

\blacksquare Physical Sciences Building - Cornell University Cornell – October 15, 2024 Future of Heavy Quark Physics

Future of Heavy Quark Physics - LepageFest **Search** Enter your search term **The Power of EFTs** Matthias Neubert, Johannes Gutenberg University Mainz

PETER LEPAGE FEST

- ▸ **A new algorithm for adaptive Monte Carlo integration** (J. Comput. Phys. 27 (1978) 192, 1446 citations)
- **Exclusive processes in QCD: Evolution equations for hadronic wave functions and the form-factors of mesons** (with S.J. Brodsky, Phys. Lett. B 87 (1979) 359, 1535 citations)
- **Exclusive processes in perturbative QCD** (with S.J. Brodsky, Phys. Rev. D 22 (1980) 2157, 4066 citations)
- ▸ **On the elimination of scale ambiguities in perturbative QCD** (with S.J. Brodsky and P.B. Mackenzie, Phys. Rev. D 28 (1983) 228, 1298 citations)
- ▸ **Effective Lagrangians for bound state problems in QED, QCD and other field theories** (with W.E. Caswell, Phys. Lett. B 167 (1986) 437, 1321 citations)
- ▸ **Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium** (with G.T. Godwin and E. Braaten, Phys. Rev. D 51 (1995) 1125, 2955 citations)
- **Heavy quark bound states in lattice QCD** (with B.A. Thacker, Phys. Rev. D 43 (1991) 196, 464 citations)
- ▸ **On the viability of lattice perturbation theory** (with P.B. Mackenzie, Phys. Rev. D 48 (1993) 2250, 1201 citations)
- ▸ **High precision lattice QCD confronts experiment** (HPQCD, UKQCD, MILC & Fermilab Lattice collaborations, Phys. Rev. Lett. 92 (2004) 022001, 466 citations)

PUSHING THE LUMINOSITY FRONTIER - GOLDEN AGE OF HEAVY-QUARK THEORY

- ▸ Tremendous experimental advances:
	- ▶ 1. generation: ARGUS & CLEO, LEP expts.
	- ▸ 2. generation: BaBar & Belle, LHC*b*, CMS, …
	- ▸ 3. generation: Belle II, LHC*b* upgrade, …
- ▸ Precise measurement of CKM elements $|V_{cb}|, |V_{ub}|, |V_{td}|, |V_{ts}|$ involving thirdgeneration quarks
- ▶ Precise determinations of angles (CP violation)
- ▸ New-physics searches using FCNC processes

PUSHING THE LUMINOSITY FRONTIER - GOLDEN AGE OF HEAVY-QUARK THEORY

- ▸ Matching the incredible precision of the *B-*factories required a revolution in theory
- ▸ Concerted effort of theory community was an important consequence Breakthrough came from using **effective field theories** (EFTs):

 \rightarrow $\mathcal{H}_{\rm eff}^{\rm weak}$, HQET, NRQCD, QCDF, SCET ▸ SCET later became a versatile tool for addressing difficult LHC theory problems eff

PUSHING THE LUMINOSITY FRONTIER - GOLDEN AGE OF HEAVY-QUARK THEORY

- ▸ Systematic method to separate shortdistance effects (weak scale and beyond) from long-distance hadronic dynamics
- ▸ Nowadays embedded into SMEFT and its low-energy variant LEFT
- ▸ **But:** challenge is to evaluate hadronic matrix elements of the quark-gluon o perators $Q_i(\mu)$ in all but simplest cases

operators only in section V , to the inclusion V and \mathcal{N} penculis in section V . Hence, in this section V

[Gilman, Wise (1979); Buras et al. (1990s)] C2 1.120 1.120 1.120 1.120 1.120 1.120 1.120 1.120 1.120 1.120 1.120 1.120 1.139 1.139 1.139 1.156 1.1139 1.11
1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.139 1.

EFFECTIVE WEAK HAM

HEAVY QUARK SYMMETRY

Hint IDENTIFY Equation
$$
B \rightarrow D^{(*)}
$$
 IDENTIFY IDENTIFY Equation $B \rightarrow D^{(*)}$ **IDENTIFY IDENTIFY Equation SET IDENTIFY IDENTIFY Equation SET EXECUTE SET EXECUTE EXECUTE**

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- ▸ Hadronic bound states containing a **flavor symmetry**
- ▸ Many predictions for spectroscopy of heavy hadrons and *A,U* ⁼ *~#y5b,* [Shuryak (1980)]
- **B** Symmetry relations among $B \to D^{(*)}$ form $\left(\langle D^*(v',\epsilon) | A^{\mu}| B(v) \rangle = 0\right)$ factors, including symmetry-breaking [Isgur, Wise (1990)]

▸ Extrapolate observed spectrum in $w = v \cdot v'$ to zero recoil:

\rightarrow Direct calculation of the $B \rightarrow Dl\nu$ form factors (HPQCD):

Fig. 1. Extraction of $|V_{cb}|$ and the Isgur–Wise function from $\bar{B}^0 \rightarrow D^{*+} \bar{\mathbb{V}}_{\bar{\mathbb{X}}}$ decays. The data are taken from ref. [16]. $\tau_{B^0} =$ 1.18 ps is assumed. $|V_{cb}|$ follows from an extrapolation of the data to $v v' = 1$. Its currently best value is indicated as a shaded area on the vertical axis.

MODEL-INDEPENDENT DETERMINATION OF |VCB|

[Eichten, Hill (1990); Georgi (1990)]

 \triangleright Firm theoretical basis for deriving he quark symmetry and its consequence

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\text{Eq} & \text{Equation (a)} \\
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\text{Eq} & \text{Equation (e)} \\
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\
$$

HEAVY QUARK EFFECTIVE THEORY (HQE

HEAVY QUARK EFFECTIVE THEORY (HQET)

▸ Firm theoretical basis for deriving heavyquark symmetry and its consequences

▸ An anecdote from 1988 …

 $\mathcal{L}_{\text{HQET}} = \bar{h}_v \, iv \cdot D \, h_v + \mathcal{O}$ (1) m_Q ◆ $+ \frac{1}{2\pi} \left[\bar{h}_v (iD)^2 h_v + \frac{g_s}{2} \bar{h}_v \, \sigma_{\mu\nu} G^{\mu\nu} h_v \right] + \ldots$ 2*m^Q* $\sqrt{ }$ $\bar{h}_v(iD)^2 h_v + \frac{g_s}{2}$ 2 $\bar{h}_v \, \sigma_{\mu\nu} G^{\mu\nu} h_v$ $\overline{}$

[Eichten, Hill (1990); Georgi (1990)]

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 \triangleright Firm theoretical basis for deriving he quark symmetry and its consequence

$$
\begin{array}{c}\n\text{Eq} \\
\text{Eq} \\
\text
$$

[Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1992, 1995)]

NONRELATIVISTIC EFFECTIVE FIELD THEORY (NRQED & NRQCD)

``*We develop a renormalization group strategy for the study of bound states in field theory. Our analysis is completely different from conventional analyses, based upon the Bethe-Salpeter equation, and it is far simpler.*"

HEAVY QUARK EFFECTIVE THEORY (HQE

HEAVY QUARK EFFECTIVE THEORY (HQET)

[Eichten, Hill (1990); Georgi (1990)]

▸ Firm theoretical basis for deriving heavy-where Gµ^ν is the gluon field-strength tensor expressed in the form of an SU(3) matrix, and q *^L*HQET ⁼ *^h*¯*^v iv · D h^v* ⁺ *^O* ✓ 1 *m^Q* ◆ ⁺ ⁺ *...* ¹ 2*m^Q* h *h*¯*^v* (*iD*) ² *^h^v* ⁺ *^g^s* 2 *h*¯*^v µ*⌫*G^µ*⌫*h^v* i The gluons and the n^f flavors of light quarks are described by the fully relativistic lagrangian ^Llight ⁼ [−]¹ tr ^GµνG^µ^ν ⁺ !q i ¯ Dq, ̸ (2.3) , where ψ is the Pauli spinor field that annihilates a heavy quark, χ is the Pauli spinor field that creates a heavy antiquark, and D^t and D are the time and space components of the gauge-covariant derivative D^µ. Color and spin indices on the fields ψ and χ have been

FICCONSTANT. THE SUM IN FURNIE CONSTANT (2.3) IN THE NEW GROUP CONSTANT α is negligible to anomaly constant α field theory for the heavy quarks and antiquarks. The relativistic effects of full QCD are reproduced through the correction term in the correction term \sim 2011 and 2.2 \sim 2011 and 2.2 \sim 2011 and 2.2 \sim 2012 and 2.2 \sim 2.2). The correction term \sim 2.2 \sim 2.2 \sim 2.2). The correction term \sim 2.2

- quark symmetry and its consequences \Box suite is DNA = ∂
- [Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1992, 1995)] **NONRELATIVISTIC EFFECTIVE FIELD THEORY (NRQED & NRQCD)**
- \rightarrow Same for $(Q\bar{Q})$ systems
- ▸ Same operators but different power counting (different scaling of energy and momenta)

 $\delta \mathcal{L}_{\rm bilinear}$

$$
\mathcal{L}_{\text{heavy}} = \psi^{\dagger} \left(iD_{t} + \frac{\mathbf{D}^{2}}{2M} \right) \psi + \chi^{\dagger} \left(iD_{t} - \frac{\mathbf{D}^{2}}{2M} \right) \chi
$$
\n
$$
\mathcal{L}_{\text{bilinear}} = \frac{c_{1}}{8M^{3}} \left(\psi^{\dagger} (\mathbf{D}^{2})^{2} \psi - \chi^{\dagger} (\mathbf{D}^{2})^{2} \chi \right)
$$
\n
$$
+ \frac{c_{2}}{8M^{2}} \left(\psi^{\dagger} (\mathbf{D} \cdot g \mathbf{E} - g \mathbf{E} \cdot \mathbf{D}) \psi + \chi^{\dagger} (\mathbf{D} \cdot g \mathbf{E} - g \mathbf{E} \cdot \mathbf{D}) \chi \right)
$$
\n
$$
+ \frac{c_{3}}{8M^{2}} \left(\psi^{\dagger} (i\mathbf{D} \times g \mathbf{E} - g \mathbf{E} \times i\mathbf{D}) \cdot \boldsymbol{\sigma} \psi + \chi^{\dagger} (i\mathbf{D} \times g \mathbf{E} - g \mathbf{E} \times i\mathbf{D}) \cdot \boldsymbol{\sigma} \chi \right)
$$
\n
$$
+ \frac{c_{4}}{2M} \left(\psi^{\dagger} (g \mathbf{B} \cdot \boldsymbol{\sigma}) \psi - \chi^{\dagger} (g \mathbf{B} \cdot \boldsymbol{\sigma}) \chi \right),
$$

THE GRAND CHALLENGE: NON-LEPTONIC DECAYS

▸ Naive factorization approach was semi-successful in describing early data,

- ▸ Georgi: *"Why we can't calculate …"*
- but lacked a firm theoretical foundation

[Georgi: *Weak Interactions and Modern Particle Theory* (1984)]

[Bauer, Stech, Wirbel (1986)]

THE GRAND CHALLENGE: NON-LEPTONIC DECAYS

▸ Naive factorization approach was semi-successful in describing early data,

[Georgi: *Weak Interactions and Modern Particle Theory* (1984)]

- ▸ Georgi: *"Why we can't calculate …"*
- but lacked a firm theoretical foundation
- ▸ **QCD factorization approach (BBNS):**
	- ▸ First model-independent calculation of $B \to M_1 M_2$ decay amplitudes from first principles (including strong- and weakinteraction phases) in heavy-quark limit

[Beneke, Buchalla, MN, Sachrajda (1999—2001)] Factorization proof at two-loop order based on method of regions, see pp. 48-79 in BBNS (2000)

[Bauer, Stech, Wirbel (1986)]

QCD FACTORIZATION IN NONLEPTONIC B DECAYS INTO LIGHT MESONS

QCD factorization theorem:

$$
\langle \pi K | Q_i | B \rangle = F_0^{B \to \pi} T_{K,i}^{\text{I}} * f_K \Phi_K + F_0^{B \to K} T_{\pi,i}^{\text{I}} * f_\pi \Phi_\pi + T_i^{\text{II}} * f_B \Phi_B
$$

$$
+ \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)
$$

QCD FACTORIZATION IN NONLEPTONIC B DECAYS INTO LIGHT MESONS

QCD factorization theorem:

$$
\langle \pi K | Q_i | B \rangle = F_0^{B \to \pi} T_{K,i}^{\text{I}} * f_K \Phi_K + F_0^{B \to K} T_{\pi,i}^{\text{I}} * f_\pi \Phi_\pi + T_i^{\text{II}} * f_B \Phi_B * f_K \Phi_K * f_\pi \Phi_\pi
$$
\nspecific for B-r

\ncontributes at

[Beneke, Buchalla, MN, Sachrajda (1999—2001)]

- $\big| * f_K \Phi_K * f_\pi \Phi_\pi$
- $\begin{equation*} \mathbb{F}_2 \longrightarrow \mathbb{F}_4 \longrightarrow \mathbb{F}_$ in particular light-cone distribution amplitudes (LCDAs), to account for hadronic dynamics
	- Spec *m^b* $\mathbf{r} = \mathbf{r} + \mathbf{r}$ ▸ Second term corresponds to **Brodsky-Lepage (1980)**, while the first term is specific for *B*-meson decays and contributes at the same order in $\Lambda_{\rm QCD}/m_b$

QCD FACTORIZATION IN NONLEPTONIC B DECAYS INTO LIGHT MESONS

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- \blacktriangleright ln 2001, fact that $\mathsf{Im}(V_{td}) \neq 0$ had been \overline{R} and $\overline{B}-\overline{B}$ and $\overline{B}-\overline{B}$ mixing and first measurements of sin 2*β* **CPN 2004** analysis: $\bar{\rho} = 0.15 \pm 0.08$, $\bar{\eta} = 0.36 \pm 0.09$ 2004 analysis: $\bar{\rho} = 0.15 \pm 0.08$, $\bar{\eta} = 0.36 \pm 0.09$ $\gamma = (67 \pm 15)^\circ, \quad \beta = (24 \pm 2)^\circ$ 2021 values: $\bar{\rho} = 0.157^{+0.009}_{-0.005}$, $\bar{\eta} = 0.347^{+0.012}_{-0.005}$
- **▶ Fact that** $Im(V_{ub}) \neq 0$ **has been established by** studying rare hadronic decays () *B* → *πK*, *ππ* in QCD factorization [BBNS (2001), here updated to 2004 data] $F = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
- ▸ **KM relation confirmed;** most stringent test of KM mechanism at the time

 $\gamma = (65.5^{+1.3}_{-1.2})^{\circ}, \quad \beta = (22.42^{+0.64}_{-0.37})^{\circ}$

hadronic B decays using QCD factorization: [BBNS 01] [CKMfitter global fit, spring 2021]

CONFIRMATION OF KM RELATION BETWEEN IM(V_{UB}) AND IM(V_{TB})

- ▸ Measuring time-dependent CP asymmetries $\sin B \to \pi\pi$ and $B \to \pi\rho$ decays one obtains an internally consistent determination of *γ*
- ▸ 2003 analysis found: *γ* = (62 ± 8) ∘
- ▸ 2021 value: $\gamma = (65.5^{+1.3}_{-1.2})$ $^{+1.5}_{-1.2})$ **o**

CONFIRMATION OF KM RELATION BETWEEN IM(V_{UB}) AND IM(V_{TB})

LIMITATIONS OF QCD FACTORIZATION

- ▸ Lots of predictive power, but uncertainties due to hadronic input quantities: form factors, decay constants, and LCDAs (reducible to some extent)
- ▶ Power corrections in $\Lambda_{\rm QCD}/m_b$ do not (naively) factorize due to endpoint divergences (⇒ different meanings of "factorization")
- ▸ In some cases, power-suppressed effects can be enhanced by large Wilson coefficients (e.g. "color-suppressed" decay modes)
- ▸ To make progress, one needed an EFT implementation of QCD factorization

SOFT-COLLINEAR EFFECTIVE THEORY (SCET) *in*¯ *· ^D iD*/ ‹ ²⁷² As a next step, one separates the large and residual momentum components by decomposing the collinear momentum into a residual momentum into a residual momentum, *p*
[Bayer (Elemina) Piriel Stewart (2001): Beneke Chanevski, Diehl, Feldmann (2002)]

▸ Firm theoretical basis for deriving QCD factorization theorems in heavy-quark

 $\xi_n(x) + \ldots$

▸ Scale separation and resummation accomplished using powerful EFT tools 2866 small momentum components is to derive the Lagrangian of SCET in position space \sim 56 . In this case \sim \blacktriangleright 3 care separation and resummation accompilshed using

Matthias Neubert - 15 289 Nathias Neubert – 15 Sepanjan is not corrected by short distance fluctuations. The physical represent distance fluctuations. The physical reason is that in the physical reason is that in the physical reason is that in

[Bauer, (Fleming,) Pirjol, Stewart (2001); Beneke, Chapovski, Diehl, Feldmann (2002)] [Bauer, (Fleming,) Pirjol, Stewart (2001); Beneke, Chapovski, Diehl, Feldmann (2002)]

- and collider physics for processes involving light energetic particles **Example 275 Depart Conservatives and** *DCD* factorization the Here the label momentum in Scenes and the label operator conserved, one operation in Section 276 and 2
- ▸ Collinear effective Lagrangian: Note that at leading order in power counting *ⁱD^µ* ²⁷⁸ *ⁿ* does not contain the soft gluon field. This leads **279 Collinear effective**

eikonal interaction, can be removed by the field redefinition $\xi_n \to S_n \xi_n^{(0)}$ ²⁸¹ latter term gives rise to the only interaction between a collinear quark and soft gluons at leading 282 **elkonal interaction, can be removed by**
The electron interactions between soft and collinear particles.

▶ Soft-collinear factorization at Lagrangian level \sim Soft collinear factorization at Lagrangian level 8 Soit-coilinear iactorization at Lagrangian lever large and the separation of the separation of the separation o

² *^Ân*(*x*)*.* (16.12) Lfinal1

$$
\mathcal{L}_n = \bar{\xi}_n(x) \left[in \cdot D_n + gn \cdot A_s + i \mathcal{D}_n^{\perp} \frac{1}{i \bar{n} \cdot \mathcal{D}_n} i \mathcal{D}_n^{\perp} \right] \frac{\vec{\eta}}{2} \xi_n(x) + \dots
$$

H $J \sqrt{J}$ J / J S

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SCET PROOF OF QCD FACTORIZATION FOR *B* → *K***γ* **DECAY**

[Becher, Hill, MN (2005)]

Two-step matching procedure $QCD \rightarrow SCET-1 \rightarrow SCET-2$:

Product/convolution of component functions each depending on a single scale:

PROTOTYPICAL SCET FACTORIZATION THEOREM

Product/convolution of component functions each depending on a single scale:

PROTOTYPICAL SCET FACTORIZATION THEOREM

▸ Extension to next-to-leading power is a hard problem, due to **endpoint-divergent**

- **convolution integrals**
- for dealing with this problem

▸ **Refactorization-based subtraction** (RBS) scheme provides a consistent framework [Liu, MN (2019, 2020); Liu, Mecaj, MN, Wang (2021); Liu, MN, Schnubel, Wang (2022)]

[Beneke et al. ; Moult et al.; Stewart et al.; Bell et al. (2018—2022)]

- ▸ at next-to-leading power in scale ratios, due to endpoint divergences
- ▸ when QED corrections are included to reach *O*(1%) accuracy, since external hadron states are in general not singlets under electromagnetism [Beneke, Bobeth, Szafron (2019); Beneke, Böer, Toelstede, Vos (2020, 2022)]
	- ▸ hadronic input (decay constants, form factors, LCDAs) need to be redefined
	- ▸ many additional hadronic matrix elements enter
	- ▸ leptonic decays become as complicated as non-leptonic decays (since leptons $ℓ[−]$ are charged)

SCET-based factorization theorems become far more complicated:

TWO FRONTIERS OF SCET FACTORIZATION

FACTORIZING THE SIMPLEST B DECAY

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:

- ▸ **Determination of |Vub|,** largely unaffected by hadronic uncertainties
- ▸ Chiral suppression offers sensitive probe of new interactions
- ▸ **Test of lepton universality** by comparing decays with different lepton flavors \Rightarrow Belle II will measure $\ell = \mu, \tau$ channels with 5-7% uncertainty [Belle II Physics Book]

B-meson decay constant

$$
\Gamma \sim m_{\ell}^2 f_{B_u}^2 |V_{ub}|^2
$$

FACTORIZING THE SIMPLEST B DECAY

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:

▸ QED corrections can be of similar magnitude or even larger, due to presence of **large logarithms** $\alpha \ln^2(m_B/m_e)$ and (m_B/m_e) and $\alpha \ln(m_B/E_\gamma) \ln(m_B/m_e)$

▸ QCD matrix element is known with <1% accuracy:

 $\langle 0|\bar{u}\gamma^{\mu}\gamma_{5}b|B^{-}(p)\rangle = i f_{B_{u}} p^{\mu}$ with $f_{B_{u}} = (189.4 \pm 1.4) \,\text{MeV}$

B-meson decay constant

$$
\Gamma \sim m_{\ell}^2 f_{B_u}^2 |V_{ub}|^2
$$

[FNAL/MILC (2017)]

FACTORIZING THE SIMPLEST B DECAY

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:

- S_n^\dagger to account for soft photon interactions with the charged lepton $\bar{u} \gamma^{\mu} P_L b$ is not gauge invariant under QED \Rightarrow *n*
- local matrix elements (LCDAs) under renormalization! *B*

 \blacktriangleright **Problem:** Defining f_B or the corresponding HQET parameter F with such a Wilson line is **incompatible** with *F* being a local parameter, since it would mix with non-[Cornella, König, MN (2022)]

B-meson decay constant

 \blacktriangleright Quark current $\bar u\,\gamma^\mu P_L\,b$ is not gauge invariant under QED \Rightarrow add a soft Wilson line

$$
\Gamma \sim m_{\ell}^2 f_{B_u}^2 |V_{ub}|^2
$$

▸ Intermediate scale range gives rise to intricate effects, as photons can resolve 2*mB* elow the scale $\Lambda_{\rm QCD}!$ $\mathbf{1}$

FACTORIZING THE SIMPLEST B DECAY The initital-state *B* meson can emit a photon and **transition** into an **excited state**.

QED effects are well under control for scales $\mu \gg m_b$ (effective weak Hamiltonian) and $\mu \ll \Lambda_{\rm OCD}^2/m_B$ (Low's theorem) $\frac{2}{\text{QCD}}/m_B$

the **inner structure of the** *B* **meson,** above and below the scale $\Lambda_{\text{QCD}}!$

JULT RUGIZATION DIEUTENT IUT VII LUAT CON ECHUNIS
ICorpolla Käpia MNI (2022) $\frac{1}{2}$ corrections at the dependent $\frac{1}{2}$ [Cornella, König, MN (2022)] SCET factorization theorem for virtual corrections Heavy-meson EFT for real emissions

enhanced wrt. the scalar case.

[Cornella, Ferré, König, MN, to appear]

The Power of Effective Field Theori**D** The Power of Effective Field Theori $B\to\mu\bar\nu$

FACTORIZING THE SIMPLEST B DECAY
 $B \to \mu \bar{\nu}$ Scales $B\to \mu\bar\nu$

While in the absence of QED effects $B^-\to\mu^-\bar{\nu}_\mu$ is governed by only 3 scales i $(m_W \gg m_b \gg \Lambda_{\rm QCD})$, with QED leff et a included 8 scales become relevant: B^- → $\mu^- \bar{\nu}_{\mu}$ tha

We have analyzed the factorization of the $B^-\to\mu^-\bar{\nu}_\mu$ amplitude including QED B^- → $\mu^- \bar{\nu}_{\mu}$

▶ Relevant modes in the EFT:

FACTORIZING THE SIMPLEST B DECAY

corrections in SCET & HPET: Rcomella, König, MN (2022)] analy

▶ hard-collinear <**---** resolve the light-cone structure of the *B* meson (à la Brodsky-Lepage)

- - ▸ hard
	-
	- ▸ soft
	- ▸ collinear
	- ▶ soft-collinear
- - ▸ ultra-soft
	-

▸ Relevant modes for real QED corrections:

▸ ultra-soft-collinear

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FACTORIZING THE SIMPLEST B DECAY

corrections in SCET & HPET: Rcomella, König, MN (2022)] analy

- - ▸ hard
	- ▸ hard-collinear
	- ▸ soft
	- ▸ collinear
	- ▶ soft-collinear
- - ▸ ultra-soft
	-

We have analyzed the factorization of the $B^-\to\mu^-\bar{\nu}_\mu$ amplitude including QED B^- → $\mu^- \bar{\nu}_{\mu}$

▶ Relevant modes in the EFT:

▶ Relevant modes for real QED corrections:

▸ ultra-soft-collinear

by /*n*. Using that /*v b^v* = *bv*, it is straightforward to derive the relation

m`, which is a power-suppressed parameter in SCET-1. Since the soft quark current scales

$$
O_1^{B(5)} = \frac{1}{\bar{n} \cdot \mathcal{P}_{hc}} \bar{u}_s \not\bar{n} P_L b_v \bar{X}_{hc}^{(\ell)} \mathcal{A}_{hc[y]}^{\perp} P_L \nu_{\bar{c}},
$$
\n
$$
O_5^{D(\frac{9}{2})} = \frac{1}{m_B^2}
$$
\n
$$
O_5^{(\frac{9}{2})} = \frac{1}{m_B^2}
$$

 $\mathcal{L}^{(0)}$

$$
O_3^{B(5)} = \frac{1}{\bar{n} \cdot \mathcal{P}_{hc}} \bar{u}_s \not\bar{n} P_L \mathcal{G}_{hc[y]}^{\perp \alpha} b_v \bar{X}_{hc}^{(\ell)} \gamma_\alpha^{\perp} P_L \nu_{\overline{c}}, \qquad O_7^{D(\frac{s}{2})} = \frac{1}{m_B^2}
$$

$$
O_1^{\ C(5)} = \frac{m_{\ell}}{m_B^2} \left[\bar{\chi}_{hc[y]}^{(u)} \left(-i \overline{\phi}_{\perp} \right) \right] P_L b_v \ \bar{\chi}_{hc}^{(\ell)} P_L \nu_{\overline{c}},
$$
\n
$$
O_1^{(5)} = \frac{1}{m_B^2} \left[\bar{\chi}_{hc[y_1]}^{(u)} \left(-i \overline{\phi}_{\perp} \right) \right] P_L b_v \ \bar{\chi}_{hc}^{(\ell)} P_L \nu_{\overline{c}},
$$
\n
$$
O_{11}^{(5)} = \frac{m_{\ell}}{m_B^2}
$$

 $\bar{\chi}^{(u)}_{hc[y]} P_R \, b_v \, \, \bar{\chi}^{(\ell)}_{hc} P_L \, \nu_{\overline{c}} \, ,$

$$
O_3^{C(4,\frac{9}{2})} = \frac{1}{m_B} \, \bar{X}_{hc[y]}^{(u)}(-i\overleftarrow{D}_s^{\alpha}) P_R \, b_v \, \, \bar{X}_{hc}^{(\ell)} \gamma^{\perp}_{\alpha} P_L \nu_{\overline{c}} \,, \qquad O_{12}^{D(5)} = \frac{m_B}{\overline{C}} \, \frac{1}{m_B} \, \frac{1}{
$$

FACTORIZING THE SIMPLEST B DECAY — SCET-1 OPERATOR BASIS rau iu *^O^A*(⁹ 2) ⁰ = ¯*us*↵ ?*P^L ^b^v* ^X¯(`) hard-collinear sector instead of the lepton mass in addition to the operators in (19). If this object carries the Lorentz index , then the quark bilinear must contain a two-index Dirac <u>i agtoimeino the oirth eeo</u>t *d'*egle derivative *in ·* @ acting on a hard-collinear field.

The large component of the total hard-collinear momentum is now shared among the total hard-collinear momentum is now shared among the total hard-collinear momentum is now shared among three fields. In the collinear momen We assign momentum fractions *y*¹ and *y*² to the up-quark and the gauge field, respectively, which implies that the charged lepton carries momentum fraction *y*³ = (1 *y*¹ *y*2). vanishes due to the first magic identity in (14). The same is true for operators containing a In an analogous way, the type-*D* operators containing two hard-collinear fermion fields and

O^D(5)

¹³ ⁼ *^m*`

X¯(*u*)

 $\overline{}$

$$
O_1^{A(\frac{11}{2})} = \frac{m_\ell}{\bar{n} \cdot \mathcal{P}_{hc}} \bar{u}_s \not\bar{n} P_L b_v \bar{X}_{hc}^{(\ell)} P_L \nu_{\bar{c}}, \qquad O_1^{D(4)} = \frac{1}{m_B} \bar{X}_{hc}^{(u)}
$$

P^L b^v X¯(`)

hc P^L ⌫*^c .*

 $\left(\varphi_{\perp}\right) \left[\varphi_{L}\nu_{\overline{c}}\right] ,$ $\bar{\chi}^{(u)}_{hc[y_1]} P_R\, b_v\, \, \bar{\chi}^{(\ell)}_{hc}\, {\cal A}^{\perp}_{hc[y_2]} P_L\, \nu_{\overline{c}}\, ,$ $\bar{\chi}^{(u)}_{hc[y_1]} \mathcal{G}^{\perp \alpha}_{hc[y_2]} P_R \, b_v \,\, \bar{\chi}^{(\ell)}_{hc} \, \gamma^{\perp}_{\alpha} P_L \, \nu_{\overline{c}} \, ,$ $\bar{\chi}^{(u)}_{hc[y_1]}(-i$ $(\overleftarrow{\partial}_{\perp})P_Lb_v\,\,\overline{\mathfrak{X}}_{hc}^{(\ell)}\mathcal{A}_{hc}^{\perp}[y_2]P_L\nu_{\overline{c}}\,,$ $\binom{u}{hc[y_1]}(-i)$ $(\overleftarrow{\partial}_{\perp})\, \mathcal{G}_{hc[y_2]}^{\perp\alpha} P_L\, b_v\, \, \bar{\chi}_{hc}^{(\ell)}\, \gamma_\alpha^\perp P_L\, \nu_{\overline{c}}\,,$ $\bar{\chi}_{hc[y_1]}^{(u)}$ ${\cal A}_{hc[y_2]}^{\perp} P_Lb_v \,\, \big[\bar{\chi}_{hc}^{(\ell)}(-i) \big]$ \overleftarrow{a} $\big[\overline{\!\!\partial\!\!\!\!/}_\bot\big)\big] P_L \nu_{\overline{c}}\,,$ $\bar{\mathfrak{X}}^{(u)}_{hc[y_1]}\mathcal{G}^{\perp}_{hc[y_2]}P_Lb_v\,\left[\bar{\mathfrak{X}}^{(\ell)}_{hc}(-i\right]$ $\overleftarrow{\partial}$ $\left[\overline{\partial}_{\bot}\right]\right]P_{L}\nu_{\overline{c}}\,,$ $\Omega^{D(\frac{9}{2})}$ a result these operators do not have $\overline{\mathbf{v}}^{(\ell)}$ in $\overline{\mathbf{v}}^{(\ell)}$ in $\overline{\mathbf{v}}^{(\ell)}$ $iP_L\nu_{\overline{c}},$ $O_7^{D(\frac{9}{2})} = \frac{1}{m_\perp^2} \bar{\mathfrak{X}}_{hc[y_1]}^{(u)}(-i\overleftarrow{\partial}_\perp^{\alpha}) \mathcal{A}_{hc[y_2]}^{\perp} P_L b_v \bar{\mathfrak{X}}_{hc}^{(\ell)} \gamma_\alpha^{\perp} P_L \nu_{\overline{c}},$ **Wilson** b $\bar{X}_{hc[y_1]}^{(u)}(-i\overleftarrow{\partial}_{\perp}^{\alpha})\mathcal{A}_{hc[y_2]}^{\perp}P_Lb_v\ \bar{X}_{hc}^{(\ell)}\gamma_{\alpha}^{\perp}P_L\nu_{\overline{c}}\,,$ **WIISON** $\int_{hc[y_1]}^{(u)}(-i\overleftarrow{\partial}_{\perp}^{\alpha})\,\mathcal{G}_{hc}^{\perp}[y_2]P_L\,b_v\,\,\bar{\mathfrak{X}}_{hc}^{(\ell)}\,\gamma_{\alpha}^{\perp}P_L\,\nu_{\overline{c}}\,,$ $\left(i\partial\!\!\!/\!\!\!\!\!{\partial_\perp}\mathcal{A}_{hc[y_2]}^{\perp\alpha}\right.$ $\int P_L b_v \, \bar{\mathfrak{X}}_{hc}^{(\ell)} \gamma^{\perp}_{\alpha} P_L \nu_{\overline{c}} \, ,$ $\left(i\partial\!\!\!/_{\perp}\mathcal{G}_{hc[y_2]}^{\perp\alpha}\right)$ $\int P_L b_v \, \bar{\mathfrak{X}}_{hc}^{(\ell)} \gamma^{\perp}_{\alpha} P_L \nu_{\overline{c}}$, $\bar{\chi}^{(u)}_{hc[y_1]}\, \mathcal{A}^{\perp}_{hc[y_2]} P_L\, b_v\, \, \bar{\chi}^{(\ell)}_{hc} P_L\, \nu_{\overline{c}}\, ,$ $\bar{\chi}^{(u)}_{hc[y_1]} \mathcal{G}^{\perp}_{hc[y_2]} P_L b_v \,\, \bar{\chi}^{(\ell)}_{hc} P_L \nu_{\overline{c}} \, .$

$$
O_2^{A(\frac{11}{2})} = \frac{m_\ell}{m_B} \bar{u}_s P_R b_v \bar{X}_{hc}^{(\ell)} P_L \nu_{\overline{c}}, \qquad O_2^{D(4)} = \frac{1}{m_B} \bar{X}_{hc}^{(u)}
$$

$$
O_3^{A(5)} = \frac{1}{\bar{n} \cdot \mathcal{P}_{hc}} \bar{u}_s \not\bar{n} P_L b_v \left[\bar{\mathfrak{X}}_{hc}^{(\ell)}(-i\overleftrightarrow{\partial}_{\perp}) \right] P_L \nu_{\overline{c}}, \qquad O_3^{D(\frac{9}{2})} = \frac{1}{m_B^2}
$$

$$
O_4^{A(5)} = \frac{1}{m_B} \bar{u}_s P_R b_v \left[\bar{\mathfrak{X}}_{hc}^{(\ell)}(-i\overleftrightarrow{\partial}_{\perp}) \right] P_L \nu_{\overline{c}}.
$$
\n
$$
O_4^{D(\frac{9}{2})} = \frac{1}{m_B^2} \bar{\mathfrak{X}}_{hc[y]}^{(u)}
$$

$$
O_2^{B(5)} = \frac{1}{m_B} \bar{u}_s P_R b_v \bar{X}_{hc}^{(\ell)} \mathcal{A}_{hc[y]}^{\perp} P_L \nu_{\overline{c}},
$$
\n
$$
O_6^{D(\frac{9}{2})} = \frac{1}{m_B^2} \bar{X}_{hc[y]}^{(u)}
$$

$$
O_{1}^{E(\frac{9}{2})} = \frac{1}{m_{B}^{2}} \bar{X}_{hc[y_{1}]}^{(u)} \mathcal{A}_{hc[y_{2}]}^{\perp} P_{L} b_{v} \bar{X}_{hc}^{(\ell)} \mathcal{A}_{hc[y_{3}]}^{\perp} P_{L} \nu_{\overline{c}},
$$

\n
$$
O_{2}^{E(\frac{9}{2})} = \frac{1}{m_{B}^{2}} \bar{X}_{hc[y_{1}]}^{(u)} \mathcal{G}_{hc[y_{2}]}^{\perp} P_{L} b_{v} \bar{X}_{hc}^{(\ell)} \mathcal{A}_{hc[y_{3}]}^{\perp} P_{L} \nu_{\overline{c}},
$$

\n
$$
O_{3}^{E(\frac{9}{2})} = \frac{1}{m_{B}^{2}} \bar{X}_{hc[y_{1}]}^{(u)} \mathcal{A}_{hc[y_{2}]}^{\perp} \mathcal{G}_{hc[y_{3}]}^{\perp \alpha} P_{L} b_{v} \bar{X}_{hc}^{(\ell)} \gamma_{\alpha}^{\perp} P_{L} \nu_{\overline{c}},
$$

\n
$$
O_{4}^{E(\frac{9}{2})} = \frac{1}{m_{B}^{2}} \bar{X}_{hc[y_{1}]}^{(u)} \mathcal{G}_{hc[y_{2}]}^{\perp} \mathcal{G}_{hc[y_{3}]}^{\perp \alpha} P_{L} b_{v} \bar{X}_{hc}^{(\ell)} \gamma_{\alpha}^{\perp} P_{L} \nu_{\overline{c}},
$$

\n
$$
O_{5}^{E(\frac{9}{2})} = \frac{1}{m_{B}^{2}} \bar{X}_{hc[y_{1}]}^{(u)} \mathcal{G}_{hc[y_{2}]}^{\perp \alpha} \mathcal{G}_{hc[y_{3}]}^{\perp} P_{L} b_{v} \bar{X}_{hc}^{(\ell)} \gamma_{\alpha}^{\perp} P_{L} \nu_{\overline{c}}.
$$

$\mathbb{E}^{(l)}$ the section on the operator basis construction $\mathbb{E}^{(l)}$ $\lambda_{hc} \gamma_\alpha L L \nu_{\overline{c}}$, $H_i(m_b, \mu)$ is the Lagrangian and soft fields would need to $H_i(m_b, \mu)$ Wilson coefficients are hard functions:

$$
O_4^{B(5)} = \frac{1}{m_B} \bar{u}_s P_R \mathcal{G}_{hc[y]}^{\perp \alpha} b_v \bar{\mathcal{X}}_{hc}^{(\ell)} \gamma_\alpha^{\perp} P_L \nu_{\overline{c}}.
$$
\n
$$
O_8^{D(\frac{9}{2})} = \frac{1}{m_B^2} \bar{\mathcal{X}}_{hc[y_1]}^{(u)} (-i\overleftarrow{\partial}_{\perp}^{\alpha}) \mathcal{G}_{hc[y_2]}^{\perp} P_L b_v \bar{\mathcal{X}}_{hc[y_3]}^{\perp}
$$

 ν ^{*z*},

$$
O_1^{C(\frac{9}{2})} = \frac{m_\ell}{m_B} \bar{\chi}_{hc[y]}^{(u)} P_R b_v \bar{\chi}_{hc}^{(\ell)} P_L \nu_{\overline{c}},
$$
\n
$$
O_1^{D(\frac{9}{2})} = \frac{1}{m_B^2} \bar{\chi}_{hc[y_1]}^{(u)}
$$
\n
$$
O_{10}^{D(\frac{9}{2})} = \frac{1}{m_Z^2} \bar{\chi}_{hc[y_1]}^{(u)}
$$
\n
$$
O_{10}^{D(\frac{9}{2})} = \frac{1}{m_Z^2} \bar{\chi}_{hc[y_1]}^{(u)}
$$

 $\overline{1}$

$$
O_4^{C(4,\frac{9}{2})} = \frac{1}{m_B} \bar{\mathfrak{X}}_{hc[y]}^{(u)} P_R b_v \left[\bar{\mathfrak{X}}_{hc}^{(\ell)}(-i \overleftrightarrow{\overline{p}}_{s\perp}) \right] P_L \nu_{\overline{c}},
$$

\n
$$
O_5^{C(\frac{9}{2})} = \frac{1}{m_B^2} \bar{\mathfrak{X}}_{hc[y]}^{(u)}(-i \overleftrightarrow{\overline{\phi}}_{\perp}) P_L b_v \left[\bar{\mathfrak{X}}_{hc}^{(\ell)}(-i \overleftrightarrow{\overline{\phi}}_{\perp}) \right] P_L \nu_{\overline{c}},
$$
\n[Corr

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 \mathbb{Z}_p

 m_B^2

 m_B^2

 $O_{12}^{D(5)} = \frac{m_{\ell}}{m^2}$

 m_B^2

 m_B^2

 m_B^2

m^B

[Cornella, Ferré, König, MN, to appear]

FACTORIZING THE SIMPLEST B DECAY — SCET-2 OPERATOR BASIS Operator Class A Operator Class C \overline{C} \mathbf{C} *OB* ² (*w*) = *^m*` *m^B* LLVI *d U* AT B BEA

contribution without gluons, only the SCET I operators *O^D* contribute to the match-

$$
\mathcal{O}_1^A = \frac{m_\ell}{(\bar{n} \cdot \mathcal{P})} \left(\bar{\mathcal{Q}}_s \bar{\boldsymbol{\psi}} P_L \mathcal{H}_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) \qquad \mathcal{O}_1^C(\boldsymbol{y}) = \frac{1}{(\bar{n} \cdot \mathcal{P})}
$$

OB

^Q¯[!]

^s PRh^v

^C ^PL(⌫)

 $\overline{}$

$$
\mathcal{O}_2^{\mathcal{A}} = \frac{m_\ell}{m_B} \left(\bar{\mathcal{Q}}_s P_R \mathcal{H}_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) \qquad \qquad \mathcal{O}_2^C(y) = \frac{1}{m_B}
$$

$$
\mathcal{O}_1^B(w) = \frac{m_\ell}{(\bar{n} \cdot \mathcal{P})} \left(\bar{\mathcal{Q}}_s^{[\omega]} \vec{\eta} P_L \mathcal{H}_v \right) \left(\bar{\chi}_C^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) \n\qquad \qquad \mathcal{O}_1^D(w, y) = \frac{1}{(\bar{n} \cdot \mathcal{P})}
$$

 $^{\circ}$ *SS*^{\cdot} *^A/* [!⁰] *s*? *PRH^v* $x + t_0$ 19 h *n/ in ·* @ *G/*[!0] *s*? i *PRH^v* ⌘⇣ ¯(`) *^c ^A/* [*y*] *^c*?*PL*(⌫) *c*¯ [Cornella, Ferré, König, MN, to appear]

$$
\mathcal{O}_2^B(w) = \frac{m_\ell}{m_B} \left(\bar{\mathcal{Q}}_s^{[\omega]} P_R h_v \right) \left(\bar{\chi}_C^{(\ell)} P_L \chi_c^{(\nu)} \right)
$$

$$
\mathcal{O}_2^D(w, y) = \frac{1}{m_B} \left(\frac{1}{\bar{\mathcal{Q}}^{[\omega]}} \left[\frac{m}{\bar{\mathcal{Q}}^{[\omega]}} \left[\frac{1}{\bar{\mathcal{Q}}^{[\omega]}} \right] P_R \mathcal{H}_v \right] \left(\bar{\chi}^{(\ell)} P_L \chi_c^{(\nu)} \right) \right)
$$

$$
\mathcal{O}_2^F(w, w', y) = \frac{1}{m_B} \left(\frac{1}{\bar{\mathcal{Q}}^{[\omega]}} \left[\frac{1}{\bar{\mathcal{Q}}^{[\omega]}} \right] \left[\frac{1}{\bar{\mathcal{Q}}^{[\omega]}} \right] P_R \mathcal{H}_v \right) \left(\bar{\chi}^{(\ell)} P_L \chi_c^{(\nu)} \right) \right)
$$

$$
\mathcal{O}_1^E(w, w') = \frac{m_\ell}{m_B} \left(\bar{\mathcal{Q}}_s^{[\omega]} \left[\frac{\mathcal{N}}{in \cdot \partial} \mathcal{A}_{s\perp}^{[\omega']} \right] P_R \mathcal{H}_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) \qquad \mathcal{O}_1^F(w, w', y) = \frac{1}{m} \mathcal{O}_2^E(w, w') = \frac{m_\ell}{m_B} \left(\bar{\mathcal{Q}}_s^{[\omega]} \left[\frac{\mathcal{N}}{in \cdot \partial} \mathcal{G}_{s\perp}^{[\omega']} \right] P_R \mathcal{H}_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right) \qquad \mathcal{O}_2^F(w, w', y) = \frac{1}{m} \mathcal{O}_2^F(w, w', y) = \frac{1}{m}
$$

OD ¹ (*w, y*) = ¹ \overline{a} \overline{a} *^Q*¯[!] *^s nP/*¯ *^LH^v* ⌘⇣¯(`) *^c ^A/* [*y*] *^c*?*PL*(⌫) *c*¯ $\frac{1}{2}$ *^Q*¯[!] *^c*?*PL*(⌫) **C**uite generically, things get *m^B* α ⁻ Ļ, *OF* ¹ (*w, w*⁰ *, y*) = ¹ *m^B* \overline{a} *, y*) = ¹ det verv *m^B* m ⌘ ▸ Quite generically, things get very messy at next-to-leading power!

OB

¹ (*w*) = *^m*`

(¯*n · P*)

^Q¯[!]

^s nP/¯ *^LH^v*

¯

^C ^PL(⌫)

OE ² (*w, w*⁰) = *^m*` *m^B* Operator Class F

 $\mathcal{O}^F(w, w', w') = -$

c¯

 $\mathcal{O}^{F}_2(w,w',y)=\frac{1}{m}$

L

 $=$ $\frac{1}{2}$

⌘⇣ ¯ (`) *^C ^PL*(⌫) *c*¯ 2 NPERATNR *^Q*¯[!] *^s PRh^v ^C ^PL*(⌫) *c*¯ $\overline{}$ *OD* ¹ (*w, y*) = ¹ ⇣ *^Q*¯[!] *^s nP/*¯ *^LH^v* ⌘⇣ ¯(`) *^c ^A/* [*y*] *^c*?*PL*(⌫) ⌘

Wilson coefficients are jet functions: $J_i(m_b\omega,\mu)$ h *n/ in ·* @ e je *s*? *PRH^v* ⌘⇣ dons: *in ·* @ ι^{n} ^b ∞ , μ ^o *^A/* [!⁰ \overline{a}

OC

² (*y*) = ¹

⇣

^Q¯*sPRH^v*

⌘⇣

^c ^A/ [*y*]

 $\overline{}$

^c?*PL*(⌫)

 \mathcal{C}_2

m^B

 m_B

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

- ▸ Hard functions: matching corrections at *μ* ∼ *mb*
- ▸ Jet functions: matching corrections at *μ* ∼ (*mb*ΛQCD)
- \rightarrow Soft functions: B -meson matrix elements (local and non-local) in HQET
- ▸ Collinear functions: leptonic matrix elements, *μ* ∼ *m^μ*

1/2

SCET-1 operators with soft SCET-1 operators with hardspectator quark collinear spectator quark

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

$$
d\omega \int_0^1 dx \, H_B(m_b, x) \, J_B(m_b \omega, x) S_B(\omega)
$$

- ▸ Endpoint-divergent convolution integrals: $\mathcal{A}_{B \to \ell \bar{\nu}}^{\mathrm{virtual}} \propto \left[H_A(\ell) \right]$ \mathcal{I}_A ($\mathcal{A}_{B\to\ell\bar\nu}^{\text{virtual}}$ \propto $\left|H_A(m_b)S_A+\right|d\omega$ $\sqrt{2}$ $H_A(m_b)S_A+$ z
Z \propto $|H_A(m_b)S_A + \int d\omega$
- ▸ Focus on second term:
	- \blacktriangleright Shared variable $x =$ collinear momentum fraction carried by the spectator quark
	- \rightarrow *H_B* ∼ *x*^{−*€*}, *J_B* ∼ *x*^{−1−*€*} ⇒ *H_B* ⊗ *J_B* is endpoint divergent
- ▶ Cannot be removed with standard RG techniques, but treatable within the **refactorization-based subtraction scheme** [Liu, MN (2019); Liu, Mecaj, MN, Wang (2020); Beneke et al. (2022)]

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

▸ Endpoint-divergent convolution integrals: $\mathcal{A}_{B \to \ell \bar{\nu}}^{\mathrm{virtual}} \propto \left[H_A(\ell) \right]$ \mathcal{I}_A ($(m_b)S_A+\int d\omega$ \int_0^1 J_{0} $dx\,H_B(m_b,x)\,J_B(m_b\omega,x)$ \mathbf{r} $\mathcal{A}_{B\to l\bar{\nu}}^{\text{virtual}}$ \propto $\left| H_A(m_b)S_A + \int d\omega \int dx H_B(m_b,x) J_B(m_b\omega,x)S_B(\omega) \right|$ $\sqrt{2}$ $H_A(m_b)S_A+$ z
Z $d\omega$ \int_0^1 0 $dx H_B(m_b, x) J_B(m_b\omega, x) S_B(\omega)$ $\overline{}$ ∝

- ▸ After refactorization, the convolutions are well defined and the **HQET decay constant F contained in SA is redefined** in such a way that it now no longer mixes with non-local matrix elements under renormalization
- ▸ Would be interesting to compute this redefined HQET parameter using lattice QCD!

reshuffle divergent terms

\overline{C} ctions at $\mathcal{O}(\alpha)$: and the parameter *F* in the MS scheme and performing $\frac{1}{2}$ where two terms in the two-second line probe two-

*^A*virtual

^B!`⌫¯ = *i*

2*G^F K*EW(*µ*)*Vub*

^p*m^B ^F*(*µ, mb, w*)

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS TAUIURIZAHUN FURMULA FUR V WE ARE NO PROTECTED TO PRESENT OUR MAIN RESULT. *B* ! *µ*⌫¯*^µ* decay amplitude including virtual QED cor-**EACTORIZE** WITH A DIVID A DIVID AND CAN BE OF A CAN BE OF A LATITLE A LATITLE CAN BE OF CAN BE OF THE LATITLE ATION I ONVIOLA I ON DIZATION FODMIII A FOD *· u*¯(*p*`)*P^L v*(*p*⌫) lem mentioned earlier, such that the anomalous dimen-**FAUIUKIZAIIUN FUKMULA FUK VIKIUAL** 2*G^F K*EW(*µ*)*Vub m*` *m^b* ^p*m^B ^F*(*µ, mb, w*) **ITATION EODMIII A EOD VIDTIIAI POD** sion of the parameter *F*, defined via *dF*(*µ,*⇤*, w*)*/d* ln *µ* = *^A*virtual *^B*!`⌫¯ = *i* $t_n = t_{n-1} - t_{n-1} - t_{n-1} - t_{n-1} - t_{n-1}$ lem mentioned earlier, such that the anomalous dimension of the parameter *F*, defined via *dF*(*µ,*⇤*, w*)*/d* ln *µ* = **PATIANA** *^A*virtual

The subtraction performance performed in \mathcal{L}_c , in conjunction with \mathcal{L}_c , in conjunction with \mathcal{L}_c

Decay amplitude including virtual QED corrections at $O(\alpha)$: with: *^F F*(*µ,*⇤*, w*), is now local and independent of IR reg- ${\cal A}^{\rm virtual}_{B \to \ell \bar\nu} = i$ Decay amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$: *^F F*(*µ,*⇤*, w*), is now local and independent of IR reg-*B* ! *µ*⌫¯*^µ* decay amplitude including virtual QED coripillude including v $\sqrt{ }$ $2\,G_F\,K_{\rm EW}(\mu) V_{ub}$ m_ℓ *m^b* ${\cal A}^{\rm virtual}_{B^{\rightarrow}\ell\bar{\nu}}=i\sqrt{2}G_{F}K_{\rm EW}(\mu)V_{ub}\frac{m_{\ell}}{\infty}\sqrt{m_{B}}\overline{F(\mu)}\ \bar{u}(p_{\ell})I_{\ell}$ *^F* = *C^F* 4⇡ 4⇡ ³ *^Q*`*Q^u* ln ⇤² *µ*2 ` *^Q*² ✓ ◆ $\frac{1}{2}$ $\frac{1}{2}$ $\sqrt{2}$ I is also possible to control the dependence on the dependence on the cuto I *F* \overline{A} ^{rint} 3↵*^s* 4⇡ $\dot{\mathcal{V}}$ $G_F K_{\text{EUV}}$ $\mathcal{A}_{B\to\ell\bar\nu}^{\rm virtual} = i\sqrt{2} G_F\,K_{\rm EW}(\mu) V_{ub} \mathop{m_{\ell}}\limits_{\infty} \sqrt{\eta}$ α *p p n l i l including virtual* OED *corrections* at $\beta(\alpha)$ ر
1112ع \overline{e} ⁺ $\frac{1}{2}$ \overline{a} $\frac{\epsilon}{\epsilon}$ $\sqrt{2}$ $\overline{2}\, G_F \, K_{\rm EW}$ $\left(\mu\right)V_{ub}\,\frac{m_{\ell}}{m_{\nu}}\,\sqrt{\,}$

$$
\mathcal{M}_{2p}(\mu) = 1 + \frac{C_F \alpha_s}{4\pi} \left[\frac{3}{2} \ln \frac{m_b^2}{\mu^2} - 2 \right]
$$

+
$$
\frac{\alpha}{4\pi} \left\{ Q_b^2 \left[\frac{3}{2} \ln \frac{m_b^2}{\mu^2} - 2 \right] - Q_\ell Q_b \left[\frac{1}{2} \ln^2 \frac{m_b^2}{\mu^2} + 2 \ln \frac{m_b^2}{\mu^2} - 3 \ln \frac{m_\ell^2}{\mu^2} + 1 + \frac{5\pi^2}{12} \right]
$$

+
$$
2Q_\ell Q_u \int_0^\infty d\omega \frac{\phi_-(\omega)}{\omega_0} \ln \frac{m_b \omega}{\mu^2} + Q_\ell^2 \left[\frac{1}{\epsilon_{\text{TR}}} \left(\ln \frac{m_B^2}{m_\ell^2} - 2 \right) + \frac{1}{2} \ln^2 \frac{m_\ell^2}{\mu^2} - \frac{1}{2} \ln \frac{m_\ell^2}{\mu^2} + 2 + \frac{5\pi^2}{12} \right] \right\}
$$

$$
\mathcal{M}_{3p}(\mu) = \frac{\alpha}{\pi} Q_\ell Q_u \int_0^\infty d\omega \int_0^\infty d\omega_g \frac{\phi_{3g}(\omega, \omega_g)}{\phi_{3g}(\omega, \omega_g)} \left[\frac{1}{\omega_g} \ln \left(1 + \frac{\omega_g}{\omega} \right) - \frac{1}{\omega + \omega_g} \right]
$$

$$
F K_{\rm EW}(\mu) V_{ub} \, \frac{m_\ell}{m_b} \sqrt{m_B} \, \mathbf{F}(\mu) \, \bar{u}(p_\ell) P_L \, v(p_\nu) \Big[\mathcal{M}_{2p}(\mu) + \mathcal{M}_{3p}(\mu) \Big]
$$

B ! *µ*⌫¯*^µ* decay amplitude including virtual QED cor-

B ! *µ*⌫¯*^µ* decay amplitude including virtual QED cor-

IR divergence cancels against real ergence cancers agair
soft photon emission where the two- two- two- two- two- and twothe divergence cancels against real
R and *B* meson. After renormalize the *B* meson. malizing the four-fermion operator in (1), the muon matrix of muon mass in \mathcal{L} $\begin{array}{ccc} 1 & m^2 & m^2 & m^2 \end{array}$ $\begin{array}{ccc} 5\pi^2 & 1 \end{array}$ t_2Q_b $\left[\frac{1}{2}\ln^2\frac{m_b}{\mu^2} + 2\ln\frac{m_b}{\mu^2} - 3\ln\frac{m_c}{\mu^2} + 1 + \frac{3m}{12}\right]$ $\frac{1}{\omega_q} \ln \left(1\right)$ $\frac{1}{1}$ 2 $\left(\frac{g}{\cdot}\right)$ - $\frac{g}{\cdot \cdot}$ *b* $-\frac{1}{\omega+\omega_g}\bigg]$ **2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000 - 2000** *b* ence cancers
ft photon em *b* eis against re
emission *^µ*² +1+ ⁵⇡² $rac{1}{2} \ln^2 \frac{m_b^2}{\mu^2} + 2$ $+2$ $\ln \frac{\eta}{2}$ $\frac{2}{2}$ $\overline{}$ 1 \mathfrak{Z} $\ln \frac{m_{\ell}^2}{2}$ $\n *p*$ ² $\n *l*$ + 1 + $\left[\frac{5\pi^2}{}\right]$ $\overline{}$ $\left[\frac{2}{2}\right]$ **1**
→ *g*_→ *g* → *g* → *g* → *g* → *g* $\frac{1}{1}$ $\frac{1}{1}$ $\bigcap_{i=1}^n$ $2 \mid + \frac{1}{2}$ $\sqrt{1}$ $\sqrt{m_p^2}$ 1 . π *J*₀ *J*₀ **i** $\frac{1}{2}$ **i** $\$ 1 2 $\ln^2\frac{m_b^2}{2}$ *b* $\frac{m_b^2}{\mu^2}+2\ln\frac{m_b^2}{\mu^2}$ *b* $\frac{m_b^2}{\mu^2}-3\ln\frac{m_\ell^2}{\mu^2}$ $\underline{\ell}$ $\frac{m_\ell^2}{\mu^2} + 1 + \frac{5\pi^2}{12}$ $\overline{}$ $\sqrt{2}$ $\boxed{1}$ $\frac{1}{2}$ $\left(\ln \frac{1}{2} \right)$ $\frac{n_B^2}{\sigma}$ $\left(-2\right) + \frac{1}{2} \ln^2$ $\begin{bmatrix} 1 \\ 2 & 3 \end{bmatrix}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{m_{\ell}^{2}}{2}$ \overline{a} the integrations over *x*, we obtain at one-loop order \ln $\frac{m_{\tilde{b}}}{\mu^2}$ - $\frac{6}{2} + 211$ $\overline{1}$ $\sqrt{\mu^2}$ ^{θ} $\frac{\eta}{1}$ $\overline{1}$ $\frac{\nu_{\ell}^{2}}{2}+1$ *b* $1 + \frac{5\pi^2}{12}$ $Q_b \left[\frac{1}{2} \ln^2 \frac{m_{\tilde{b}}^2}{\mu^2} + 2 \ln \frac{m_{\tilde{b}}^2}{\mu^2} - 3 \ln \frac{m_{\tilde{\ell}}^2}{\mu^2} + 1 + \frac{5\pi^2}{12} \right]$ ℓ $\biggl[\frac{1}{\epsilon_{\rm IR}} \, \biggl(\ln \frac{m_{P}^2}{m_{\ell}^2}$ *B* m_ℓ^2 -2 ◆ $+$ 1 2 $\ln^2\frac{m_\ell^2}{2}$ $\underline{\ell}$ $\frac{m_\ell^2}{\mu^2} - \frac{1}{2}$ $\ln \frac{m_\ell^2}{2}$ $\underline{\ell}$ $\frac{m_\ell^2}{\mu^2} + 2 + \frac{5\pi^2}{12}$ \bigcap 1 !*g* $\sqrt{1 + \frac{\omega_g}{\omega_g}}$ $\ddot{}$ $\frac{1}{1}$ IR divergence cancels against real $\frac{1}{2}$ $\ln^2\frac{\mu^2}{\mu^2}$ μ^2 $\frac{1}{2}$ $]$ μ ² $3 \ln$ $\frac{\ell}{\mu^2}$ + $+1 + \frac{1}{12}$ *^µ*² 3 ln *^m*² *^µ*² +1+ ⁵⇡² $\theta_\ell^2 \left[\frac{1}{\epsilon_{\rm IR}} \left(\ln \frac{n}{n} \right) \right]$ *B m*² ` $\frac{\overline{2}}{\ell}$ – \rightarrow \int \vdash $\frac{1}{2} \ln^2 \frac{n}{\mu}$ \overline{a} $\frac{m_{\ell}}{\mu^2}$ – $\frac{1}{2}$ $\ln \frac{n}{\iota}$ \overline{a} $\frac{m_{\tilde{\ell}}}{\mu^2} + 2 + \frac{5n}{1}$ $\begin{array}{c} 2 \ \blacksquare \end{array}$ *,* (22) ω_g $\ln\left(1+\frac{\omega_g}{\sigma}\right)$ ω $\bigg) - \frac{1}{\omega + \frac{1}{\omega}}$ $\omega + \omega_g$ $\overline{}$

\overline{C} ctions at $\mathcal{O}(\alpha)$: and the parameter *F* in the MS scheme and performing $\frac{1}{2}$ where two terms in the two-second line probe two-*· u*¯(*p*`)*P^L v*(*p*⌫)

*^A*virtual

^B!`⌫¯ = *i*

2*G^F K*EW(*µ*)*Vub*

^p*m^B ^F*(*µ, mb, w*)

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS TAUIURIZAHUN FURMULA FUR V WE ARE NO PROTECTED TO PRESENT OUR MAIN RESULT. *B* ! *µ*⌫¯*^µ* decay amplitude including virtual QED cor-**EACTORIZE** WITH A DIVID A DIVID AND CAN BE OF A CAN BE OF A LATITLE A LATITLE CAN BE OF CAN BE OF THE LATITLE ATION I ONVIOLA I ON DIZATION FODMIII A FOD *· u*¯(*p*`)*P^L v*(*p*⌫) lem mentioned earlier, such that the anomalous dimen-**FAUIUKIZAIIUN FUKMULA FUK VIKIUAL** 2*G^F K*EW(*µ*)*Vub* **MIZATION EARMIII A EAR VIRTIIAI CARRECTIONS** *m^b* ^p*m^B ^F*(*µ, mb, w*) sion of the parameter *F*, defined via *dF*(*µ,*⇤*, w*)*/d* ln *µ* = *^A*virtual *^B*!`⌫¯ = *i* $t_n = t_{n-1} - t_{n-1} - t_{n-1} - t_{n-1} - t_{n-1}$ lem mentioned earlier, such that the anomalous dimension of the parameter *F*, defined via *dF*(*µ,*⇤*, w*)*/d* ln *µ* = **PATIANA** *^A*virtual

The subtraction performance performed in \mathcal{L}_c , in conjunction with \mathcal{L}_c , in conjunction with \mathcal{L}_c

Decay amplitude including virtual QED corrections at $O(\alpha)$: with: *^F F*(*µ,*⇤*, w*), is now local and independent of IR reg- ${\cal A}^{\rm virtual}_{B \to \ell \bar\nu} = i$ Decay amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$: *^F F*(*µ,*⇤*, w*), is now local and independent of IR reg-*B* ! *µ*⌫¯*^µ* decay amplitude including virtual QED coripillude including v $\sqrt{ }$ $2\,G_F\,K_{\rm EW}(\mu) V_{ub}$ m_ℓ *m^b* ${\cal A}^{\rm virtual}_{B^{\rightarrow}\ell\bar{\nu}}=i\sqrt{2}G_{F}K_{\rm EW}(\mu)V_{ub}\frac{m_{\ell}}{\infty}\sqrt{m_{B}}\overline{F(\mu)}\ \bar{u}(p_{\ell})I_{\ell}$ *^F* = *C^F* 4⇡ 4⇡ ³ *^Q*`*Q^u* ln ⇤² *µ*2 ` *^Q*² ✓ $\frac{1}{2}$ $\frac{1}{2}$ $\sqrt{2}$ I is also possible to control the dependence on the dependence on the cuto I *F* \overline{A} ^{rint} 3↵*^s* 4⇡ $\dot{\mathcal{V}}$ $G_F K_{\text{EUV}}$ $\mathcal{A}_{B\to\ell\bar\nu}^{\rm virtual} = i\sqrt{2} G_F\,K_{\rm EW}(\mu) V_{ub} \mathop{m_{\ell}}\limits_{\infty} \sqrt{\eta}$ α amplitude including virtual OED corrections at $\theta(\alpha)$ ر
1112ع \overline{e} ⁺ $\frac{1}{2}$ \overline{a} $\frac{\epsilon}{\epsilon}$ $\sqrt{2}$ $\overline{2}\, G_F \, K_{\rm EW}$ $\left(\mu\right)V_{ub}\,\frac{m_{\ell}}{m_{\nu}}\,\sqrt{\,}$

$$
\mathcal{M}_{2p}(\mu) = 1 + \frac{C_F \alpha_s}{4\pi} \left[\frac{3}{2} \ln \frac{m_b^2}{\mu^2} - 2 \right]
$$

+
$$
\frac{\alpha}{4\pi} \left\{ Q_b^2 \left[\frac{3}{2} \ln \frac{m_b^2}{\mu^2} - 2 \right] - Q_\ell Q_b \left[\frac{1}{2} \ln^2 \frac{m_b^2}{\mu^2} + 2 \ln \frac{m_b^2}{\mu^2} - 3 \ln \frac{m_b^2}{\mu^2} \right] \right\}
$$

+
$$
2Q_\ell Q_u \int_0^\infty d\omega \frac{\phi_-(\omega)}{\phi_-(\omega)} \ln \frac{m_b \omega}{\mu^2} + Q_\ell^2 \left[\frac{1}{\epsilon_{\text{IR}}} \left(\ln \frac{m_B^2}{m_\ell^2} - 2 \right) + \frac{1}{2} \ln \frac{m_b \omega}{\mu^2} \right]
$$

$$
\mathcal{M}_{3p}(\mu) = \frac{\alpha}{\pi} Q_\ell Q_u \int_0^\infty d\omega \int_0^\infty d\omega_g \frac{\phi_{3g}(\omega, \omega_g)}{\phi_{3g}(\omega, \omega_g)} \left[\frac{1}{\omega_g} \ln \left(1 + \frac{\omega_g}{\omega} \right) - \frac{1}{\omega + \omega_g} \right]
$$

 J_{0}

 $\overline{0}$

$$
F K_{\rm EW}(\mu) V_{ub} \, \frac{m_\ell}{m_b} \sqrt{m_B} \, \mathbf{F}(\mu) \, \bar{u}(p_\ell) P_L \, v(p_\nu) \Big[\mathcal{M}_{2p}(\mu) + \mathcal{M}_{3p}(\mu) \Big]
$$

B ! *µ*⌫¯*^µ* decay amplitude including virtual QED cor-

B ! *µ*⌫¯*^µ* decay amplitude including virtual QED cor-

0

 π

²

REAL PHOTON EMISSIONS

Structure-dependent QED corrections below Λ_{OCD}:

- ▸ contribution becomes relevant *B* → *B***γ* $for E_{\gamma} \gtrsim (m_{B^*} - m_{B}) \approx 46 \text{ MeV}$
- ▸ Contributions of higher excited states are power suppressed for E_{γ} ≪ $\Lambda_{\rm QCD}$ (real photon emission)
- ▸ The looser the cut on additional radiation, the more important the *B** and higher contributions are

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