Precision Møller Scattering at Low Energies

Charles Epstein

Intense Electron Beams Workshop, Cornell University

June 18, 2015
Outline

1. Applications at Low Energies
2. Radiative Corrections at Low Energy
3. Upcoming Measurements
Outline

1 Applications at Low Energies
2 Radiative Corrections at Low Energy
3 Upcoming Measurements
Møller Scattering at Low Energy

This talk will cover the process

\[ e^- + e^- \rightarrow e^- + e^- + (\gamma) \]

- Precision calculations including radiative corrections
  - Unpolarized, QED only
- Relevance to future experiments
- Upcoming new measurements
• Baseline process for nuclear physics experiments, e.g.:
  • **OLYMPUS**\(^1\): measure two-photon exchange in lepton-proton scattering via \(e^+ p/e^- p\) cross-section ratio @ 2 GeV
  • Separate \(e^+/e^-\) beams: normalize to Møller/Bhabha

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Møller Scattering at Low Energy

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  - PRad\(^2\): measure proton form factor at low \( Q^2 \): charge radius
  - Normalize \( ep \) to Møller at forward angles

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- High-rate noise background: e.g. **DarkLight** @ 100 MeV

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Potential searches for new physics via vacuum polarization diagrams

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Measure $e^- p \rightarrow e^- p e^+ e^-$ at 100 MeV, $\mathcal{L} = 6 \times 10^{35}$ cm$^{-2}$s$^{-1}$

- Sweep Møller $e^-$ with solenoid: still significant backscattering
- Møller photons produce high rate of secondaries
Outline

1 Applications at Low Energies
2 Radiative Corrections at Low Energy
3 Upcoming Measurements
Theoretical Goals

Desired and Produced:

- Unpolarized Møller (and Bhabha) scattering
- Full calculation including NLO radiative corrections (QED only)
  - No peaking, ultra-relativistic (etc) approximations
- Full treatment of bremsstrahlung
- C++ event generator → drive simulations
Approach to the radiative corrections

Separate “elastic” from radiative events by an $E_\gamma$ cutoff: $\Delta E$

Treat types of events separately

- $E_\gamma > \Delta E$: Hard-photon bremsstrahlung
  - Exact $e\,e \to e\,e\,\gamma$ cross-section
- $E_\gamma < \Delta E$: “Close enough” to elastic kinematics
  - Apply soft-photon radiative corrections
Soft Radiative Corrections: $E_\gamma < \Delta E$

- Integrate soft photon over all $\Omega$ and up to $E_\gamma = \Delta E$
- Soft-photon + loop-level IR divergences cancel

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Soft Radiative Corrections: $E_\gamma < \Delta E$

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$$\frac{d\sigma}{d\Omega} = (1 + \delta) \frac{d\sigma}{d\Omega} \bigg|_{\text{Born}}$$

$$\delta = \delta(\Delta E, s, t, u)$$

Expect that as $\Delta E$ gets smaller, $\delta$ gets smaller

- Many formulations use ultra-relativistic approximations: $m_e \to 0$
- Recently worked out beyond URA for Møller$^3$, Møller/Bhabha$^4$

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Ultra-relativistic Approximations

- Higher-order $m_e$ terms often neglected
  - For high beam energies, typically a good approximation

- Effects visible at low energy:
  at angles where $m_e^2/t$ or $m_e^2/u$ no longer $\ll 1$

  $$\delta \text{ grows as } \Delta E \text{ gets smaller}$$

- **Unphysical** $\rightarrow$ implies higher rate at smaller energy windows
Lab-frame angles at which $m_e^2/t = 0.1$
Improper behavior of $\delta$: $\sqrt{s} = 10.16$ MeV (DL)

\[(\Delta E, \theta)
\]

(Tsai, 1960)
What $\delta$ should look like

$\delta(\Delta E, \theta)$

(Kaiser, 2010)
Hard-photon Bremsstrahlung: $E_\gamma > \Delta E$

Calculated exactly using FeynArts/FormCalc

$$e^-_1 + e^-_2 \rightarrow e^-_3 + e^-_4 + \gamma$$
Bremsstrahlung Cross-Section (CM System)

\[
\frac{d^5 \sigma}{dE_\gamma d\Omega_\gamma d\Omega_3} = \frac{1}{32m_e^3(2\pi)^5} \frac{E_\gamma}{2EpR_c} \sum_\nu p_{3\nu}^2 \langle |M|^2 \rangle \quad[^5]
\]

[^5]: Haug & Nakel, 2004
Bremsstrahlung Cross-Section (CM System)

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\]

\[
E_3 = \frac{2E(E - E_\gamma)(2E - E_\gamma) \mp E_\gamma m_e^2 \cos \alpha R_c}{(2E - E_\gamma)^2 - E_\gamma^2 \cos^2 \alpha}
\]

Typically only upper sign is permitted.

Above a photon energy \( E_{\gamma 0} \), both become allowed.

\[5\] Haug & Nakel, 2004
Lepton Constraints

Two values allowed for:

\[ E_{\gamma_0} > 2E(E - m_e)/(2E - m_e) \]

Additional constraint:

\[ \cos \alpha < -\frac{1}{E_\gamma} \sqrt{(2E - E_\gamma)^2 - 4E^2(E - E_\gamma)^2/m_e^2} \]
Identical Møller electrons: a complication

Cross-section:

\[ \frac{d\sigma}{d\Omega_3} = \left( \frac{1}{2} \right)_{\text{Symmetry}} \times \langle |M|^2 \rangle \times [\text{Phase Space}] \]
Identical Møller electrons: a complication

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- Must integrate over “interest” and “recoil” regions
- Easy with elastic kinematics

Region of Interest

“Recoil” Region

(CM system)
Identical Møller electrons: a complication

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- Radiative: “recoil” region isn’t simple
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\]

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Generator: check each event for double-counting
Bremsstrahlung Comparison

Soft-photon corrections and bremsstrahlung should agree near $E_\gamma \sim \Delta E$

Rearrange:

\[
\frac{d^3\sigma}{d\Omega_3 dE_\gamma} = \left. \frac{d\sigma}{d\Omega_3} \right|_{\text{Born}} \times \frac{d}{d\Delta E} \{ \delta(\Omega_3, \Delta E) \}
\]

**Soft-Photon**

\[
\frac{d^3\sigma}{d\Omega_3 dE_\gamma} = \frac{d\sigma}{d\Omega_3} \times \frac{d}{d\Delta E} \{ \delta(\Omega_3, \Delta E) \}
\]

**Bremsstrahlung**

\[
\frac{d^3\sigma}{d\Omega_3 dE_\gamma} = \int_{4\pi} d\Omega_3 \frac{d^5\sigma}{dE_\gamma d\Omega_\gamma} d\Omega_\gamma
\]
Electron Cross-Section at fixed angles (2 GeV)

Møller Cross Section [pb / MeV / radian]

CM Photon Energy [MeV]

$\Delta E$

$20^\circ_{CM}$

$40^\circ_{CM}$

$90^\circ_{CM}$

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Ratio of hard/soft cross-sections

![Graph showing the ratio of hard Bremsstrahlung to soft photon as a function of CM photon energy.](image)

- Ratio: Hard Bremsstrahlung / Soft Photon
- CM Photon Energy [MeV]
- ∆E

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Low-Energy Möller Scattering
June 18, 2015
Electron Cross-Section at high photon energies

Møller Cross Section \([\text{pb} / \text{MeV} / \text{radian}]\)

CM Photon Energy \([\text{MeV}]\)

\(E_{\gamma_0}\)

Charles Epstein (MIT)

Low-Energy Møller Scattering

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18 / 25
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Why measure unpolarized low-energy Møller scattering?

Quantities with few precision data

- Distribution of $E$ at fixed $\theta$: **radiative tail**
- Verify bremsstrahlung calculation
Møller Scattering at 100 MeV

Why measure unpolarized low-energy Møller scattering?

Quantities with few precision data

- Distribution of $E$ at fixed $\theta$: radiative tail
- Verify bremsstrahlung calculation
- Precise electron-electron cross-section vs $\theta$
- Verify soft-photon radiative corrections → beyond URA
Why measure unpolarized low-energy Møller scattering?

Quantities with few precision data

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  - Verify bremsstrahlung calculation
- Precise electron-electron cross-section vs $\theta$
  - Verify soft-photon radiative corrections → beyond URA

Requirements

- Measure electrons with energy 1-5 MeV/c
- Momentum resolution $\frac{\delta p}{p} \sim 1\%$
- Scattering angles 25°-45°
The radiative Møller process will be measured in tandem with the DarkLight experiment

- Jefferson Lab LERF/ERL: 100 MeV electrons
- Energy measurement with magnetic spectrometers
- Map out electron energy spectrum at various $\theta$
- $\sim$100 settings
• Pointlike target: 5\(\mu\)m MicroMatter diamond-like carbon foil
• Measure electron \(p\) and \(\theta\); normalize to e-C scattering
- $25^\circ$ to $45^\circ$ every $5^\circ$
- Entirely vacuum until detector: eliminate multiple scattering
Cross-Sections: Inelastic

\[ e^- e^- \rightarrow e^- e^- \gamma \] single-\(e^-\) cross-section: \(25 \pm 0.5^\circ\)
$e^- e^- \rightarrow e^- e^- \gamma$ single-$e^-$ cross-section: $45 \pm 0.5^\circ$
Detector Concept

2D array of crossed scintillating fibers

- Two 2-layer arrays at 90°
- Detector Size: 15cm × 5cm
- Light collection: SiPM array or multianode PMT

Summary

Calculated

- Single-photon bremsstrahlung to extend Møller/Bhabha soft-photon corrections
- Beyond URA → optimal for low beam energies
- Incorporated into a new Monte Carlo event generator

Plan to Measure

- Møller electrons: 25°-45°
  - Map out energy distribution every 5°
- Magnetic spectrometers: 30cm, 90° dipole
  - Potential: 2D scintillating fiber detector
Backup
## Target and Beam Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>$\sim 2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$</td>
</tr>
<tr>
<td>Target</td>
<td>5 µm diamond-like carbon</td>
</tr>
<tr>
<td>Target $e^-$ thickness</td>
<td>$\sim 3 \times 10^{20} \text{ cm}^{-2} @ 5 \mu\text{m}$</td>
</tr>
<tr>
<td>Beam</td>
<td>100 MeV, 0.01 mA</td>
</tr>
<tr>
<td>Beam power deposition</td>
<td>0.027 Watts / 0.01 mA</td>
</tr>
</tbody>
</table>
## Spectrometer Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detectors</td>
<td>Two dipole spectrometers + readout planes</td>
</tr>
<tr>
<td>Dipole radius</td>
<td>30 cm</td>
</tr>
<tr>
<td>Dipole angle</td>
<td>90°</td>
</tr>
<tr>
<td>Momentum acceptance</td>
<td>$\Delta p/p \sim 10%$</td>
</tr>
<tr>
<td>Momentum resolution</td>
<td>$\delta p/p \sim 10^{-3} - 10^{-2}$</td>
</tr>
<tr>
<td>$\theta$ acceptance</td>
<td>$\pm 0.5^\circ$</td>
</tr>
<tr>
<td>$\phi$ acceptance</td>
<td>$1^\circ$</td>
</tr>
<tr>
<td>$\theta$ pos. resolution</td>
<td>$&lt; 0.1^\circ$</td>
</tr>
<tr>
<td>Luminosity Angle</td>
<td>$\sim 25^\circ$</td>
</tr>
<tr>
<td>Signal Angles</td>
<td>$25^\circ$ to $45^\circ$ (in $5^\circ$ steps)</td>
</tr>
<tr>
<td>Luminosity electron momentum</td>
<td>$\sim 100$ MeV/c</td>
</tr>
<tr>
<td>Luminosity detector field</td>
<td>$\sim 1.2$ T: existing magnet</td>
</tr>
<tr>
<td>Signal electron momentum</td>
<td>$1 - 5$ MeV/c</td>
</tr>
<tr>
<td>Signal detector field</td>
<td>$120 - 600$ Gauss</td>
</tr>
</tbody>
</table>
Target Parameter Estimates

- ERL transverse emittance $\sim 10 \text{ mm-mrad}$ (normalized)
  - Multiple scattering $\sim 1 \text{ mrad}$
  - Beam spot $\sim 0.1 \text{ mm} \rightarrow \sim 0.5 \text{ mrad}$ angular spread
  - MS increases emittance $\sim 2 \times$ (OK)

- Power deposition in 5 $\mu$m C: $\sim 0.027$ Watts at 100 MeV, 0.01 mA
  - $\Delta T \sim \frac{P/t}{4\pi\lambda}[1 + \ln(R/r)]$ ($\lambda \sim 70 \text{ W/m-K}$)
  - At $R = 1 \text{ cm}$, $r = 0.1 \text{ mm}$, $t = 5$ microns, $\Delta T \sim 34^\circ\text{C}$
  - $P = \epsilon\sigma AT^4$: $A \sim 2 \times \pi \cdot (1 \text{ mm})^2$, $\epsilon = 0.7 \rightarrow T \sim 574 \text{ K}$
Positron Cross-Section at fixed angles

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Ratio of hard/soft cross-sections (Bhabha)

![Graph showing the ratio of hard Bremsstrahlung to soft photon as a function of CM Photon Energy [MeV]. The graph includes data points for different CM angles: 20°, 40°, 90°, and 120°. The y-axis represents the ratio, and the x-axis represents the CM Photon Energy in MeV.](image-url)
Positron Cross-Section at high photon energies

Bhabha Cross Section [pb / MeV / radian]

CM Photon Energy [MeV]

E_{\gamma_0}

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Rate of 100 MeV Elastic, Moller Scattering in 1° Bins

Elastic vs Møller Cross-Sections

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Low-Energy Møller Scattering

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