Monte Carlo simulations of a solenoid spectrometer for Project P2

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Institute for Nuclear Physics, JGU Mainz
- Project P2 @ MESA:
  A new high precision determination of the electroweak mixing angle at low momentum transfer

- P2 main detector concept:
  Monte Carlo simulations of a solenoid spectrometer

- Monte Carlo simulations regarding a precision measurement of the weak mixing angle at higher beam energies and beam current
Project P2 @ MESA:

- New high precision determination of the proton weak charge $Q_W(p)$ at low $Q^2 \sim 6 \cdot 10^{-3}$ GeV$^2$/c$^2$
- Precision goal: $\Delta Q_W(p) = 1.9 \%$
  $\Delta \sin^2 \theta_W = 0.15 \%$
- Measurement of $Q_W(p)$ through parity violation in elastic e-p scattering
Access to the weak mixing angle

\[ h = \frac{\vec{s} \cdot \vec{p}}{|\vec{s} \cdot \vec{p}|} = \pm 1 \]

Parity violating asymmetry in elastic e-p scattering:

\[ A^{PV} = -\frac{G_F Q^2}{4 \sqrt{2} \pi \alpha} \left[ Q_w(p) - F(Q^2) \right] \]

- \( Q_w(p) \): Proton weak charge, \( Q_w(p) = 1 - 4 \cdot \sin^2(\theta_w) \) (tree level)
- \( F(Q^2) \): Nucleon structure contribution, small at low \( Q^2 \)

Parity violating asymmetry, averaged over solid angle

- Beam energy = 150 MeV
- Detector acceptance = 20 deg

\[ A^{PV} \sim \sin^2 \theta_w \]
Prediction of achievable precision and choice of kinematics

- Monte Carlo approach to error propagation calculation

- Assumption of back angle measurement of axial and strange magnetic form factor in P2
  → Reduction of form factor uncertainty by factor 4

- $A^{PV} = -39.80 \text{ ppb} 
  \pm 0.54 \text{ ppb (stat.)} 
  \pm 0.34 \text{ ppb (other)}$

\[
\Delta \sin^2 \theta_w = 3.2 \cdot 10^{-4}
\]

Form factor parametrizations: P. Larin and S. Baunack
\textit{y-Z-box according to}: Gorchtein, Horowitz, Ramsey-Musolf 1102.3910 [nucl-th]

Beam energy: 150 MeV
Central scattering angle: 35 deg
Detector acceptance: 20 deg
The new M.E.S.A. facility in Mainz

**Mainz Energy recovering Superconducting Accelerator:**

- Normal-conducting injector LINAC
- Superconducting cavities in re-circulations

**Energy recovering mode:** Unpolarized beam, 10 mA, 100 MeV, pseudo-internal gas-target, \( L \sim 10^{35} \text{cm}^{-2} \text{s}^{-1} \)

**External beam mode:** \( P = 85\% \pm 0.5\% \), 150 µA, 155 MeV, \( L \sim 10^{39} \text{cm}^{-2} \text{s}^{-1} \), \( \langle \Delta A_{\text{app}} \rangle / \Delta t = 0.1 \text{ ppb} \)
Experimental setup under investigation

- Superconducting solenoid
- Lead shielding
- PMTs
- Quartz bars (Cherenkov)
- Unauthorized hall access

- e- beam, 150 MeV
- B = 0.6 T
- 60 cm liquid hydrogen target

(CAD-drawing by D. Rodriguez)
Raytrace simulations in the magnetic field

Beam energy = 155 MeV
Moller, $\theta \in [0^\circ, 90^\circ]$
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$

Magnetic field: OFF
Raytrace simulations in the magnetic field

Magnetic field: 0.06 T

Beam energy = 155 MeV
Moller, $\theta \in [0°, 90°]$
Elastic $e-p$, $\theta \in [25°, 45°]$
Elastic $e-p$, $\theta \in [0°, 90°]$
Raytrace simulations in the magnetic field

Magnetic field:
0.12 T

Beam energy = 155 MeV
Moller, $\theta \in [0^\circ, 90^\circ]$  
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$  
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$
Raytrace simulations in the magnetic field

Magnetic field: 0.18 T

Beam energy = 155 MeV
Moller, \( \theta \in [0^\circ, 90^\circ] \)
Elastic e-p, \( \theta \in [25^\circ, 45^\circ] \)
Elastic e-p, \( \theta \in [0^\circ, 90^\circ] \)
Raytrace simulations in the magnetic field

Beam energy = 155 MeV

Moller, θ ∈ [0°, 90°]

Elastic e-p, θ ∈ [25°, 45°]

Magnetic field: 0.24 T
Raytrace simulations in the magnetic field

Magnetic field: 0.3 T

Beam energy = 155 MeV
Moller, $\theta \in [0^\circ, 90^\circ]$
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$
Raytrace simulations in the magnetic field

Beam energy = 155 MeV
Moller, $\theta \in [0^\circ, 90^\circ]$
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$

Magnetic field: 0.36 T

Target center @ $z = -700$ mm
$E_{\text{beam}} = 155.0$ MeV
el. e-p-scattering: $\theta \in [25.00 \text{ deg}, 45.00 \text{ deg}]$
el. e-p-scattering: $\theta \in [0.00 \text{ deg}, 90.00 \text{ deg}]$
el. e-e-scattering: $\theta \in [0.00 \text{ deg}, 90.00 \text{ deg}]$
Raytrace simulations in the magnetic field

Magnetic field: 0.42 T

Beam energy = 155 MeV
Møller, $\theta \in [0^\circ, 90^\circ]$
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$

Beam energy = 155 MeV
Møller, $\theta \in [0^\circ, 90^\circ]$
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$

FOPI_real Solenoid, $B_{\text{max}} = 0.60$ T
$B = 0.70 B_{\text{max}}$
Target center @ $z = -700$ mm
$E_{\text{beam}} = 155.0$ MeV
el. e-p-scattering: $\theta \in [25.00 \text{ deg}, 45.00 \text{ deg}]$
el. e-p-scattering: $\theta \in [0.00 \text{ deg}, 90.00 \text{ deg}]$
el. e-e-scattering: $\theta \in [0.00 \text{ deg}, 90.00 \text{ deg}]$
Raytrace simulations in the magnetic field

Beam energy = 155 MeV
Moller, \( \theta \in [0^\circ, 90^\circ] \)
Elastic e-p, \( \theta \in [25^\circ, 45^\circ] \)
Elastic e-p, \( \theta \in [0^\circ, 90^\circ] \)

Magnetic field: 0.48 T
Beam energy = 155 MeV

Moller, $\theta \in [0^\circ, 90^\circ]$  
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$  
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$  

Magnetic field: 0.54 T  

Raytrace simulations in the magnetic field
Raytrace simulations in the magnetic field

Magnetic field: 0.6 T

Beam energy = 155 MeV
Moller, $\theta \in [0^\circ, 90^\circ]$
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$
Beam energy = 155 MeV
Möller, $\theta \in [0^\circ, 90^\circ]$
Elastic e-p, $\theta \in [25^\circ, 45^\circ]$
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$

Magnetic field:
0.6 T

Raytrace simulations in the magnetic field
Raytrace simulations in the magnetic field

Beam energy = 155 MeV
Moller, $\theta \in [0°, 90°]$
Elastic e-p, $\theta \in [25°, 45°]$
Elastic e-p, $\theta \in [0°, 90°]$

Magnetic field:
0.6 T

Quartz
Shielding
Geant4 Simulation of beam-target-interaction

Radial projection of spatial vertex distribution

Energy deposition in target volume

- Coherent simulation of elastic e-p scattering for P2 is impossible with Geant4

- Sample initial state distribution for elastic e-p scattering → To be used with event generator

- Use tree-level event generator for primary event-generation

- Prototype of event generator with radiative corrections available and currently under evaluation
Geant4 Simulation of detector module response

Tracking of optical photons in detector module

Create parametrization of photo electron yield for different
- Active materials
- Geometries
- Particle types
- Particle energies
- Impact angles

Photo electron yield distribution, $E = 155$ MeV

(K. Gerz)
Geant4 Simulation of experimental setup

- Use initial state distribution with tree level event generator to simulate elastic e-p scattering
- Tracking in realistic map of magnetic field, CAD-interface for definition of geometry
- Use parametrization of detector response to predict distribution of photo electrons
- Use $Q^2$ distribution in error propagation calculation to predict the achievable precision in the weak mixing angle

Photo electron rate distribution
- e-p, $\theta$ not in $[25^\circ, 45^\circ]$  
- e-p, $\theta$ in $[25^\circ, 45^\circ]$  

$$A^{PV} = \frac{-G_F Q^2}{4 \sqrt{2\pi} \alpha} \left[ Q_W(p) - F(Q^2) \right]$$

$E = 155$ MeV, $I = 150$ µA
The following results are based on error propagation calculations including the results of the Geant4 simulation of the experimental setup:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>155 MeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam current</td>
<td>150 µA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polarization</td>
<td>85 %</td>
<td>± 0.425 %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target</td>
<td>60 cm liquid hydrogen</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detector acceptance</td>
<td>2π·20°</td>
<td>θ ∈ [25°, 45°]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detector rate</td>
<td>0.5 THz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement time</td>
<td>1e4 h</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∥Q²∥</td>
<td>4.49e-3 GeV²/c²</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A^exp</td>
<td>-28.35 ppb</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Statistics</th>
<th>Polarization</th>
<th>Apparative</th>
<th>Form factors</th>
<th>Re(□YZA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆sin²(θ_w)</td>
<td>3.1e-4</td>
<td>2.6e-4</td>
<td>9.7e-5</td>
<td>7.0e-5</td>
<td>1.4e-4</td>
<td>6e-5</td>
</tr>
<tr>
<td></td>
<td>(0.13 %)</td>
<td>(0.11 %)</td>
<td>(0.04 %)</td>
<td>(0.03 %)</td>
<td>(0.04 %)</td>
<td>(0.03 %)</td>
</tr>
<tr>
<td>∆A^exp/ppb</td>
<td>0.44</td>
<td>0.38</td>
<td>0.14</td>
<td>0.10</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(1.5 %)</td>
<td>(1.34 %)</td>
<td>(0.49 %)</td>
<td>(0.35 %)</td>
<td>(0.38 %)</td>
<td>(0.32 %)</td>
</tr>
</tbody>
</table>
Achievable precision @ higher energies/beam current

Beam current: 1 mA
Polarization: 85 % ± 0.425 %
Target material: liquid hydrogen
Target: 60 cm
Measurement time: 10000 h
Detector acceptance: 2π·20°
ΔA_{app}: 0.1 ppb

Beam energy: 300 MeV
Central scattering angle: 19°
A^{PV} = (-30.8 ± 0.34) ppb
<Q^2> = 4.84e-3 GeV²/c²
Rate elastic e-p: 1.8 THz

Beam energy: 500 MeV
Central scattering angle: 14°
A^{PV} = (-24.8 ± 0.36) ppb
<Q^2> = 3.82e-3 GeV²/c²
Rate elastic e-p: 3.6 THz

Δsin^2θ_w = 2.14 · 10^{-4}
Δsin^2θ_w = 2.95 · 10^{-4}
A very first idea for 300 MeV

Beam energy: 300 MeV
Beam current: 150 µA
Central magnetic field: 1.8 Tesla

Moller, $\theta \in [0^\circ, 90^\circ]$  
Elastic e-p, $\theta \in [9^\circ, 29^\circ]$  
Elastic e-p, $\theta \in [0^\circ, 90^\circ]$

Rate prediction @ $z = 3000$ mm

Elastic e-p, $\theta$ in $[9^\circ, 29^\circ]$  
Elastic e-p, $\theta$ not in $[9^\circ, 29^\circ]$  
Moller, e-p  
Moller, background  
Positrons, e-p  
Positrons, background  
Photons, e-p  
Photons, background
A very first idea for 500 MeV

Beam energy: 500 MeV
Beam current: 150 µA
Central magnetic field: 3 Tesla

Moller, \( \theta \in [0°, 90°] \)
Elastic e-p, \( \theta \in [4°, 24°] \)
Elastic e-p, \( \theta \in [0°, 90°] \)

Rate prediction @ z = 3000 mm

Elastic e-p, \( \theta \) in \([4°, 24°] \)
Elastic e-p, \( \theta \) not in \([4°, 24°] \)
Moller, e-p
Moller, background
Positrons, e-p
Positrons, background
Photons, e-p
Photons, background
Summary

• **Project P2 @ MESA:**
  A new measurement of the weak mixing angle with precision goal:
  \[ \Delta Q_w(p) = 1.9 \% \]
  \[ \Delta \sin^2 \theta_w = 0.15 \% \]

• **P2 main detector concept study:**
  Solenoid spectrometer and 2π-Cherenkov-detector
  \[ \rightarrow \Delta \sin^2 \theta_w = 0.13 \% \]

• **Measurement at higher beam energies and beam current:**
  \[ \rightarrow \] Very high precision in \( \sin^2 \theta_w \) at small scattering angles for 300 MeV
  \[ \rightarrow \] Most important contributions from gamma-Z-box and form factors
  \[ \rightarrow \] Experiment may be difficult to perform with a solenoid because of small scattering angles
  \[ \rightarrow \] Toroid may be better choice due to lower dependence on counting statistics
BACKUP SLIDES
Include simulated response of detector modules

Use results of detector module simulation to transform event rates into photo electron rates:

**Event rate distribution:**

<table>
<thead>
<tr>
<th>Event from elastic e-p scattering:</th>
<th>Event from target shower reconstruction:</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal electrons (25 deg &lt; θ &lt; 45 deg)</td>
<td>electrons</td>
</tr>
<tr>
<td>primary electrons</td>
<td>photons</td>
</tr>
<tr>
<td>secondary electrons</td>
<td>positrons</td>
</tr>
<tr>
<td>secondary photons</td>
<td></td>
</tr>
<tr>
<td>secondary positrons</td>
<td></td>
</tr>
<tr>
<td>primary protons</td>
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**Photo electron rate distribution:**

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<tr>
<td>primary electrons</td>
<td>photons</td>
</tr>
<tr>
<td>secondary electrons</td>
<td>positrons</td>
</tr>
<tr>
<td>secondary photons</td>
<td></td>
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<tr>
<td>secondary positrons</td>
<td></td>
</tr>
<tr>
<td>primary protons</td>
<td></td>
</tr>
</tbody>
</table>

**Monte Carlo results:**

\[ R_{\text{total}}^{ep} = 0.19 \text{ THz} \]

\[ \langle A^{PV} \rangle_{L,\Delta\Omega} = -39.8 \text{ ppb} \]

\[ I_{\text{total}}^{\text{cathode}} = 1 \mu\text{A} \]

\[ \langle A^{PV} \rangle_{L,\Delta\Omega} = -33.5 \text{ ppb} \]
Weapon of choice: Solenoid or Toroid?

**Solenoid:**
- Full azimuthal coverage
- Compact setup
- Superconducting coils

**Toroid:**
- Loss of ~50% azimuth → double measurement time
- Larger setup
- Copper coils
We would like to use a superconducting solenoid...

A promising candidate: The FOPI solenoid (GSI, Darmstadt)

- Field strength: 0.6 T
- Coil current: 725 A
- Stored energy: 3.4 MJ
- Material: Cu/Nb-Ti
- Cable length: 22.5 km
- Inner diameter: 2.4 m
- Total length: 3.8 m
- Total weight: 108.7 tons
- l-He consumption: 0.02 g/s, 0.6 l/h
- l-N consumption: 3 g/s, 13 l/h (perm. cooling)

(Courtesy Y. Leifels)

Use fieldmap with Geant4 to simulate the P2 experiment
\[ A^{\text{exp}} \sim \sin^2(\theta_W) \]

\[ \sin^2(\theta_W) = Z(A^{\text{exp}}, A^{\text{app}}, E, P, L, \Delta \Omega, \text{Re}(\Boxyz), \{f_i\}) \]

**Monte Carlo approach:**

Sample distribution for \( \sin^2(\theta_W) \) by assigning Gaussian distributions to each parameter \( \zeta_i \in \{A^{\text{exp}}, A^{\text{app}}, E, P, L, \Delta \Omega, \{f_i\}\} \).

\[ \sin^2(\theta_W) + \delta \sin^2(\theta_W) = Z(\zeta_i ' + \delta \zeta_i) \]

N sampling-iterations yield \( \sin^2(\theta_W) \)-distribution.

Extract \( \Delta \sin^2(\theta_W) \) as standard deviation.

**Parameters:**
- \( E = 150 \text{ MeV} \)
- \( I = 150 \mu \text{A} \)
- \( P = 0.85 \)
- \( \theta = 25^\circ \pm 10^\circ \)
- \( L = 60 \text{ cm} \)
- \( T = 10000 \text{ h} \)
Choice of new weighting function:
Detection yield distribution

\[ \langle A_{PV}^{\Delta\Omega} \rangle_{L,\Delta\Omega} = \frac{\int_0^L dz \int_{\Delta\Omega} d\Omega \left[ \left( \frac{d\sigma}{d\Omega} \right)^{Ros} \cdot \epsilon \cdot A_{PV}^{\Delta\Omega} \right]}{\int_0^L dz \int_{\Delta\Omega} d\Omega \left[ \left( \frac{d\sigma}{d\Omega} \right)^{Ros} \cdot \epsilon \right]} \]

\[ \epsilon(z, \theta) \equiv \text{Rate of photo electrons in detector, produced in target at position } z \text{ with angle } \theta \]

Event rate according to Rosenbluth formula, produced in target at position \( z \) with angle \( \theta \)
What is the number of detected e-p events?

To determine $\Delta \sin^2(\theta_W)$, we sample the mapping:

$$\sin^2(\theta_W) = Z(A^{exp}, A^{app}, E, P, L, \Delta \Omega, \text{Re}(\mathbb{Y}_Z), \{f_i\})$$

with

$$\Delta A^{exp} \approx 1/\sqrt{N} \quad \text{and} \quad N : \text{Total number of detected e-p events}$$

$$N = \Phi \cdot \rho \cdot T \cdot \langle \frac{d\sigma}{d\Omega} \rangle_{L, \Delta \Omega} \cdot \Delta L_{eff} \cdot \Delta \Omega_{eff}$$

$$\langle \frac{d\sigma}{d\Omega} \rangle_{L, \Delta \Omega} = \int_0^L dz \int_{\Delta \Omega} d\Omega \left[ \frac{d\sigma}{d\Omega} \right]^{Ros} \cdot \epsilon$$

$$\int_0^L dz \int_{\Delta \Omega} d\Omega \left[ \epsilon \right]$$

$$\Delta L_{eff} \cdot \Delta \Omega_{eff} = \int_0^L dz \int_{\Delta \Omega} d\Omega \left[ y(z, \theta) \right]$$

$y(z, \theta)$: Detection efficiency distribution

Probability for an elastic e-p event with $(z, \theta)$ to produce a signal in the virtual detector
Measured the yield of photo electrons for different

- materials (quartzes, wrappings, lightguids, PMTs)
- geometries
- impact positions
- angles of incidence
ΔA_{tot} = 0.25 \text{ ppb}, T = 20 \text{ K}, E_{beam} = 200.0 \text{ MeV}, P = 85.0 \% , \theta_{mean} = 20 \text{ deg}, \Delta \theta = 20 \text{ deg}
Low $Q^2$?

$A^{PV}$ is dominated by $Q_W(p)$ at low values of $Q^2$.

$$Q^2 = 4EE' \sin^2 \left( \frac{\theta_{lab}}{2} \right)$$

Low $Q^2$: Low beam energy and large angle or vice versa?

$\gamma$-$Z$-box correction to $Q_w(p)$

At low beam energies: Uncertainty of $\gamma$-$Z$-box contribution to $\sin^2(\theta_w)$ is negligible.