Precision Møller Scattering at Low Energies

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Intense Electron Beams Workshop, Cornell University

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Outline

1. Applications at Low Energies
2. Radiative Corrections at Low Energy
3. Upcoming Measurements

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Møller Scattering at Low Energy

This talk will cover the process

$$e^- + e^- \to e^- + e^- + (\gamma)$$

- Precision calculations including radiative corrections
  - Unpolarized, QED only
- Relevance to future experiments
- Upcoming new measurements

Møller Scattering at Low Energy

- Baseline process for nuclear physics experiments, e.g.:
  - **OLYMPUS**[1]: measure two-photon exchange in lepton-proton scattering via e^+p/e^-p cross-section ratio @ 2 GeV
    - Separate e^+/e^- beams: normalize to Møller/Bhabha

[1]R. Milner, D. K. Hasell, et al., Nucl. Instrum. Methods Phys. Res. A 741, 1 (2014).
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- High-rate noise background: e.g. **DarkLight** @ 100 MeV

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- High-rate noise background: e.g. **DarkLight** @ 100 MeV
- Potential searches for new physics via vacuum polarization diagrams

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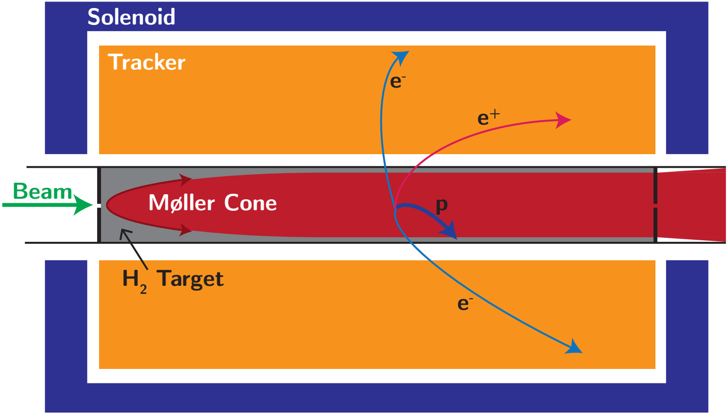
DarkLight

Measure $e^- p \to e^- p e^+ e^-$ at 100 MeV, $\mathcal{L} = 6 \times 10^{35}$ cm^{-2}s^{-1}



- Sweep Møller e^- with solenoid: still significant backscattering
- Møller photons produce high rate of secondaries

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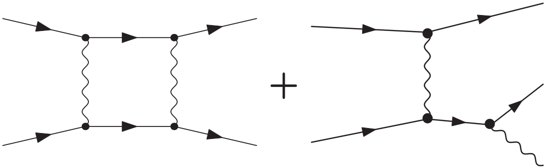
Theoretical Goals



Desired and Produced:

- Unpolarized Møller (and Bhabha) scattering
- Full calculation including NLO radiative corrections (QED only)
  - No peaking, ultra-relativistic (etc) approximations
- Full treatment of bremsstrahlung
- C++ event generator → drive simulations

Approach to the radiative corrections

Separate "elastic" from radiative events by an E_γ cutoff: ΔE

Treat types of events separately

- $E_\gamma > \Delta E$: Hard-photon bremsstrahlung
  - Exact $e\ e \to e\ e\ \gamma$ cross-section
- $E_\gamma < \Delta E$: "Close enough" to elastic kinematics
  - Apply soft-photon radiative corrections

Soft Radiative Corrections: $E_\gamma < \Delta E$

- Integrate soft photon over all Ω and up to $E_\gamma = \Delta E$
- Soft-photon $+$ loop-level IR divergences cancel

[3] I. Akishevich, H. Gao, A. Ilyichev, and M. Meziane, Euro Phys Journal A 51 (2015).

[4] N. Kaiser, J. Phys. G: Nucl. Part. Phys. 37 (2010).

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- Integrate soft photon over all Ω and up to $E_\gamma = \Delta E$
- Soft-photon + loop-level IR divergences cancel

$$\frac{d\sigma}{d\Omega} = (1+\delta)\frac{d\sigma}{d\Omega}\bigg|_{\text{Born}}$$

$$\delta = \delta(\mathbf{\Delta E}, \mathrm{s}, \mathrm{t}, \mathrm{u})$$

Expect that as ΔE gets smaller, δ gets smaller

- Many formulations use ultra-relativistic approximations: $m_e \to 0$
- Recently worked out beyond URA for Møller[3], Møller/Bhabha[4]

[3] I. Akishevich, H. Gao, A. Ilyichev, and M. Meziane, Euro Phys Journal A 51 (2015).
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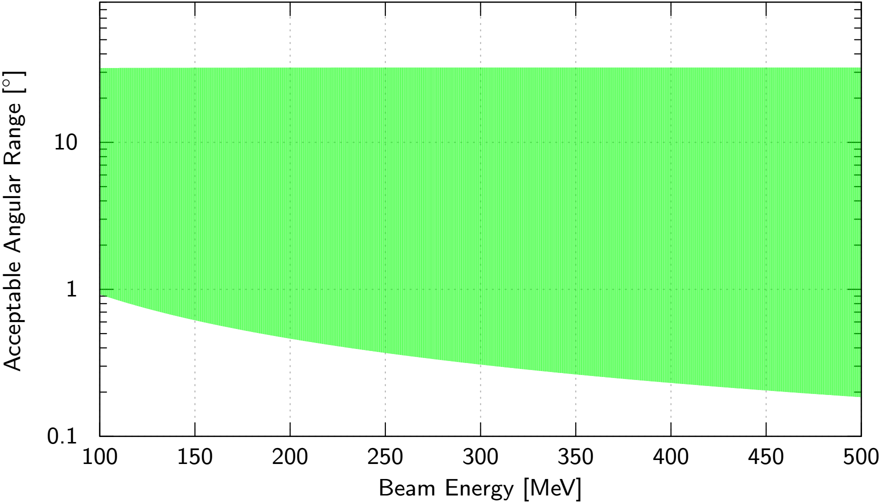
Ultra-relativistic Approximations

- Higher-order m_e terms often neglected
  - For high beam energies, typically a good approximation
- Effects visible at low energy:
  at angles where m_e^2/t or m_e^2/u no longer $\ll 1$

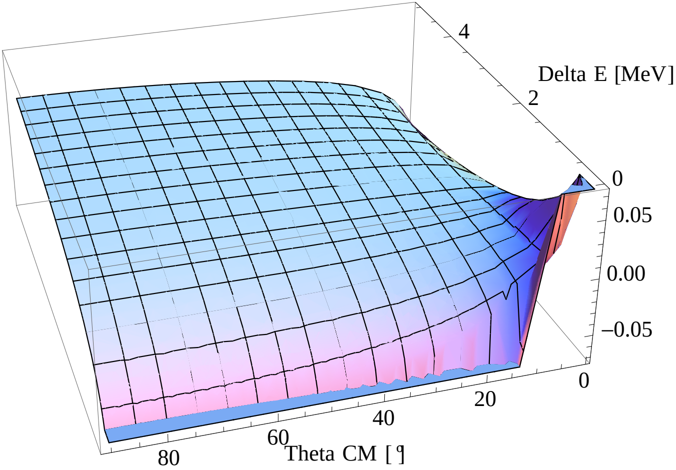
δ grows as ΔE gets smaller

- **Unphysical** $\rightarrow$ implies higher rate at smaller energy windows

Lab-frame angles at which $m_e^2/t = 0.1$



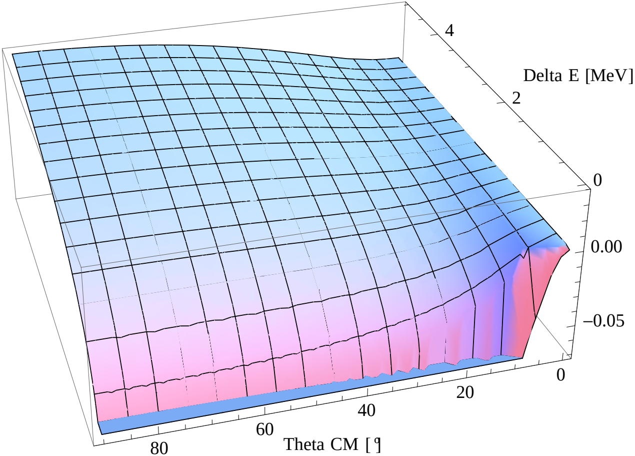
Improper behavior of δ: $\sqrt{s} = 10.16$ MeV (DL)



$$\delta(\Delta E, \theta)$$

(Tsai, 1960)

What δ should look like



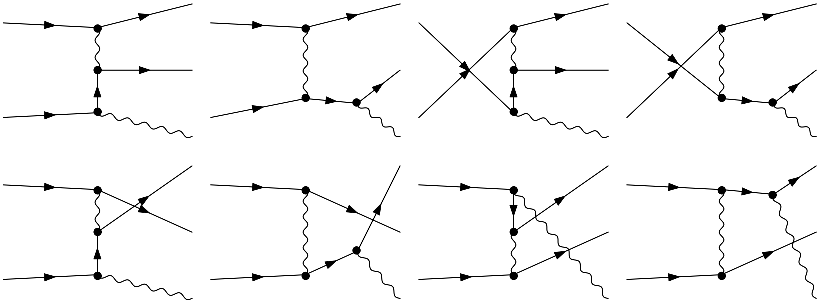
$\delta(\Delta E, \theta)$

(Kaiser, 2010)

Hard-photon Bremsstrahlung: $E_\gamma > \Delta E$

Calculated exactly using `FeynArts`/`FormCalc`

$$e_1^- + e_2^- \to e_3^- + e_4^- + \gamma$$



Bremsstrahlung Cross-Section (CM System)

$$\frac{d^5\sigma}{\mathbf{dE}_\gamma \; d\Omega_\gamma \; d\Omega_3} = \frac{1}{32 m_e^3 (2\pi)^5} \frac{E_\gamma}{2EpR_c} \sum_\nu p_{3\nu}^2 \langle |M|^2 \rangle$$ [5]

[5] Haug & Nakel, 2004

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$$\mathbf{E_3} = \frac{2E(E-E_\gamma)(2E-E_\gamma) \mp E_\gamma m_e^2 \cos\alpha R_c}{(2E-E_\gamma)^2 - E_\gamma^2\cos^2\alpha}$$

Typically only upper sign is permitted.
Above a photon energy E_{γ_0}, both become allowed

[5] Haug & Nakel, 2004

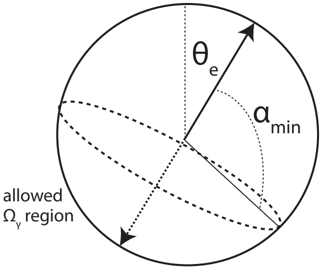
Lepton Constraints

Two values allowed for:

$$E_{\gamma_0} > 2E(E - m_e)/(2E - m_e)$$

Additional constraint:

$$\cos\alpha < -\frac{1}{E_\gamma}\sqrt{(2E - E_\gamma)^2 - 4E^2(E - E_\gamma)^2/m_e^2}$$



Identical Møller electrons: a complication

Cross-section:

$$\frac{d\sigma}{d\Omega_3} = \left(\frac{1}{2}\right)_{\text{Symmetry}} \times \langle |M|^2 \rangle \times [\text{Phase Space}]$$

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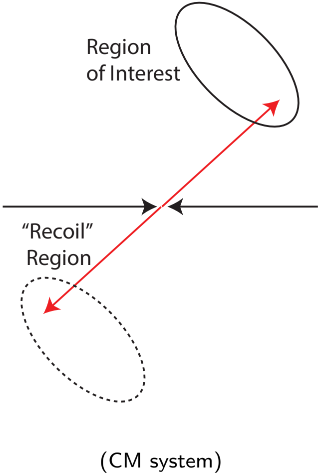

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- Easy with elastic kinematics

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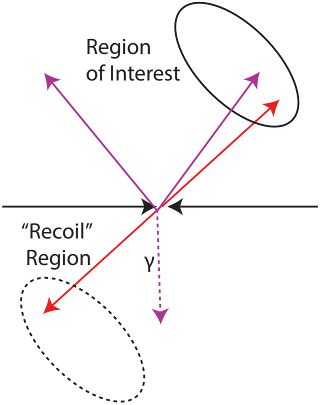

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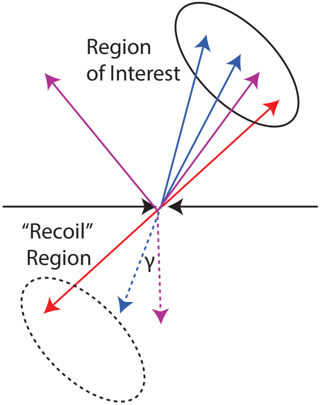

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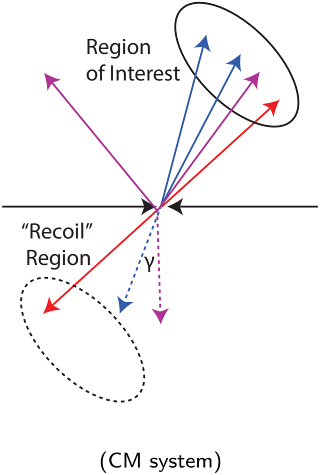

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Generator: check each event for double-counting

Bremsstrahlung Comparison

Soft-photon corrections and bremsstrahlung
should agree near $E_\gamma \sim \Delta E$

Rearrange:

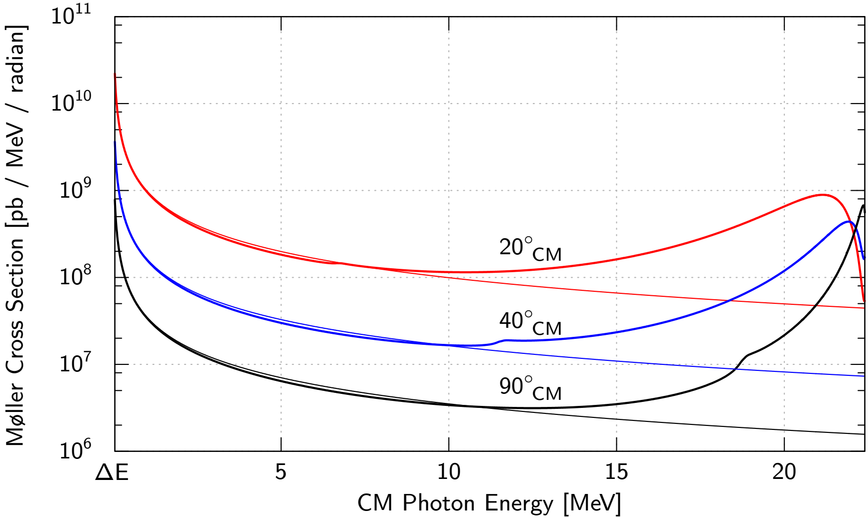
Soft-Photon

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_3\ \mathrm{d}E_\gamma} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_3}\right|_{\mathrm{Born}} \times \frac{\mathrm{d}}{\mathrm{d}\Delta E}\left\{\delta(\Omega_3, \Delta E)\right\}$$

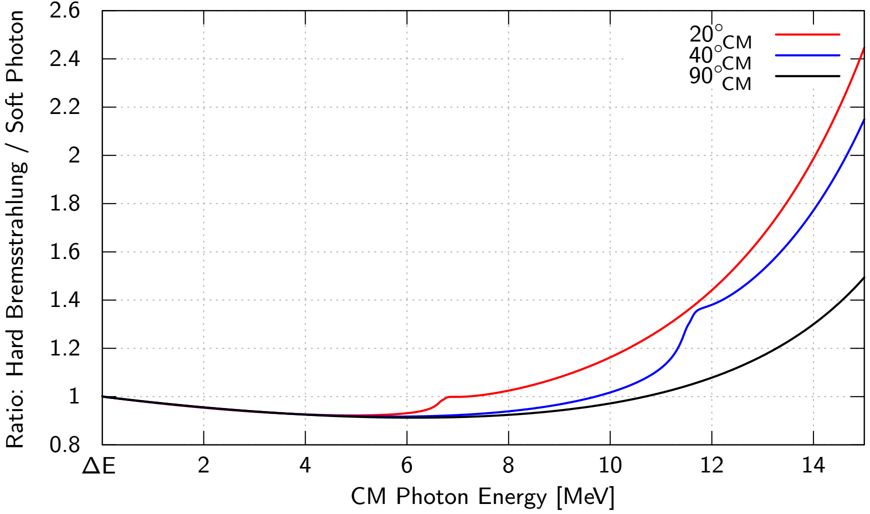
Bremsstrahlung

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_3\ \mathrm{d}E_\gamma} = \int_{4\pi} \frac{\mathrm{d}^5\sigma}{\mathrm{d}\Omega_3\ \mathrm{d}E_\gamma\ \mathrm{d}\Omega_\gamma}\ \mathrm{d}\Omega_\gamma$$

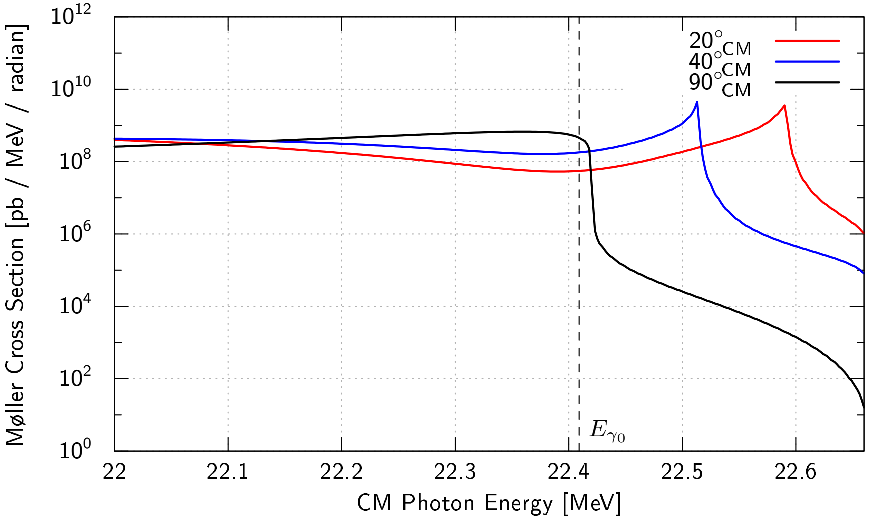
Electron Cross-Section at fixed angles (2 GeV)



Ratio of hard/soft cross-sections



Electron Cross-Section at high photon energies



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Møller Scattering at 100 MeV

Why measure unpolarized low-energy Møller scattering?

Quantities with few precision data

- Distribution of E at fixed θ: **radiative tail**
  - Verify bremsstrahlung calculation

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  - Verify soft-photon radiative corrections $\rightarrow$ beyond URA

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Requirements

- Measure electrons with energy 1-5 MeV/c
- Momentum resolution $\delta p/p \sim 1\%$
- Scattering angles 25°-45°

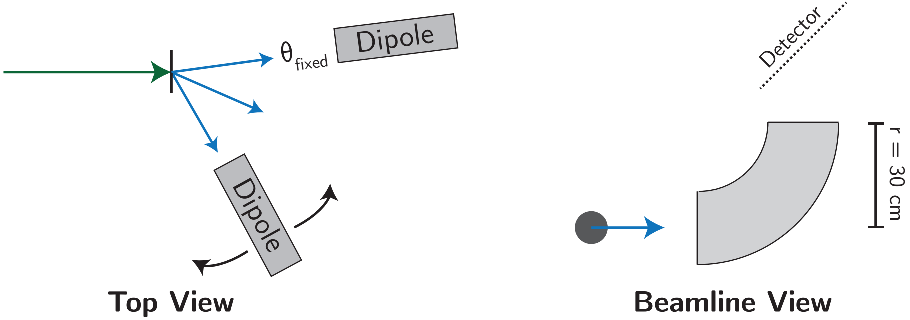
Measurement Summary

The radiative Møller process will be measured in tandem with the DarkLight experiment

- Jefferson Lab LERF/ERL: 100 MeV electrons
- Energy measurement with magnetic spectrometers
- Map out electron energy spectrum at various θ
- $\sim$100 settings

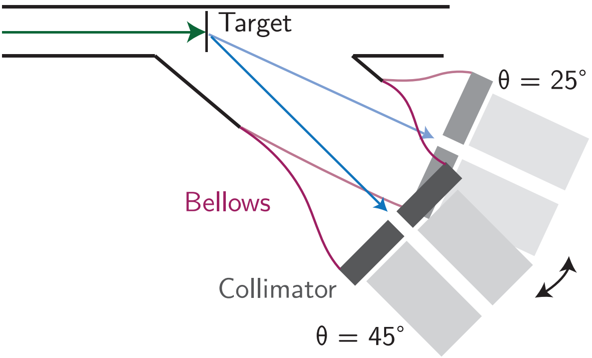
Møller Measurement Layout

Two-Spectrometer Layout



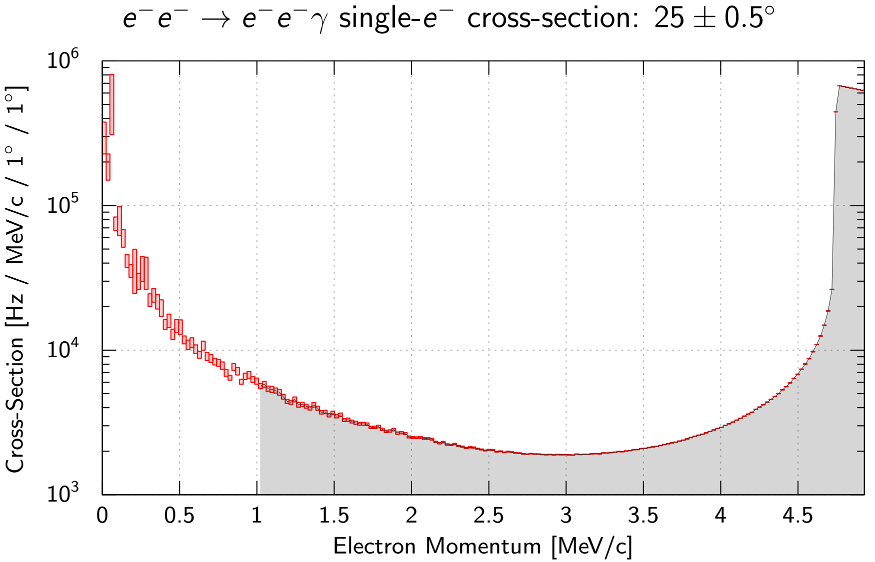
- Pointlike target: 5μm MicroMatter diamond-like carbon foil
- Measure electron *p* and θ; normalize to *e*-*C* scattering

Møller Measurement Layout



- 25° to 45° every 5°
- Entirely vacuum until detector: eliminate multiple scattering

Cross-Sections: Inelastic



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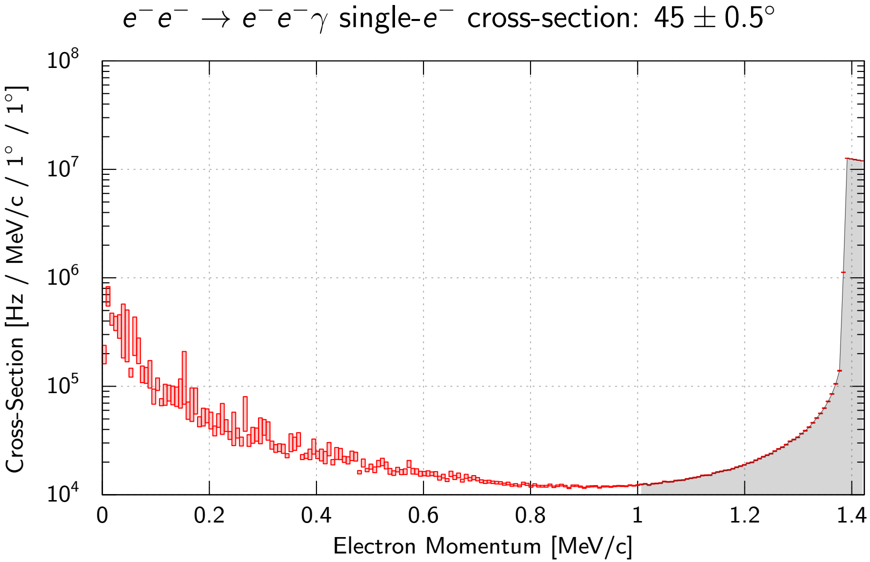
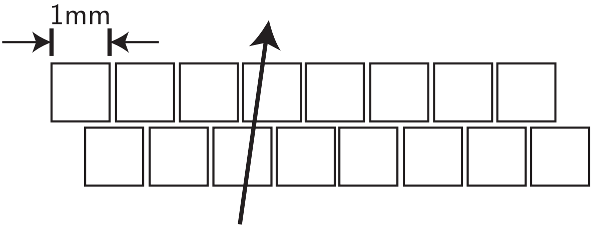


Detector Concept

2D array of crossed scintillating fibers



- Two 2-layer arrays at 90°
- Detector Size: $15\text{cm} \times 5\text{cm}$
- Light collection: SiPM array or multianode PMT

arXiv:1011.0226, https://userweb.jlab.org/~yez/Work/SFT/SFT_Status.pdf

Summary

Calculated

- Single-photon bremsstrahlung to extend Møller/Bhabha soft-photon corrections
- Beyond URA $\rightarrow$ optimal for low beam energies
- Incorporated into a new Monte Carlo event generator

Plan to Measure

- Møller electrons: 25°-45°
  - Map out energy distribution every 5°
- Magnetic spectrometers: 30cm, 90° dipole
  - Potential: 2D scintillating fiber detector

Backup

Target and Beam Parameters

Luminosity	$\sim 2 \times 10^{34}$ cm^{-2}s^{-1}
Target	5 μm diamond-like carbon
Target e$^-$ thickness	$\sim 3 \times 10^{20}$ cm^{-2} @ 5 μm
Beam	100 MeV, 0.01 mA
Beam power deposition	0.027 Watts / 0.01 mA

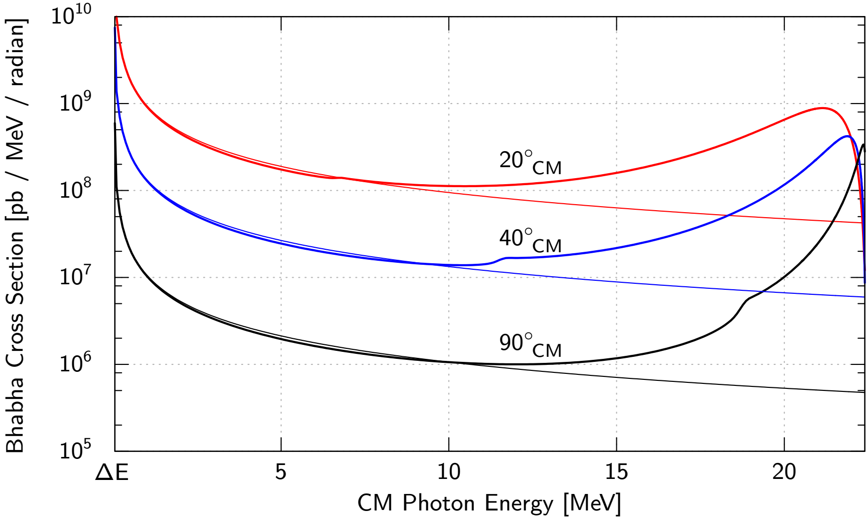
Spectrometer Parameters

Detectors	Two dipole spectrometers + readout planes
Dipole radius	30 cm
Dipole angle	90°
Momentum acceptance	$\Delta p/p \sim$10%
Momentum resolution	$\delta p/p \sim 10^{-3}$-10^{-2}
θ acceptance	$\pm 0.5^\circ$
ϕ acceptance	1°
θ pos. resolution	$< 0.1^\circ$
Luminosity Angle	$\sim 25^\circ$
Signal Angles	25° to 45° (in 5° steps)
Luminosity electron momentum	$\sim$100 MeV/c
Luminosity detector field	$\sim$1.2 T: existing magnet
Signal electron momentum	$1 - 5$ MeV/c
Signal detector field	$120 - 600$ Gauss

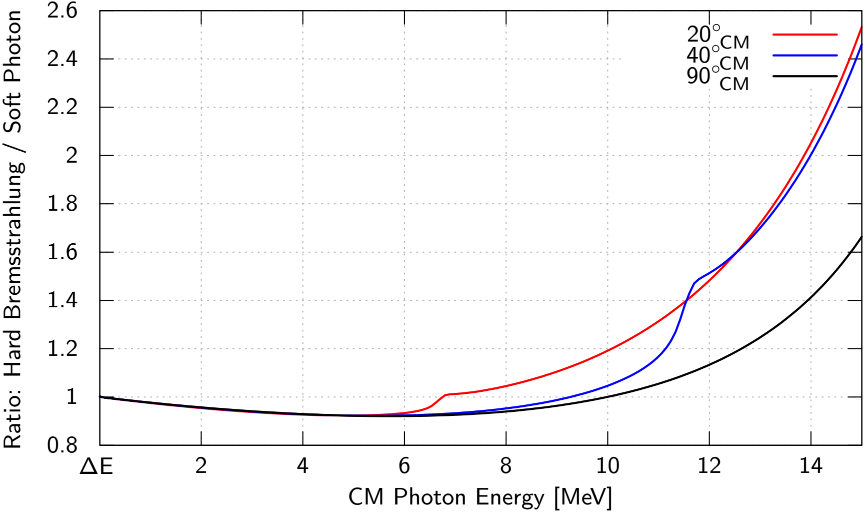
Target Parameter Estimates

- ERL transverse emittance $\sim$ 10 mm-mrad (normalized)
  - Multiple scattering $\sim$1 mrad
  - Beam spot $\sim$0.1 mm $\rightarrow$ $\sim$0.5 mrad angular spread
  - MS increases emittance $\sim 2\times$ (OK)
- Power deposition in 5 µm C: $\sim$ 0.027 Watts at 100 MeV, 0.01 mA
  - $\Delta T \sim \frac{P/t}{4\pi\lambda}[1 + \ln(R/r)]$ ($\lambda \sim 70$ W/m-K)
  - At $R = 1$ cm, $r = 0.1$ mm, $t = 5$ microns, $\Delta T \sim 34°$C
  - $P = \epsilon\sigma A T^4$: $A \sim 2 \times \pi \cdot (1 \text{ mm})^2$, $\epsilon = 0.7 \rightarrow$ T $\sim$ 574 K

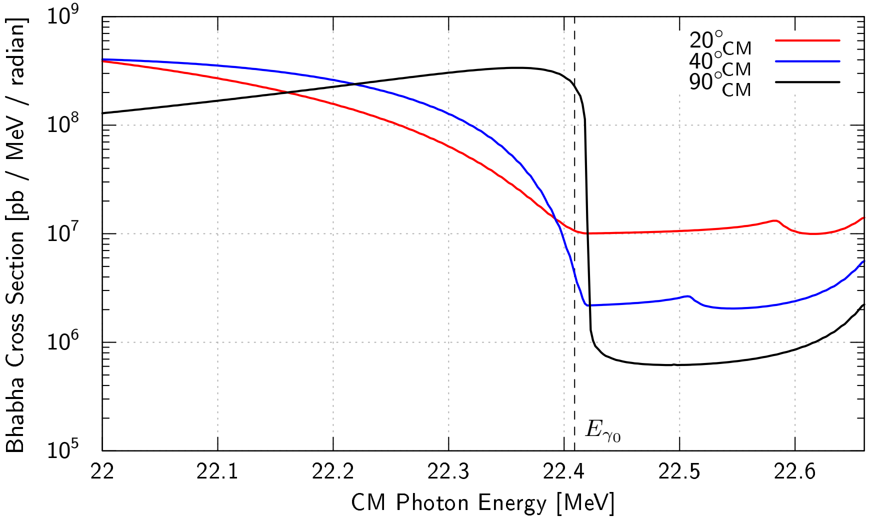
Positron Cross-Section at fixed angles



Ratio of hard/soft cross-sections (Bhabha)



Positron Cross-Section at high photon energies



Elastic vs Møller Cross-Sections