The Power of EFTs

Future of Heavy Quark Physics

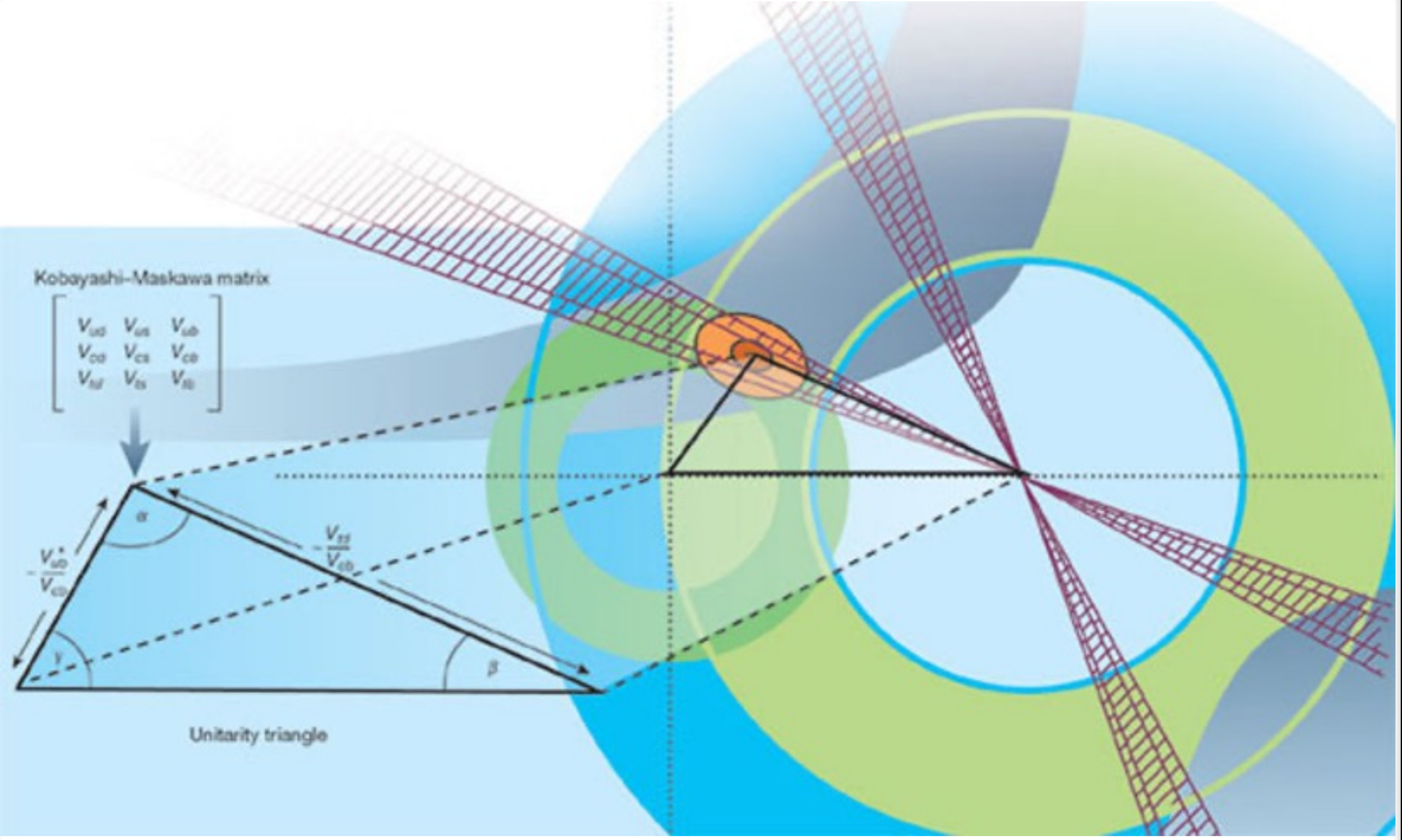
Cornell – October 15, 2024

Matthias Neubert, Johannes Gutenberg University Mainz

- **A new algorithm for adaptive Monte Carlo integration** (J. Comput. Phys. 27 (1978) 192, 1446 citations)
- **Exclusive processes in QCD: Evolution equations for hadronic wave functions and the form-factors of mesons** (with S.J. Brodsky, Phys. Lett. B 87 (1979) 359, 1535 citations)
- **Exclusive processes in perturbative QCD** (with S.J. Brodsky, Phys. Rev. D 22 (1980) 2157, 4066 citations)
- **On the elimination of scale ambiguities in perturbative QCD** (with S.J. Brodsky and P.B. Mackenzie, Phys. Rev. D 28 (1983) 228, 1298 citations)
- **Effective Lagrangians for bound state problems in QED, QCD and other field theories** (with W.E. Caswell, Phys. Lett. B 167 (1986) 437, 1321 citations)
- **Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium** (with G.T. Godwin and E. Braaten, Phys. Rev. D 51 (1995) 1125, 2955 citations)
- **Heavy quark bound states in lattice QCD** (with B.A. Thacker, Phys. Rev. D 43 (1991) 196, 464 citations)
- **On the viability of lattice perturbation theory** (with P.B. Mackenzie, Phys. Rev. D 48 (1993) 2250, 1201 citations)
- **High precision lattice QCD confronts experiment** (HPQCD, UKQCD, MILC & Fermilab Lattice collaborations, Phys. Rev. Lett. 92 (2004) 022001, 466 citations)

PUSHING THE LUMINOSITY FRONTIER – GOLDEN AGE OF HEAVY-QUARK THEORY

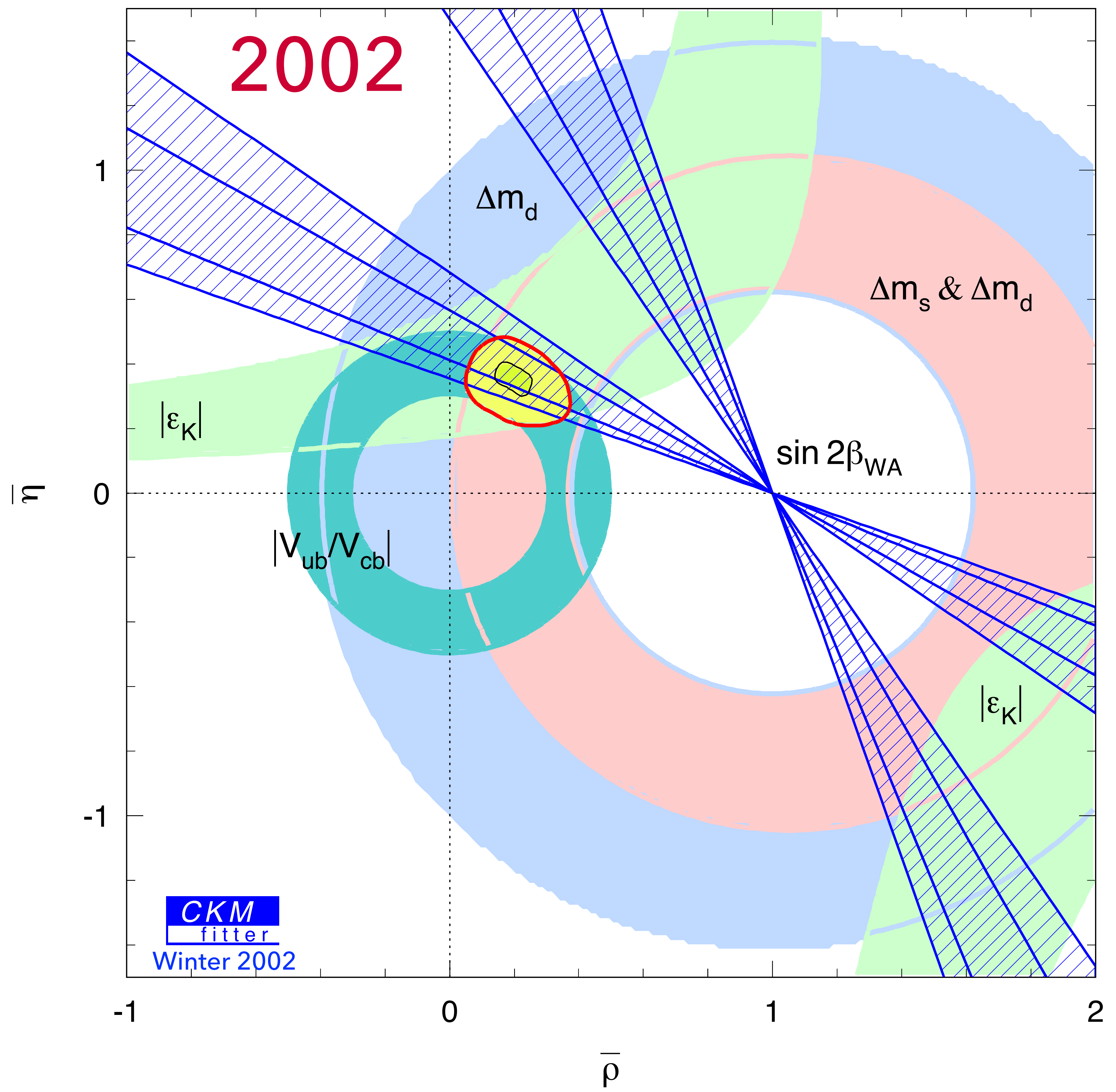
- Tremendous experimental advances:
  - 1. generation: ARGUS & CLEO, LEP expts.
  - 2. generation: BaBar & Belle, LHC*b*, CMS, …
  - 3. generation: Belle II, LHC*b* upgrade, …
- Precise measurement of CKM elements $|V_{cb}|, |V_{ub}|, |V_{td}|, |V_{ts}|$ involving third-generation quarks
- Precise determinations of angles (CP violation)
- New-physics searches using FCNC processes

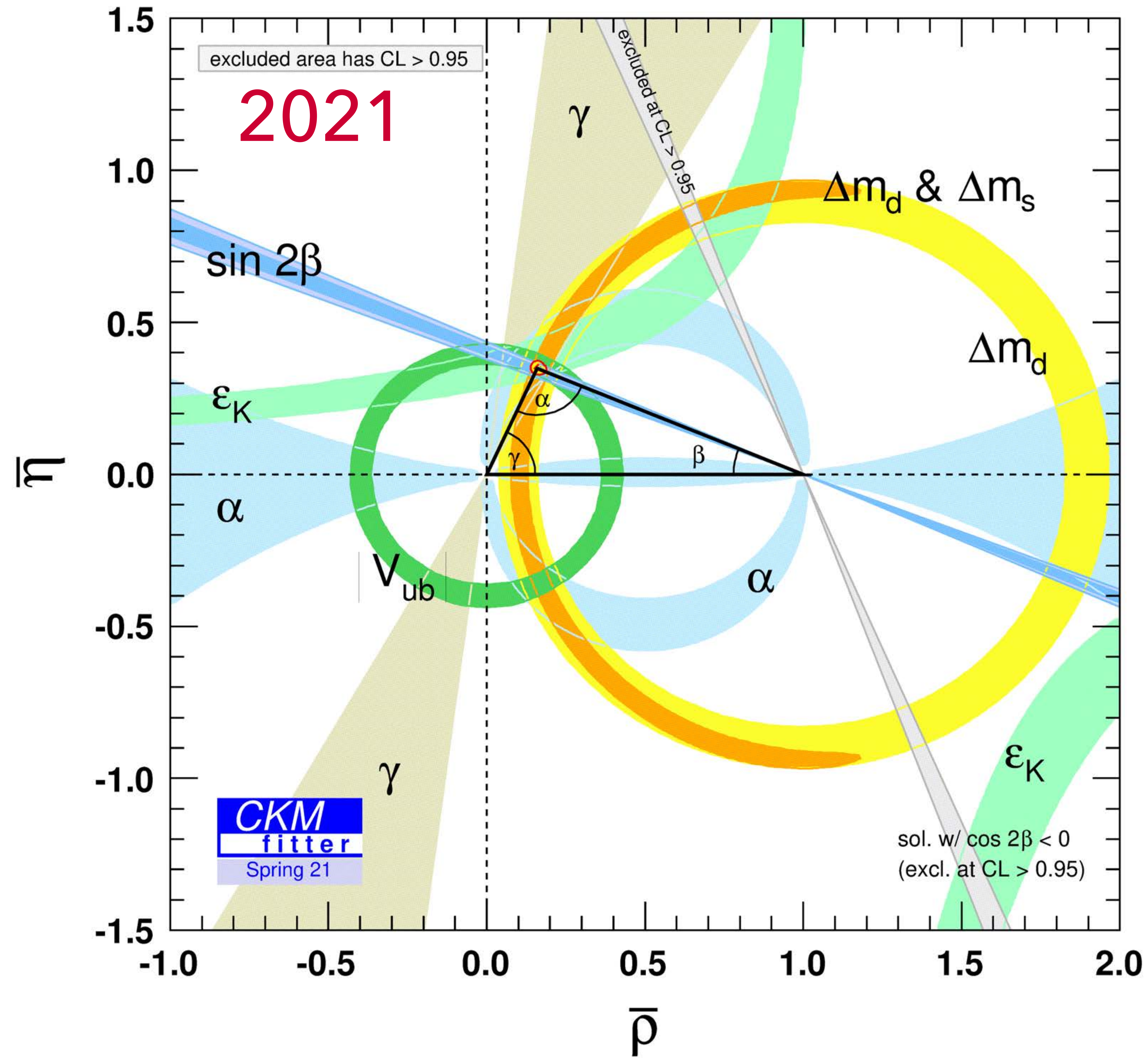


$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}. \quad (13)$$

Then, we have CP-violating effects through the interference among these different current components.

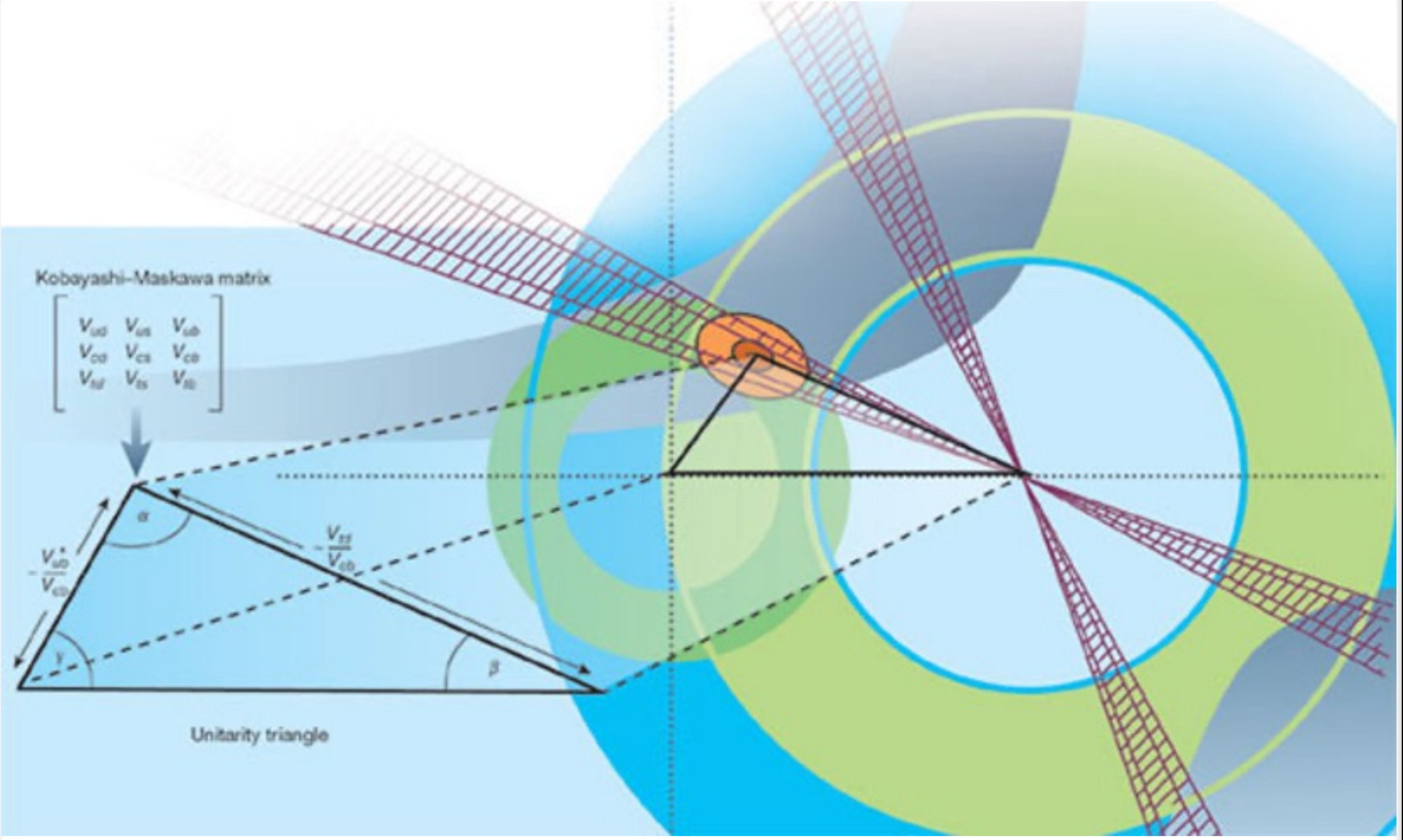
PUSHING THE LUMINOSITY FRONTIER – GOLDEN AGE OF HEAVY-QUARK THEORY





PUSHING THE LUMINOSITY FRONTIER – GOLDEN AGE OF HEAVY-QUARK THEORY

- Matching the incredible precision of the *B*-factories required a revolution in theory
- Concerted effort of theory community was an important consequence
  Breakthrough came from using **effective field theories** (EFTs):
  - $\mathscr{H}_{\rm eff}^{\rm weak}$, HQET, NRQCD, QCDF, SCET
- SCET later became a versatile tool for addressing difficult LHC theory problems

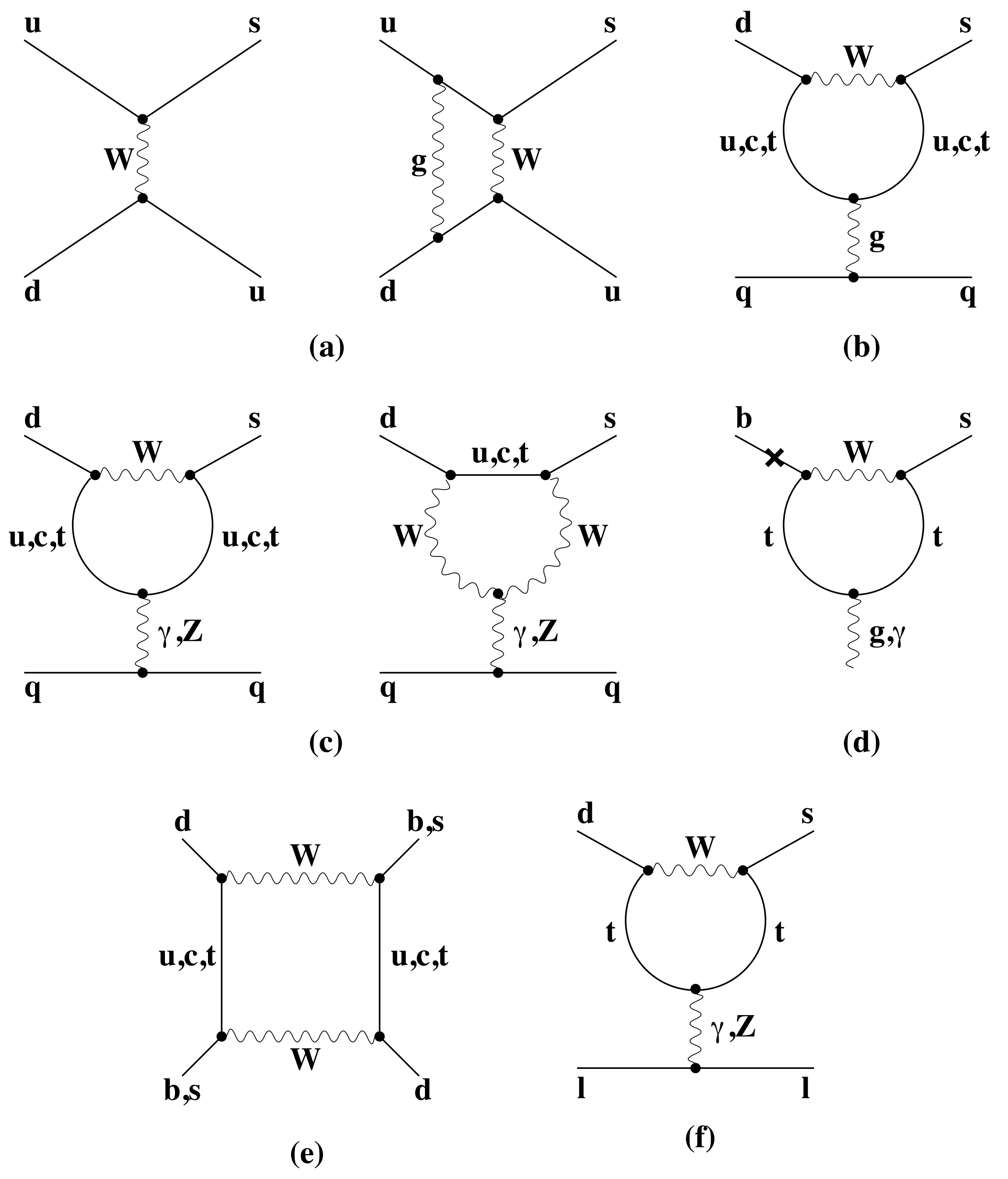


$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\cos\theta_3e^{i\delta} \end{pmatrix}. \tag{13}$$

Then, we have *CP*-violating effects through the interference among these different current components.

EFFECTIVE WEAK HAMILTONIAN

- Systematic method to separate short-distance effects (weak scale and beyond) from long-distance hadronic dynamics
- Nowadays embedded into SMEFT and its low-energy variant LEFT
- **But:** challenge is to evaluate hadronic matrix elements of the quark-gluon operators $Q_i(\mu)$ in all but simplest cases



$$Q_1^q = \left(\bar b_i q_j\right)_{\rm V-A} \left(\bar q_j d_i\right)_{\rm V-A}$$
$$Q_2^q = \left(\bar b q\right)_{\rm V-A} \left(\bar q d\right)_{\rm V-A}$$
$$Q_3 = \left(\bar b d\right)_{\rm V-A} \sum_q \left(\bar q q\right)_{\rm V-A}$$
$$Q_4 = \left(\bar b_i d_j\right)_{\rm V-A} \sum_q \left(\bar q_j q_i\right)_{\rm V-A}$$
$$Q_5 = \left(\bar b d\right)_{\rm V-A} \sum_q \left(\bar q q\right)_{\rm V+A}$$
$$Q_6 = \left(\bar b_i d_j\right)_{\rm V-A} \sum_q \left(\bar q_j q_i\right)_{\rm V+A}$$
$$Q_7 = \frac{3}{2} \left(\bar s d\right)_{\rm V-A} \sum_q e_q \left(\bar q q\right)_{\rm V+A}$$
$$Q_8 = \frac{3}{2} \left(\bar s_i d_j\right)_{\rm V-A} \sum_q e_q \left(\bar q_j q_i\right)_{\rm V+A}$$
$$Q_9 = \frac{3}{2} \left(\bar s d\right)_{\rm V-A} \sum_q e_q \left(\bar q q\right)_{\rm V-A}$$
$$Q_{10} = \frac{3}{2} \left(\bar s_i d_j\right)_{\rm V-A} \sum_q e_q \left(\bar q_j q_i\right)_{\rm V-A}$$

$$\mathcal{H}_{\rm eff}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} V_{\rm us}^* V_{\rm ud} \sum_{i=1}^{10} \left(z_i(\mu) + \tau\, y_i(\mu)\right) Q_i(\mu)$$

[Gilman, Wise (1979); Buras et al. (1990s)]

HEAVY QUARK SYMMETRY

- Hadronic bound states containing a heavy quark obey an approximate **spin-flavor symmetry**
- Many predictions for spectroscopy of heavy hadrons [Shuryak (1980)]
- Symmetry relations among $B \to D^{(*)}$ form factors, including symmetry-breaking corrections $\sim \alpha_s(m_Q)$ or $\Lambda_{\rm QCD}/m_Q$ [Isgur, Wise (1990)]

Relations between level spacings in bottom and charm systems, e.g.:

- $m_{B^*}^2 - m_B^2 \approx 0.49\,\text{GeV}^2$ vs. $m_{D^*}^2 - m_D^2 \approx 0.55\,\text{GeV}^2$
- $m_{B_s} - m_B \approx m_{D_s} - m_d \approx 0.10\,\text{GeV}$
- $m_{B_2^*}^2 - m_{B_1}^2 \approx m_{D_2^*}^2 - m_{D_1}^2 \approx 0.17\,\text{GeV}^2$

Form-factor relations:

$$\langle D(v')|V^\mu|B(v)\rangle = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu$$

$$\langle D^*(v',\epsilon)|V^\mu|B(v)\rangle = i h_V(w)\,\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* v'_\alpha\, v_\beta$$

$$\langle D^*(v',\epsilon)|A^\mu|B(v)\rangle = h_{A_1}(w)(w+1)\epsilon^{*\mu} - [h_{A_2}(w)v^\mu + h_{A_3}(w)v'^\mu]\,\epsilon^*\cdot v$$

with ($w = v\cdot v'$):

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w) \quad \text{and} \quad \xi(1) = 1$$

$$h_-(w) = h_{A_2}(w) = 0$$

MODEL-INDEPENDENT DETERMINATION OF $|V_{CB}|$

- Extrapolate observed spectrum in $w = v \cdot v'$ to zero recoil:

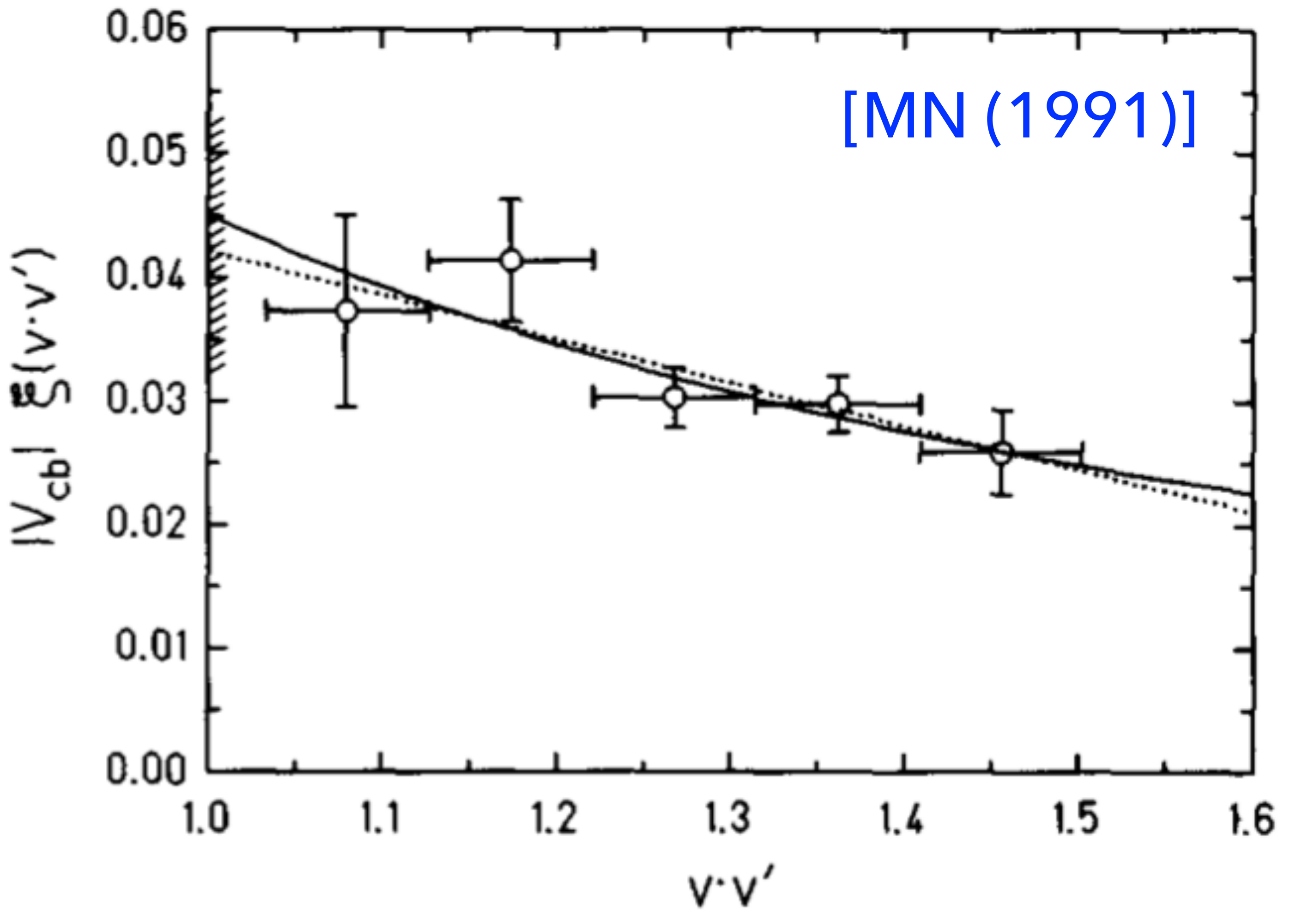
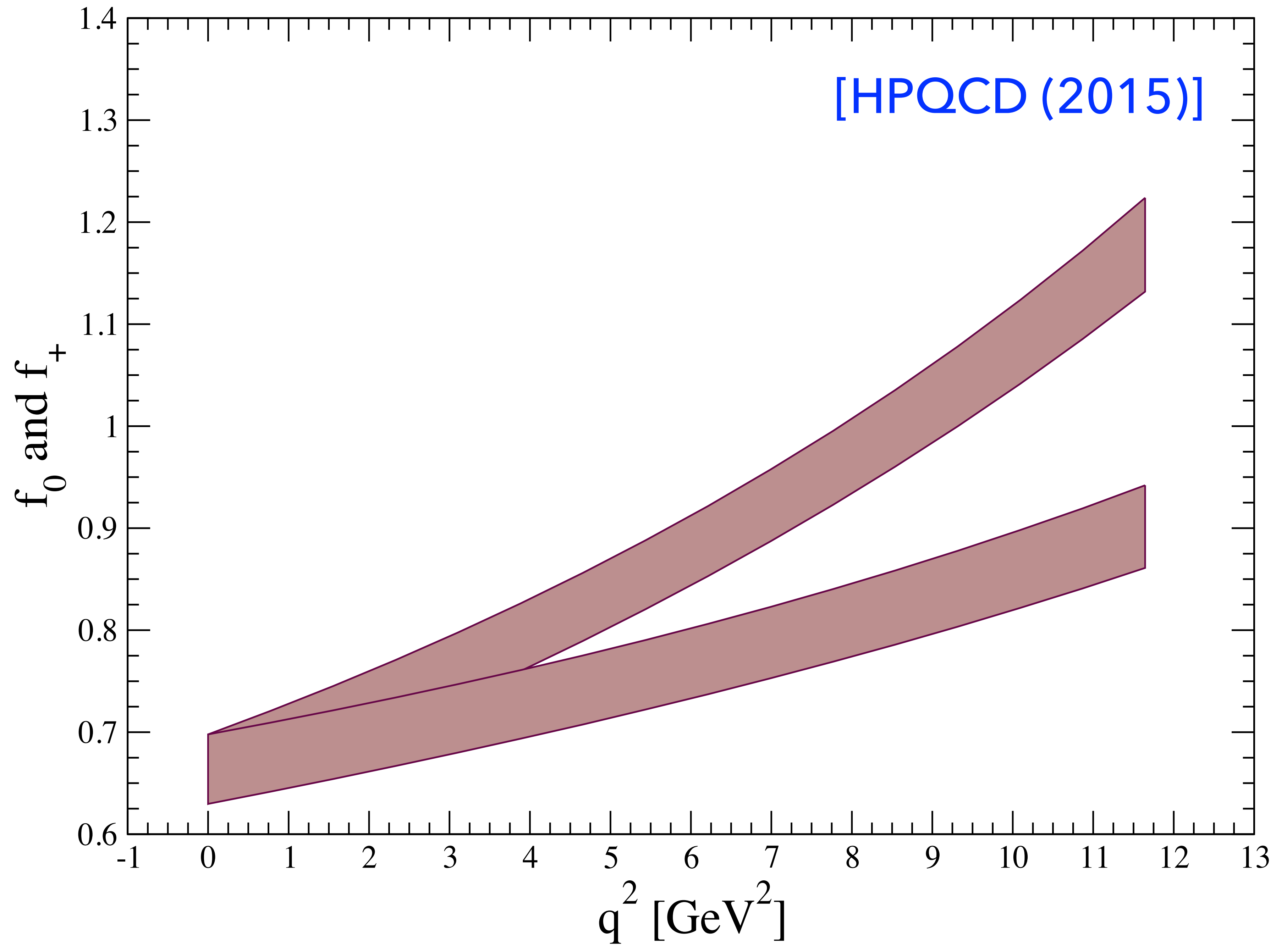


Fig. 1. Extraction of $|V_{cb}|$ and the Isgur–Wise function from $\bar{B}^0 \to D^{*+} \ell \bar{\nu}_\ell$ decays. The data are taken from ref. [16]. $\tau_{B^0} =$ 1.18 ps is assumed. $|V_{cb}|$ follows from an extrapolation of the data to $v \cdot v' = 1$. Its currently best value is indicated as a shaded area on the vertical axis.

- Direct calculation of the $B \to D l \nu$ form factors (HPQCD):



HEAVY QUARK EFFECTIVE THEORY (HQET)

[Eichten, Hill (1990); Georgi (1990)]

- Firm theoretical basis for deriving heavy-quark symmetry and its consequences

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v\, iv\cdot D\, h_v + \mathcal{O}\left(\frac{1}{m_Q}\right)$$

$$+\frac{1}{2m_Q}\left[\bar{h}_v\,(iD)^2\,h_v + \frac{g_s}{2}\,\bar{h}_v\,\sigma_{\mu\nu}G^{\mu\nu}h_v\right] + \dots$$

HEAVY QUARK EFFECTIVE THEORY (HQET)

[Eichten, Hill (1990); Georgi (1990)]

- Firm theoretical basis for deriving heavy-quark symmetry and its consequences

$$\mathcal{L}_{\rm HQET} = \bar{h}_v\, iv\cdot D\, h_v + \mathcal{O}\left(\frac{1}{m_Q}\right)$$
$$+\frac{1}{2m_Q}\left[\bar{h}_v\,(iD)^2\,h_v + \frac{g_s}{2}\,\bar{h}_v\,\sigma_{\mu\nu}G^{\mu\nu}h_v\right] + \dots$$

- An anecdote from 1988 …

HEAVY QUARK EFFECTIVE THEORY (HQET)

[Eichten, Hill (1990); Georgi (1990)]

- Firm theoretical basis for deriving heavy-quark symmetry and its consequences

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v\, iv\cdot D\, h_v + \mathcal{O}\left(\frac{1}{m_Q}\right)$$
$$+\frac{1}{2m_Q}\left[\bar{h}_v\,(iD)^2\,h_v + \frac{g_s}{2}\,\bar{h}_v\,\sigma_{\mu\nu}G^{\mu\nu}h_v\right] + \dots$$

NONRELATIVISTIC EFFECTIVE FIELD THEORY (NRQED & NRQCD)

[Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1992, 1995)]

"We develop a renormalization group strategy for the study of bound states in field theory. Our analysis is completely different from conventional analyses, based upon the Bethe-Salpeter equation, and it is far simpler."

HEAVY QUARK EFFECTIVE THEORY (HQET)

[Eichten, Hill (1990); Georgi (1990)]

- Firm theoretical basis for deriving heavy-quark symmetry and its consequences

$$\mathcal{L}_{\rm HQET} = \bar{h}_v\, iv\cdot D\, h_v + \mathcal{O}\left(\frac{1}{m_Q}\right)$$
$$+\frac{1}{2m_Q}\left[\bar{h}_v\,(iD)^2\,h_v + \frac{g_s}{2}\,\bar{h}_v\,\sigma_{\mu\nu}G^{\mu\nu}h_v\right] + \dots$$

NONRELATIVISTIC EFFECTIVE FIELD THEORY (NRQED & NRQCD)

[Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1992, 1995)]

- Same for $(Q\bar{Q})$ systems
- Same operators but different power counting (different scaling of energy and momenta)

$$\begin{aligned}
\mathcal{L}_{\rm heavy} &= \psi^\dagger\left(iD_t + \frac{\mathbf{D}^2}{2M}\right)\psi + \chi^\dagger\left(iD_t - \frac{\mathbf{D}^2}{2M}\right)\chi \\
\delta\mathcal{L}_{\rm bilinear} &= \frac{c_1}{8M^3}\left(\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right) \\
&+ \frac{c_2}{8M^2}\left(\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right) \\
&+ \frac{c_3}{8M^2}\left(\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right) \\
&+ \frac{c_4}{2M}\left(\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right),
\end{aligned}$$

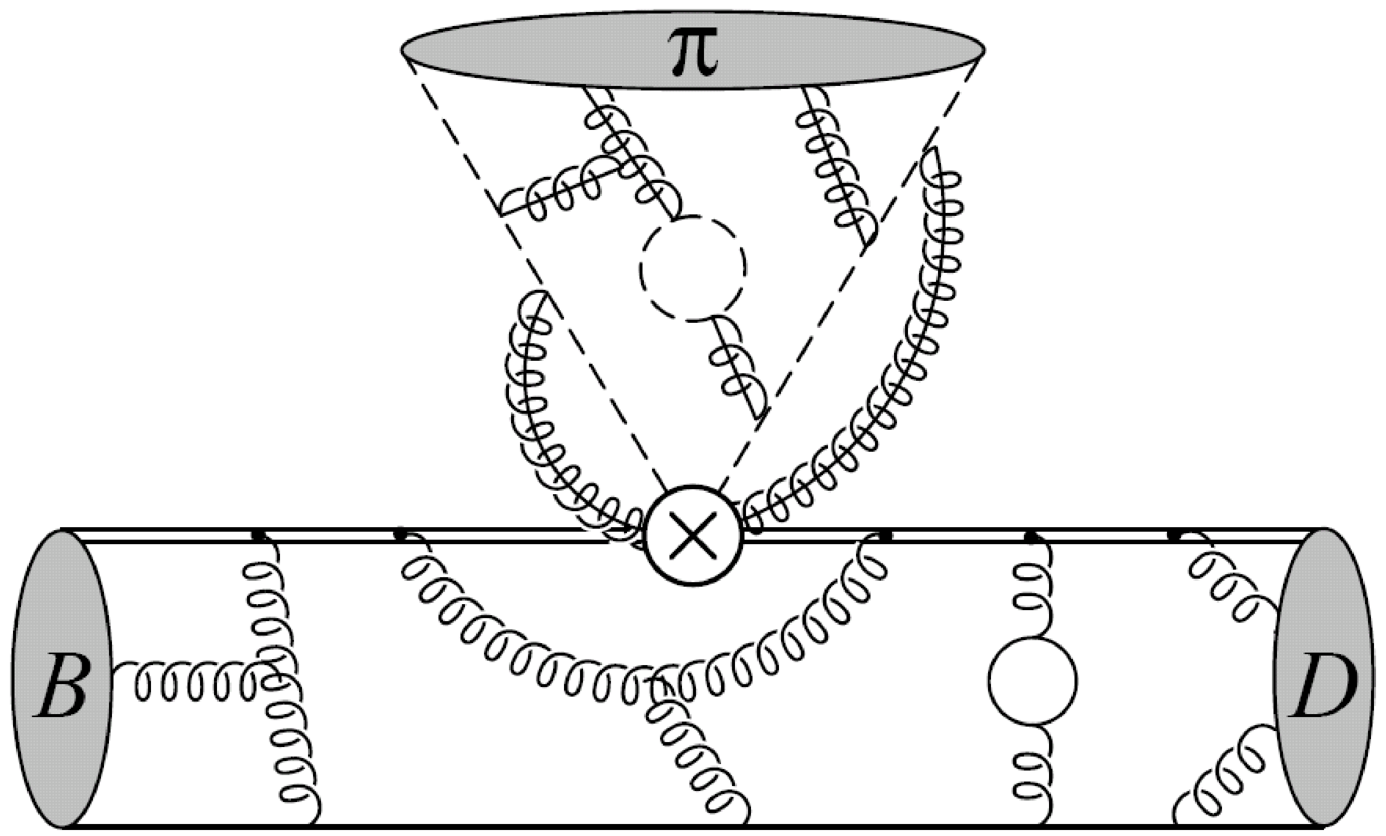
THE GRAND CHALLENGE: NON-LEPTONIC DECAYS

- Georgi: *"Why we can't calculate …"* [Georgi: *Weak Interactions and Modern Particle Theory* (1984)]
- Naive factorization approach was semi-successful in describing early data, but lacked a firm theoretical foundation [Bauer, Stech, Wirbel (1986)]

THE GRAND CHALLENGE: NON-LEPTONIC DECAYS

- Georgi: *"Why we can't calculate …"* [Georgi: *Weak Interactions and Modern Particle Theory* (1984)]
- Naive factorization approach was semi-successful in describing early data, but lacked a firm theoretical foundation [Bauer, Stech, Wirbel (1986)]
- **QCD factorization approach (BBNS):**
  - First model-independent calculation of $B \to M_1 M_2$ decay amplitudes from first principles (including strong- and weak-interaction phases) in heavy-quark limit

[Beneke, Buchalla, MN, Sachrajda (1999–2001)]

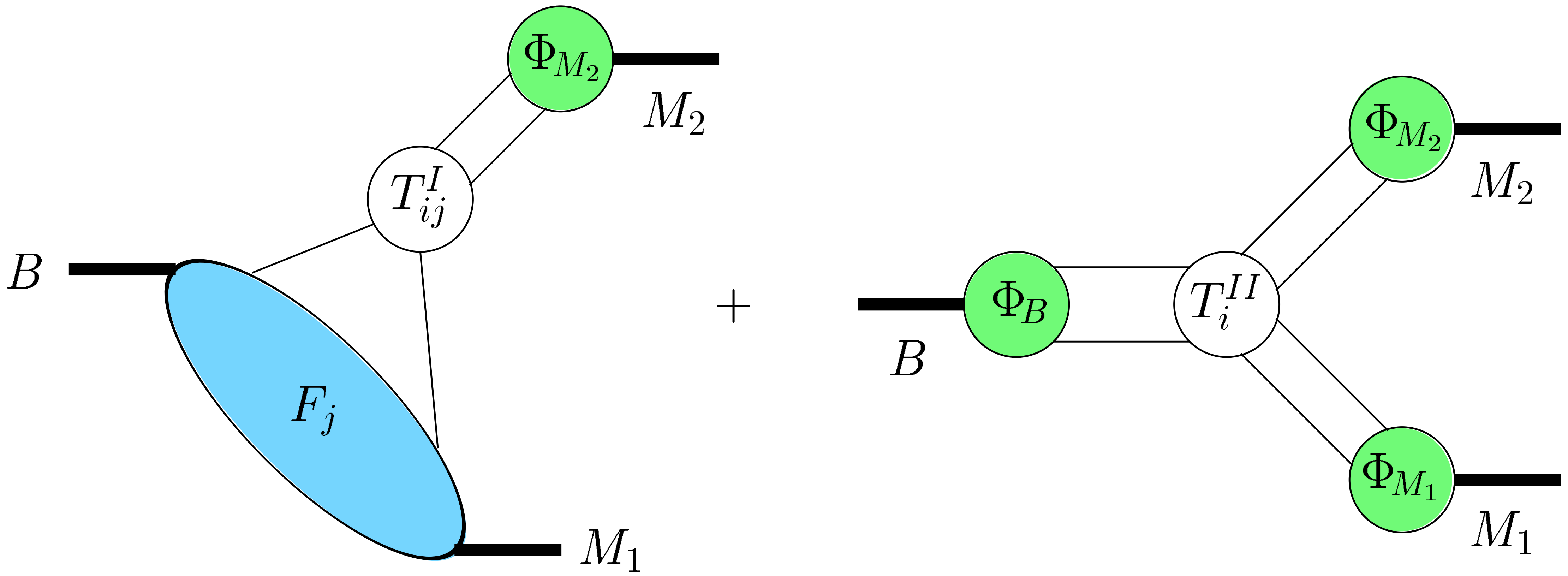


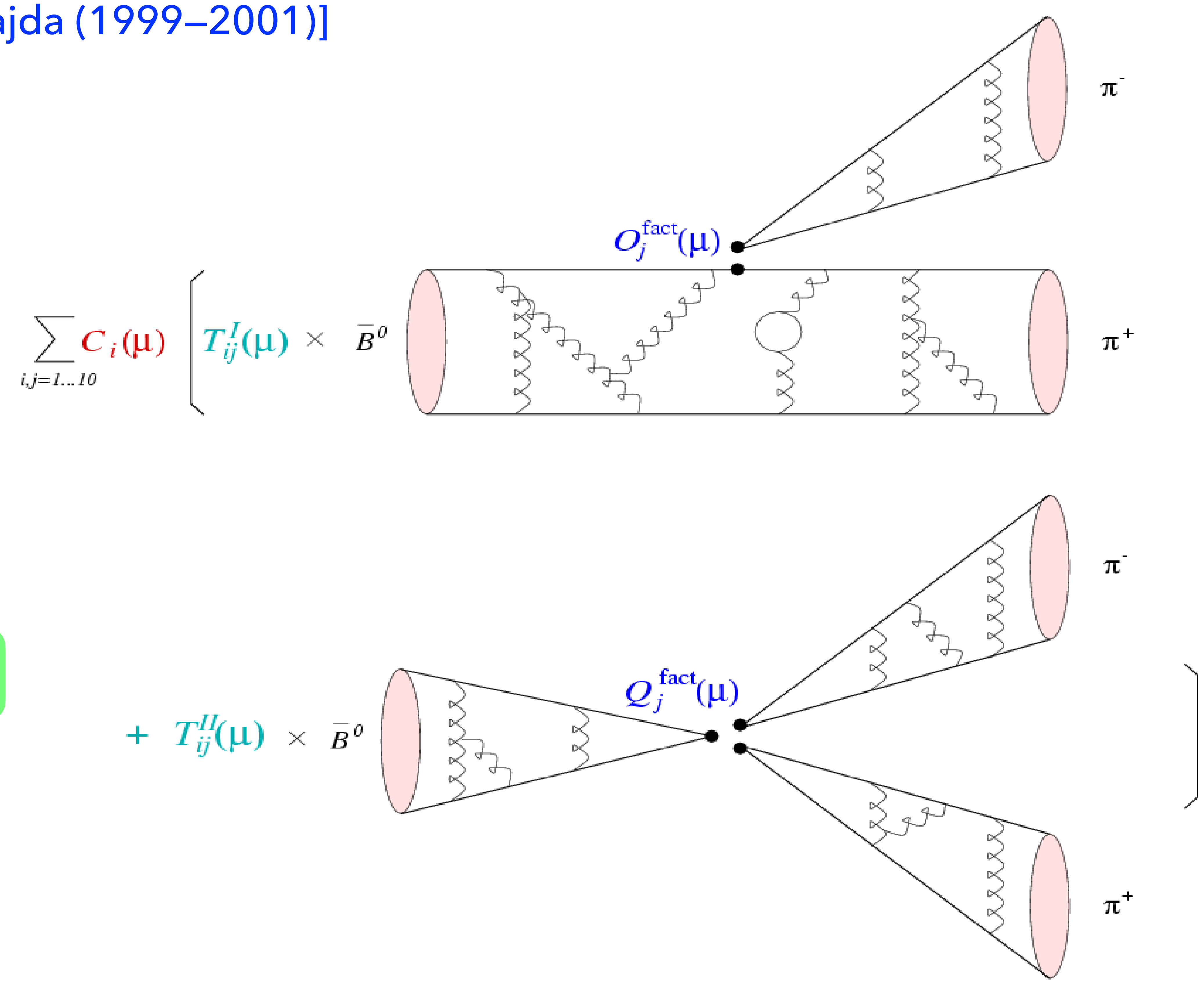
Factorization proof at two-loop order based on method of regions, see pp. 48-79 in BBNS (2000)

QCD FACTORIZATION IN NONLEPTONIC B DECAYS INTO LIGHT MESONS

QCD factorization theorem:

[Beneke, Buchalla, MN, Sachrajda (1999–2001)]





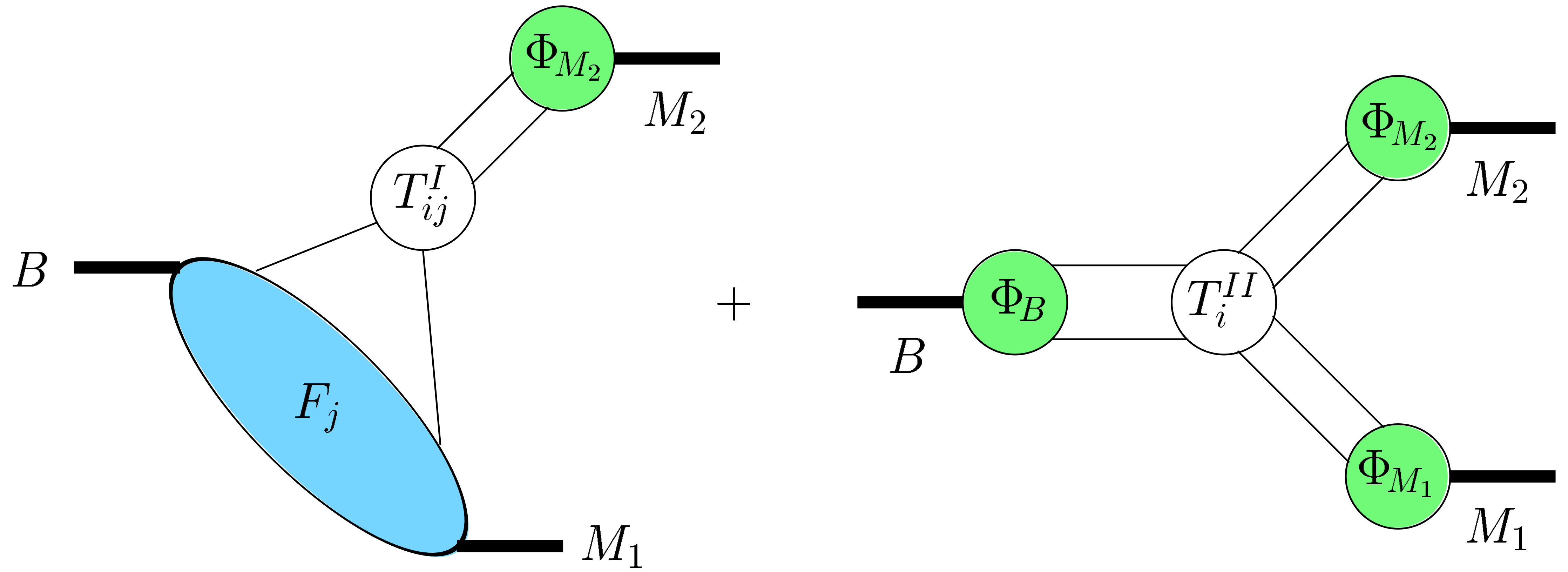
$$\langle \pi K | Q_i | B \rangle = F_0^{B\to\pi}\, T^{\rm I}_{K,i} * f_K \Phi_K + F_0^{B\to K}\, T^{\rm I}_{\pi,i} * f_\pi \Phi_\pi + T^{\rm II}_i * f_B \Phi_B * f_K \Phi_K * f_\pi \Phi_\pi$$

$$+\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

QCD FACTORIZATION IN NONLEPTONIC B DECAYS INTO LIGHT MESONS

QCD factorization theorem:

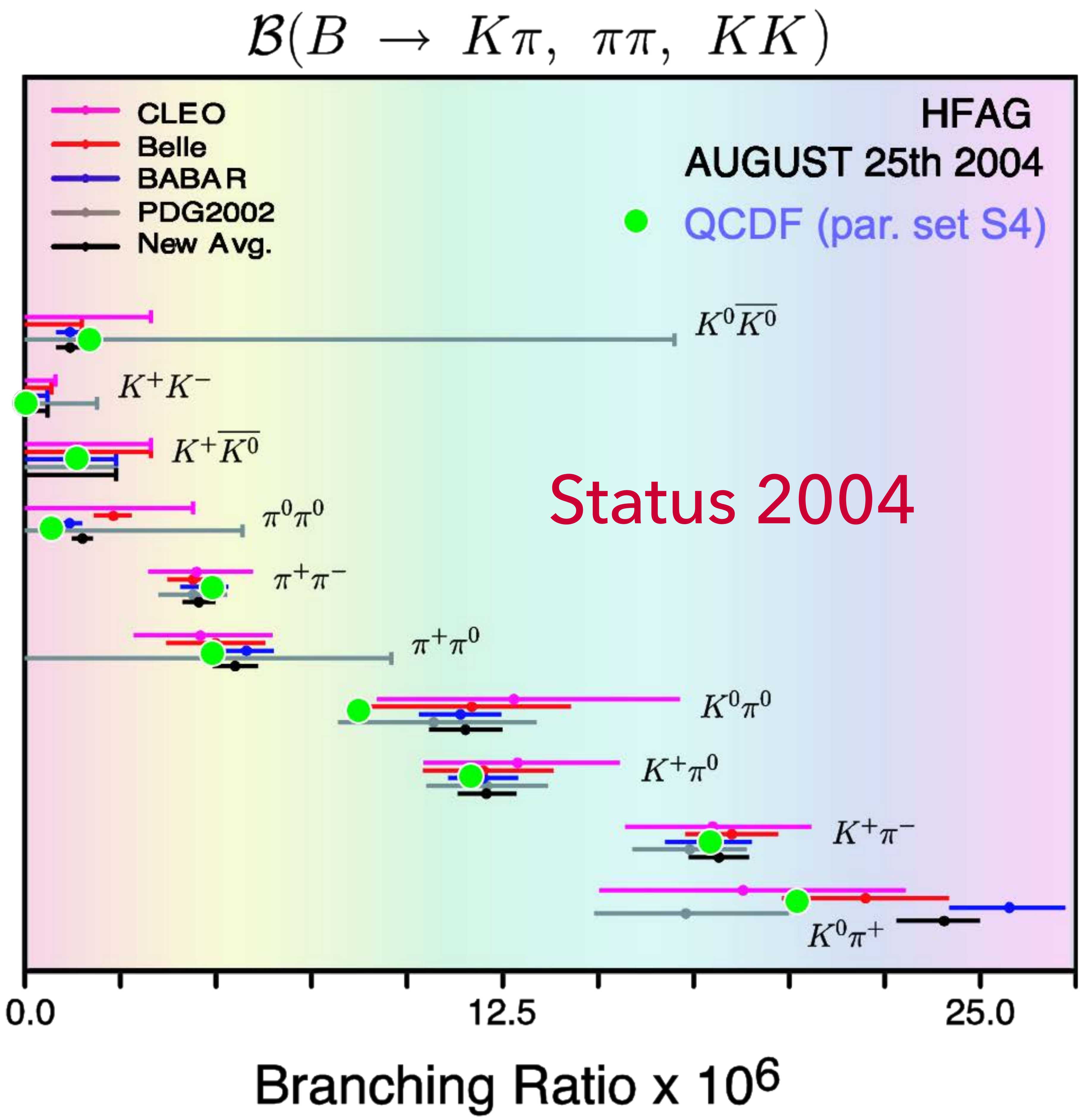
[Beneke, Buchalla, MN, Sachrajda (1999–2001)]

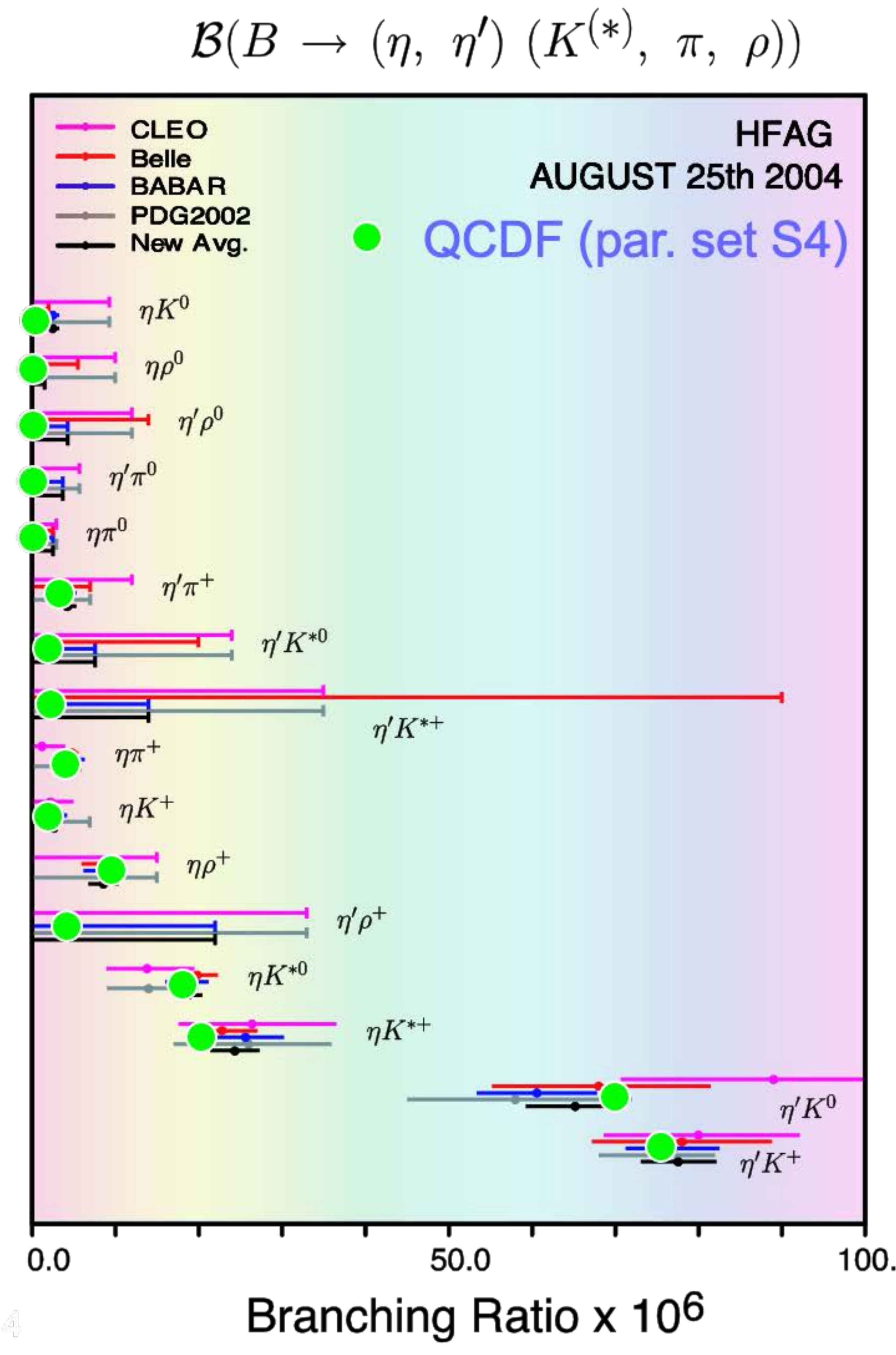


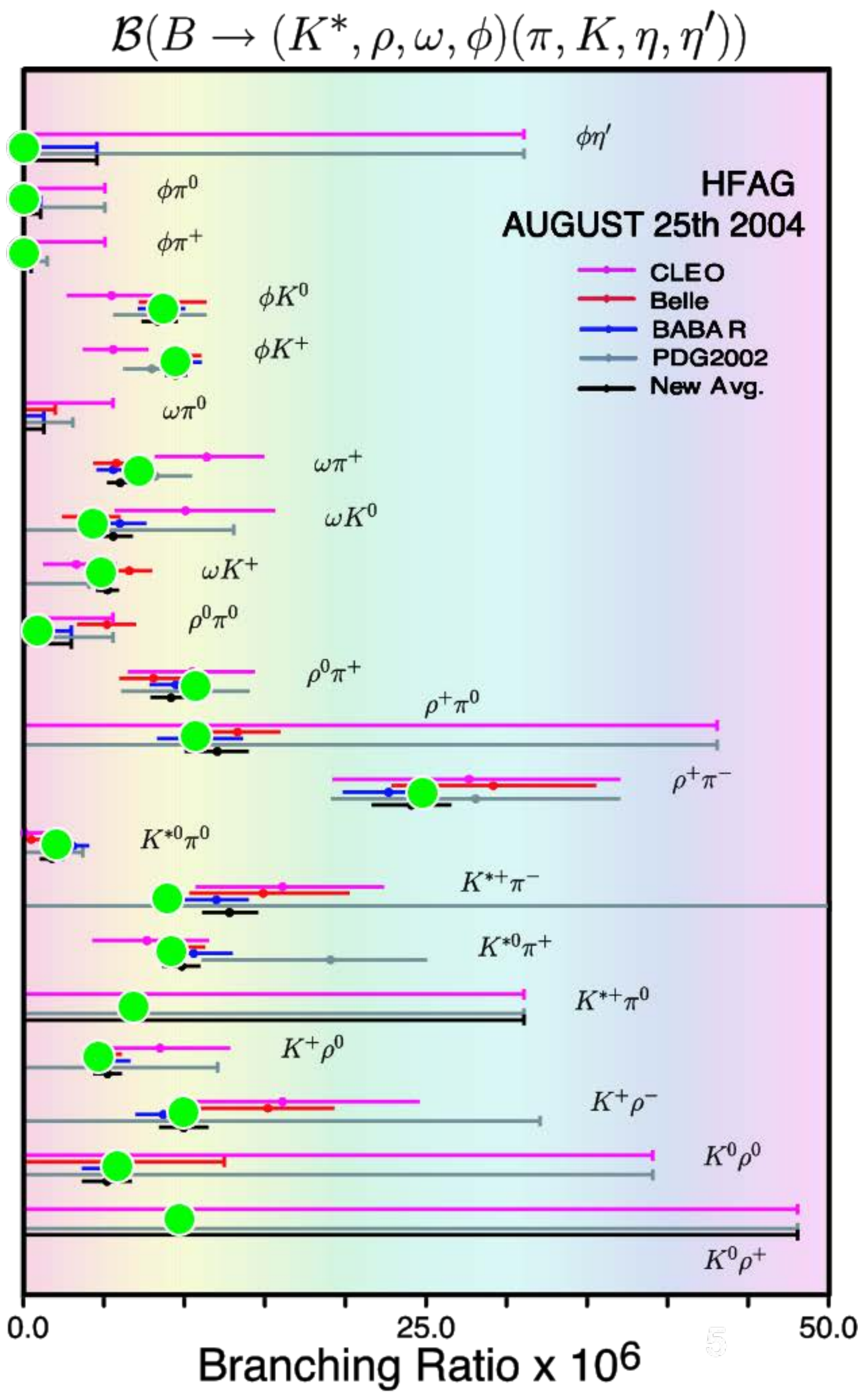
$$\langle \pi K | Q_i | B \rangle = F_0^{B\to\pi} \, T^{\rm I}_{K,i} * f_K \Phi_K + F_0^{B\to K} \, T^{\rm I}_{\pi,i} * f_\pi \Phi_\pi + T^{\rm II}_i * f_B \Phi_B * f_K \Phi_K * f_\pi \Phi_\pi + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

- Importance of **non-local matrix elements**, in particular light-cone distribution amplitudes (LCDAs), to account for hadronic dynamics
- Second term corresponds to **Brodsky-Lepage (1980)**, while the first term is specific for *B*-meson decays and contributes at the same order in $\Lambda_{\rm QCD}/m_b$

QCD FACTORIZATION IN NONLEPTONIC B DECAYS INTO LIGHT MESONS







[Beneke, MN (2003)]

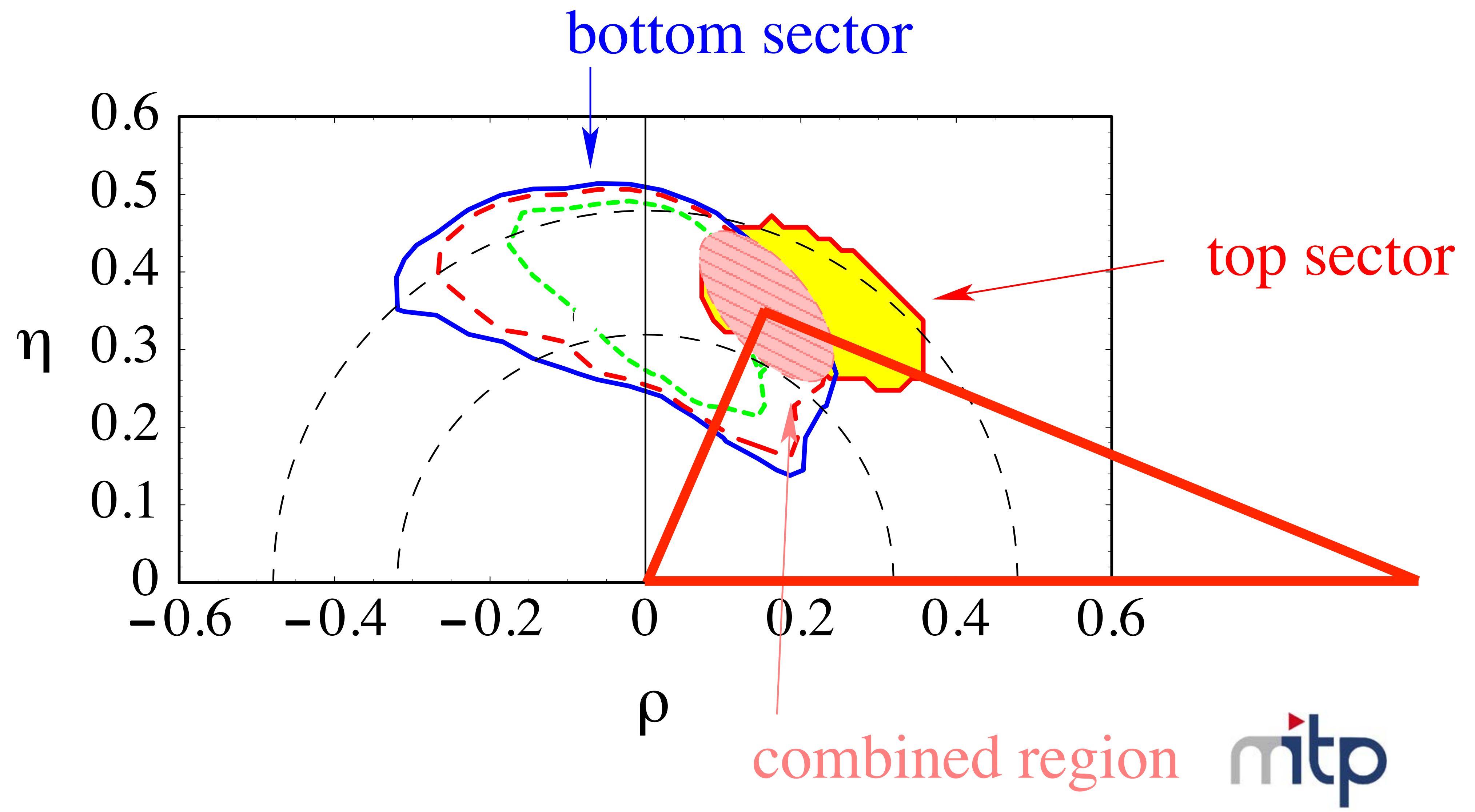
CONFIRMATION OF KM RELATION BETWEEN IM(V_{UB}) AND IM(V_{TB})

- In 2001, fact that Im$(V_{td}) \neq 0$ had been established by studies of $K - \bar{K}$ and $B - \bar{B}$ mixing and first measurements of $\sin 2\beta$
- Fact that Im$(V_{ub}) \neq 0$ has been established by studying rare hadronic decays $(B \to \pi K, \pi\pi)$ in QCD factorization [BBNS (2001), here updated to 2004 data]
- **KM relation confirmed;** most stringent test of KM mechanism at the time

2004 analysis: $\bar{\rho} = 0.15 \pm 0.08,\ \bar{\eta} = 0.36 \pm 0.09$
$\gamma = (67 \pm 15)^\circ,\quad \beta = (24 \pm 2)^\circ$

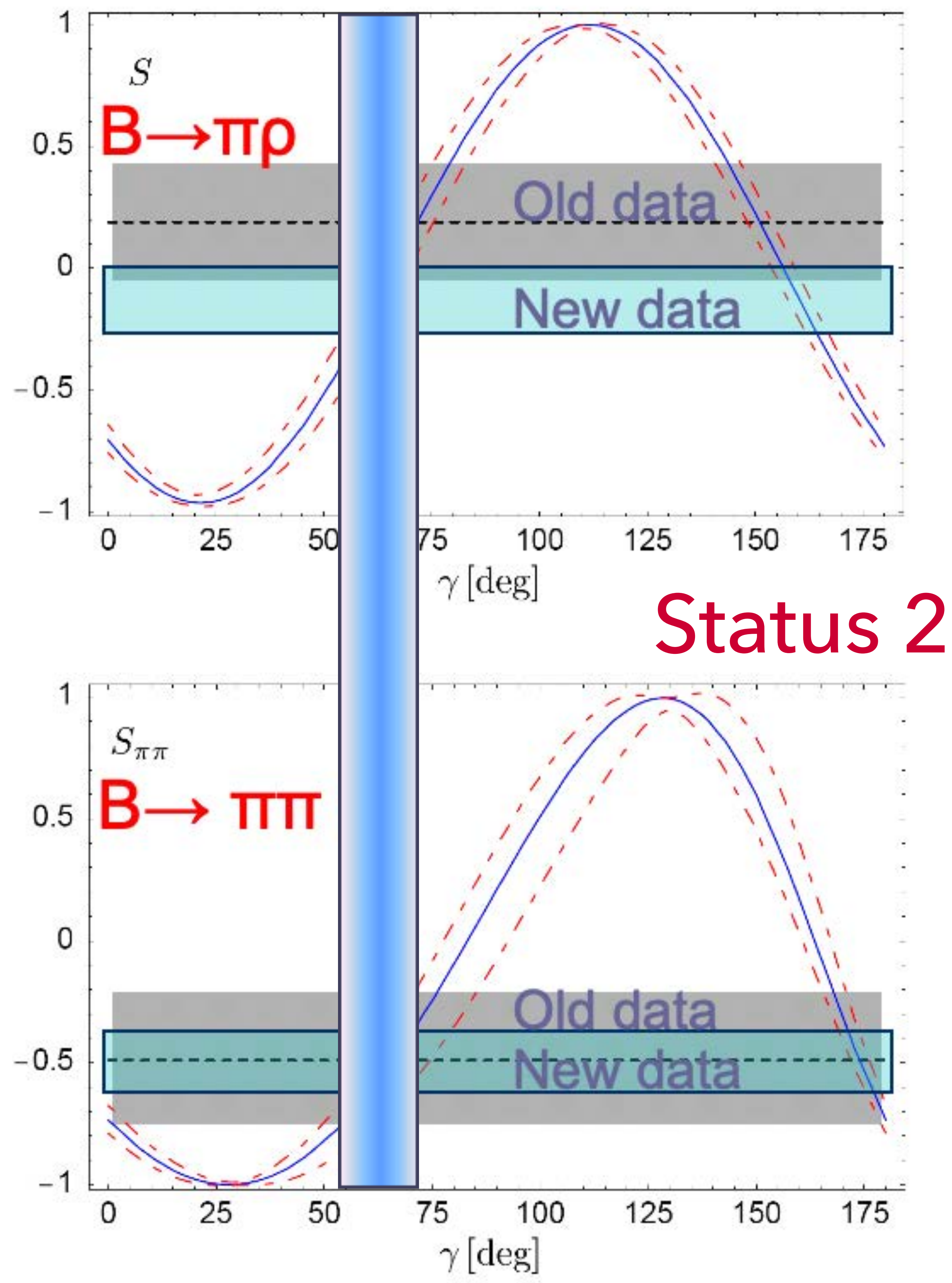
2021 values: $\bar{\rho} = 0.157^{+0.009}_{-0.005},\ \bar{\eta} = 0.347^{+0.012}_{-0.005}$
$\gamma = (65.5^{+1.3}_{-1.2})^\circ,\quad \beta = (22.42^{+0.64}_{-0.37})^\circ$

[CKMfitter global fit, spring 2021]



CONFIRMATION OF KM RELATION BETWEEN IM(V_{UB}) AND IM(V_{TB})

- Measuring time-dependent CP asymmetries in $B \to \pi\pi$ and $B \to \pi\rho$ decays one obtains an internally consistent determination of γ
- 2003 analysis found: $\gamma = (62 \pm 8)^\circ$
- 2021 value: $\gamma = (65.5^{+1.3}_{-1.2})^\circ$



Status 2003

[Beneke, MN (2003)]

LIMITATIONS OF QCD FACTORIZATION

- Lots of predictive power, but uncertainties due to hadronic input quantities: form factors, decay constants, and LCDAs (reducible to some extent)
- **Power corrections in** Λ_{QCD}/m_b do not (naively) factorize due to endpoint divergences ($\Rightarrow$ **different meanings of "factorization"**)
- In some cases, power-suppressed effects can be enhanced by large Wilson coefficients (e.g. "color-suppressed" decay modes)
- To make progress, one needed an EFT implementation of QCD factorization

SOFT-COLLINEAR EFFECTIVE THEORY (SCET)

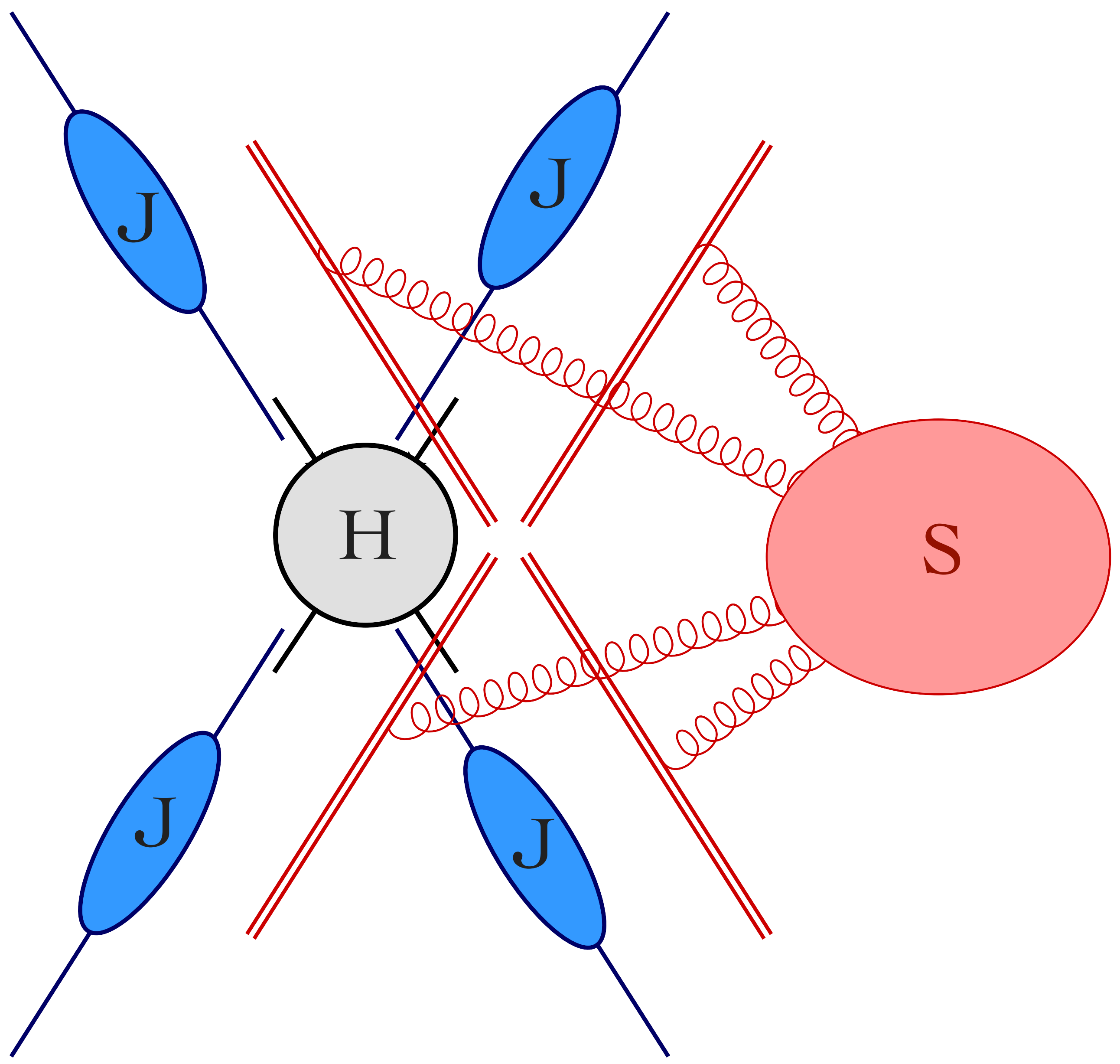
[Bauer, (Fleming,) Pirjol, Stewart (2001); Beneke, Chapovski, Diehl, Feldmann (2002)]

- Firm theoretical basis for deriving QCD factorization theorems in heavy-quark and collider physics for processes involving light energetic particles

- Collinear effective Lagrangian:

$$\mathcal{L}_n = \bar{\xi}_n(x) \left[in \cdot D_n + gn \cdot A_s + i\not{\mathcal{D}}_n^{\perp} \frac{1}{i\bar{n} \cdot \mathcal{D}_n} i\not{\mathcal{D}}_n^{\perp} \right] \frac{\not{\bar{n}}}{2} \xi_n(x) + \ldots$$

eikonal interaction, can be removed by the field redefinition $\xi_n \to S_n \xi_n^{(0)}$



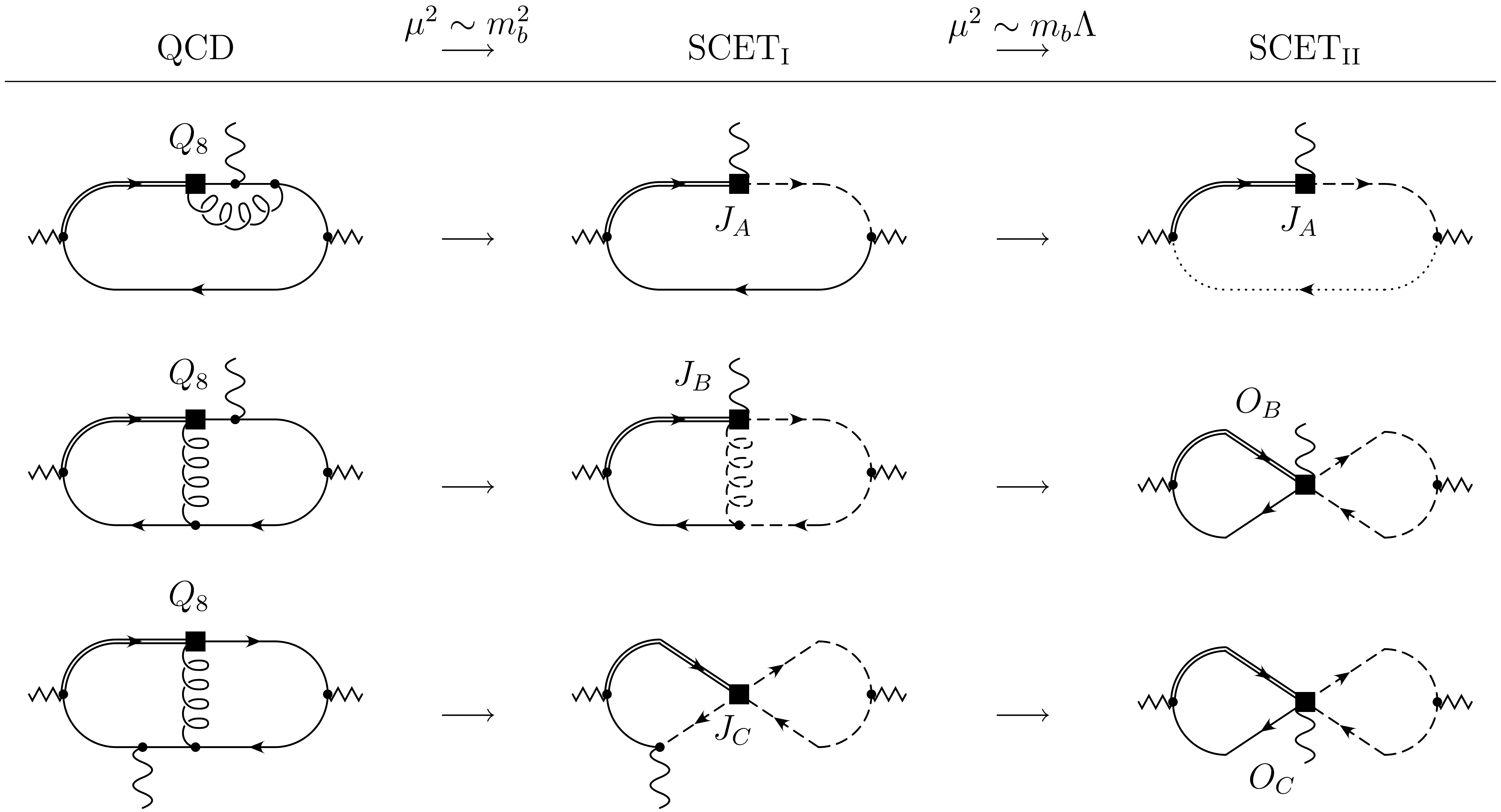
- Soft-collinear factorization at Lagrangian level

- Scale separation and resummation accomplished using powerful EFT tools

SCET PROOF OF QCD FACTORIZATION FOR $B \to K^*\gamma$ DECAY

[Becher, Hill, MN (2005)]

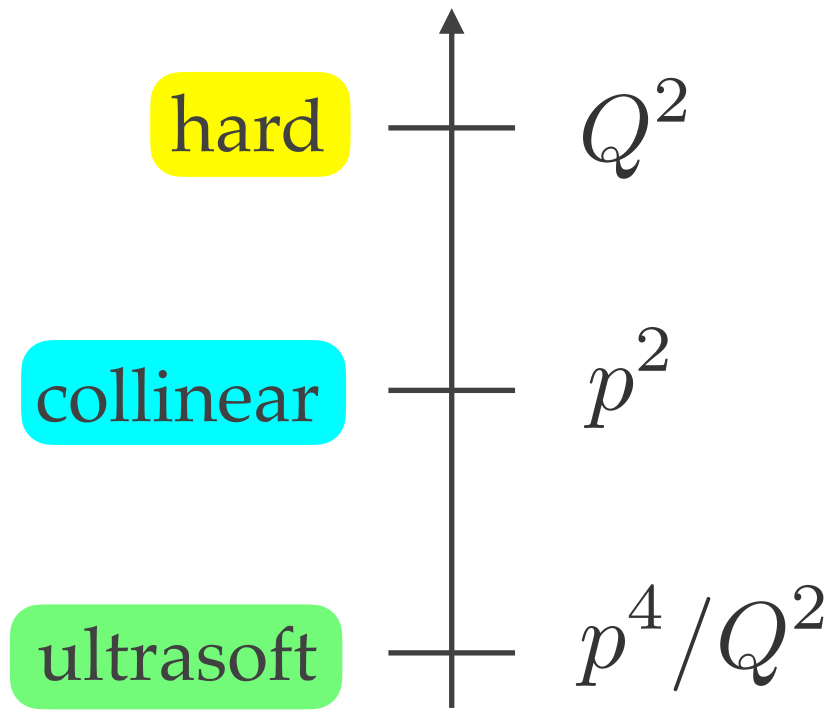
Two-step matching procedure QCD → SCET-1 → SCET-2:



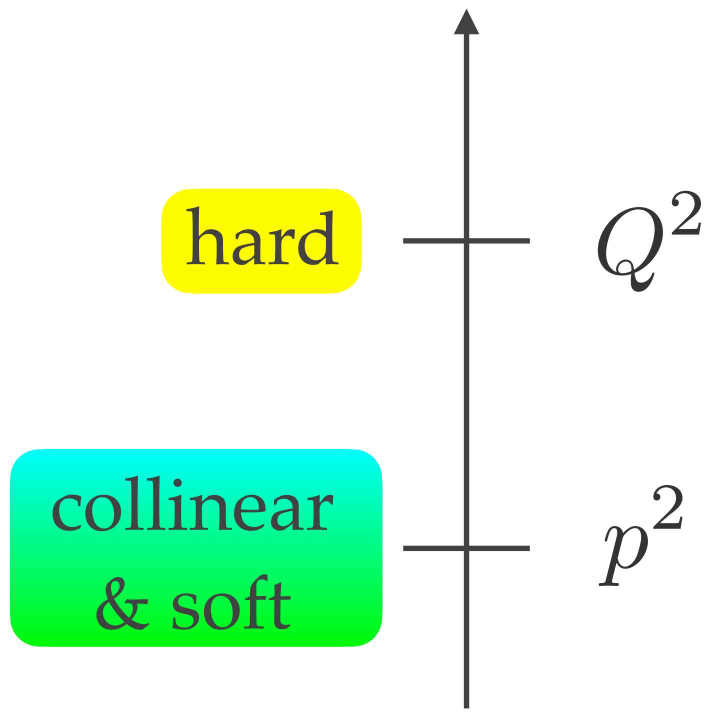
PROTOTYPICAL SCET FACTORIZATION THEOREM

Product/convolution of component functions each depending on a single scale:

$$\sigma \sim \underbrace{H}_{\text{hard}} \int \underbrace{J \otimes J}_{\text{collinear}} \otimes \underbrace{S}_{\text{soft}}$$



SCET-1

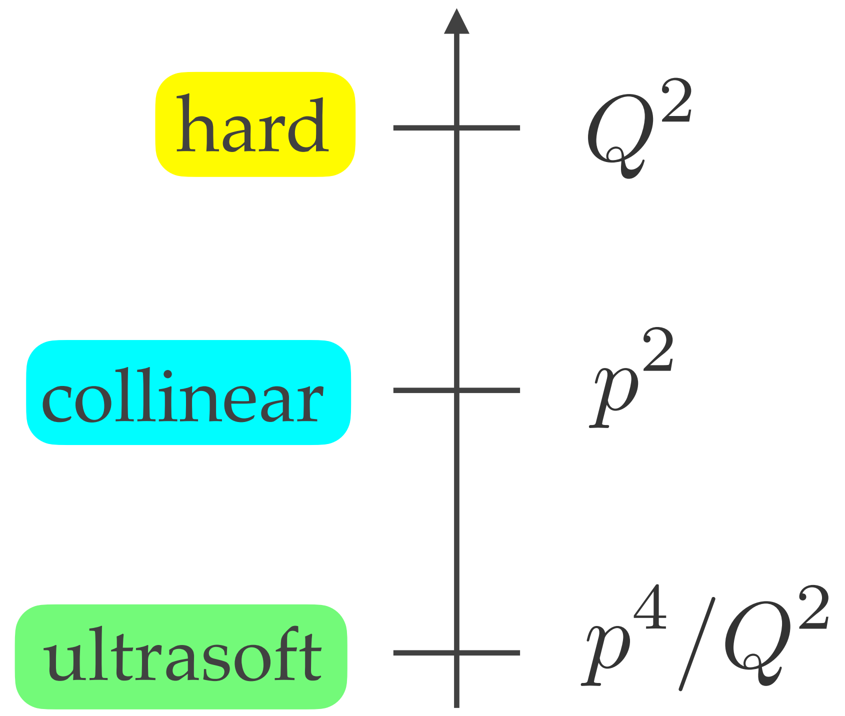


SCET-2

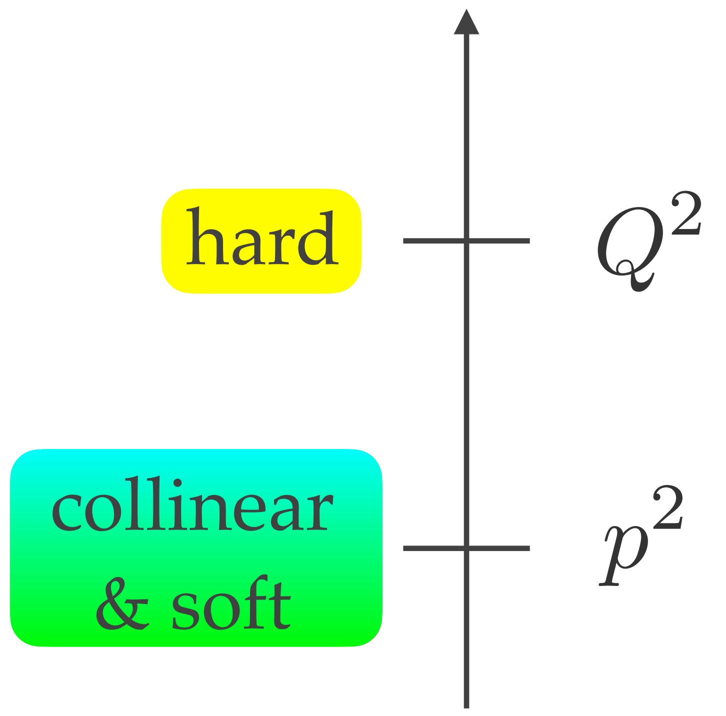
PROTOTYPICAL SCET FACTORIZATION THEOREM

Product/convolution of component functions each depending on a single scale:

$$\sigma \sim \underbrace{H}_{\text{hard}} \int \underbrace{J \otimes J}_{\text{collinear}} \otimes \underbrace{S}_{\text{soft}}$$



SCET-1



SCET-2

- Extension to next-to-leading power is a hard problem, due to **endpoint-divergent convolution integrals** [Beneke et al. ; Moult et al.; Stewart et al.; Bell et al. (2018–2022)]
- **Refactorization-based subtraction** (RBS) scheme provides a consistent framework for dealing with this problem [Liu, MN (2019, 2020); Liu, Mecaj, MN, Wang (2021); Liu, MN, Schnubel, Wang (2022)]

TWO FRONTIERS OF SCET FACTORIZATION

SCET-based factorization theorems become far more complicated:

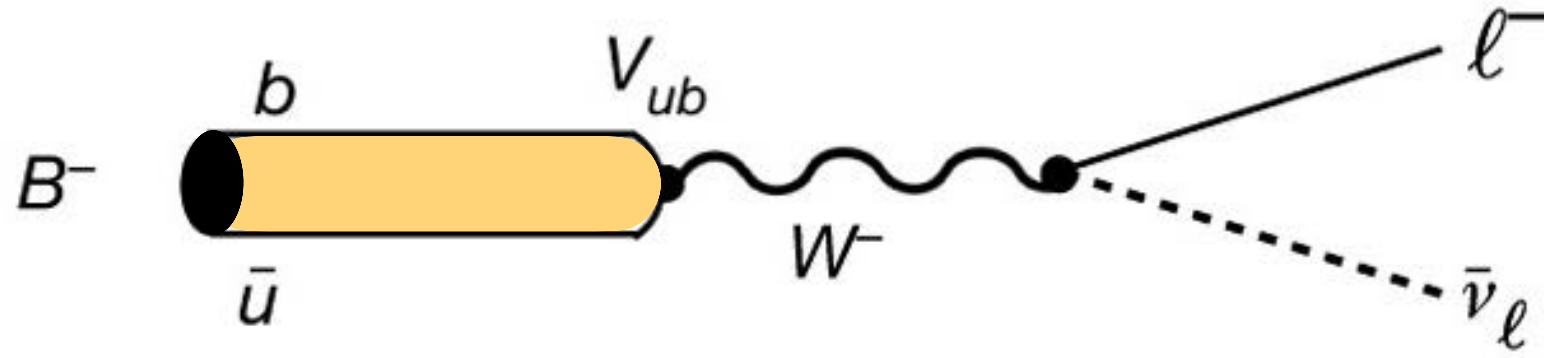
- at next-to-leading power in scale ratios, due to endpoint divergences
- when QED corrections are included to reach $O(1\%)$ accuracy, since external hadron states are in general not singlets under electromagnetism

[Beneke, Bobeth, Szafron (2019); Beneke, Böer, Toelstede, Vos (2020, 2022)]

  - hadronic input (decay constants, form factors, LCDAs) need to be redefined
  - many additional hadronic matrix elements enter
  - leptonic decays become as complicated as non-leptonic decays (since leptons ℓ^- are charged)

FACTORIZING THE SIMPLEST B DECAY

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:



$$\Gamma \sim m_\ell^2 \, f_{B_u}^2 \, |V_{ub}|^2$$

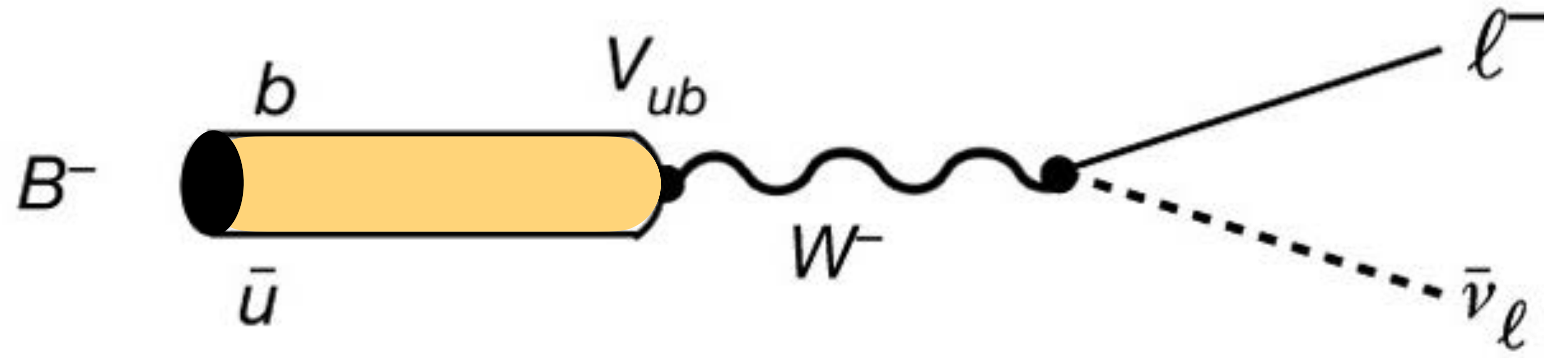
B-meson decay constant

- **Determination of $|V_{ub}|$,** largely unaffected by hadronic uncertainties
- Chiral suppression offers sensitive probe of new interactions
- **Test of lepton universality** by comparing decays with different lepton flavors

  ⇒ Belle II will measure $\ell = \mu, \tau$ channels with 5-7% uncertainty [Belle II Physics Book]

FACTORIZING THE SIMPLEST B DECAY

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:



$$\Gamma \sim m_\ell^2 \, f_{B_u}^2 \, |V_{ub}|^2$$

B-meson decay constant

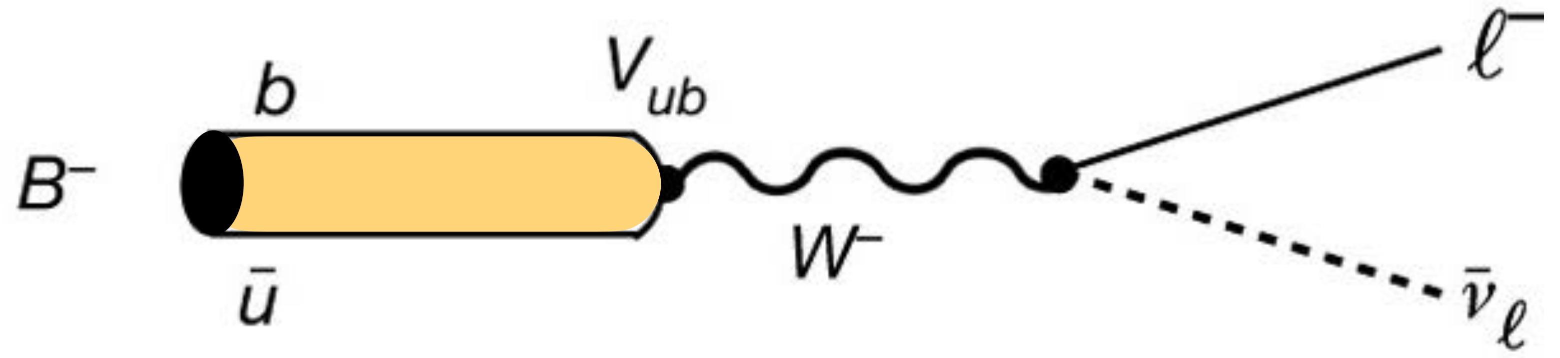
- QCD matrix element is known with <1% accuracy: [FNAL/MILC (2017)]

$$\langle 0 | \, \bar{u} \gamma^\mu \gamma_5 b \, | B^-(p) \rangle = i f_{B_u} p^\mu \quad \text{with} \quad f_{B_u} = (189.4 \pm 1.4)\,\text{MeV}$$

- QED corrections can be of similar magnitude or even larger, due to presence of **large logarithms** $\alpha \ln^2(m_B/m_\ell)$ and $\alpha \ln(m_B/E_\gamma) \, \ln(m_B/m_\ell)$

FACTORIZING THE SIMPLEST B DECAY

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:



$$\Gamma \sim m_\ell^2 \, f_{B_u}^2 \, |V_{ub}|^2$$

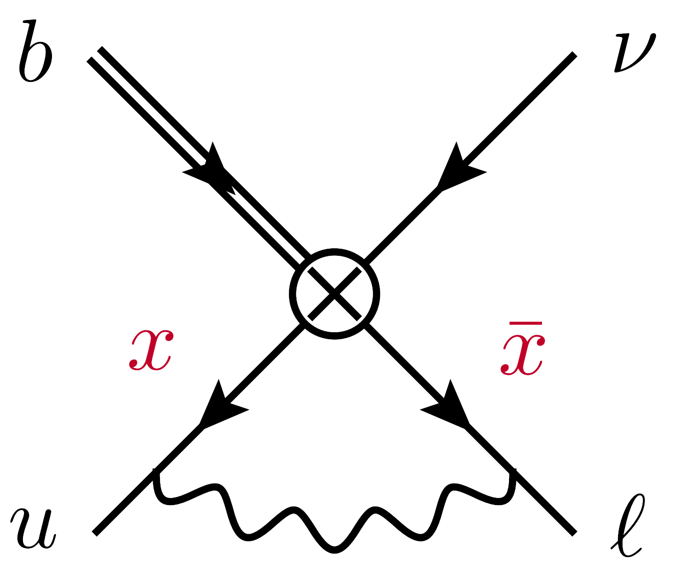
B-meson decay constant

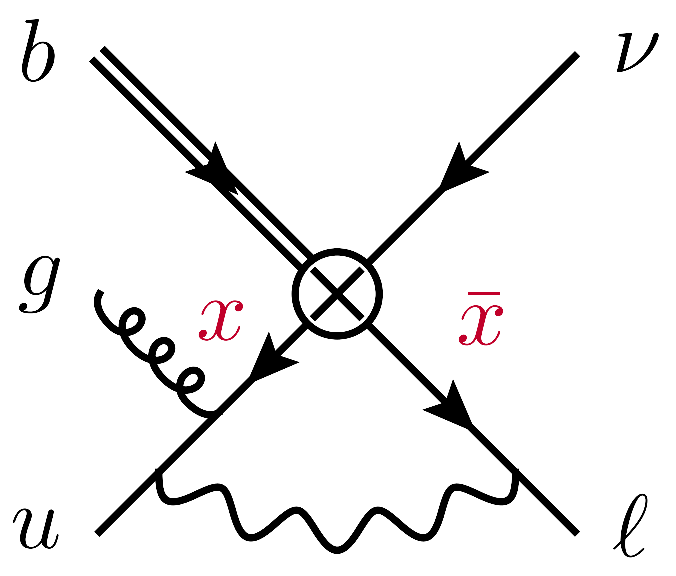
- Quark current $\bar{u}\,\gamma^\mu P_L\, b$ is not gauge invariant under QED ⇒ add a soft Wilson line $S_n^\dagger$ to account for soft photon interactions with the charged lepton
- **Problem:** Defining f_B or the corresponding HQET parameter *F* with such a Wilson line is **incompatible** with *F* being a local parameter, since it would mix with non-local matrix elements (LCDAs) under renormalization! [Cornella, König, MN (2022)]

FACTORIZING THE SIMPLEST B DECAY

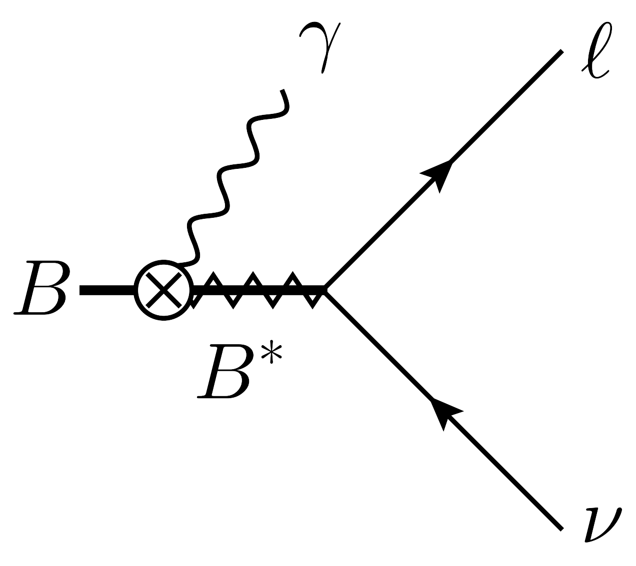
QED effects are well under control for scales $\mu \gg m_b$ (effective weak Hamiltonian) and $\mu \ll \Lambda_{\rm QCD}^2/m_B$ (Low's theorem)

- Intermediate scale range gives rise to intricate effects, as photons can resolve the **inner structure of the *B* meson,** above and below the scale $\Lambda_{\rm QCD}$!





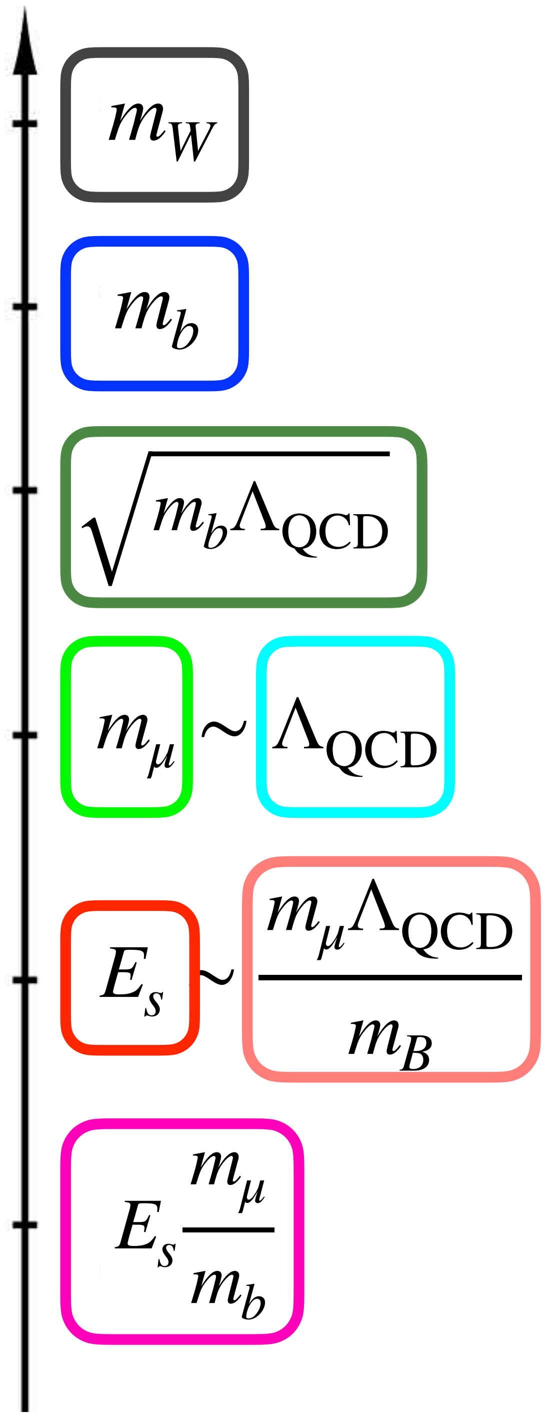
SCET factorization theorem for virtual corrections
[Cornella, König, MN (2022)]

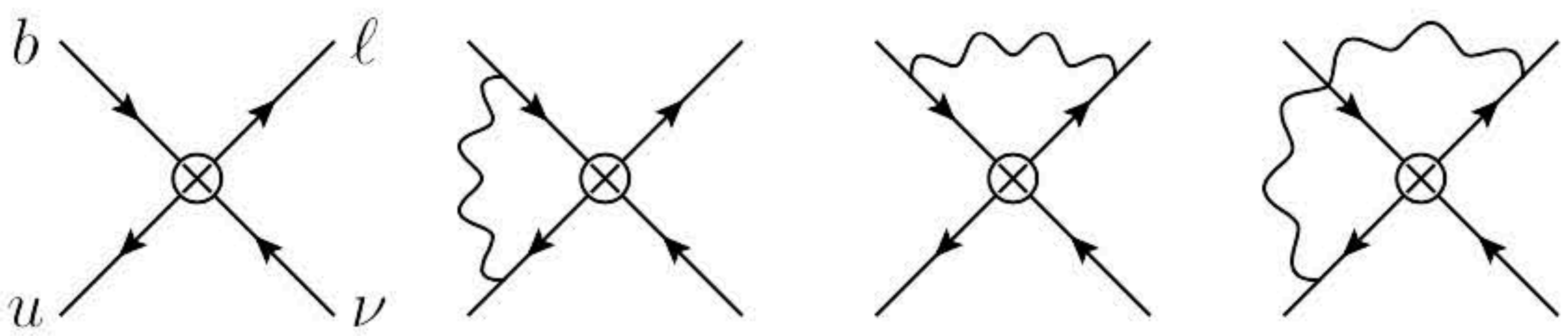


Heavy-meson EFT for real emissions
[Cornella, Ferré, König, MN, to appear]

FACTORIZING THE SIMPLEST B DECAY

While in the absence of QED effects $B^- \to \mu^- \bar{\nu}_\mu$ is governed by only 3 scales ($m_W \gg m_b \gg \Lambda_{\rm QCD}$), with QED effects included 8 scales become relevant:

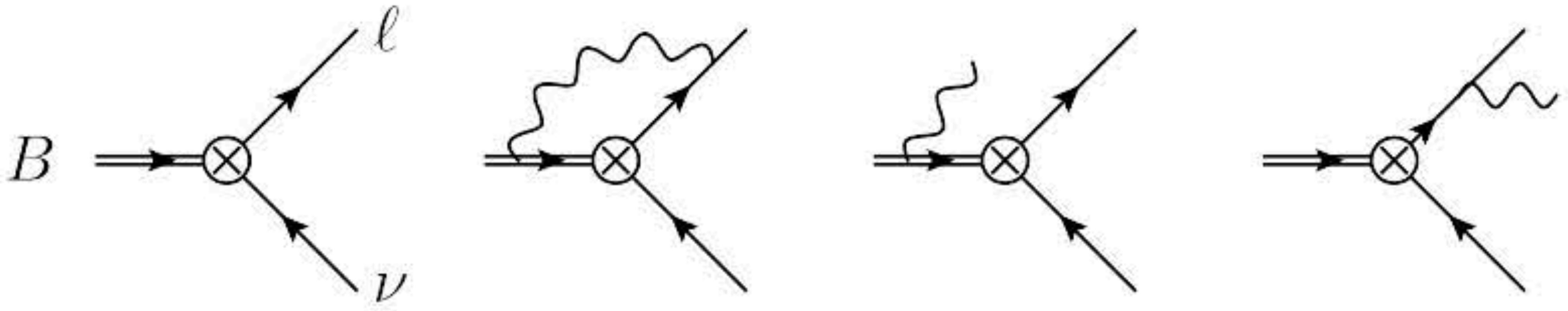




Fock-state description of B meson: $|\bar{u}b\rangle + |\bar{u}gb\rangle + \ldots$

? [Beneke, Bobeth, Szafron (2019)]

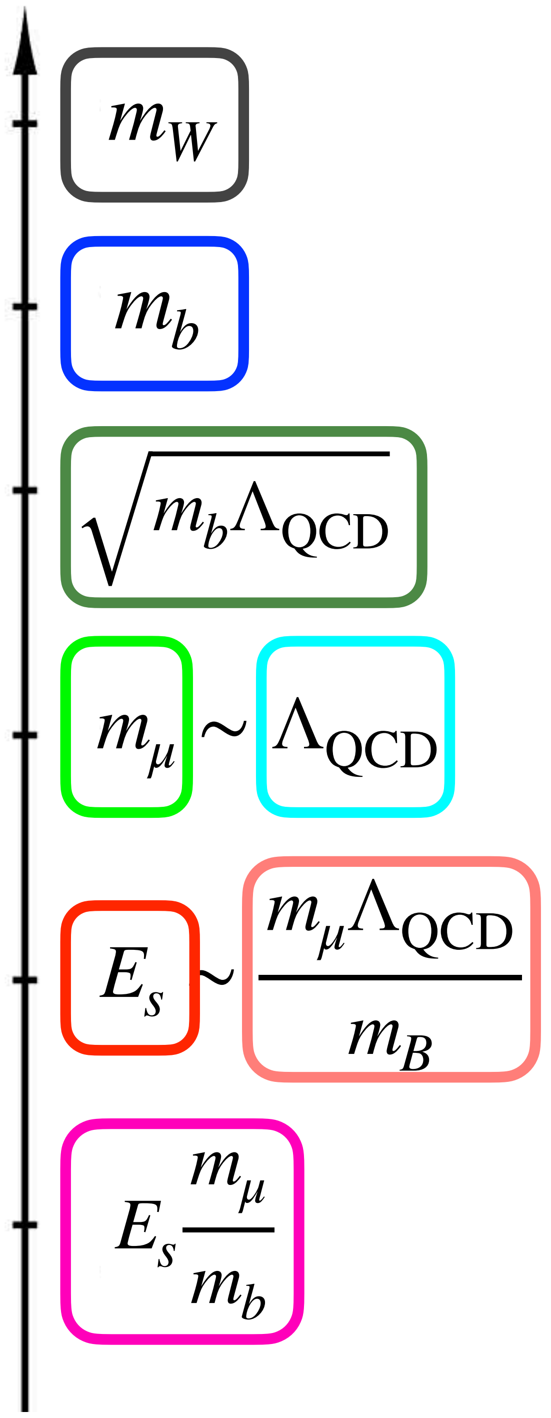
B meson described as a point-like pseudo-scalar boson



[e.g.: Isidori, Nabeebaccus, Zwicky (2020); Dai, Kim, Leibovich (2021)]

FACTORIZING THE SIMPLEST B DECAY

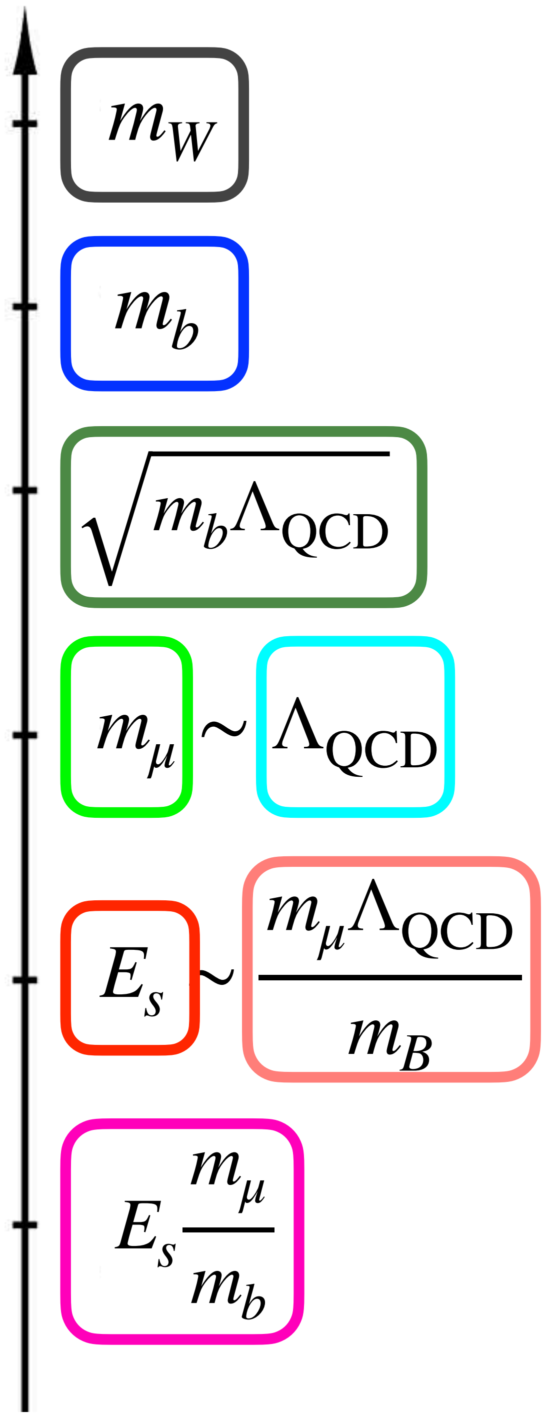
We have analyzed the factorization of the $B^- \to \mu^- \bar{\nu}_\mu$ amplitude including QED corrections in SCET & HPET: [Cornella, König, MN (2022)]



- Relevant modes in the EFT:
  - hard
  - hard-collinear ⟵ resolve the light-cone structure of the *B* meson (à la Brodsky-Lepage)
  - soft
  - collinear
  - soft-collinear
- Relevant modes for real QED corrections:
  - ultra-soft
  - ultra-soft-collinear

FACTORIZING THE SIMPLEST B DECAY

We have analyzed the factorization of the $B^- \to \mu^- \bar{\nu}_\mu$ amplitude including QED corrections in SCET & HPET: [Cornella, König, MN (2022)]



- Relevant modes in the EFT:
  - hard
  - hard-collinear
  - soft
  - collinear
  - soft-collinear
- Relevant modes for real QED corrections:
  - ultra-soft
  - ultra-soft-collinear

Effective weak Hamiltonian

↓

SCET-1

↓

SCET-2

↓

HMEFT

FACTORIZING THE SIMPLEST B DECAY — SCET-1 OPERATOR BASIS

$$O_1^{A(\frac{11}{2})} = \frac{m_\ell}{\bar n\cdot\mathcal{P}_{hc}}\,\bar u_s \not{\bar n} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} P_L \nu_{\bar c}\,,$$
$$O_2^{A(\frac{11}{2})} = \frac{m_\ell}{m_B}\,\bar u_s P_R b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} P_L \nu_{\bar c}\,,$$
$$O_3^{A(5)} = \frac{1}{\bar n\cdot\mathcal{P}_{hc}}\,\bar u_s \not{\bar n} P_L b_v\,\big[\bar{\mathcal{X}}_{hc}^{(\ell)}(-i\overleftarrow{\not\partial}_\perp)\big] P_L \nu_{\bar c}\,,$$
$$O_4^{A(5)} = \frac{1}{m_B}\,\bar u_s P_R b_v\,\big[\bar{\mathcal{X}}_{hc}^{(\ell)}(-i\overleftarrow{\not\partial}_\perp)\big] P_L \nu_{\bar c}\,.$$

$$O_1^{B(5)} = \frac{1}{\bar n\cdot\mathcal{P}_{hc}}\,\bar u_s \not{\bar n} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \not{\mathcal{A}}^\perp_{hc[y]} P_L \nu_{\bar c}\,,$$
$$O_2^{B(5)} = \frac{1}{m_B}\,\bar u_s P_R b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \not{\mathcal{A}}^\perp_{hc[y]} P_L \nu_{\bar c}\,,$$
$$O_3^{B(5)} = \frac{1}{\bar n\cdot\mathcal{P}_{hc}}\,\bar u_s \not{\bar n} P_L \mathcal{G}^{\perp\alpha}_{hc[y]} b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_4^{B(5)} = \frac{1}{m_B}\,\bar u_s P_R\, \mathcal{G}^{\perp\alpha}_{hc[y]} b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,.$$

$$O_1^{C(\frac{9}{2})} = \frac{m_\ell}{m_B}\,\bar{\mathcal{X}}^{(u)}_{hc[y]} P_R b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} P_L \nu_{\bar c}\,,$$
$$O_2^{C(5)} = \frac{m_\ell}{m_B^2}\,\big[\bar{\mathcal{X}}^{(u)}_{hc[y_1]}(-i\overleftarrow{\not\partial}_\perp)\big] P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} P_L \nu_{\bar c}\,,$$
$$O_3^{C(4,\frac{9}{2})} = \frac{1}{m_B}\,\bar{\mathcal{X}}^{(u)}_{hc[y]}(-i\overleftarrow{D}^\alpha_{s\perp}) P_R b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_4^{C(4,\frac{9}{2})} = \frac{1}{m_B}\,\bar{\mathcal{X}}^{(u)}_{hc[y]} P_R b_v\,\big[\bar{\mathcal{X}}_{hc}^{(\ell)}(-i\overleftarrow{\not D}_{s\perp})\big] P_L \nu_{\bar c}\,,$$
$$O_5^{C(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y]}(-i\overleftarrow{\not\partial}_\perp) P_L b_v\,\big[\bar{\mathcal{X}}_{hc}^{(\ell)}(-i\overleftarrow{\not\partial}_\perp)\big] P_L \nu_{\bar c}\,,$$

$$O_1^{D(4)} = \frac{1}{m_B}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} P_R b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \not{\mathcal{A}}^\perp_{hc[y_2]} P_L \nu_{\bar c}\,,$$
$$O_2^{D(4)} = \frac{1}{m_B}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \mathcal{G}^{\perp\alpha}_{hc[y_2]} P_R b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_3^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]}(-i\overleftarrow{\not\partial}_\perp) P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \not{\mathcal{A}}^\perp_{hc[y_2]} P_L \nu_{\bar c}\,,$$
$$O_4^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]}(-i\overleftarrow{\not\partial}_\perp)\, \mathcal{G}^{\perp\alpha}_{hc[y_2]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_5^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{A}}^\perp_{hc[y_2]} P_L b_v\,\big[\bar{\mathcal{X}}_{hc}^{(\ell)}(-i\overleftarrow{\not\partial}_\perp)\big] P_L \nu_{\bar c}\,,$$
$$O_6^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{G}}^\perp_{hc[y_2]} P_L b_v\,\big[\bar{\mathcal{X}}_{hc}^{(\ell)}(-i\overleftarrow{\not\partial}_\perp)\big] P_L \nu_{\bar c}\,,$$
$$O_7^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]}(-i\overleftarrow{\partial}^\alpha_\perp) \not{\mathcal{A}}^\perp_{hc[y_2]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_8^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]}(-i\overleftarrow{\partial}^\alpha_\perp) \not{\mathcal{G}}^\perp_{hc[y_2]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_9^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]}\big(i\not\partial_\perp \mathcal{A}^{\perp\alpha}_{hc[y_2]}\big) P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_{10}^{D(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]}\big(i\not\partial_\perp \mathcal{G}^{\perp\alpha}_{hc[y_2]}\big) P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_{11}^{D(5)} = \frac{m_\ell}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{A}}^\perp_{hc[y_2]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} P_L \nu_{\bar c}\,,$$
$$O_{12}^{D(5)} = \frac{m_\ell}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{G}}^\perp_{hc[y_2]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} P_L \nu_{\bar c}\,.$$

$$O_1^{E(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{A}}^\perp_{hc[y_2]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \not{\mathcal{A}}^\perp_{hc[y_3]} P_L \nu_{\bar c}\,,$$
$$O_2^{E(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{G}}^\perp_{hc[y_2]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \not{\mathcal{A}}^\perp_{hc[y_3]} P_L \nu_{\bar c}\,,$$
$$O_3^{E(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{A}}^\perp_{hc[y_2]} \mathcal{G}^{\perp\alpha}_{hc[y_3]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_4^{E(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \not{\mathcal{G}}^\perp_{hc[y_2]} \mathcal{G}^{\perp\alpha}_{hc[y_3]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,,$$
$$O_5^{E(\frac{9}{2})} = \frac{1}{m_B^2}\,\bar{\mathcal{X}}^{(u)}_{hc[y_1]} \mathcal{G}^{\perp\alpha}_{hc[y_2]} \not{\mathcal{G}}^\perp_{hc[y_3]} P_L b_v\,\bar{\mathcal{X}}_{hc}^{(\ell)} \gamma^\perp_\alpha P_L \nu_{\bar c}\,.$$

Wilson coefficients are hard functions:

$$H_i(m_b,\mu)$$

[Cornella, Ferré, König, MN, to appear]

FACTORIZING THE SIMPLEST B DECAY — SCET-2 OPERATOR BASIS

$$\mathcal{O}_1^A = \frac{m_\ell}{(\bar{n}\cdot\mathcal{P})}\Big(\bar{\mathcal{Q}}_s \not{\bar{n}} P_L \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$
$$\mathcal{O}_2^A = \frac{m_\ell}{m_B}\Big(\bar{\mathcal{Q}}_s P_R \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$

$$\mathcal{O}_1^C(y) = \frac{1}{(\bar{n}\cdot\mathcal{P})}\Big(\bar{\mathcal{Q}}_s \not{\bar{n}} P_L \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[y]} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$
$$\mathcal{O}_2^C(y) = \frac{1}{m_B}\Big(\bar{\mathcal{Q}}_s P_R \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[y]} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$

HQET decay constant *F*

$$\mathcal{O}_1^B(w) = \frac{m_\ell}{(\bar{n}\cdot\mathcal{P})}\Big(\bar{\mathcal{Q}}_s^{[w]} \not{\bar{n}} P_L \mathcal{H}_v\Big)\Big(\bar{\chi}_C^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$
$$\mathcal{O}_2^B(w) = \frac{m_\ell}{m_B}\Big(\bar{\mathcal{Q}}_s^{[w]} P_R h_v\Big)\Big(\bar{\chi}_C^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$

$$\mathcal{O}_1^D(w,y) = \frac{1}{(\bar{n}\cdot\mathcal{P})}\Big(\bar{\mathcal{Q}}_s^{[w]} \not{\bar{n}} P_L \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[y]} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$
$$\mathcal{O}_2^D(w,y) = \frac{1}{m_B}\Big(\bar{\mathcal{Q}}_s^{[w]} P_R \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[y]} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$

2-particle LCDAs

$$\mathcal{O}_1^E(w,w') = \frac{m_\ell}{m_B}\Big(\bar{\mathcal{Q}}_s^{[w]}\Big[\frac{\not{n}}{in\cdot\partial}\not{\mathcal{A}}_{s\perp}^{[w']}\Big] P_R \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$
$$\mathcal{O}_2^E(w,w') = \frac{m_\ell}{m_B}\Big(\bar{\mathcal{Q}}_s^{[w]}\Big[\frac{\not{n}}{in\cdot\partial}\not{\mathcal{G}}_{s\perp}^{[w']}\Big] P_R \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$

$$\mathcal{O}_1^F(w,w',y) = \frac{1}{m_B}\Big(\bar{\mathcal{Q}}_s^{[w]}\Big[\frac{\not{n}}{in\cdot\partial}\not{\mathcal{A}}_{s\perp}^{[w']}\Big] P_R \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[y]} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$
$$\mathcal{O}_2^F(w,w',y) = \frac{1}{m_B}\Big(\bar{\mathcal{Q}}_s^{[w]}\Big[\frac{\not{n}}{in\cdot\partial}\not{\mathcal{G}}_{s\perp}^{[w']}\Big] P_R \mathcal{H}_v\Big)\Big(\bar{\chi}_c^{(\ell)} \not{\mathcal{A}}_{c\perp}^{[y]} P_L \chi_{\bar{c}}^{(\nu)}\Big)$$

3-particle LCDAs

Wilson coefficients are jet functions:

$$J_i(m_b\,\omega,\mu)$$

- Quite generically, things get very messy at next-to-leading power!

[Cornella, Ferré, König, MN, to appear]

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

convolution

$$\mathscr{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = \sum_j \underbrace{H_j\, S_j\, K_j}_{} + \sum_i \underbrace{H_i \otimes J_i \otimes S_i \otimes K_i}_{}$$

SCET-1 operators with soft spectator quark

SCET-1 operators with hard-collinear spectator quark

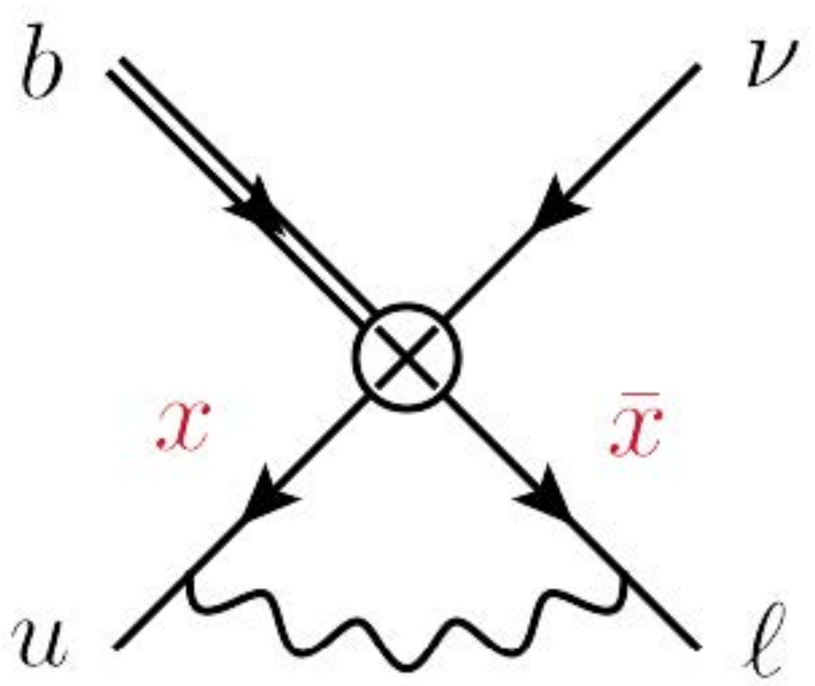
- **Hard functions:** matching corrections at $\mu \sim m_b$
- **Jet functions:** matching corrections at $\mu \sim (m_b \Lambda_{\text{QCD}})^{1/2}$
- **Soft functions:** B-meson matrix elements (local and non-local) in HQET
- **Collinear functions:** leptonic matrix elements, $\mu \sim m_\mu$

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

- Endpoint-divergent convolution integrals:

$$\mathcal{A}_{B\to\ell\bar{\nu}}^{\text{virtual}} \propto \left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b,x)\, J_B(m_b\omega,x) S_B(\omega) \right]$$

- Focus on second term:
  - Shared variable x = collinear momentum fraction carried by the spectator quark
  - $H_B \sim x^{-\epsilon}, J_B \sim x^{-1-\epsilon} \Rightarrow$ $H_B \otimes J_B$ is **endpoint divergent**



- Cannot be removed with standard RG techniques, but treatable within the **refactorization-based subtraction scheme** [Liu, MN (2019); Liu, Mecaj, MN, Wang (2020); Beneke et al. (2022)]

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

- Endpoint-divergent convolution integrals:

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} \propto \left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b,x)\, J_B(m_b\omega,x) S_B(\omega) \right]$$

reshuffle divergent terms

- After refactorization, the convolutions are well defined and the **HQET decay constant F contained in S_A is redefined** in such a way that it now no longer mixes with non-local matrix elements under renormalization
- Would be interesting to compute this redefined HQET parameter using lattice QCD!

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

Decay amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$:

$$\mathcal{A}_{B\to\ell\bar{\nu}}^{\text{virtual}} = i\sqrt{2}G_F\,K_{\text{EW}}(\mu)\,V_{ub}\,\frac{m_\ell}{m_b}\,\sqrt{m_B}\,F(\mu)\;\bar{u}(p_\ell)P_L\,v(p_\nu)\Big[\mathcal{M}_{2p}(\mu)+\mathcal{M}_{3p}(\mu)\Big]$$

with:

$$\begin{aligned}
\mathcal{M}_{2p}(\mu) &= 1 + \frac{C_F\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right] \\
&\quad + \frac{\alpha}{4\pi}\left\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right] - Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2}+2\ln\frac{m_b^2}{\mu^2}-3\ln\frac{m_\ell^2}{\mu^2}+1+\frac{5\pi^2}{12}\right]\right. \\
&\quad \left. +2Q_\ell Q_u\int_0^\infty d\omega\,\phi_-(\omega)\ln\frac{m_b\omega}{\mu^2} + Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2}-2\right)+\frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2}-\frac{1}{2}\ln\frac{m_\ell^2}{\mu^2}+2+\frac{5\pi^2}{12}\right]\right\} \\
\mathcal{M}_{3p}(\mu) &= \frac{\alpha}{\pi}\,Q_\ell Q_u\int_0^\infty d\omega\int_0^\infty d\omega_g\,\phi_{3g}(\omega,\omega_g)\left[\frac{1}{\omega_g}\ln\left(1+\frac{\omega_g}{\omega}\right)-\frac{1}{\omega+\omega_g}\right]
\end{aligned}$$

FACTORIZATION FORMULA FOR VIRTUAL CORRECTIONS

Decay amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$:

$$\mathcal{A}_{B\to\ell\bar{\nu}}^{\text{virtual}} = i\sqrt{2}G_F\,K_{\text{EW}}(\mu)V_{ub}\,\frac{m_\ell}{m_b}\sqrt{m_B}\,F(\mu)\;\bar{u}(p_\ell)P_L\,v(p_\nu)\Big[\mathcal{M}_{2p}(\mu)+\mathcal{M}_{3p}(\mu)\Big]$$

with:

IR divergence cancels against real soft photon emission

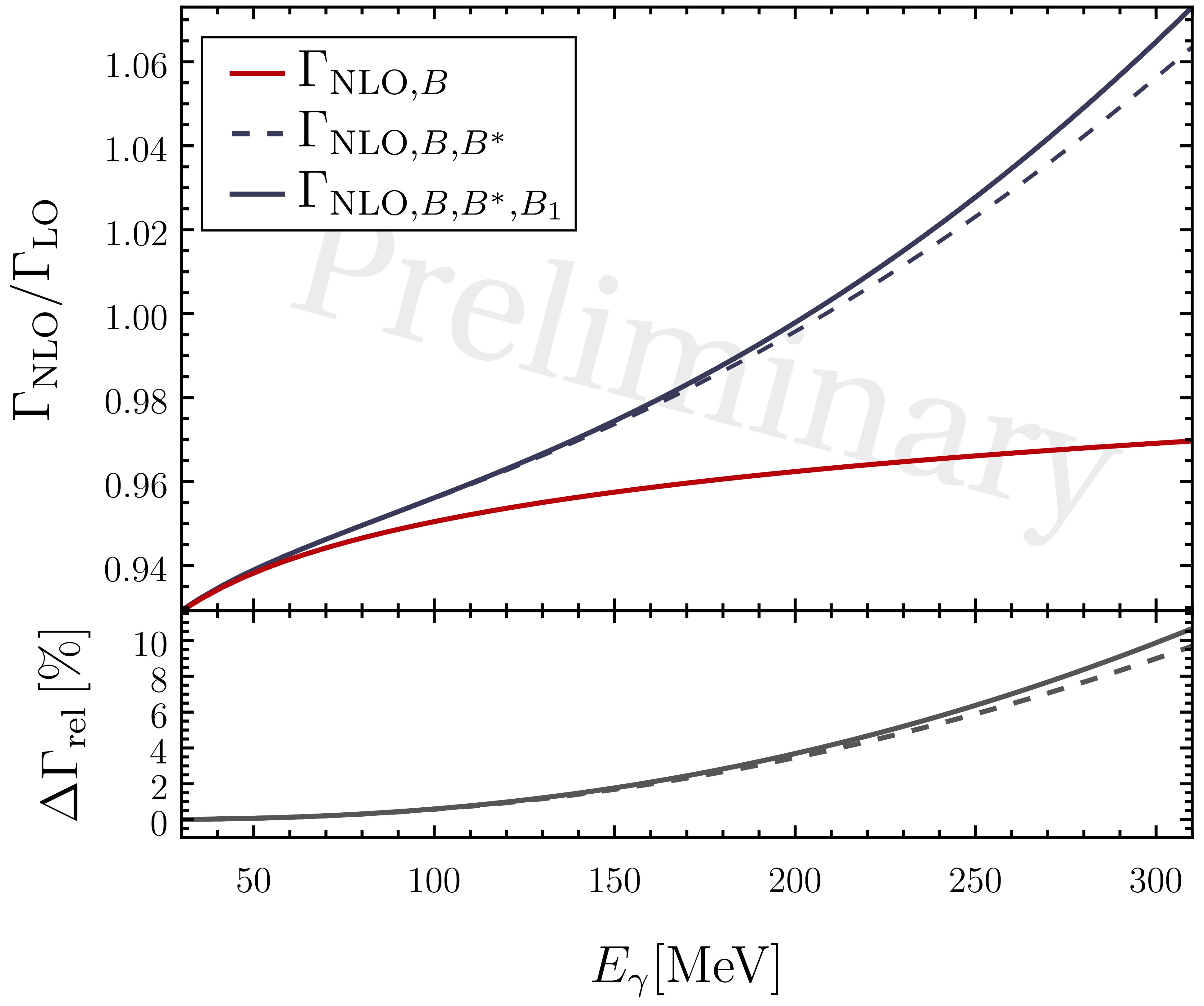
$$\begin{aligned}\mathcal{M}_{2p}(\mu) &= 1+\frac{C_F\,\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right]\\ &\quad+\frac{\alpha}{4\pi}\left\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right]-Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2}+2\ln\frac{m_b^2}{\mu^2}-3\ln\frac{m_\ell^2}{\mu^2}+1+\frac{5\pi^2}{12}\right]\right.\\ &\quad\left.+2Q_\ell Q_u\int_0^\infty d\omega\,\phi_-(\omega)\ln\frac{m_b\,\omega}{\mu^2}+Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2}-2\right)+\frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2}-\frac{1}{2}\ln\frac{m_\ell^2}{\mu^2}+2+\frac{5\pi^2}{12}\right]\right\}\end{aligned}$$

$$\mathcal{M}_{3p}(\mu) = \frac{\alpha}{\pi}\,Q_\ell Q_u\int_0^\infty d\omega\int_0^\infty d\omega_g\,\phi_{3g}(\omega,\omega_g)\left[\frac{1}{\omega_g}\ln\left(1+\frac{\omega_g}{\omega}\right)-\frac{1}{\omega+\omega_g}\right]$$

REAL PHOTON EMISSIONS

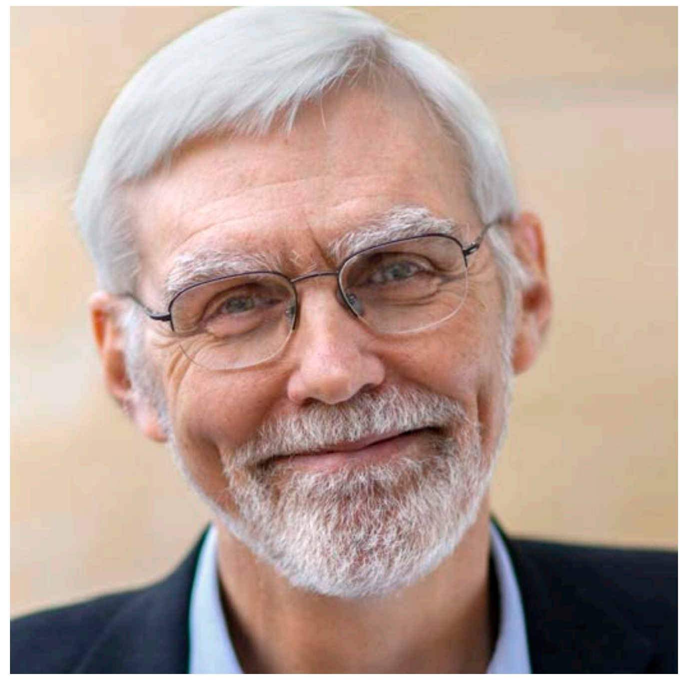
Structure-dependent QED corrections below $\Lambda_{\rm QCD}$:

- $B \to B^*\gamma$ contribution becomes relevant for $E_\gamma \gtrsim (m_{B^*} - m_B) \approx 46\,\text{MeV}$
- Contributions of higher excited states are power suppressed for $E_\gamma \ll \Lambda_{\rm QCD}$ (real photon emission)
- The looser the cut on additional radiation, the more important the B^* and higher contributions are

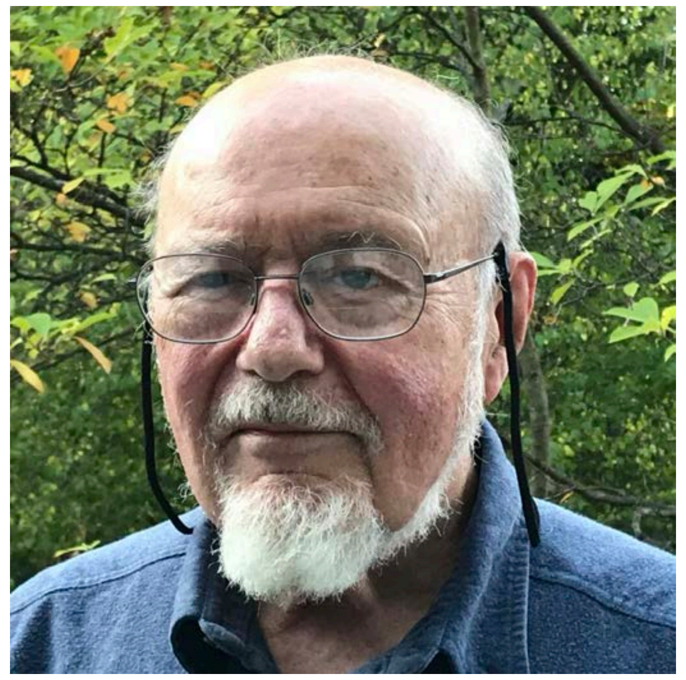


NYS Baroque

Board of Directors (2024-2025)

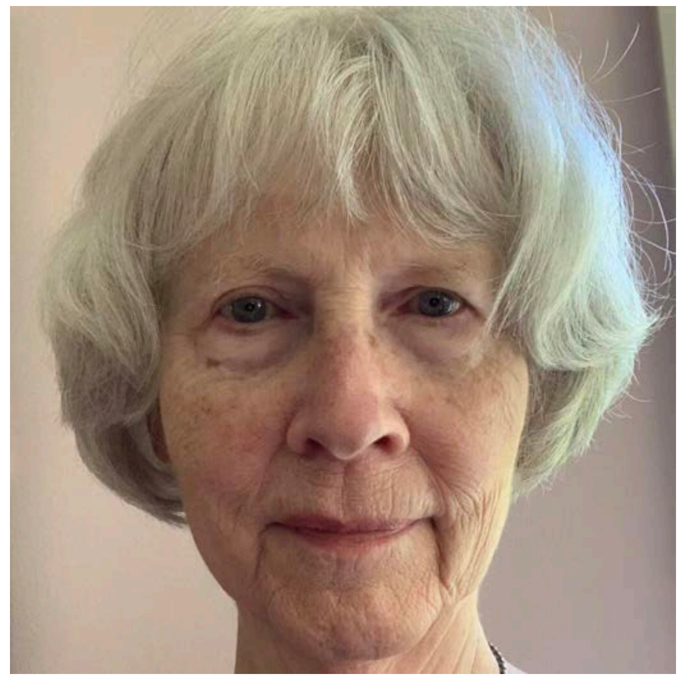


Peter Lepage, President
Cornell University

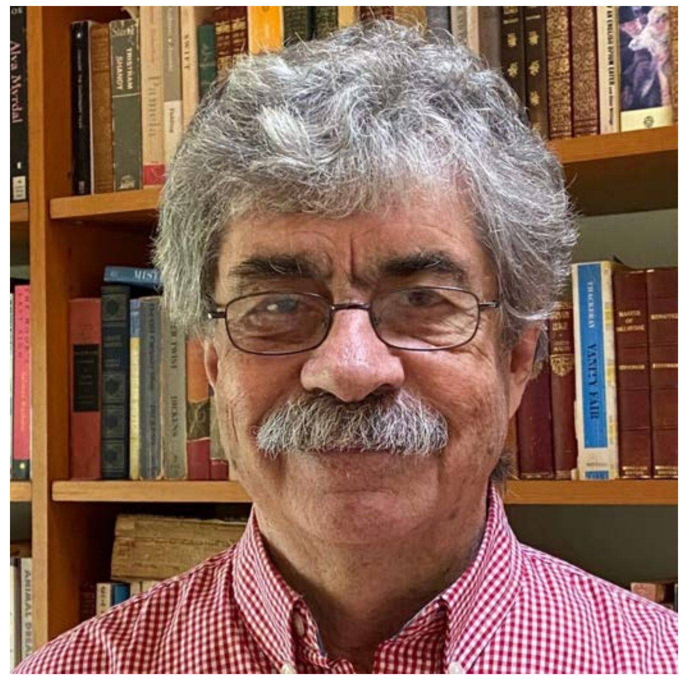


Maury Tigner, Treasurer
Cornell University

Walter Freeman,
Secretary
Syracuse University



Libby Hedrick, Founder,
co-director emeritus,
photographer, Ithaca



Ron Harris-Warrick,
Board Member
Cornell University

K. Alice Lindsay, Board
Member
Syracuse University

Peggy Liuzzi, Board
Member
Syracuse

Holly Sammons, Board
Member
Syracuse