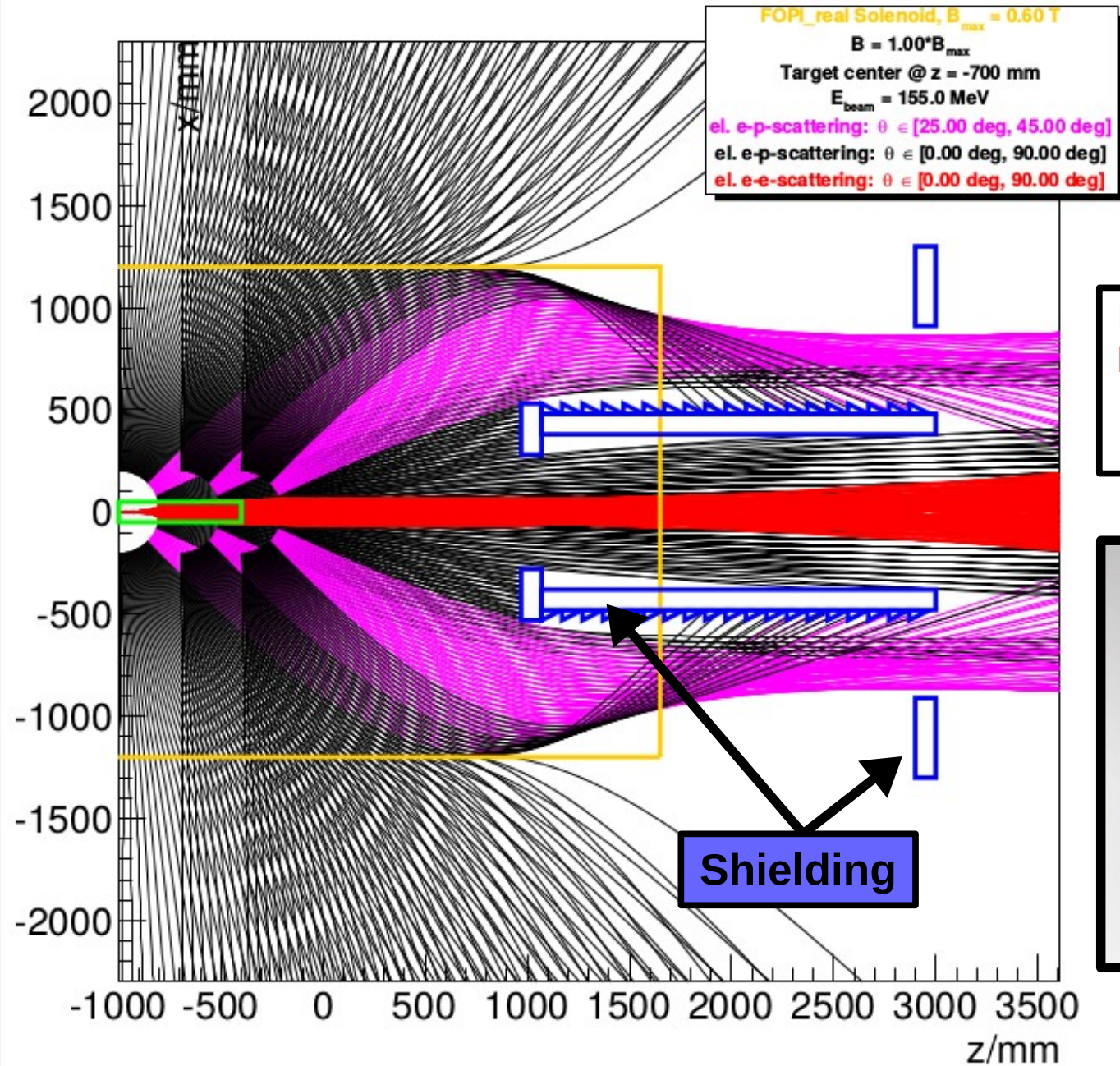


**Magnetic field:**  
**0.6 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

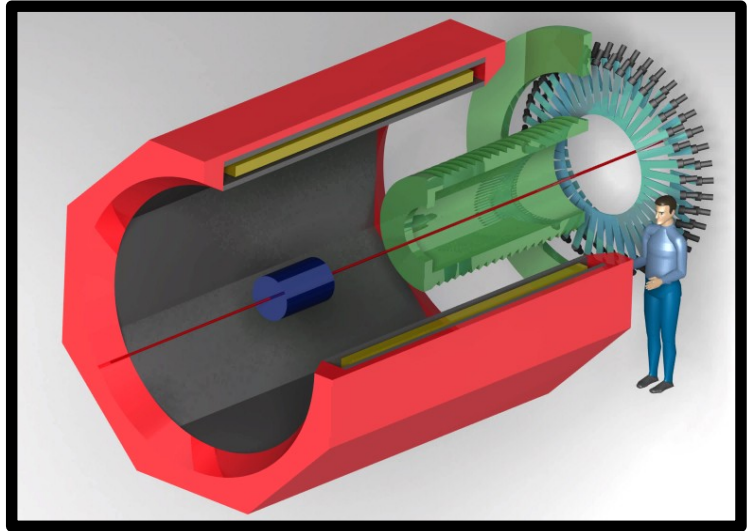


# Raytrace simulations in the magnetic field

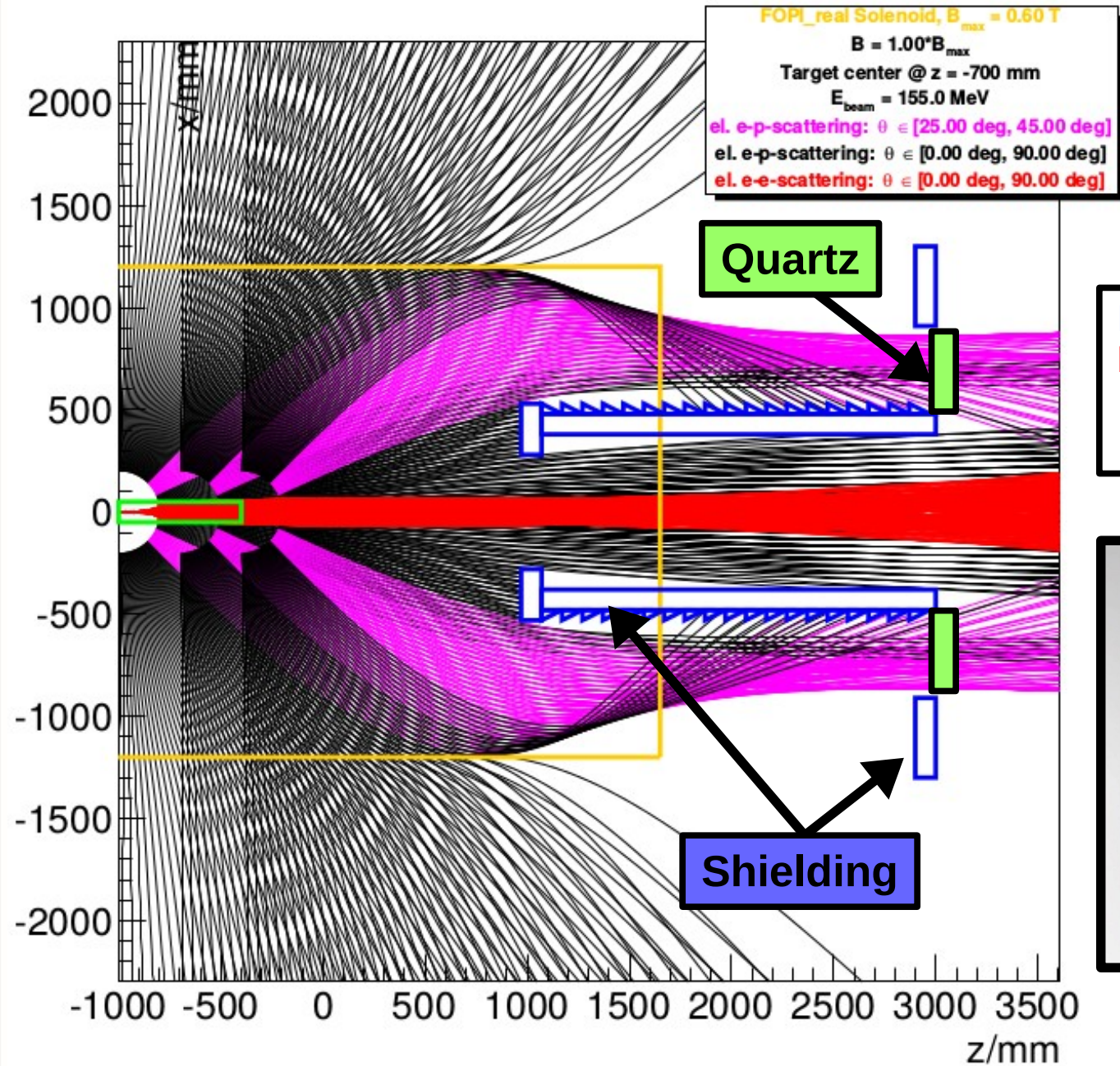


**Magnetic field:**  
**0.6 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

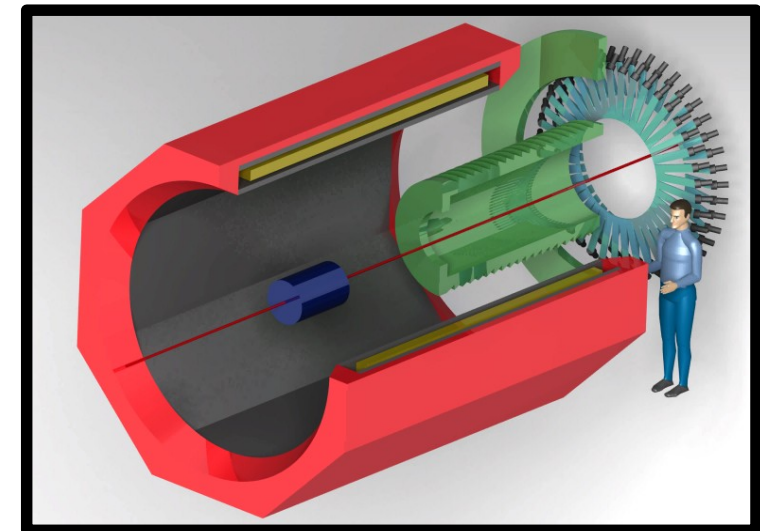


# Raytrace simulations in the magnetic field



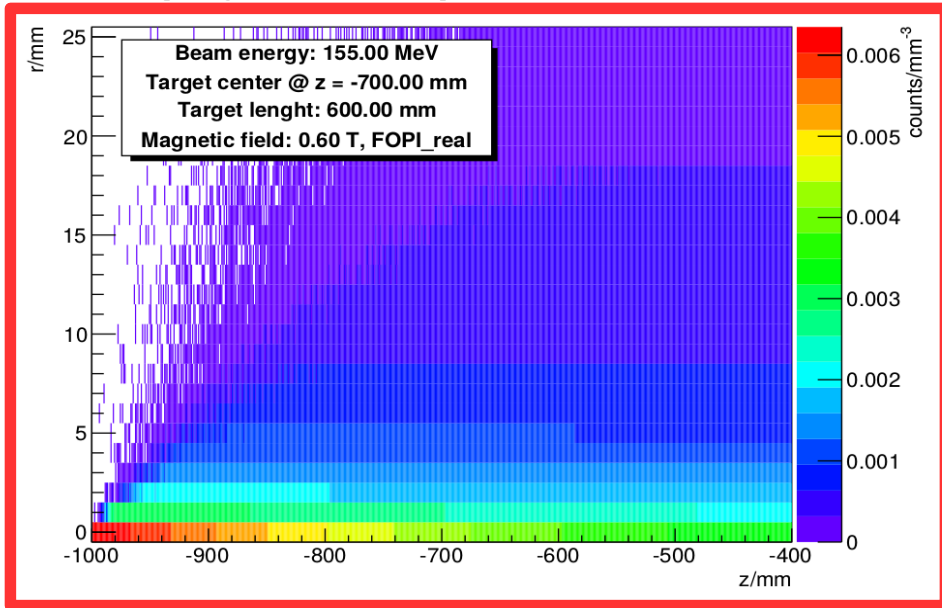
**Magnetic field:**  
**0.6 T**

Beam energy = 155 MeV  
Moller,  $\theta \in [0^\circ, 90^\circ]$   
Elastic e-p,  $\theta \in [25^\circ, 45^\circ]$   
Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$

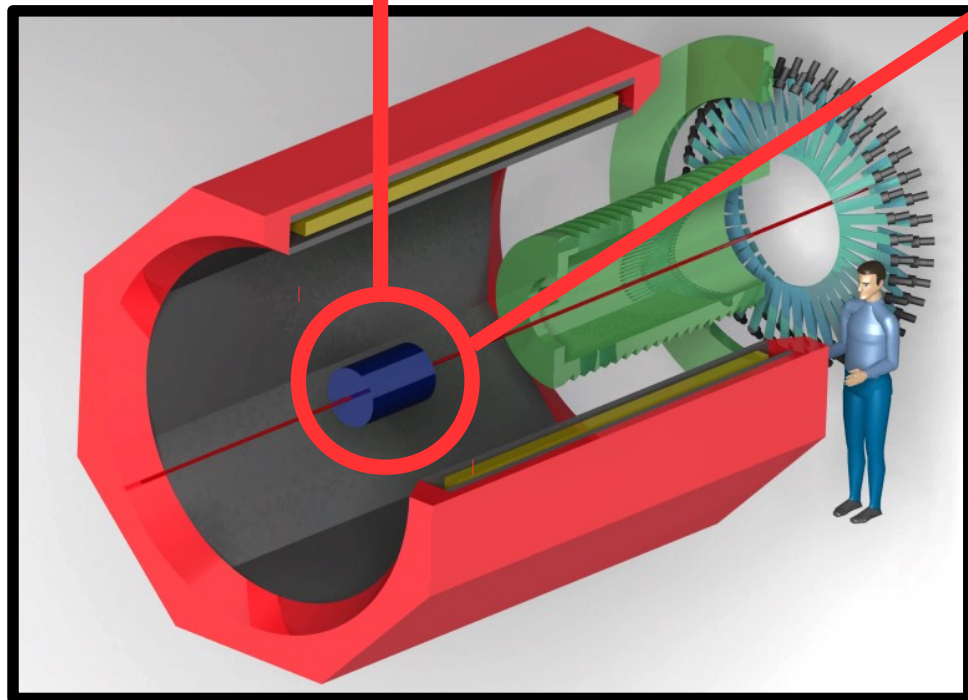
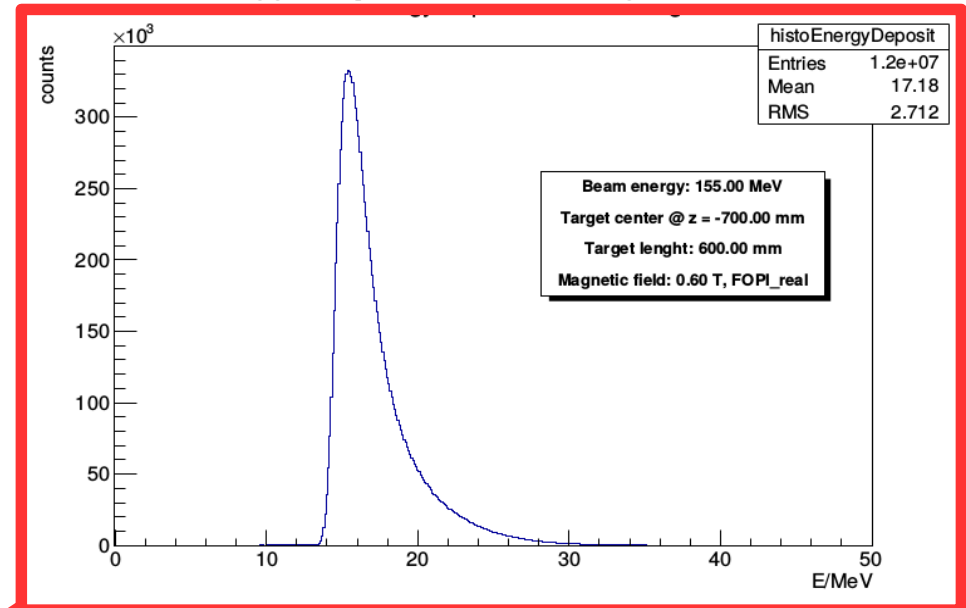


# Geant4 Simulation of beam-target-interaction

## Radial projection of spatial vertex distribution



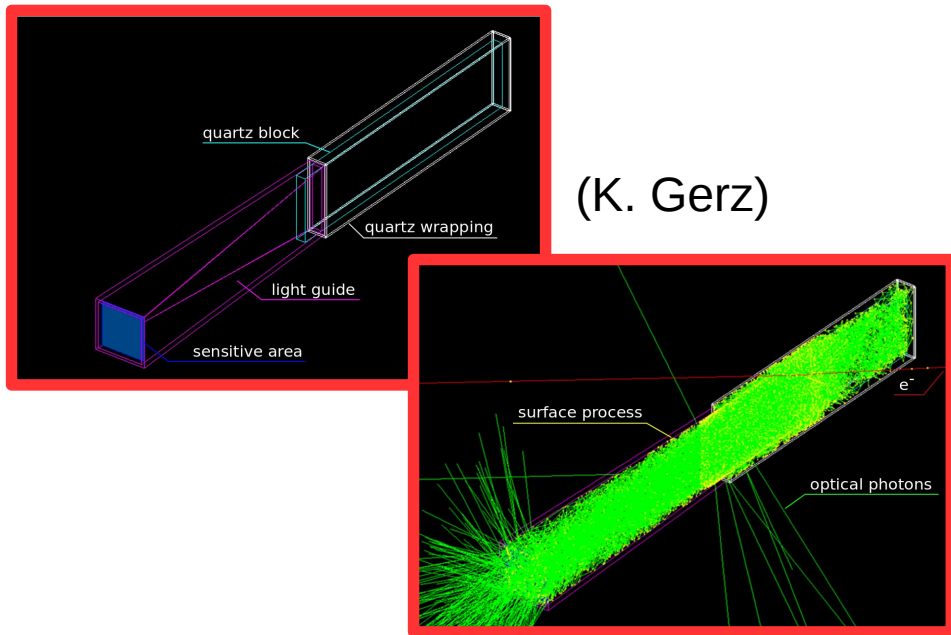
## Energy deposition in target volume



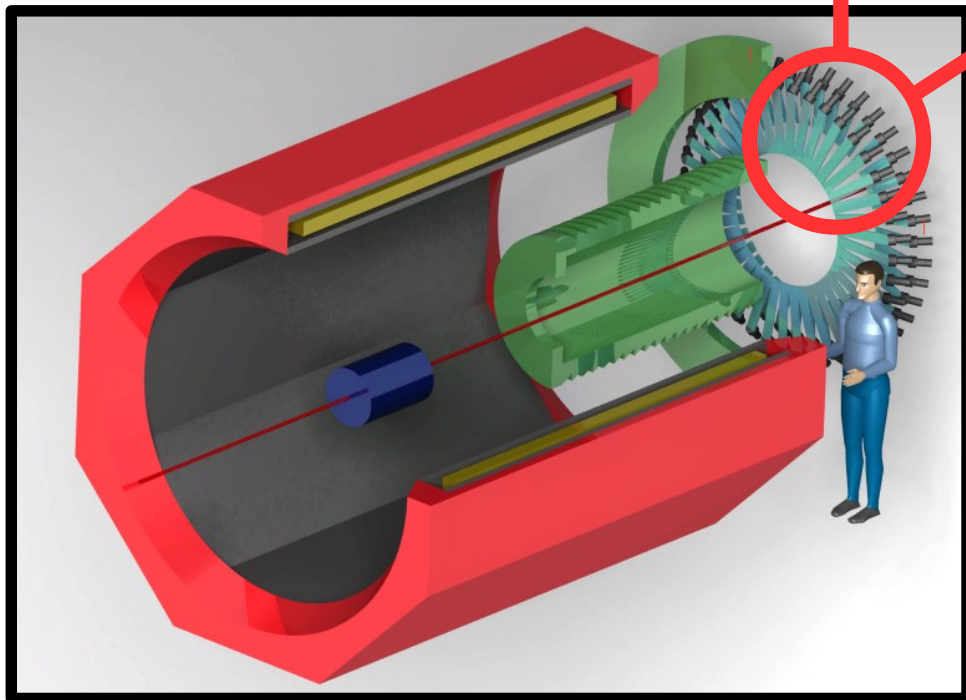
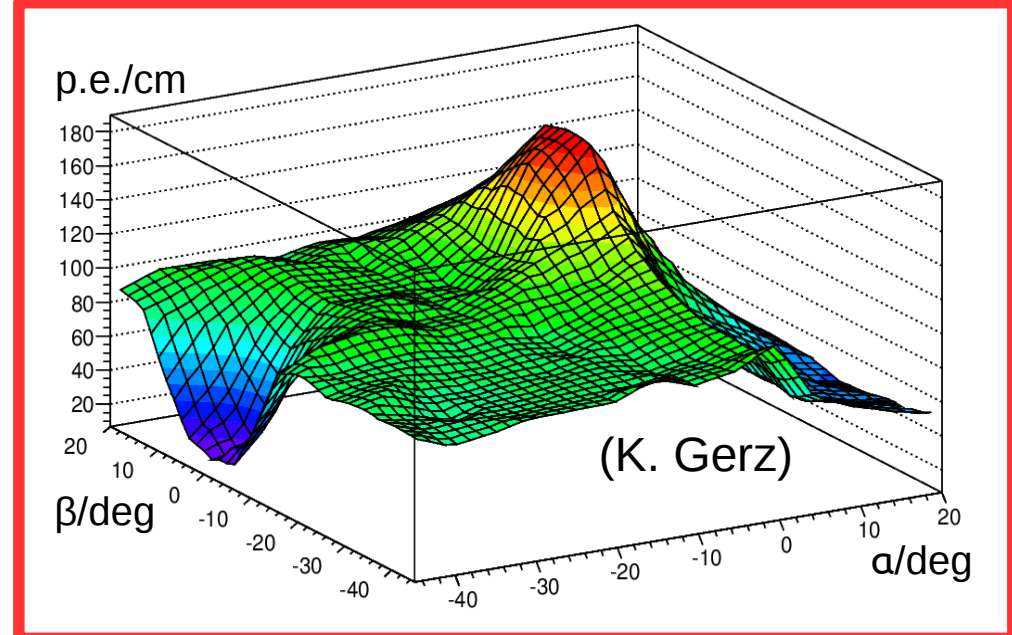
- Coherent simulation of elastic e-p scattering for P2 is impossible with Geant4
- Sample initial state distribution for elastic e-p scattering  
→ To be used with event generator
- Use tree-level event generator for primary event-generation
- Prototype of event generator with radiative corrections available and currently under evaluation

# Geant4 Simulation of detector module response

## Tracking of optical photons in detector module



## Photo electron yield distribution, $E = 155$ MeV

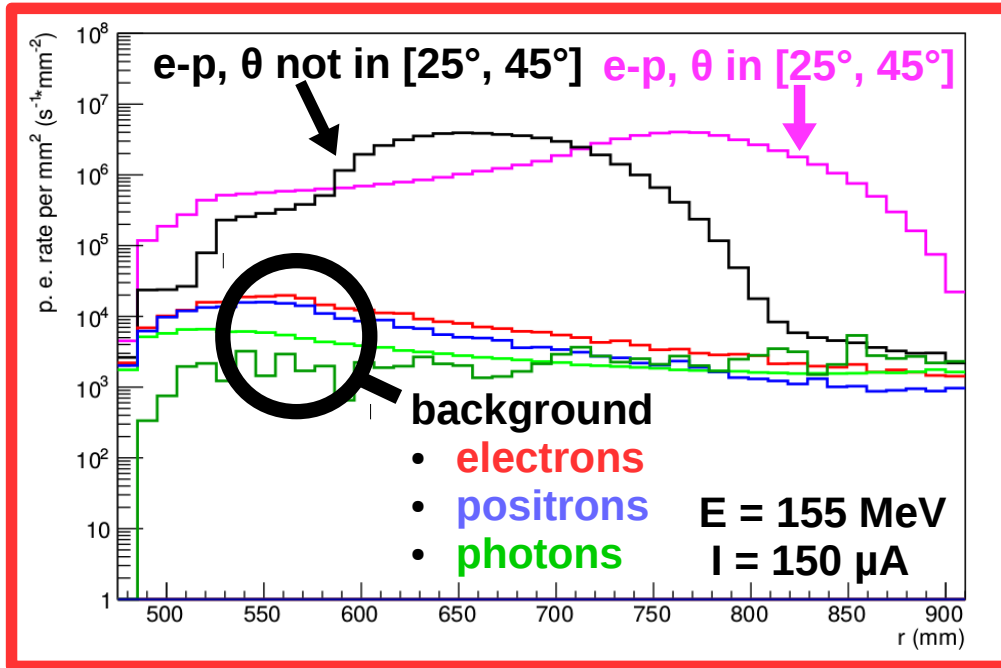


## Create parametrization of photo electron yield for different

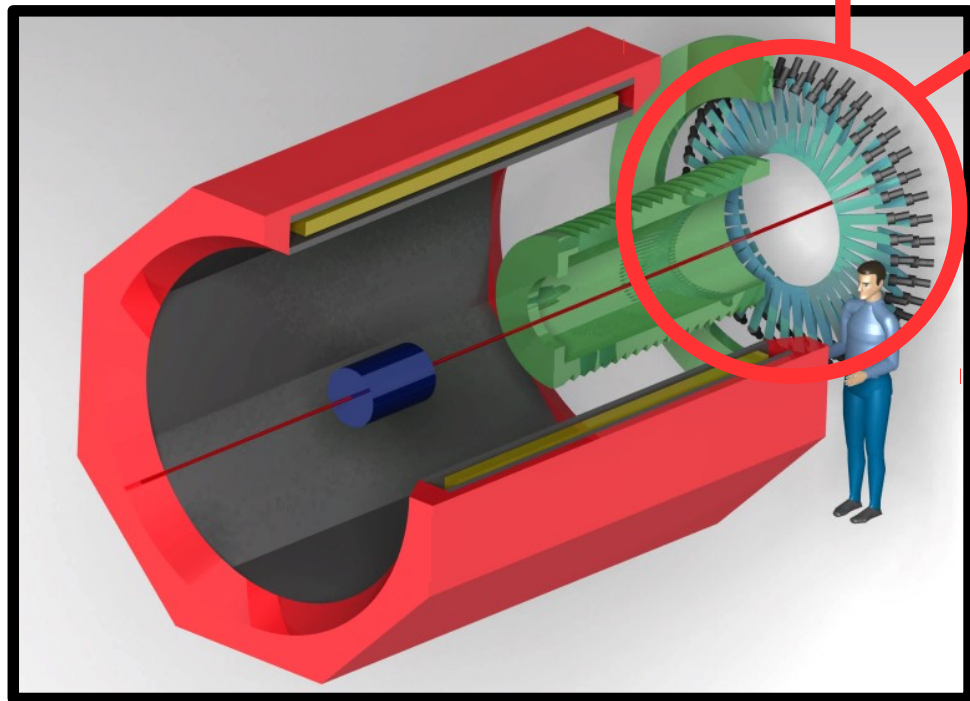
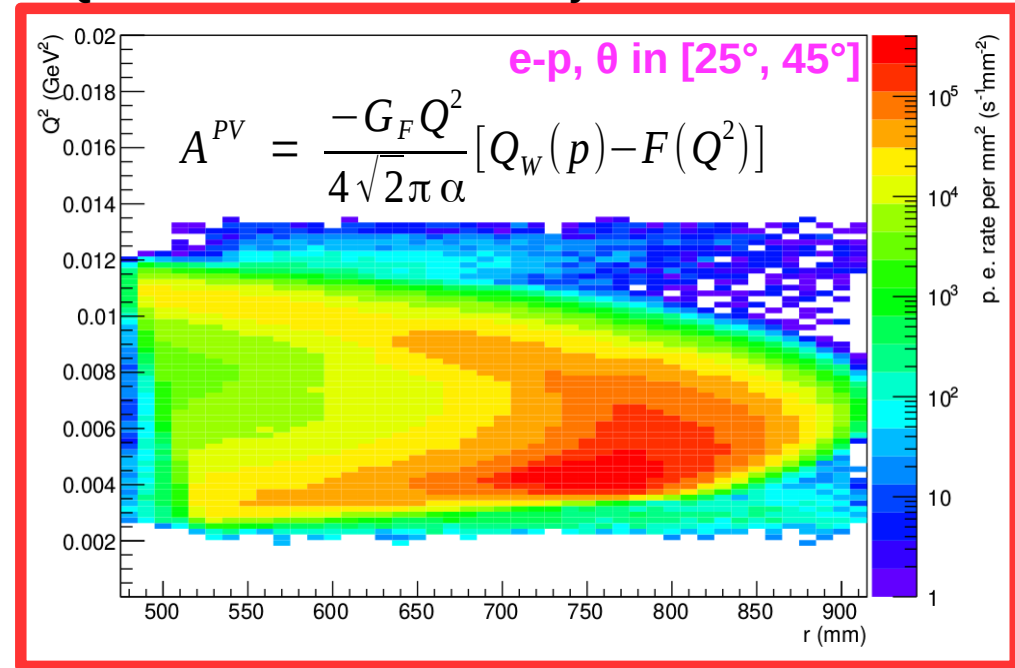
- Active materials
- Geometries
- Particle types
- Particle energies
- Impact angles

# Geant4 Simulation of experimental setup

## Photo electron rate distribution



## Q<sup>2</sup> distribution of elastically scattered electrons



- Use initial state distribution with tree level event generator to simulate elastic e-p scattering
- Tracking in realistic map of magnetic field, CAD-interface for definition of geometry
- Use parametrization of detector response to predict distribution of photo electrons
- Use Q<sup>2</sup> distribution in error propagation calculation to predict the achievable precision in the weak mixing angle

# Facts and Figures

The following results are based on error propagation calculations **including** the results of the Geant4 simulation of the experimental setup:

<b>Beam energy</b>	155 MeV	
<b>Beam current</b>	150 $\mu\text{A}$	
<b>Polarization</b>	85 %	$\pm 0.425 \%$
<b>Target</b>	60 cm	liquid hydrogen
<b>Detector acceptance</b>	$2\pi \cdot 20^\circ$	$\theta \in [25^\circ, 45^\circ]$
<b>Detector rate</b>	0.5 THz	
<b>Measurement time</b>	1e4 h	
<b><math>\langle Q^2 \rangle</math></b>	4.49e-3 $\text{GeV}^2/c^2$	
<b><math>A^{\text{exp}}</math></b>	-28.35 ppb	

	<b>Total</b>	<b>Statistics</b>	<b>Polarization</b>	<b>Apparative</b>	<b>Form factors</b>	<b><math>\text{Re}(\chi_{\text{YZA}})</math></b>
<b><math>\Delta \sin^2(\theta_w)</math></b>	<b>3.1e-4</b> <b>(0.13 %)</b>	2.6e-4 (0.11 %)	9.7e-5 (0.04 %)	7.0e-5 (0.03 %)	1.4e-4 (0.04 %)	6e-5 (0.03 %)
<b><math>\Delta A^{\text{exp}}/\text{ppb}</math></b>	<b>0.44</b> <b>(1.5 %)</b>	0.38 (1.34 %)	0.14 (0.49 %)	0.10 (0.35 %)	0.11 (0.38 %)	0.09 (0.32 %)



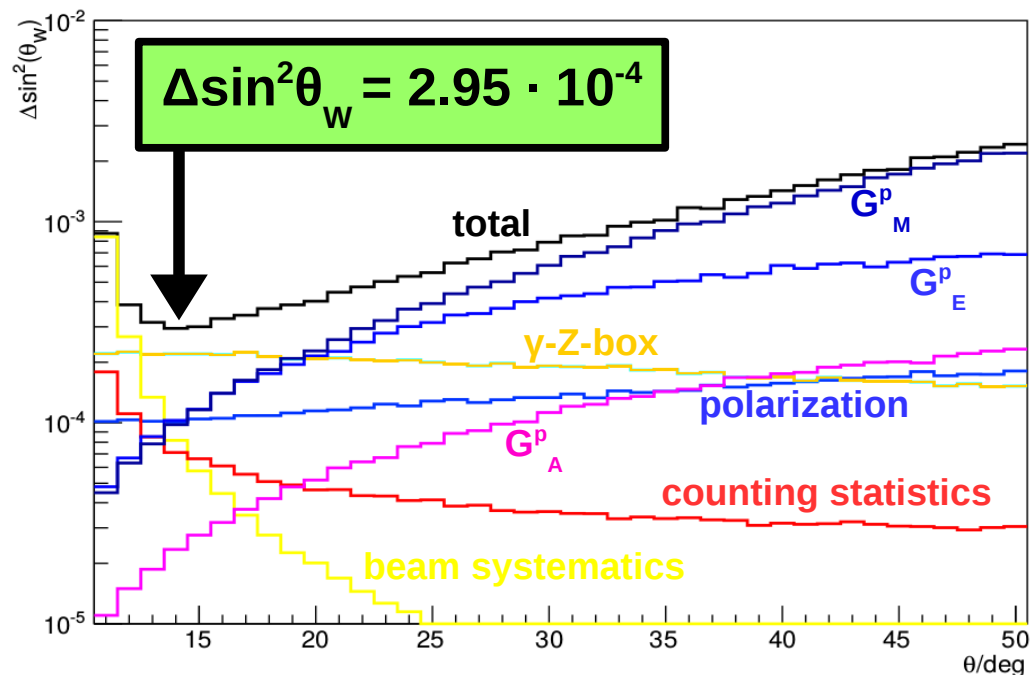
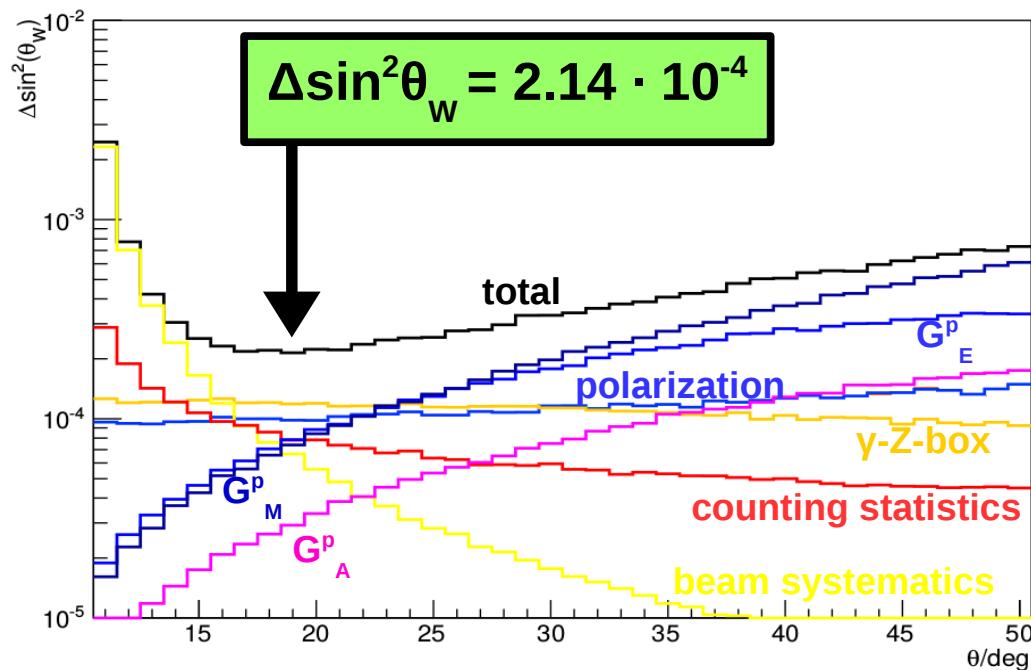
# Achievable precision @ higher energies/beam current

Beam current:  
Polarization:  
Target material:  
Target:  
Measurement time:  
Detector acceptance:  
 $\Delta A^{\text{app}}$ :

1 mA  
85 %  $\pm$  0.425 %  
liquid hydrogen  
60 cm  
10000 h  
 $2\pi \cdot 20^\circ$   
0.1 ppb

Beam energy: 300 MeV  
Central scattering angle:  $19^\circ$   
 $A^{\text{PV}} = (-30.8 \pm 0.34)$  ppb  
 $\langle Q^2 \rangle = 4.84e-3 \text{ GeV}^2/c^2$   
Rate elastic e-p: 1.8 THz

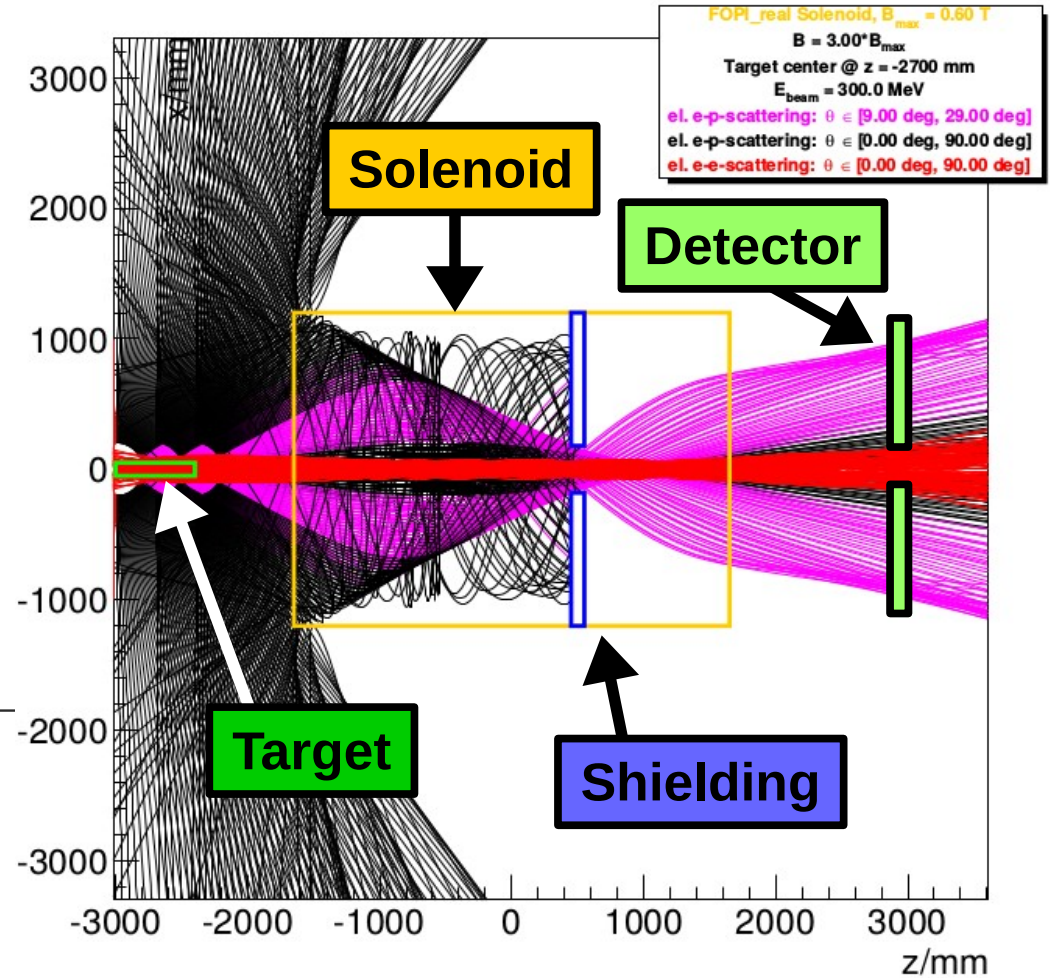
Beam energy: 500 MeV  
Central scattering angle:  $14^\circ$   
 $A^{\text{PV}} = (-24.8 \pm 0.36)$  ppb  
 $\langle Q^2 \rangle = 3.82e-3 \text{ GeV}^2/c^2$   
Rate elastic e-p: 3.6 THz



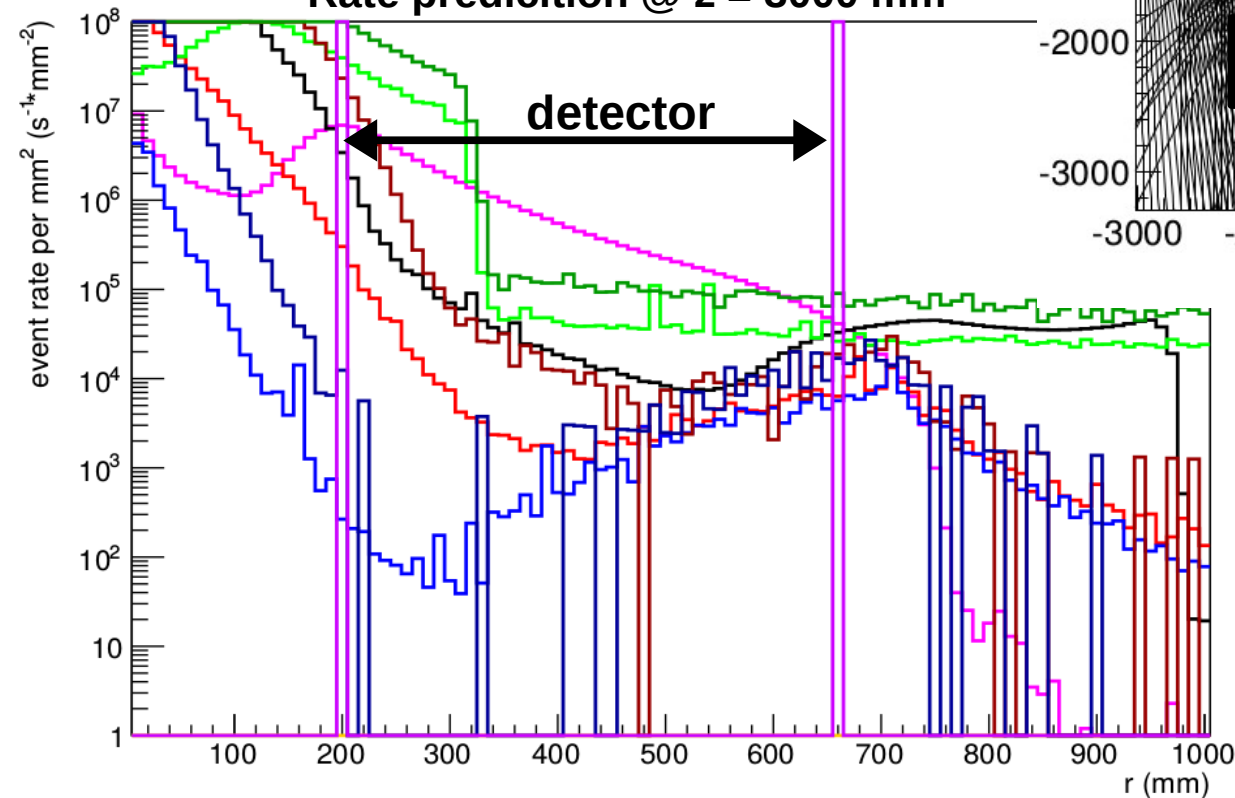
# A very first idea for 300 MeV

Beam energy: 300 MeV  
 Beam current : 150  $\mu$ A  
 Central magnetic field: 1.8 Tesla

Moller,  $\theta \in [0^\circ, 90^\circ]$   
 Elastic e-p,  $\theta \in [9^\circ, 29^\circ]$   
 Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$



Rate prediction @  $z = 3000$  mm

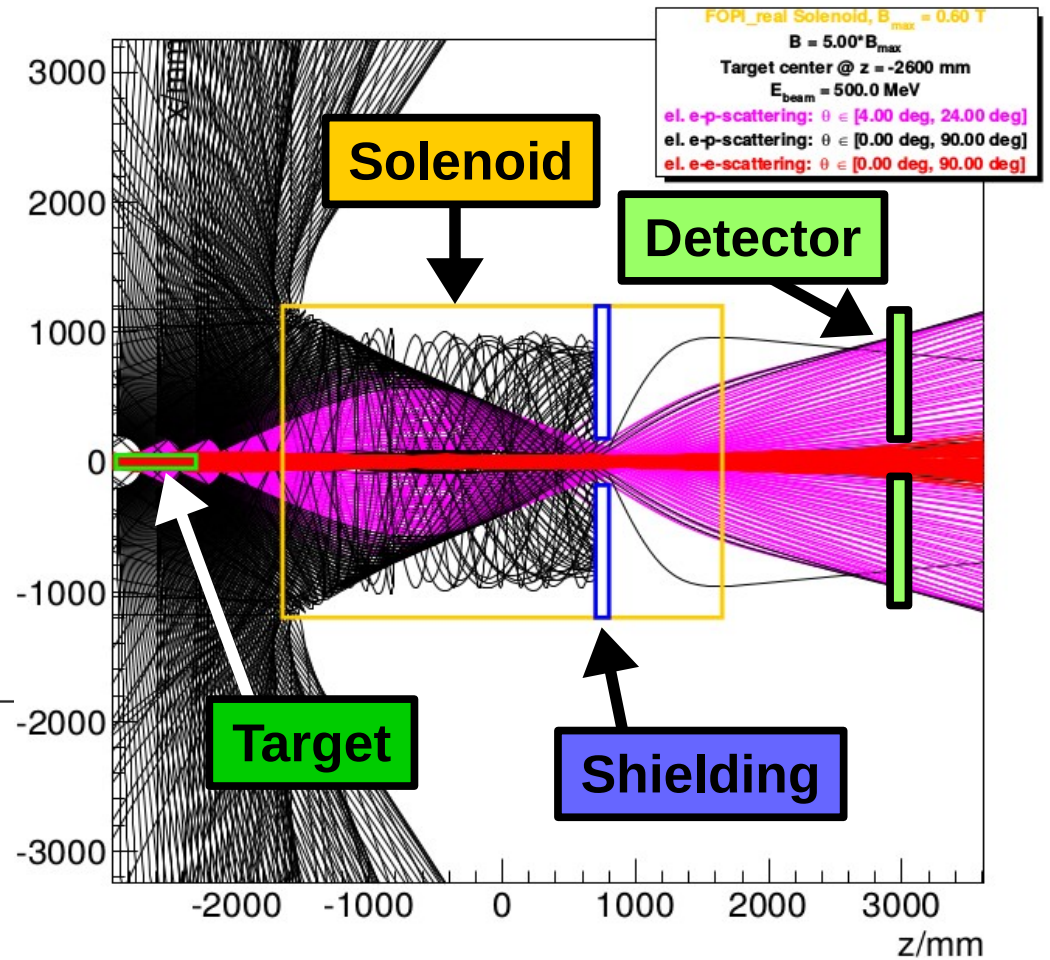


Elastic e-p,  $\theta$  in  $[9^\circ, 29^\circ]$   
 Elastic e-p,  $\theta$  not in  $[9^\circ, 29^\circ]$   
 Moller, e-p  
 Moller, background  
 Positrons, e-p  
 Positrons, background  
 Photons, e-p  
 Photons, background

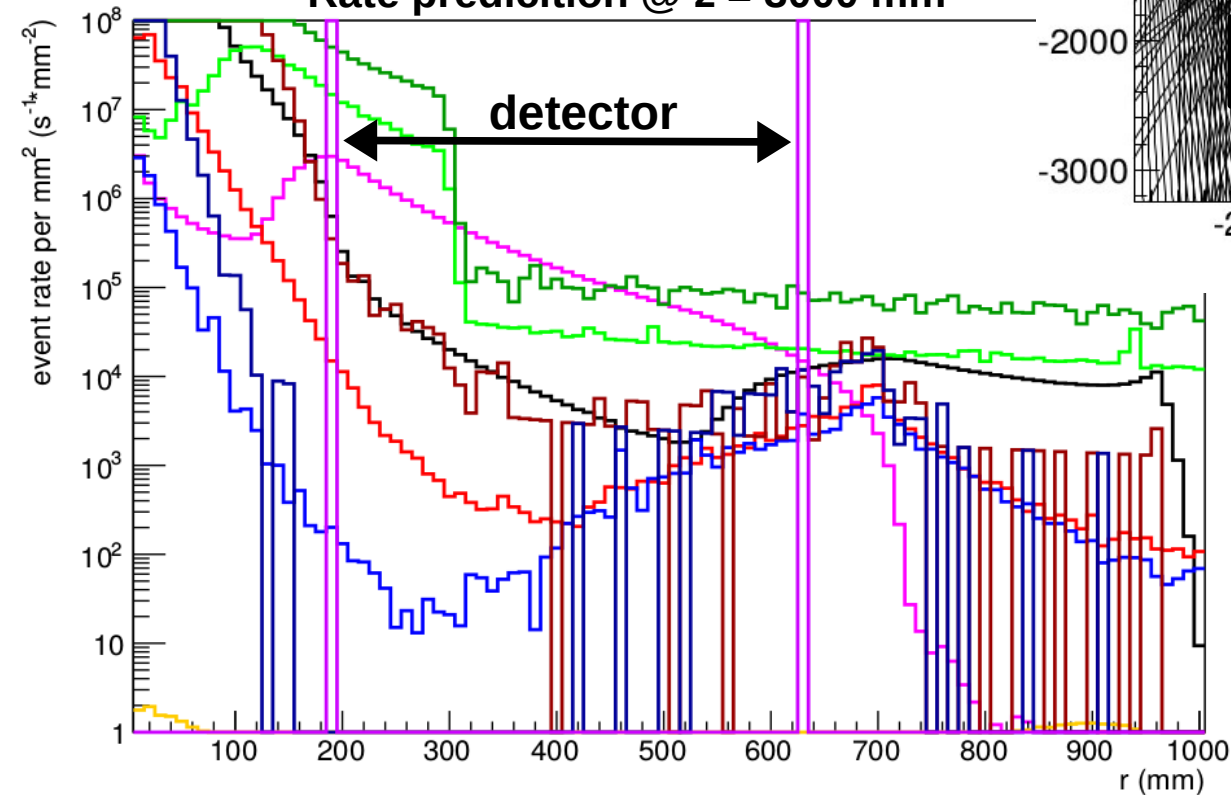
# A very first idea for 500 MeV

**Beam energy: 500 MeV**  
**Beam current: 150  $\mu$ A**  
**Central magnetic field: 3 Tesla**

**Moller,  $\theta \in [0^\circ, 90^\circ]$**   
**Elastic e-p,  $\theta \in [4^\circ, 24^\circ]$**   
**Elastic e-p,  $\theta \in [0^\circ, 90^\circ]$**



Rate prediction @ z = 3000 mm



**Elastic e-p,  $\theta$  in  $[4^\circ, 24^\circ]$**   
**Elastic e-p,  $\theta$  not in  $[4^\circ, 24^\circ]$**   
**Moller, e-p**  
**Moller, background**  
**Positrons, e-p**  
**Positrons, background**  
**Photons, e-p**  
**Photons, background**

# Summary

- **Project P2 @ MESA:**

A new measurement of the weak mixing angle with precision goal:

$$\Delta Q_w(p) = 1.9 \%$$

$$\Delta \sin^2 \theta_w = 0.15 \%$$

- **P2 main detector concept study:**

Solenoid spectrometer and  $2\pi$ -Cherenkov-detector

→  $\Delta \sin^2 \theta_w = 0.13 \%$

- **Measurement at higher beam energies and beam current:**

→ Very high precision in  $\sin^2 \theta_w$  at small scattering angles for 300 MeV

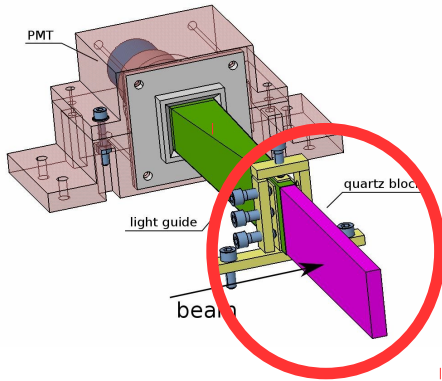
→ Most important contributions from gamma-Z-box and form factors

→ Experiment may be difficult to perform with a solenoid because of small scattering angles

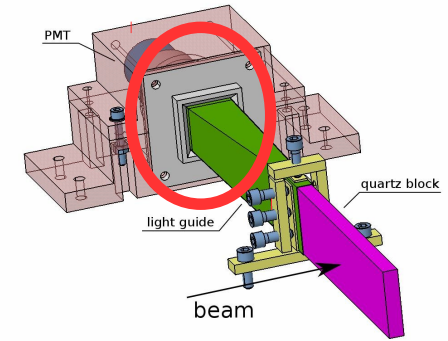
→ Toroid may be better choice due to lower dependence on counting statistics

**BACKUP SLIDES**

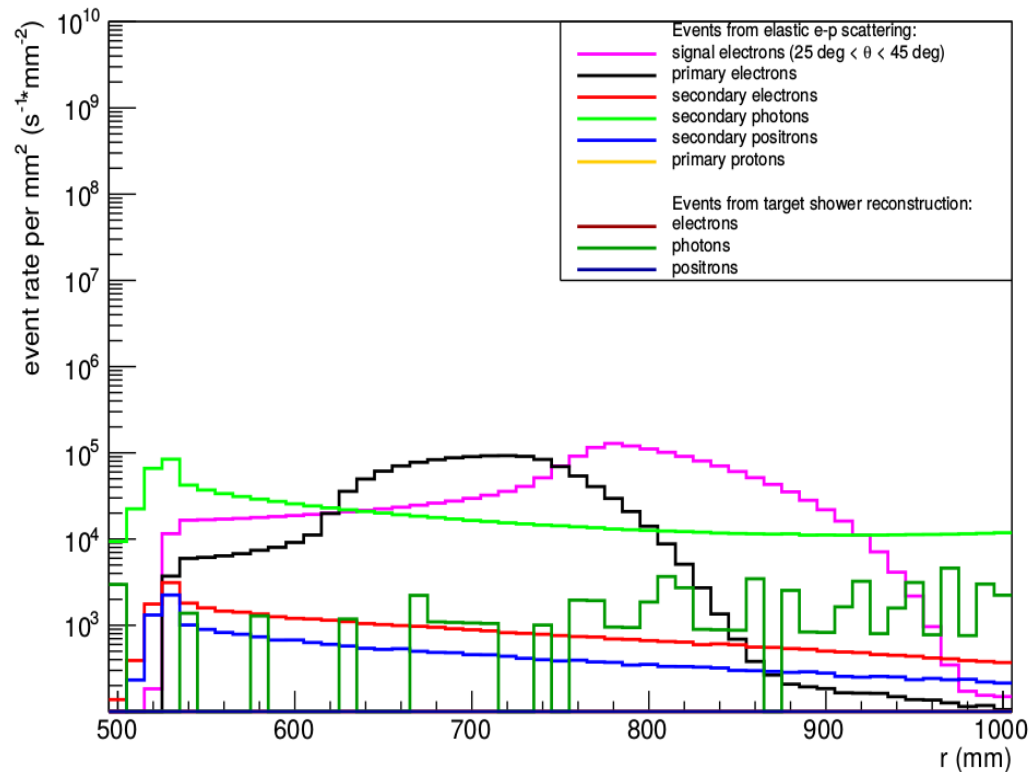
# Include simulated response of detector modules



Use results of detector module simulation to transform event rates into photo electron rates:



## Event rate distribution:

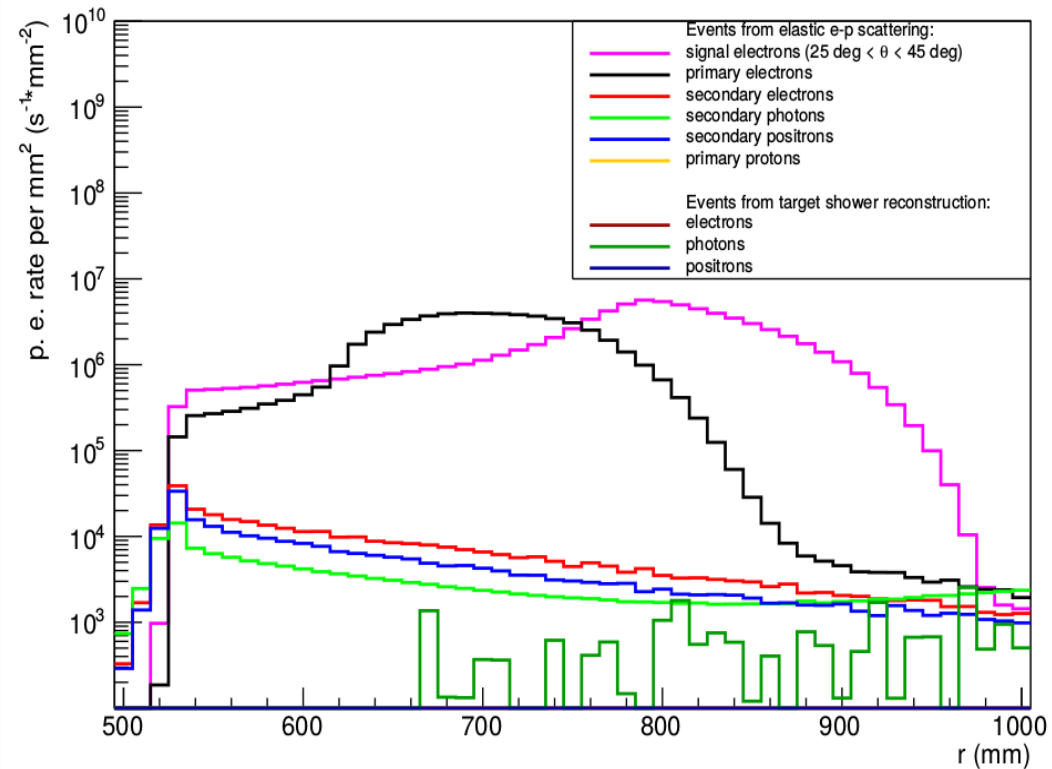


### Monte Carlo results:

$$R_{total}^{ep} = 0.19 \text{ THz}$$

$$\langle A^{PV} \rangle_{L, \Delta\Omega} = -39.8 \text{ ppb}$$

## Photo electron rate distribution:

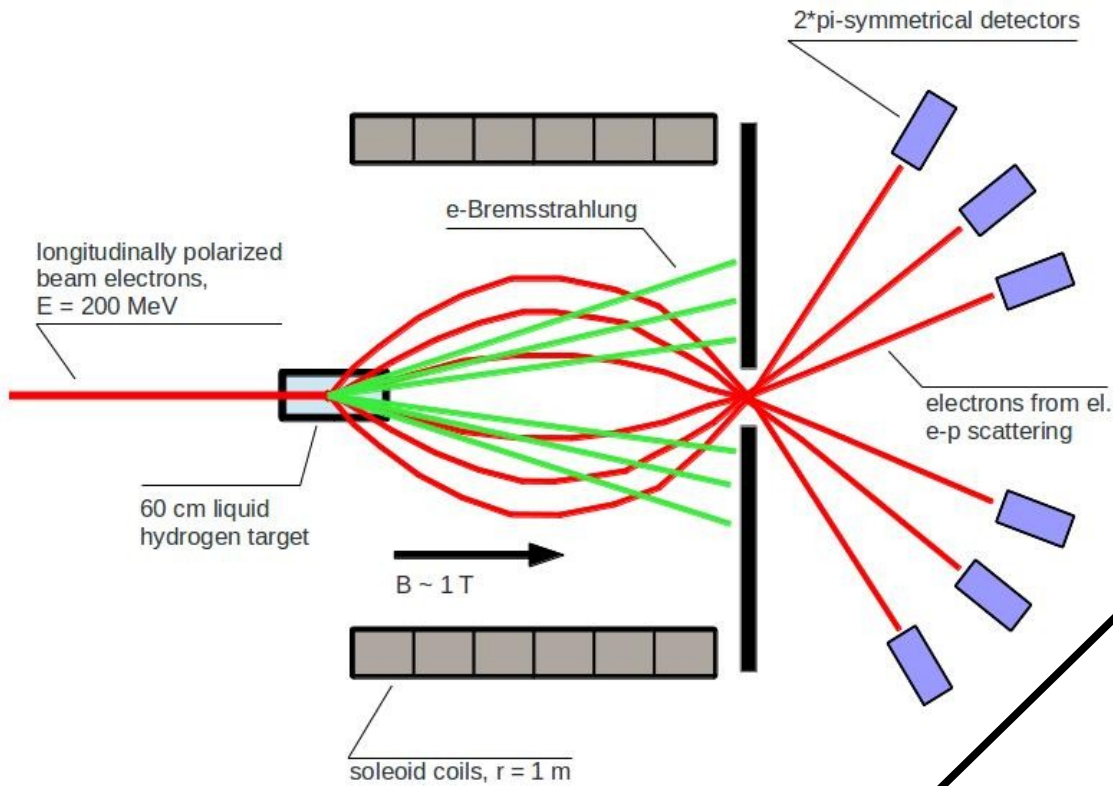


### Monte Carlo results:

$$I_{total}^{cathode} = 1 \mu\text{A}$$

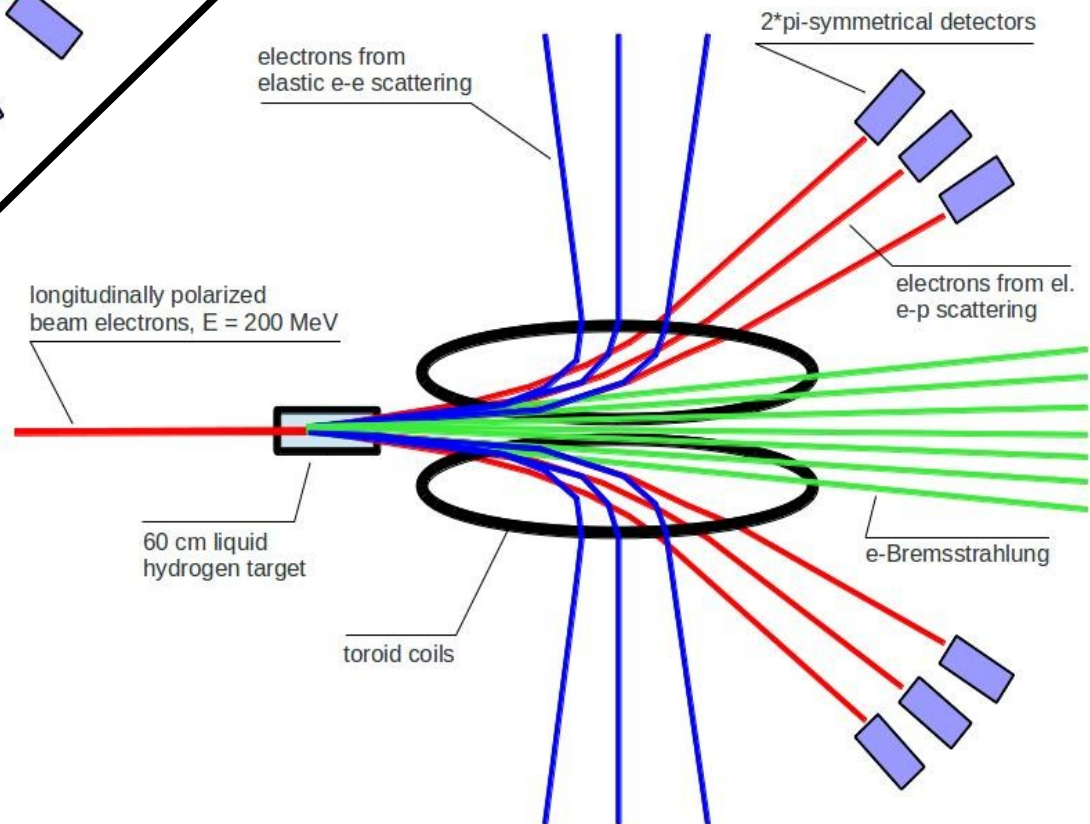
$$\langle A^{PV} \rangle_{L, \Delta\Omega} = -33.5 \text{ ppb}$$

# Weapon of choice: Solenoid or Toroid?



## Solenoid:

- Full azimuthal coverage
- Compact setup
- Superconducting coils



## Toroid:

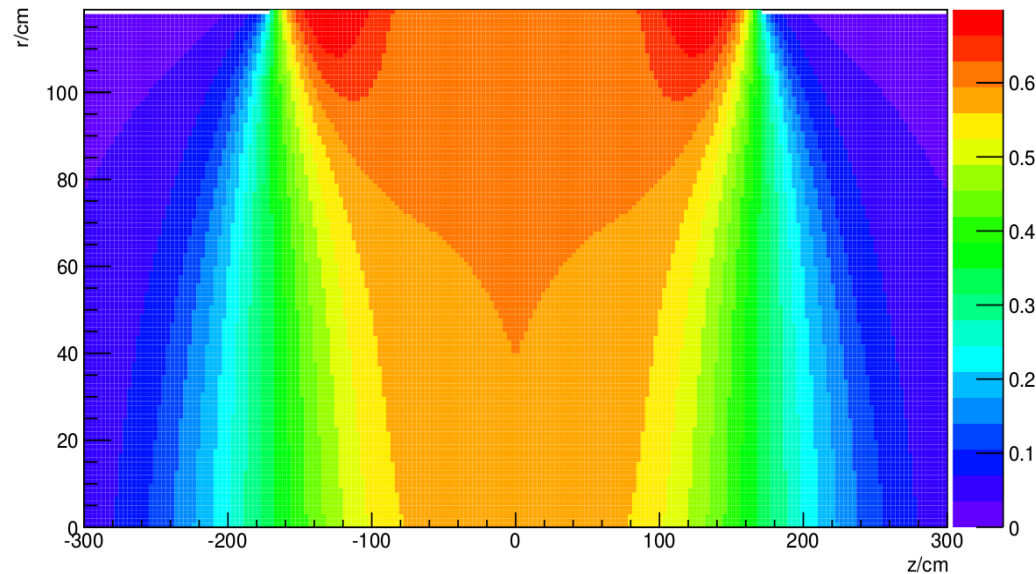
- Loss of  $\sim 50\%$  azimuth  $\rightarrow$  double measurement time
- Larger setup
- Copper coils

# We would like to use a superconducting solenoid...

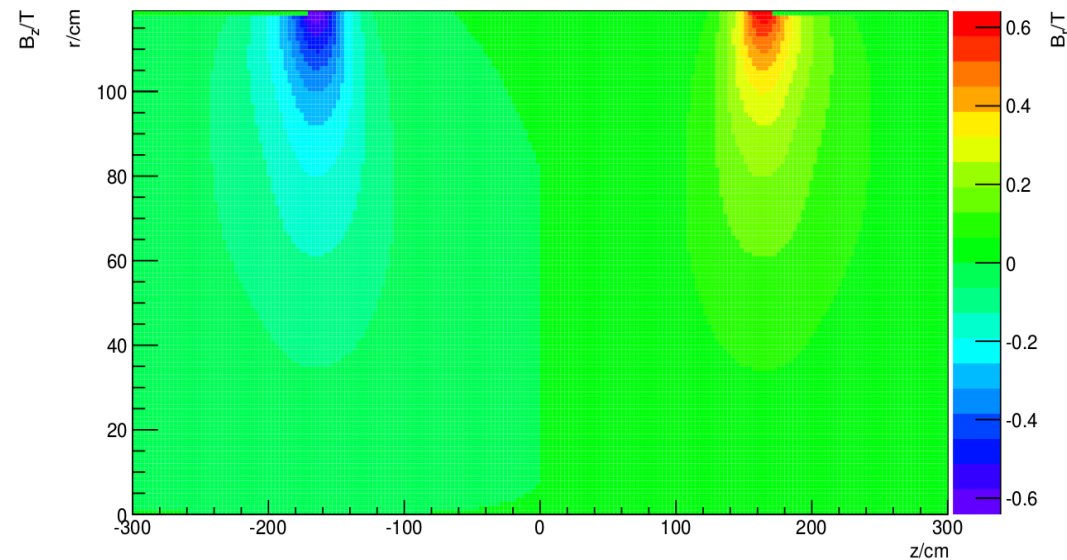
## A promising candidate: The FOPI solenoid (GSI, Darmstadt)

- Field strength: 0.6 T
- Coil current: 725 A
- Stored energy: 3.4 MJ
- Material: Cu/Nb-Ti
- Cable length: 22.5 km
- Inner diameter: 2.4 m
- Total length: 3.8 m
- Total weight: 108.7 tons
- I-He consumption: 0.02 g/s, 0.6 l/h
- I-N consumption: 3 g/s, 13 l/h (perm. cooling)

z-component of FOPI fieldmap



r-component of FOPI fieldmap



(Courtesy Y. Leifels)

**Use fieldmap with Geant4 to simulate the P2 experiment**



$$A^{\text{exp}} \sim \sin^2(\theta_W)$$

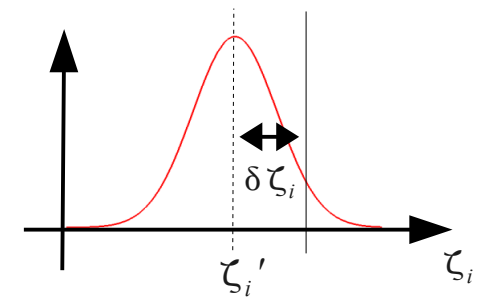
$$\longrightarrow \sin^2(\theta_W) = Z(A^{\text{exp}}, A^{\text{app}}, E, P, L, \Delta\Omega, \text{Re}(\square_{yz}), \{f_i\})$$

↑  
 Set of form factor  
 fit parameters

**Monte Carlo approach:**

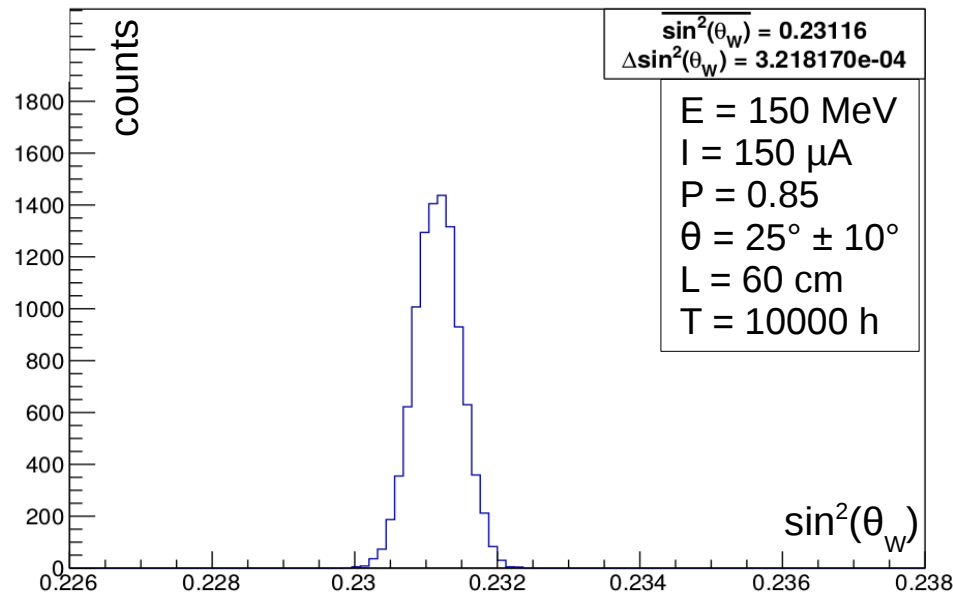
Sample distribution for  $\sin^2(\theta_W)$  by assigning Gaussian distributions to each parameter  $\zeta_i \in \{A^{\text{exp}}, A^{\text{app}}, E, P, L, \Delta\Omega, \{f_i\}\}$ .

$$\longrightarrow \sin^2(\theta_W) + \delta \sin^2(\theta_W) = Z(\zeta_i' + \delta \zeta_i)$$



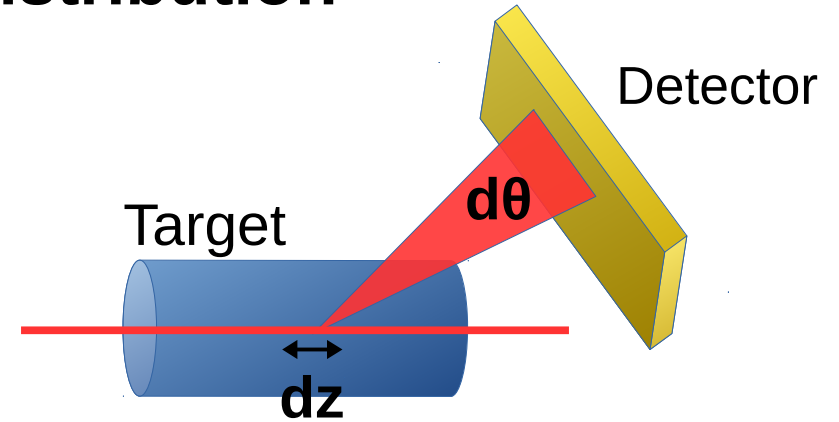
N sampling-iterations  
 yield  $\sin^2(\theta_W)$ -distribution.

Extract  $\Delta \sin^2(\theta_W)$  as  
 standard deviation.



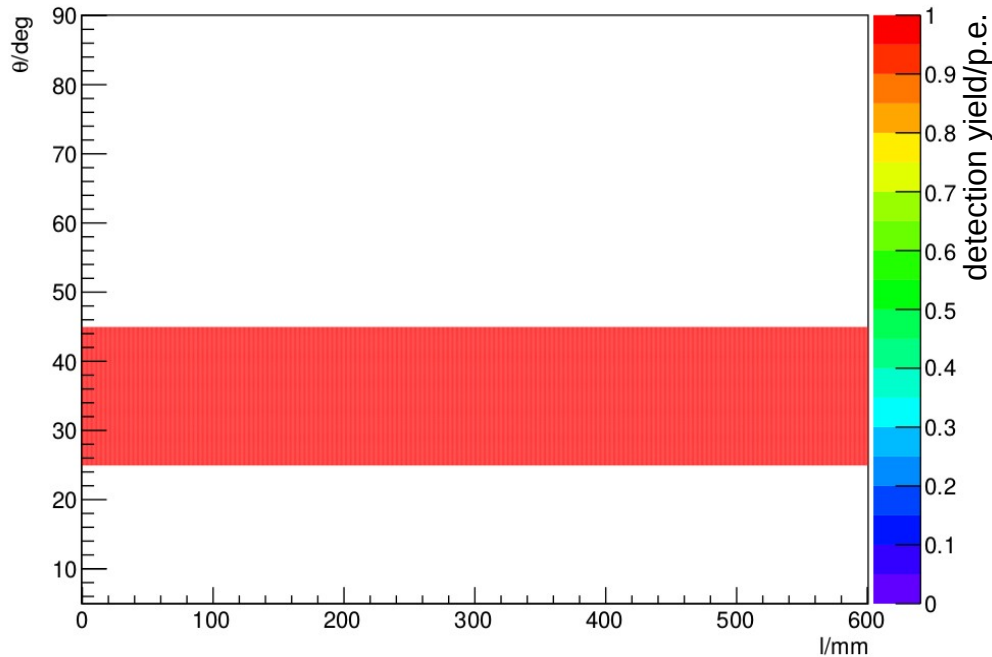
# Choice of new weighting function: Detection yield distribution

$$\langle A^{PV} \rangle_{L, \Delta\Omega} = \frac{\int_0^L dz \int_{\Delta\Omega} d\Omega \left[ \left( \frac{d\sigma}{d\Omega} \right)^{Ros} \cdot \epsilon \cdot A^{PV} \right]}{\int_0^L dz \int_{\Delta\Omega} d\Omega \left[ \left( \frac{d\sigma}{d\Omega} \right)^{Ros} \cdot \epsilon \right]}$$

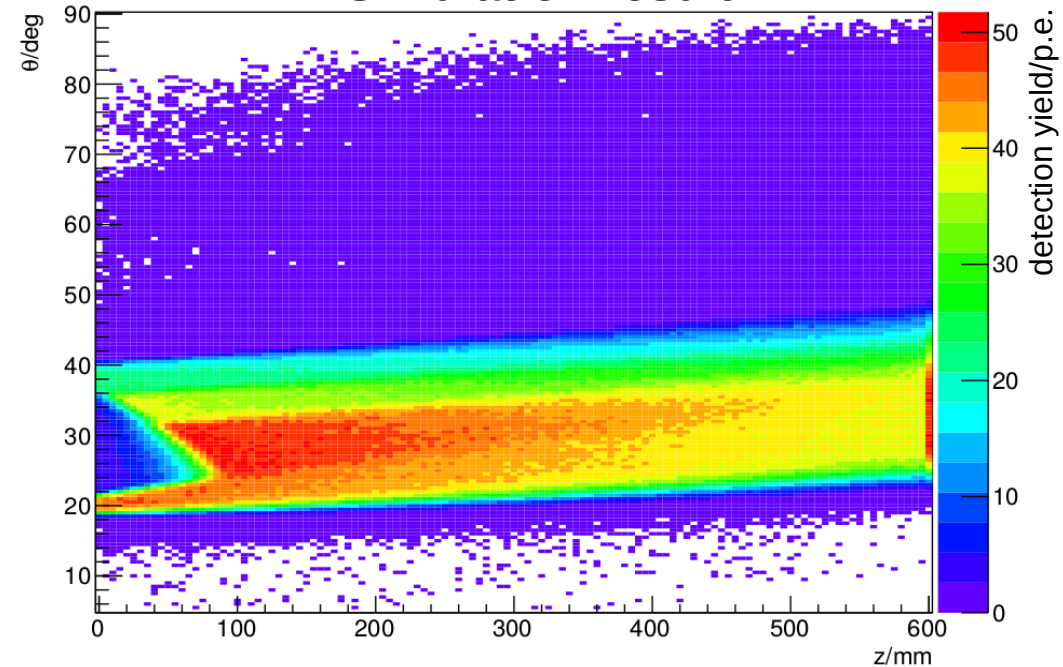


$\epsilon(z, \theta) \equiv \frac{\text{Rate of photo electrons in detector, produced in target at position } z \text{ with angle } \theta}{\text{Event rate according to Rosenbluth formula, produced in target at position } z \text{ with angle } \theta}$

**Ideal case**



**Simulation result**



# What is the number of detected e-p events?

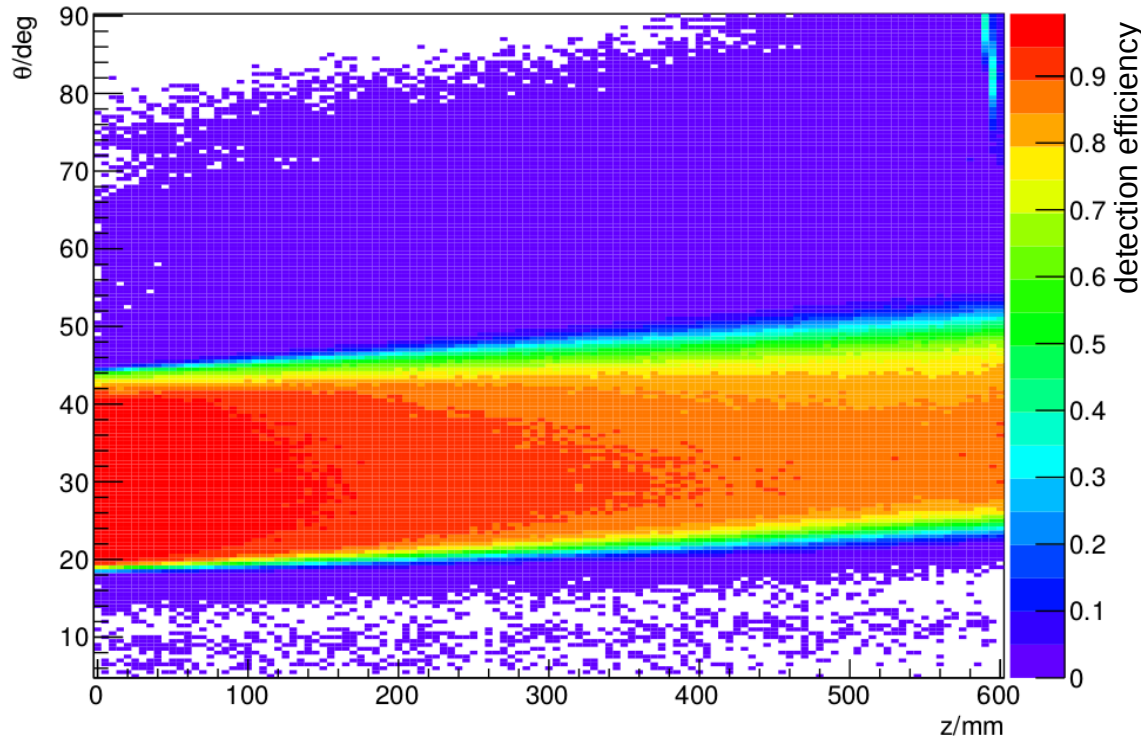
To determine  $\Delta \sin^2(\theta_w)$ , we sample the mapping:

$$\sin^2(\theta_w) = Z(A^{\text{exp}}, A^{\text{app}}, E, P, L, \Delta\Omega, \text{Re}(\square_{yz}), \{f_i\})$$

with

$$\Delta A^{\text{exp}} \approx 1/\sqrt{N} \quad \text{and} \quad N: \text{Total number of detected e-p events}$$

$$N = \Phi \cdot \rho \cdot T \cdot \left\langle \frac{d\sigma}{d\Omega} \right\rangle_{L, \Delta\Omega} \cdot \Delta L_{\text{eff}} \cdot \Delta \Omega_{\text{eff}} \quad \text{with} \quad \left\langle \frac{d\sigma}{d\Omega} \right\rangle_{L, \Delta\Omega} = \frac{\int_0^L dz \int_{\Delta\Omega} d\Omega \left[ \left( \frac{d\sigma}{d\Omega} \right)^{\text{Ros}} \cdot \epsilon \right]}{\int_0^L dz \int_{\Delta\Omega} d\Omega [\epsilon]}$$

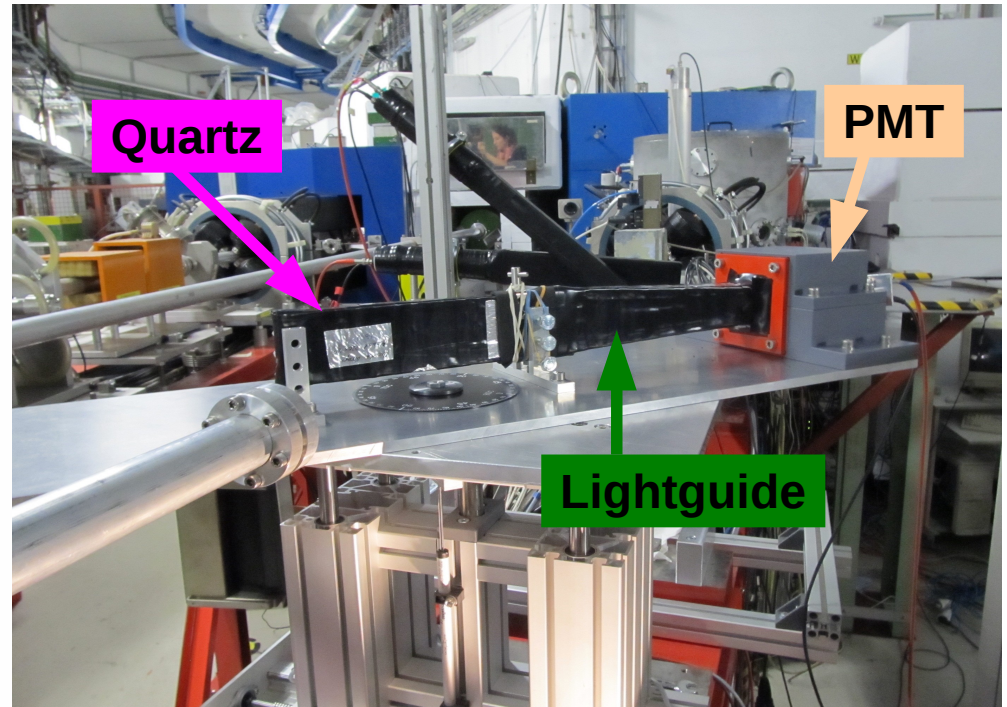
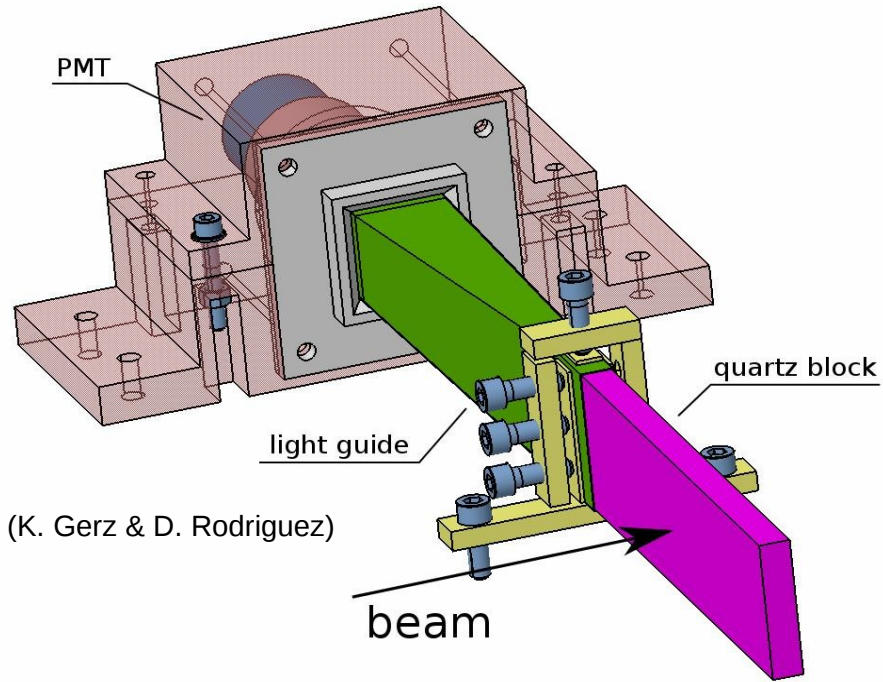


$$\Delta L_{\text{eff}} \cdot \Delta \Omega_{\text{eff}} = \int_0^L dz \int_{\Delta\Omega} d\Omega [y(z, \theta)]$$

$y(z, \theta)$ :

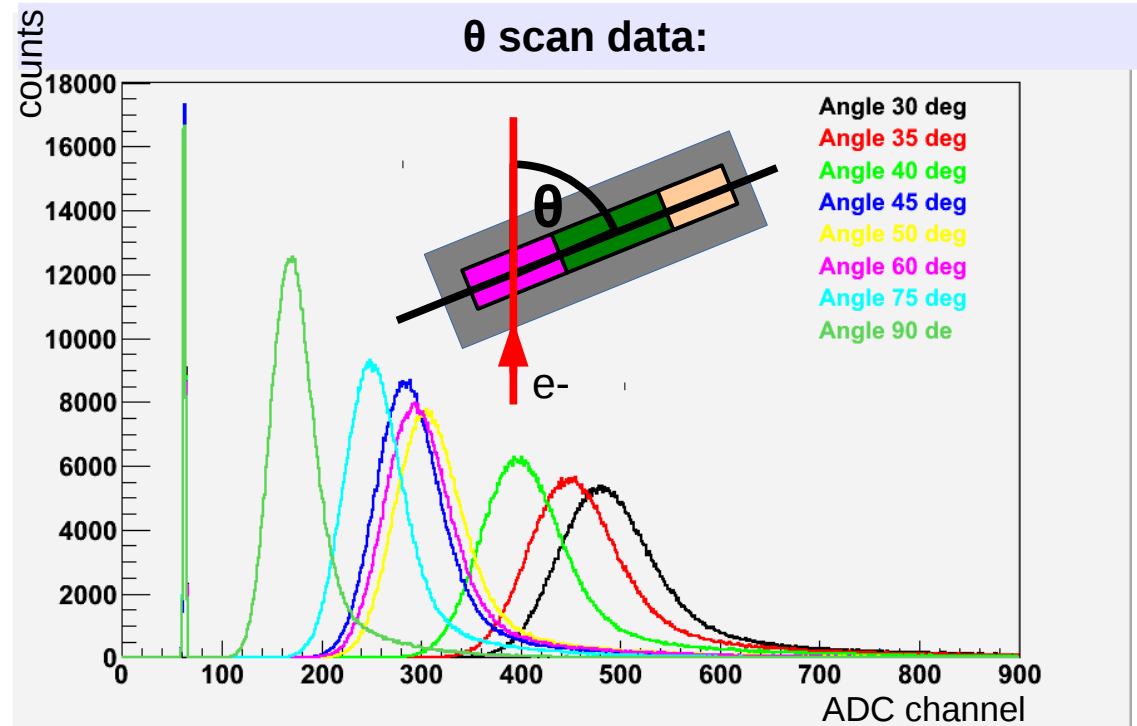
**Detection efficiency distribution**  
Probability for an elastic e-p event with  $(z, \theta)$  to produce a signal in the virtual detector

# Prototype tests @ MAMI

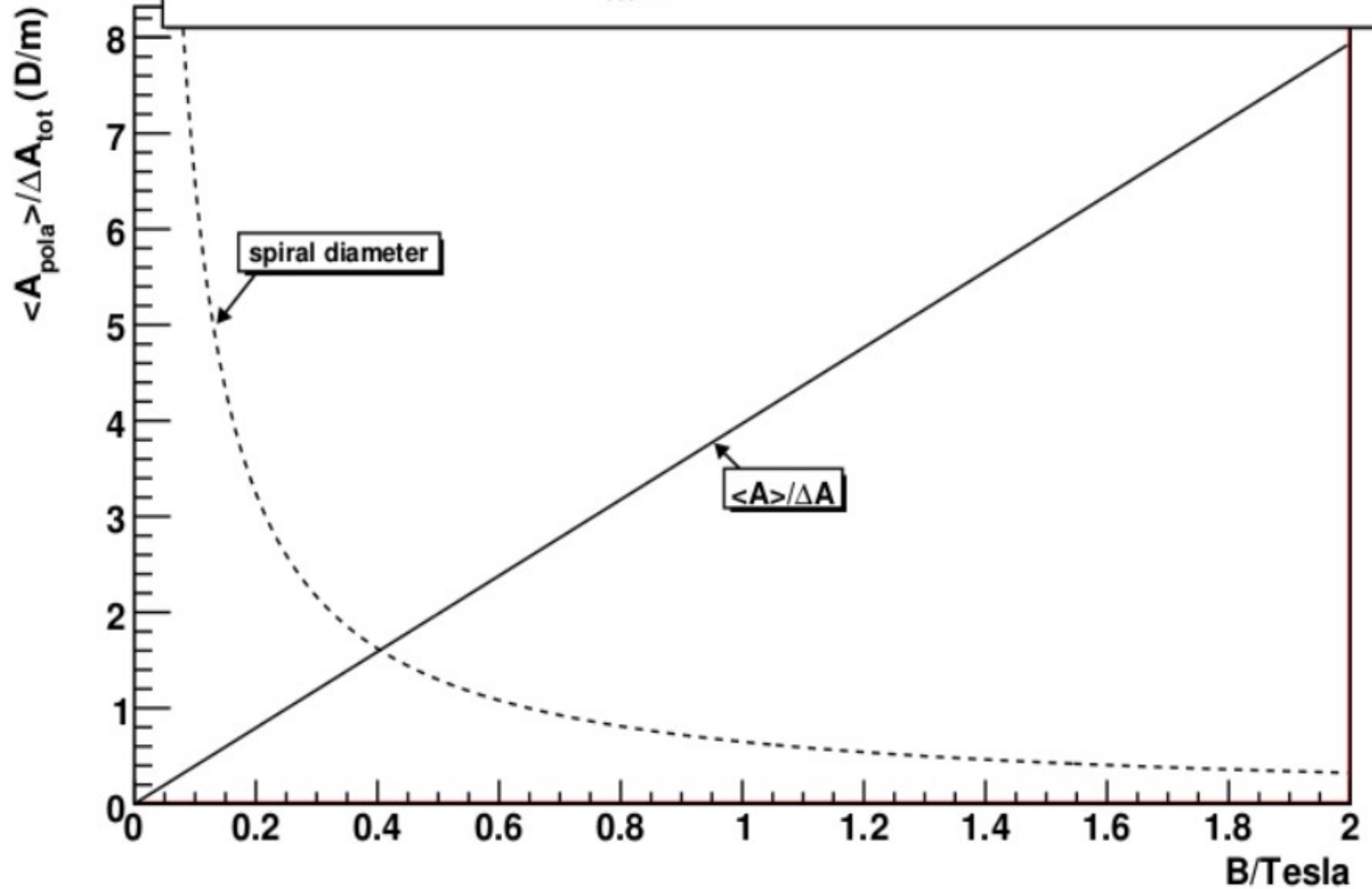


Measured the yield of photo electrons for different

- materials (quartzes, wrappings, lightguides, PMTs)
- geometries
- impact positions
- angles of incidence



$\Delta A_{\text{tot}}=0.25$  ppb,  $T=20$  K,  $E_{\text{beam}}=200.0$  MeV,  $P=85.0$  %,  $\theta_{\text{mean}}=20$  deg,  $\Delta\theta=20$  deg

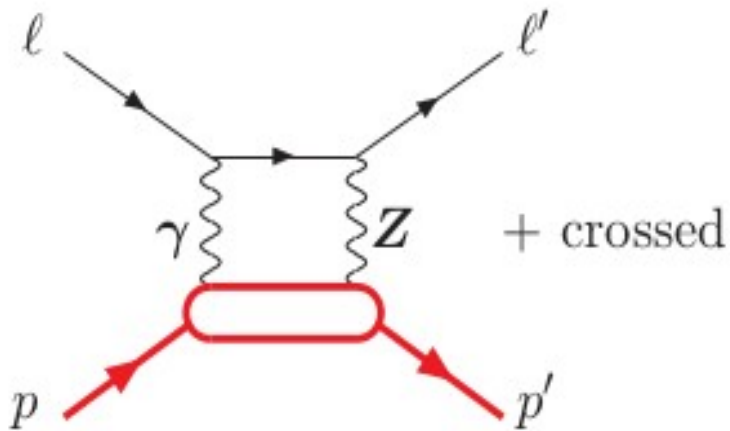


# Low $Q^2$ ?

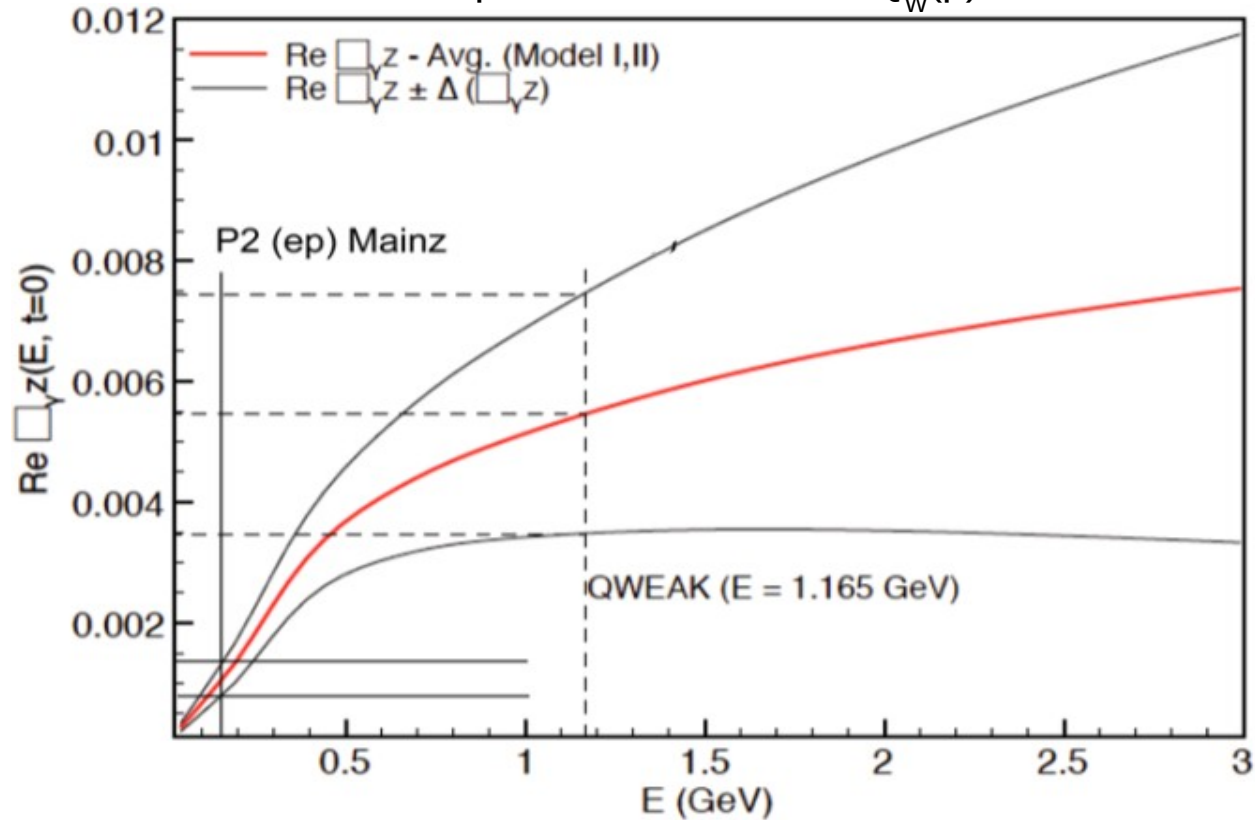
$A^{PV}$  is dominated by  $Q_W(p)$  at low values of  $Q^2$ .

$$Q^2 = 4EE' \sin^2(\theta_{lab}/2)$$

Low  $Q^2$ : Low beam energy and large angle or vice versa?



$\gamma$ -Z-box correction to  $Q_W(p)$



Gorchtein, Horowitz, Ramsey-Musolf 1102.3910 [nucl-th]

**At low beam energies: Uncertainty of  $\gamma$ -Z-box contribution to  $\sin^2(\theta_w)$  is negligible.**