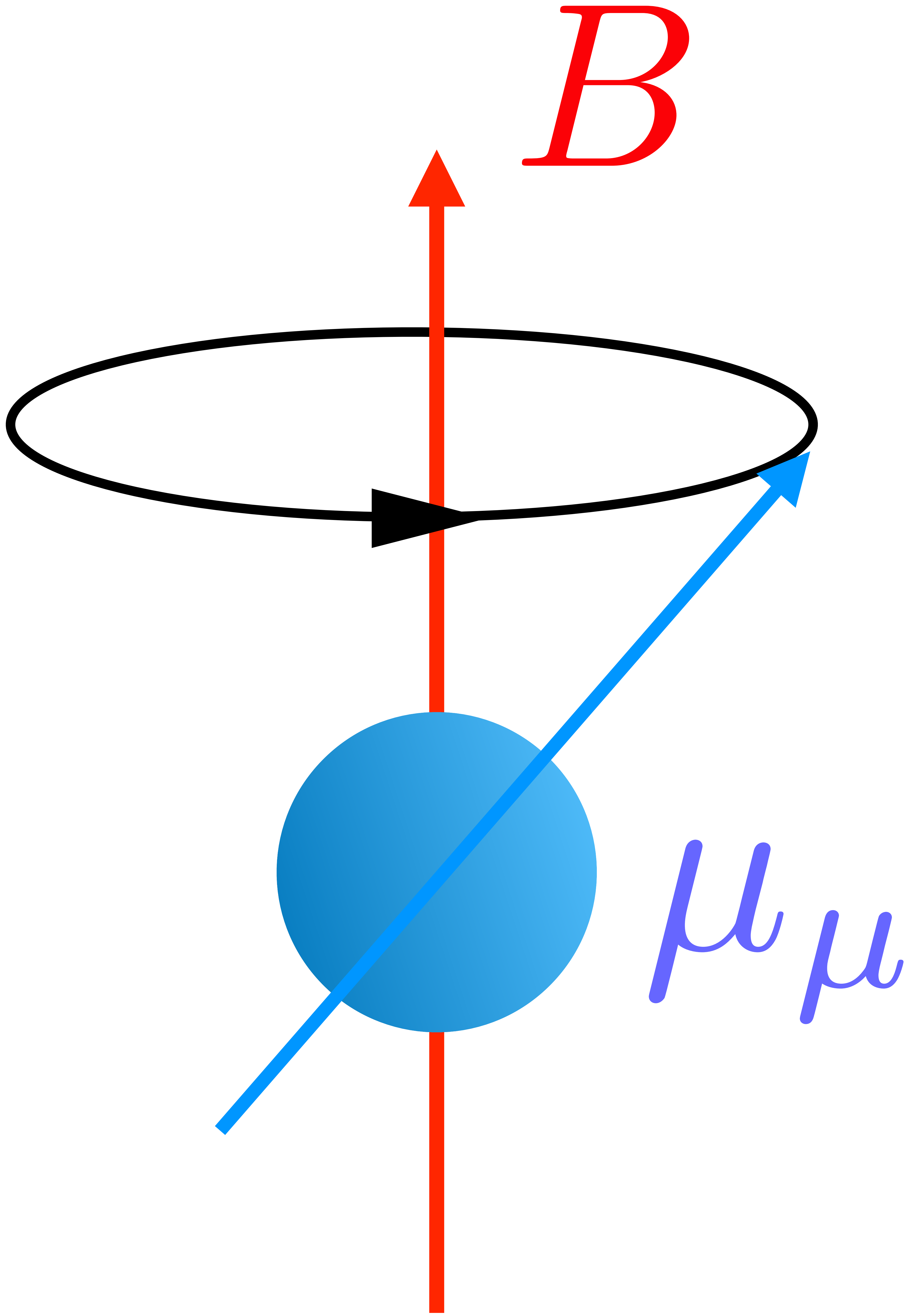
Muon g-2

Christine Davies

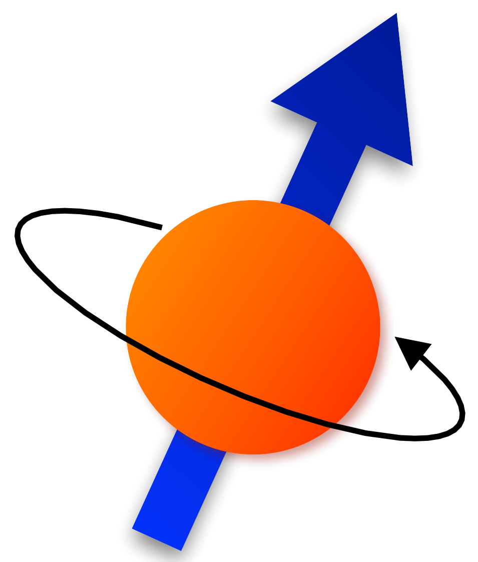
University of Glasgow



Lepagefest, Cornell, October 2024

Muon magnetic moment

$$\vec{\mu}_\mu = g_\mu \left(\frac{e}{2m_\mu}\right) \vec{S}$$



Anomalous magnetic moment

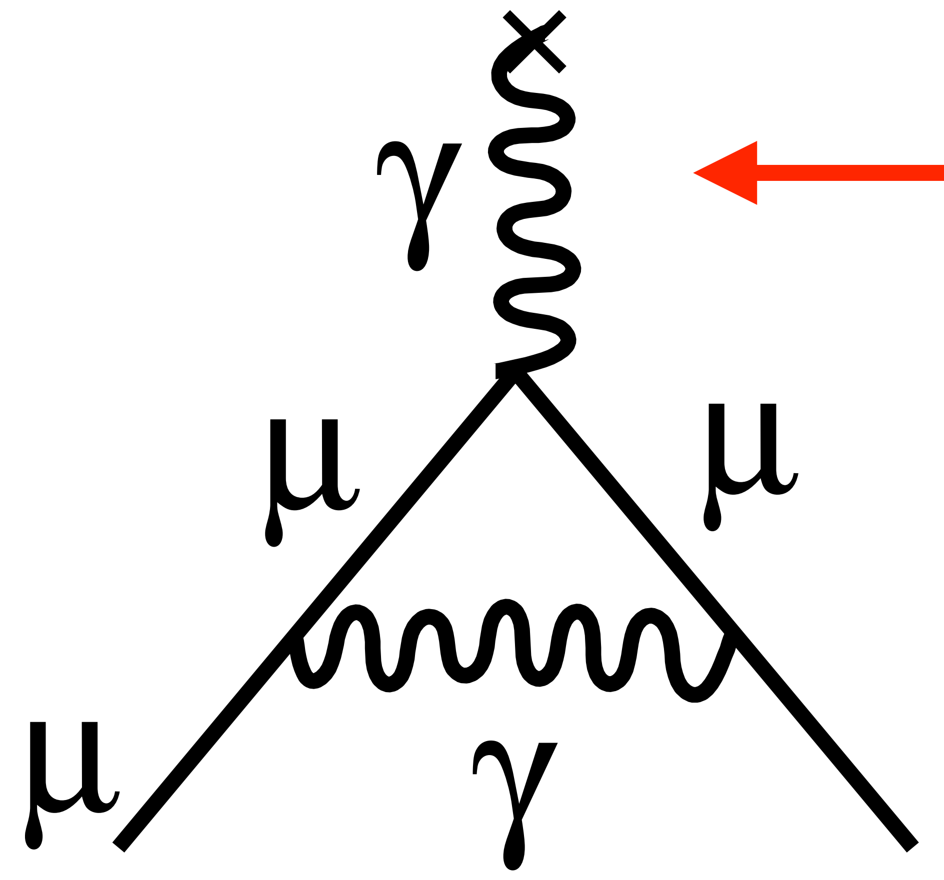
$$a_\mu = \frac{g_\mu - 2}{2}$$

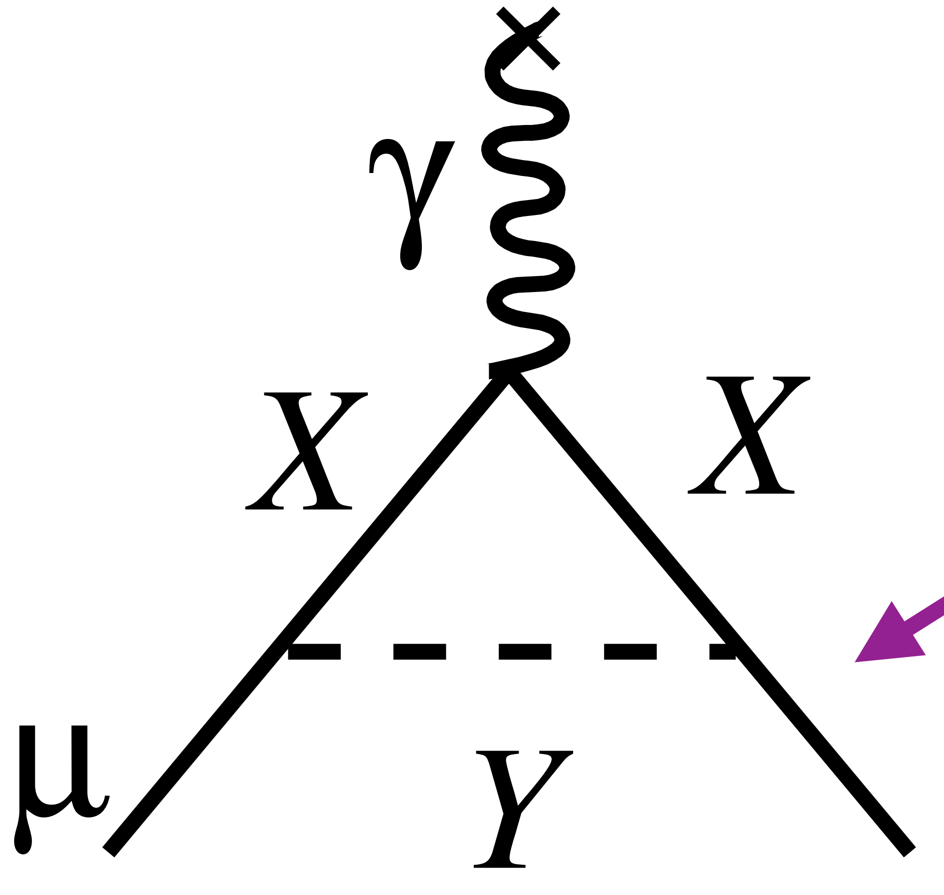
B field

Leading, O(α), contribn is $\frac{\alpha}{2\pi} = 0.00116\ldots$

Schwinger 1948

+ many higher order pieces …..



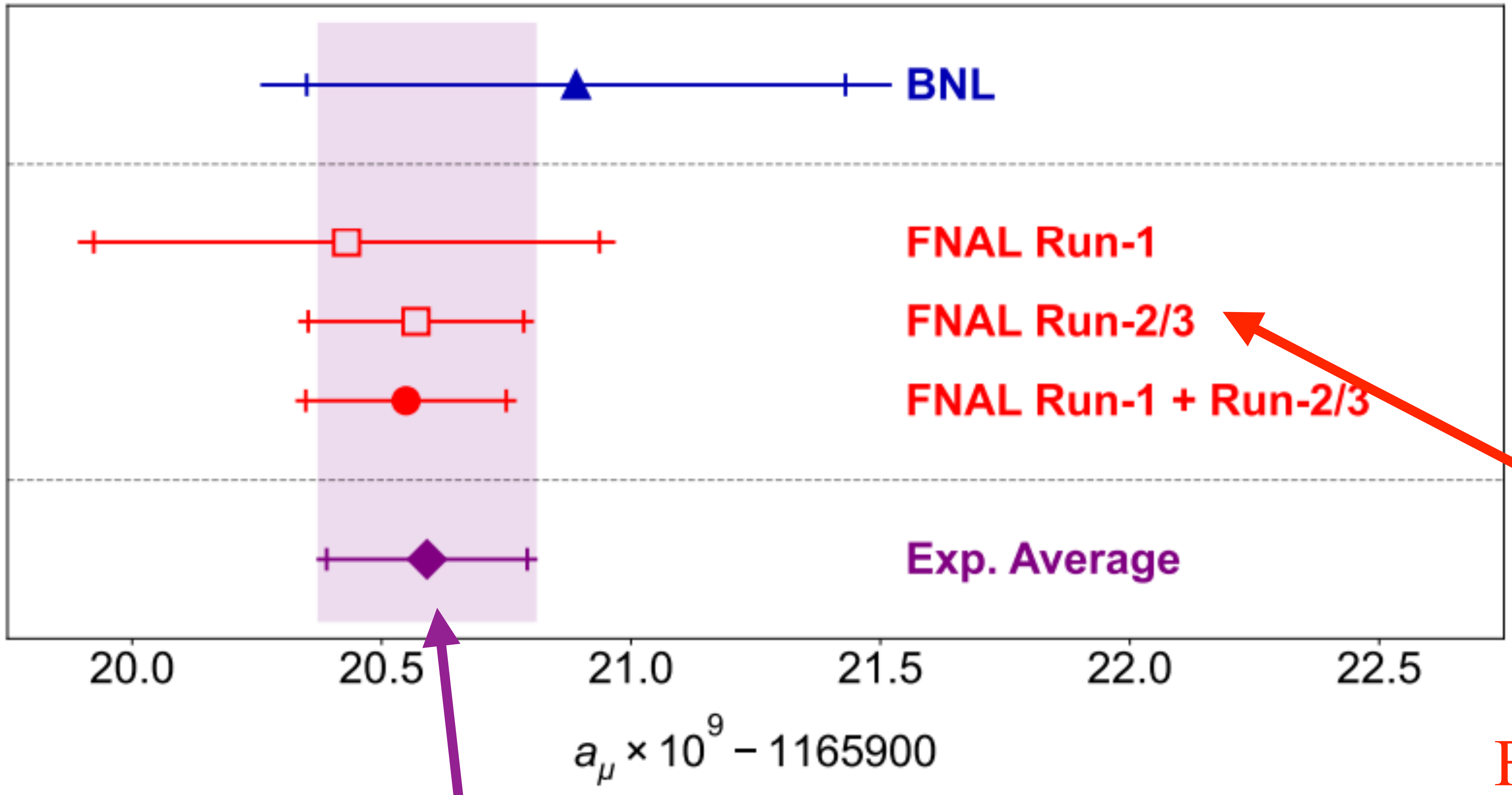


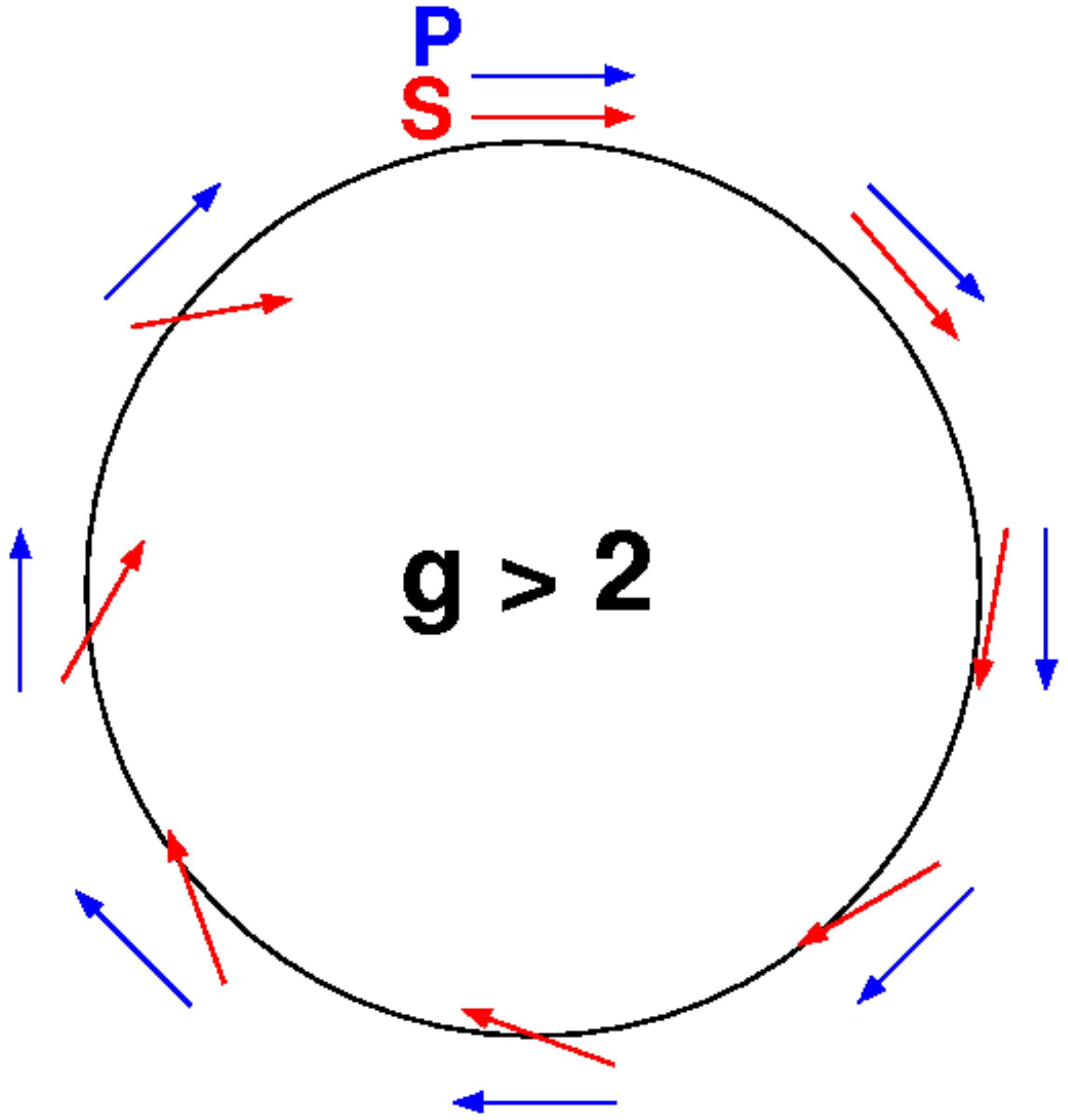
New physics would give SM/expt discrepancy

$$\delta a_l^{\text{new heavy physics}} \propto \frac{m_l^2}{M_X^2}$$

motivates study of μ rather than e.

(Anti-)Muon g-2 experiment @ Fermilab





measure $\vec{S}$ from $\mu^+ \to e^+ \overline{\nu}_\mu \nu_e$

New result - August 2023 : arXiv:2308.06230

$$a_\mu(\text{expt}) = 11659205.9(2.2) \times 10^{-10}$$

Final result, inc. runs 4, 5 & 6, mid-2025. Further factor of 3 in stats: reduce total uncertainty to ~1.6 x10-10.

J-PARC@KEK, muon g-2 and EDM using compact magnetic ring, low momentum μ^+
Data-taking to start in 2028 - 2 years running to get BNL uncertainties.
Muonium ($\mu^+ e^-$) spectroscopy from MUSEUM@KEK can also determine μ_μ 2106.11998

Comparison to the Standard Model

Current status $10^{10} a_\mu = 11659205.9(2.2)$ ← Experiment - Muon g-2@FNAL PRL131:161802 (2023)

$10^{10} a_\mu = 11659181.0(4.3)$ ← Theory white paper: Phys. Rep. 887:1 (2020)

Difference $= 24.9(4.8) \times 10^{-10}$

5σ ! NO!

QCD contributions need more work ….

Theory white paper: Phys. Rep. 887:1 (2020)

10^{10} x contribution:

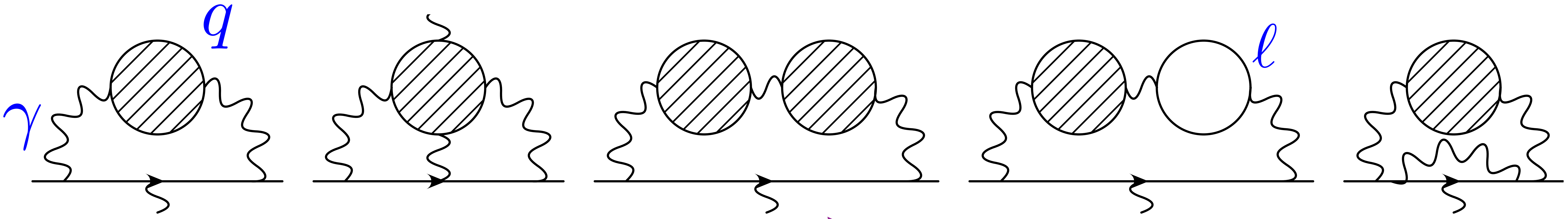
QED:	11658471.8931(104)
EW:	15.36(10)
QCD:	693.7(4.3)

Uncertainty in SM a_μ almost entirely from QCD.

Lattice QCD is important here

QCD contributions

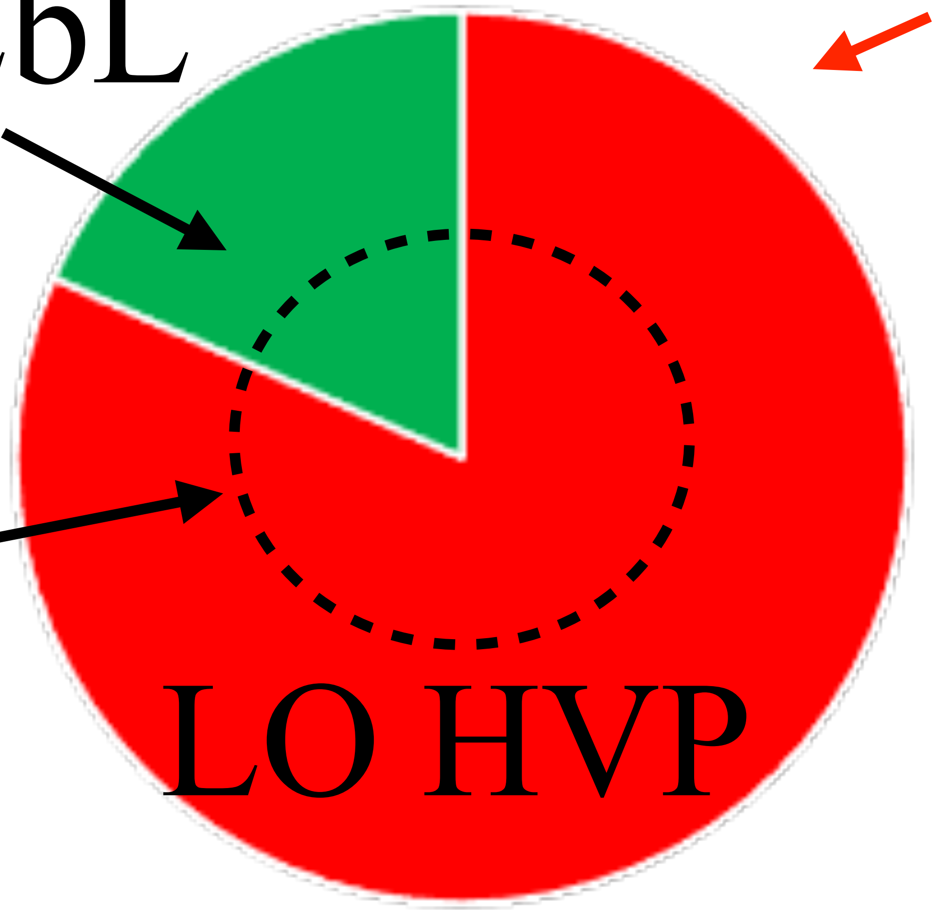
Blum et al, 1301.2607



Leading-order (α^2) hadronic vacuum polarisation, LOHVP

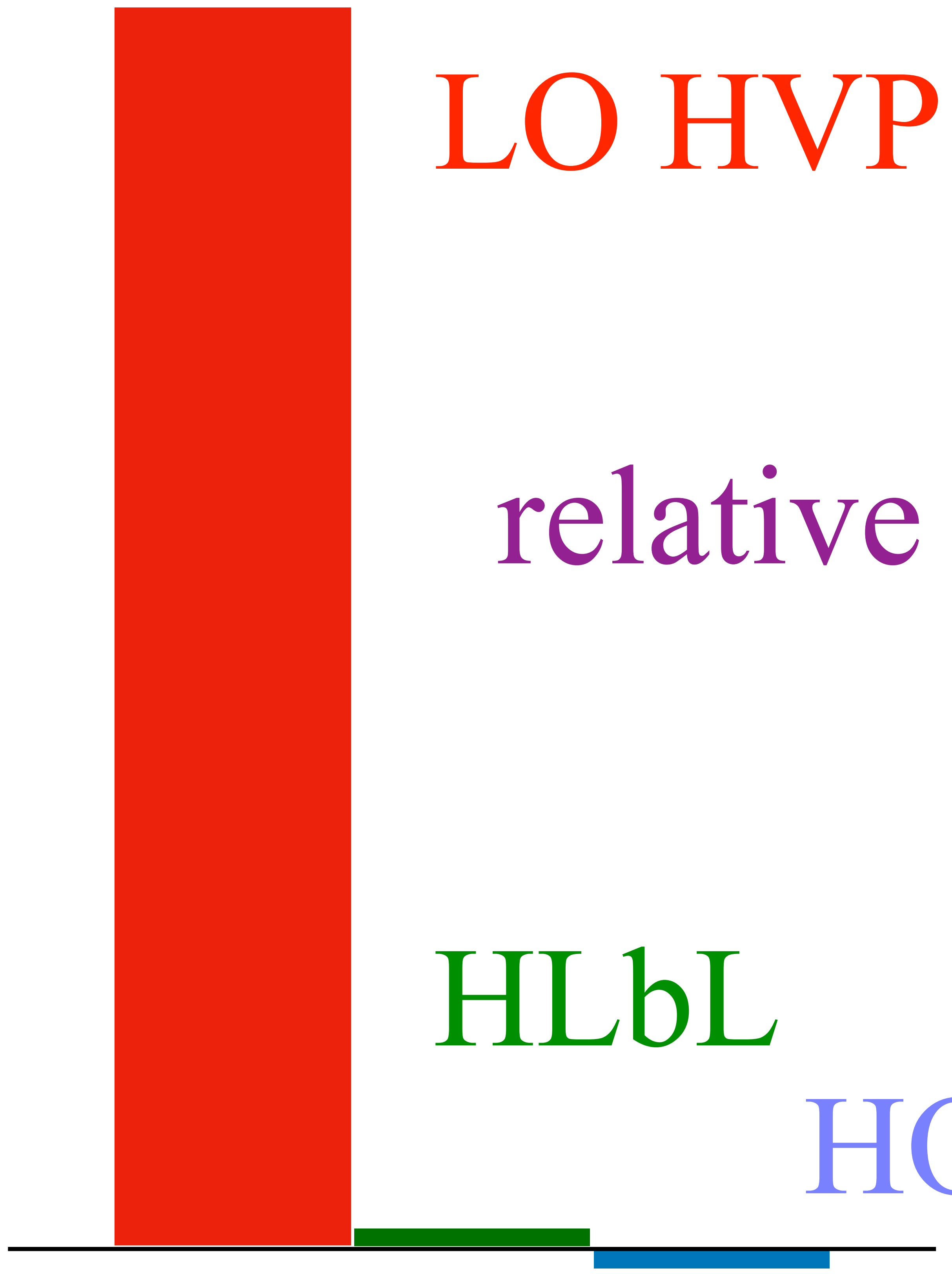
Hadronic light-by-light, HLbL (α^3)

Higher order HVP



expt 2024

relative variance in WP20

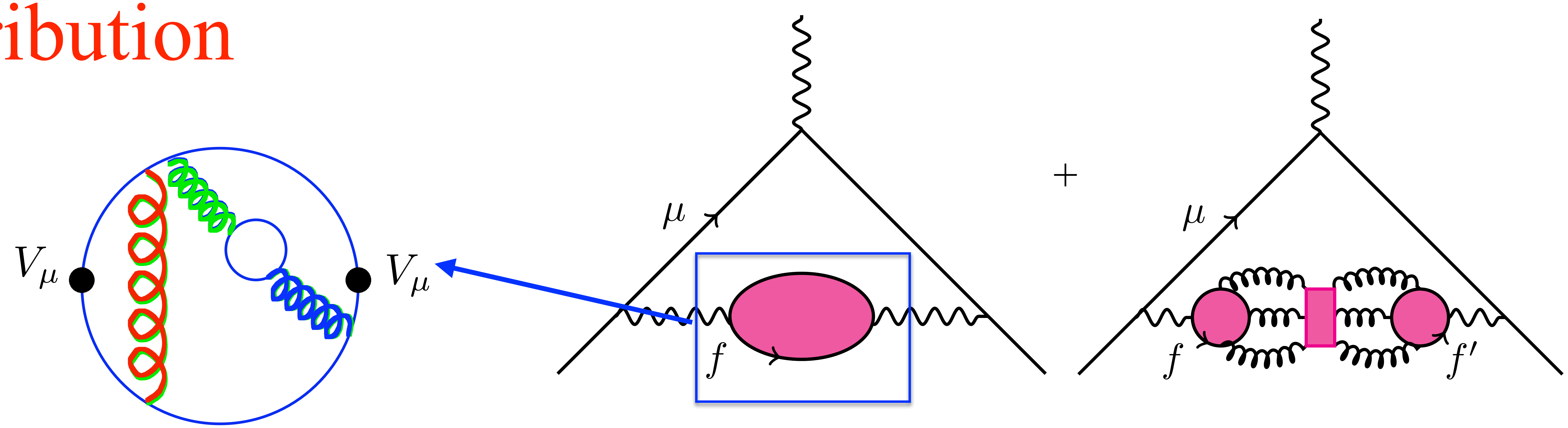


Theory white paper: Phys. Rep. 887:1 (2020)

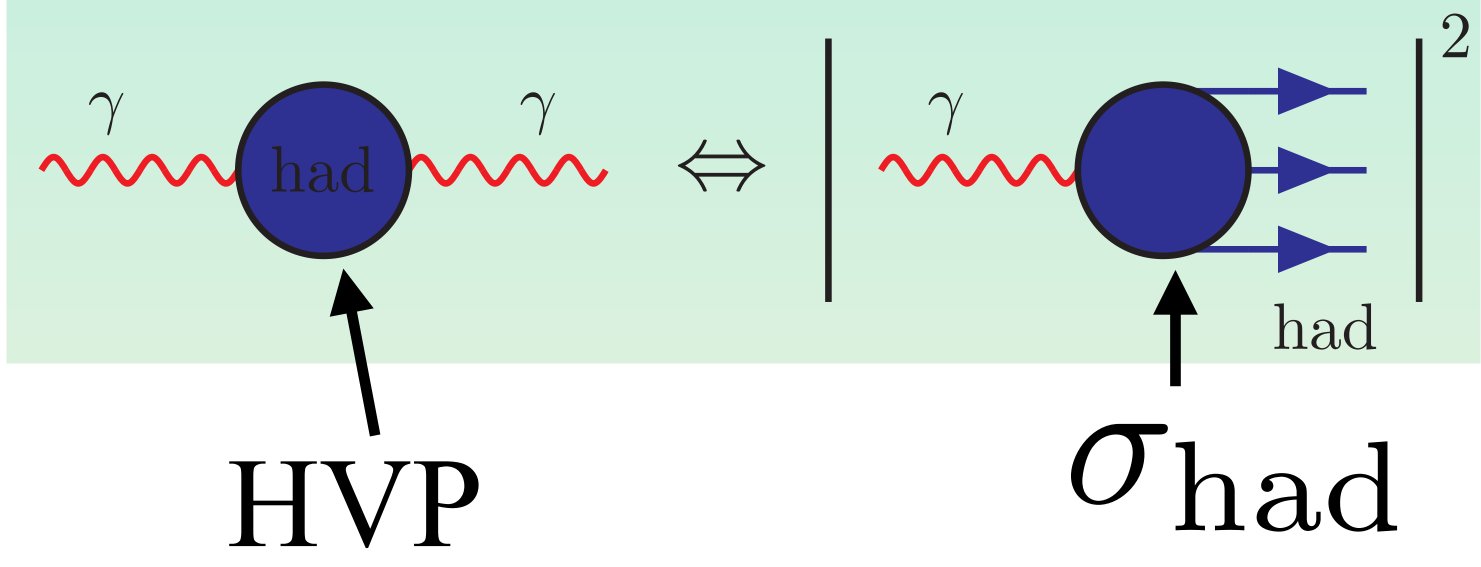
The LOHVP contribution

Key ingredient is quark bubble connected to a photon at either side

$$V_\mu = e_f \overline{\psi}_f \gamma_\mu \psi_f$$



‘Data-driven’ method



Relate HVP to $\sigma(e^+e^- \longrightarrow \gamma^* \longrightarrow$ hadrons) and input experimental data.

WP20 HVP number uses this since has been most accurate.

See Keshavarzi, Lat2023 talk, for details of this method

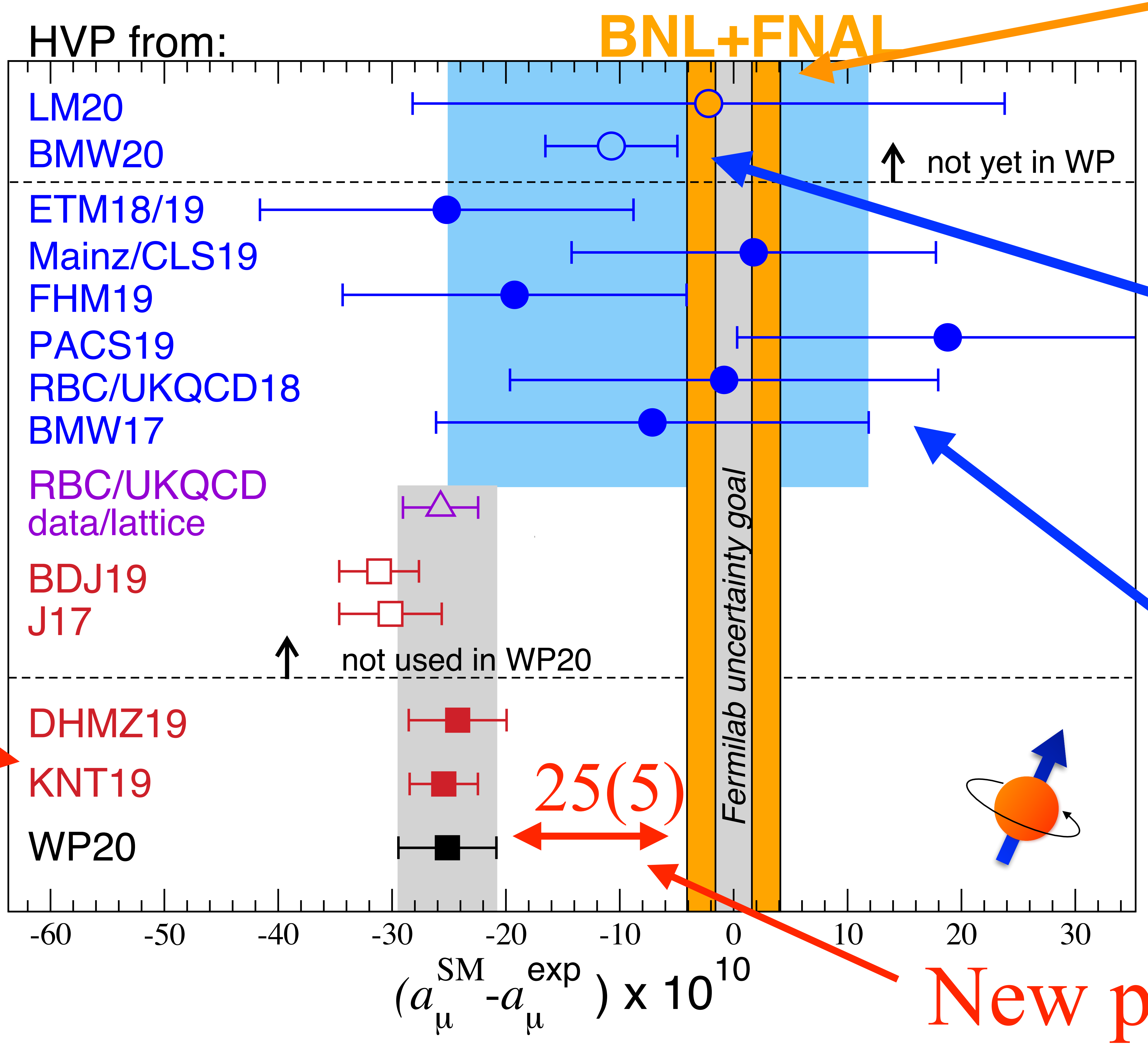
Lattice QCD

Direct computation of the vector-vector correlation function for u, d, s and c quarks in Lattice QCD.

Need connected + disconnected correlators + QED + isospin-breaking corrections.

Impact of LOHVP on SM-experiment comparison for a_μ

Snowmass, 2203.15810
BMW20, 2002.12347



Expt uncertainty now halved - 2308.06230

BMW20 first complete lattice calc. 0.8% uncertainty. 2σ above data-driven.

Other lattice results - 2% uncertainty

Data-driven results for WP20 - 0.6% uncertainty

New physics? size ~ 4% of LOHVP

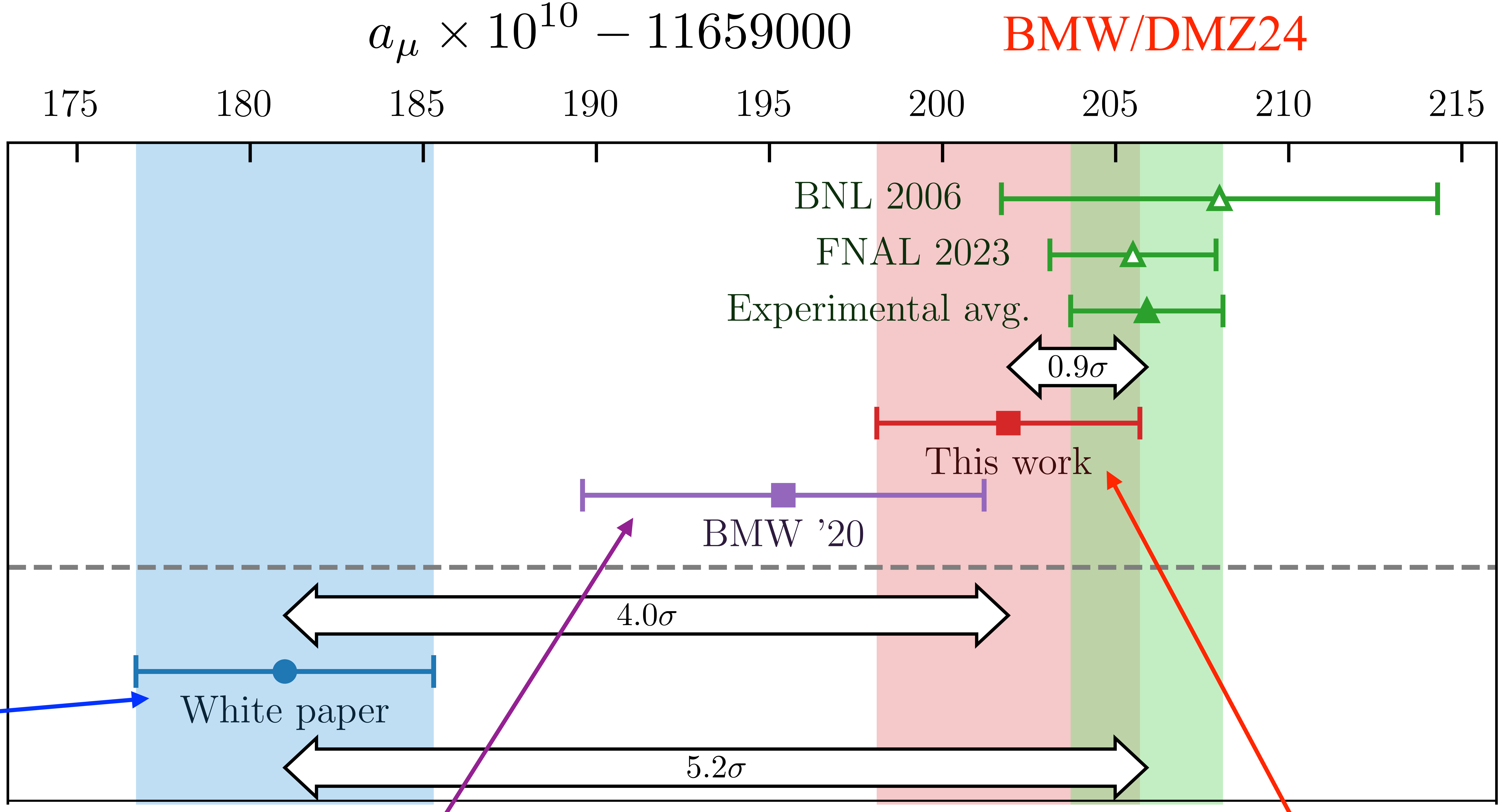
BUT: do data-driven and lattice QCD HVP agree?

CRITICAL to conclusion on new physics evidence

2024 update

BMW/DMZ24, 2407.10913

adds 0.048fm ensemble, reduces finite L/T error. Uses data-driven for large-t tail. Blinded analysis.

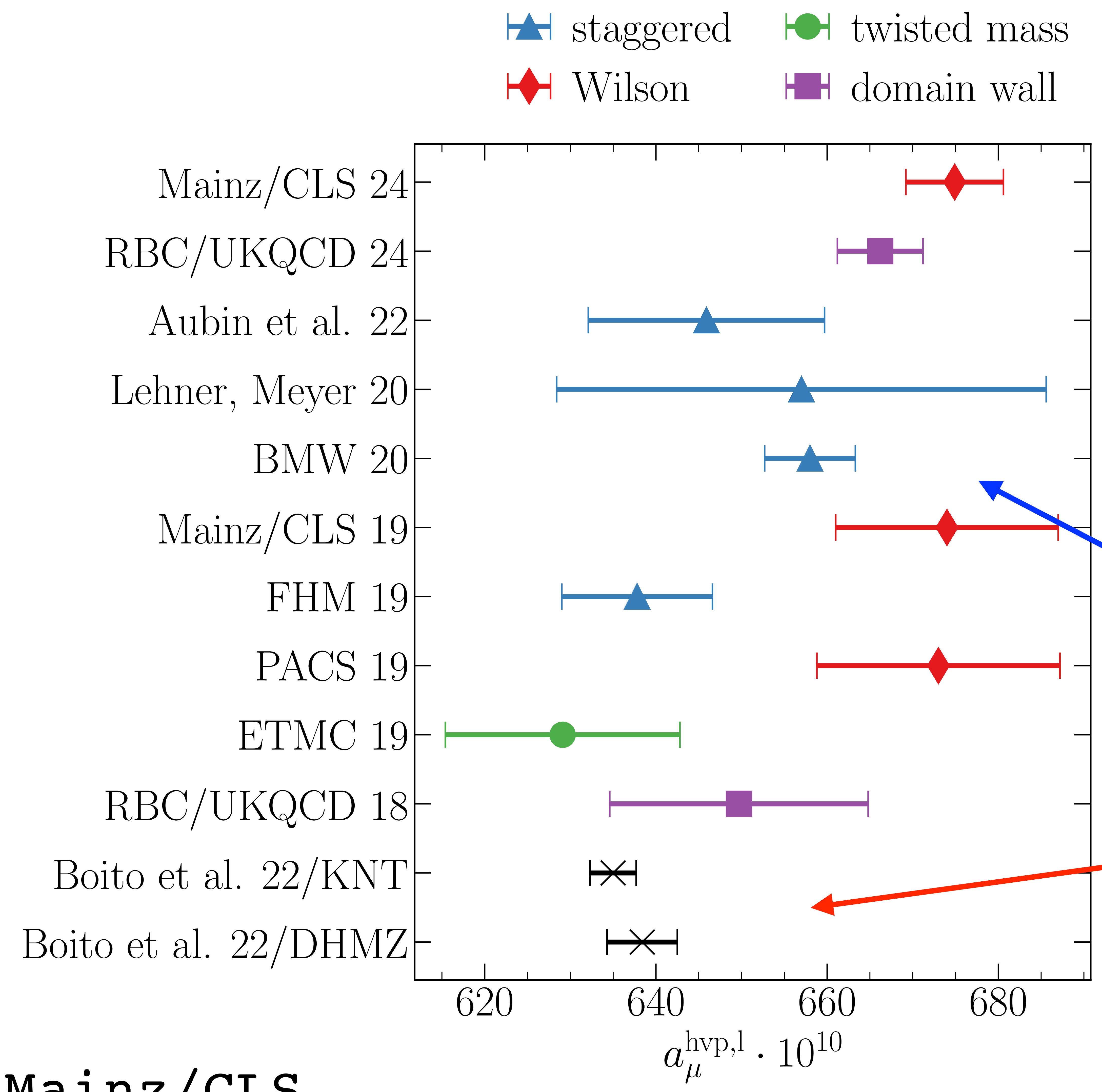


WP20 data-driven: 693.1(4.0)

BMW20: $10^{10} a_\mu^{\mathrm{LOHVP}} = 707.5(5.5)$

BMW/DMZ24: $10^{10} a_\mu^{\mathrm{LOHVP}} = 714.1(3.3)$

2024 update



RBC/UKQCD and Mainz/CLS updates of light-quark connected LOHVP from blinded analyses

$$a_\mu^{\rm LOHVP,lqc} = 666.2(5.0) \times 10^{-10}$$

RBC/UKQCD, Lehner, LAT24

0.75% uncertainty

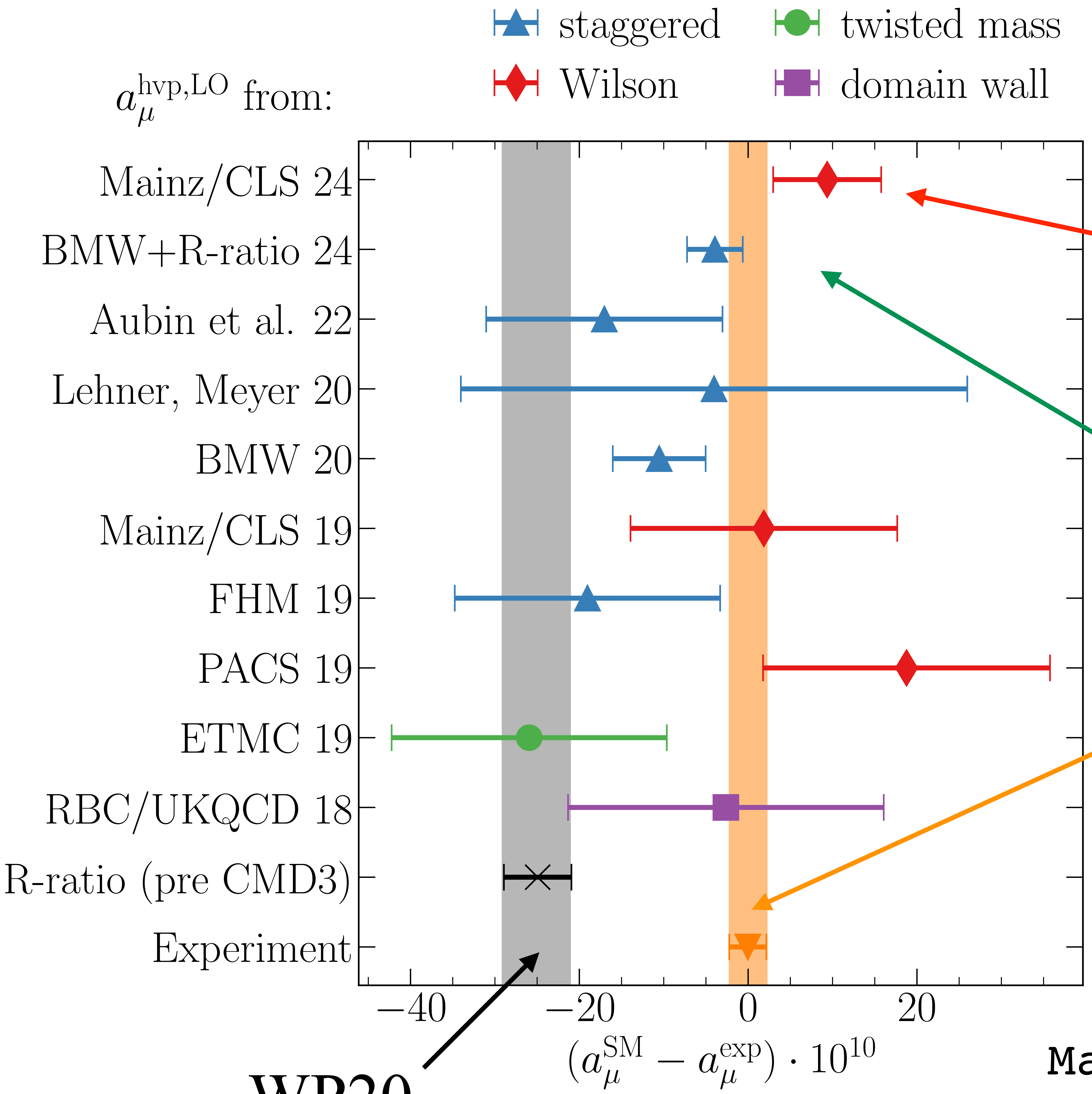
Agree with BMW20

5.4σ higher than BBGKMP result based on KNT19

Consistent picture to that from BMW/DMZ24

Mainz/CLS, Kuberski, KEK, Sept24

2024 update



WP20

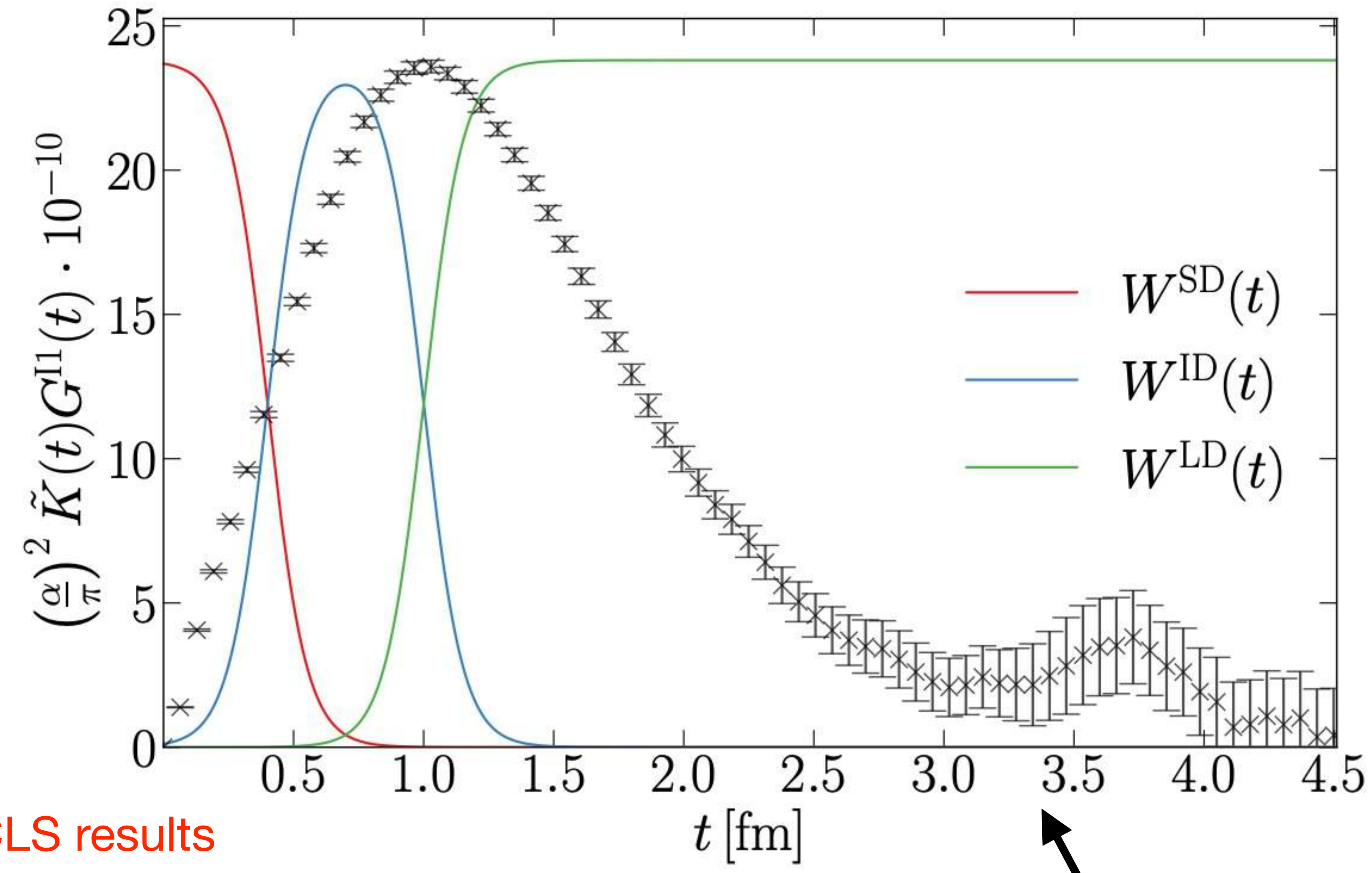
Mainz/CLS preliminary update of full HVP (includes estimate of isospin-breaking corrections)

Agrees with BMW/DMZ24

Supports scenario in which SM agrees with experiment for muon g-2 i.e. no significant sign of new physics

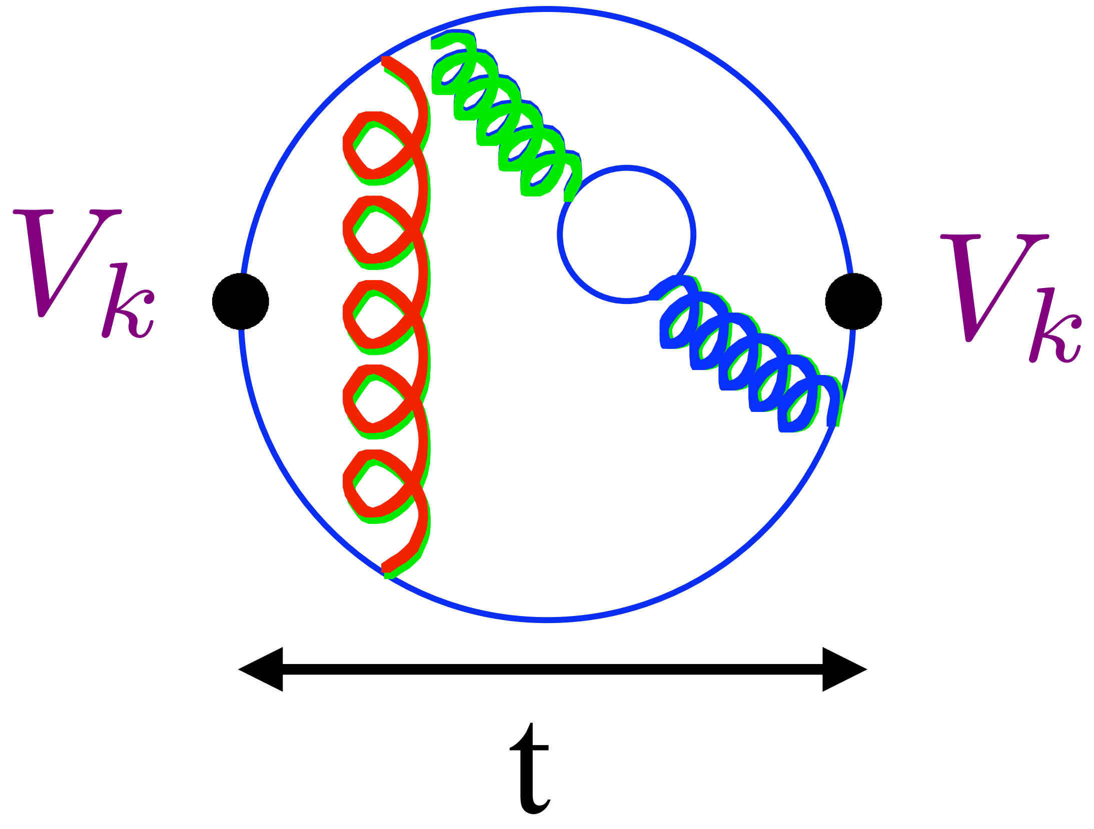
Mainz/CLS, Kuberski,KEK, Sept24

Lattice HVP



CLS results

$$a_\mu^{\mathrm{LOHVP}} = (\frac{\alpha}{\pi})^2 \int_0^\infty dt\, G(t) \tilde{K}(t)$$

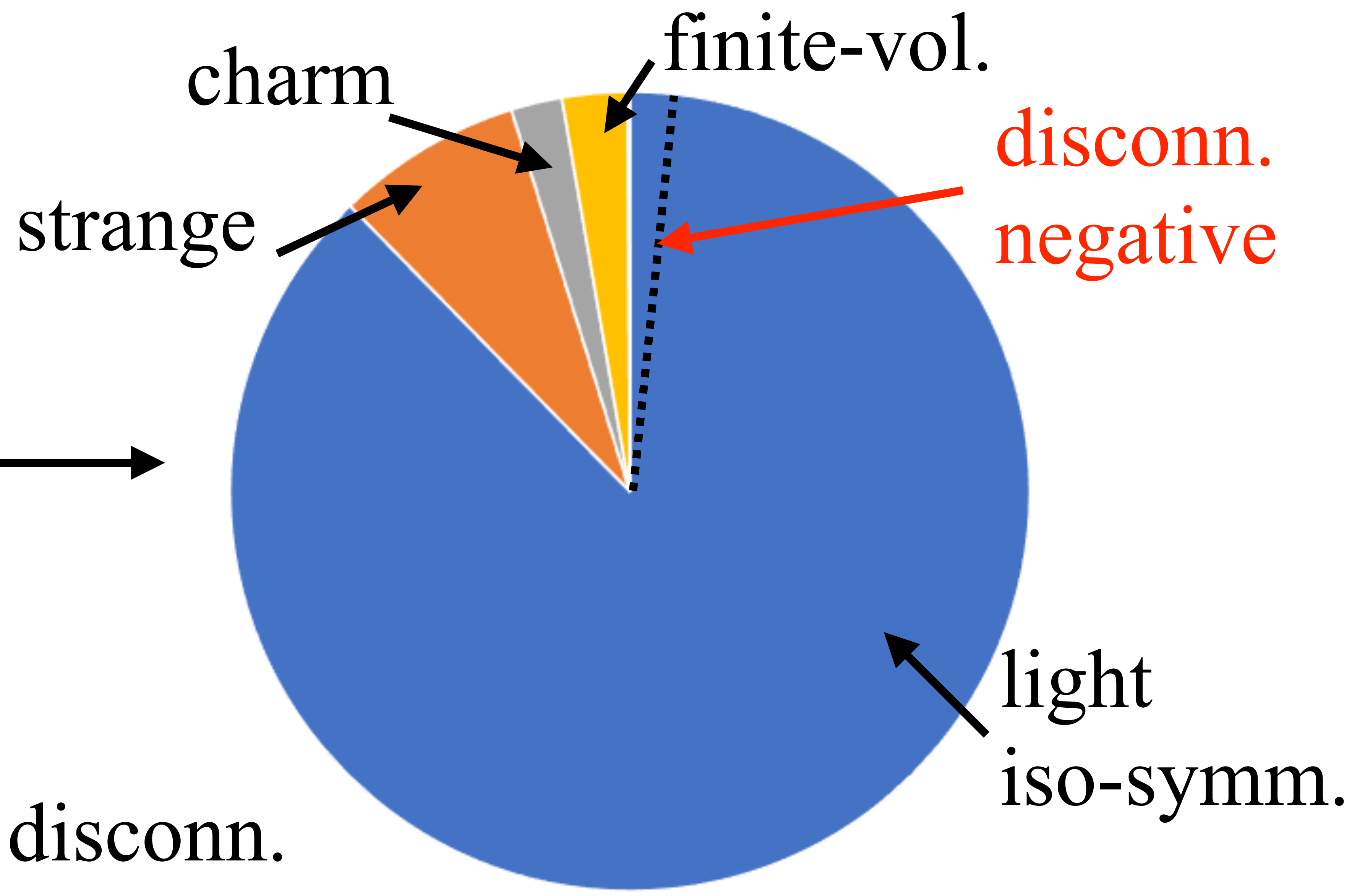


Simple zero-momentum vector-vector 2-point correlator for each flavour (+ the disconnected case).

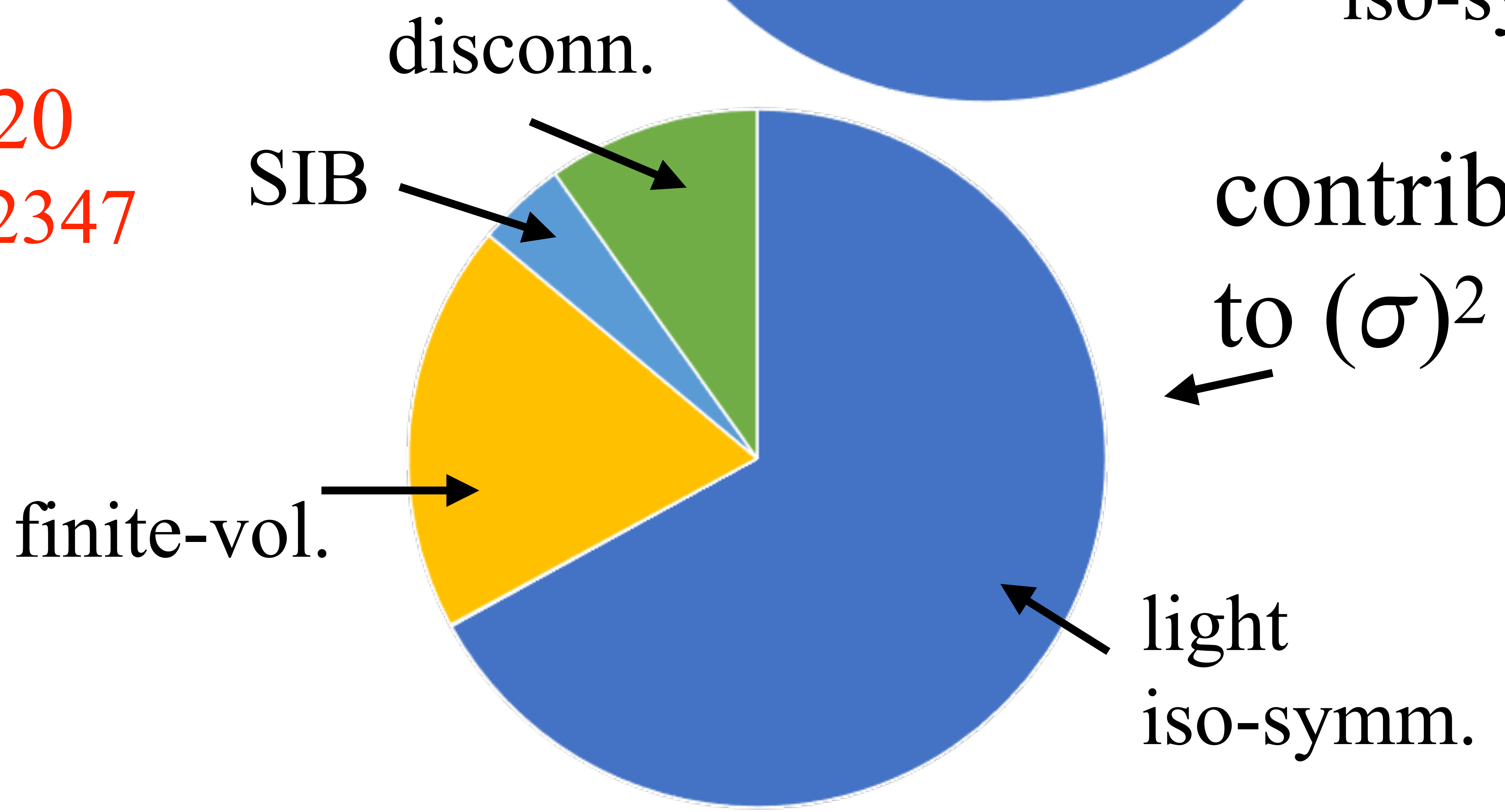
$$V_k^{\mathrm{em}} = \sum_f e_f \overline{\psi}_f \gamma_k \psi_f$$

contributions to LOHVP

from BMW20 2002.12347



contribns to $(\sigma)^2$



Key contribution is light connected.

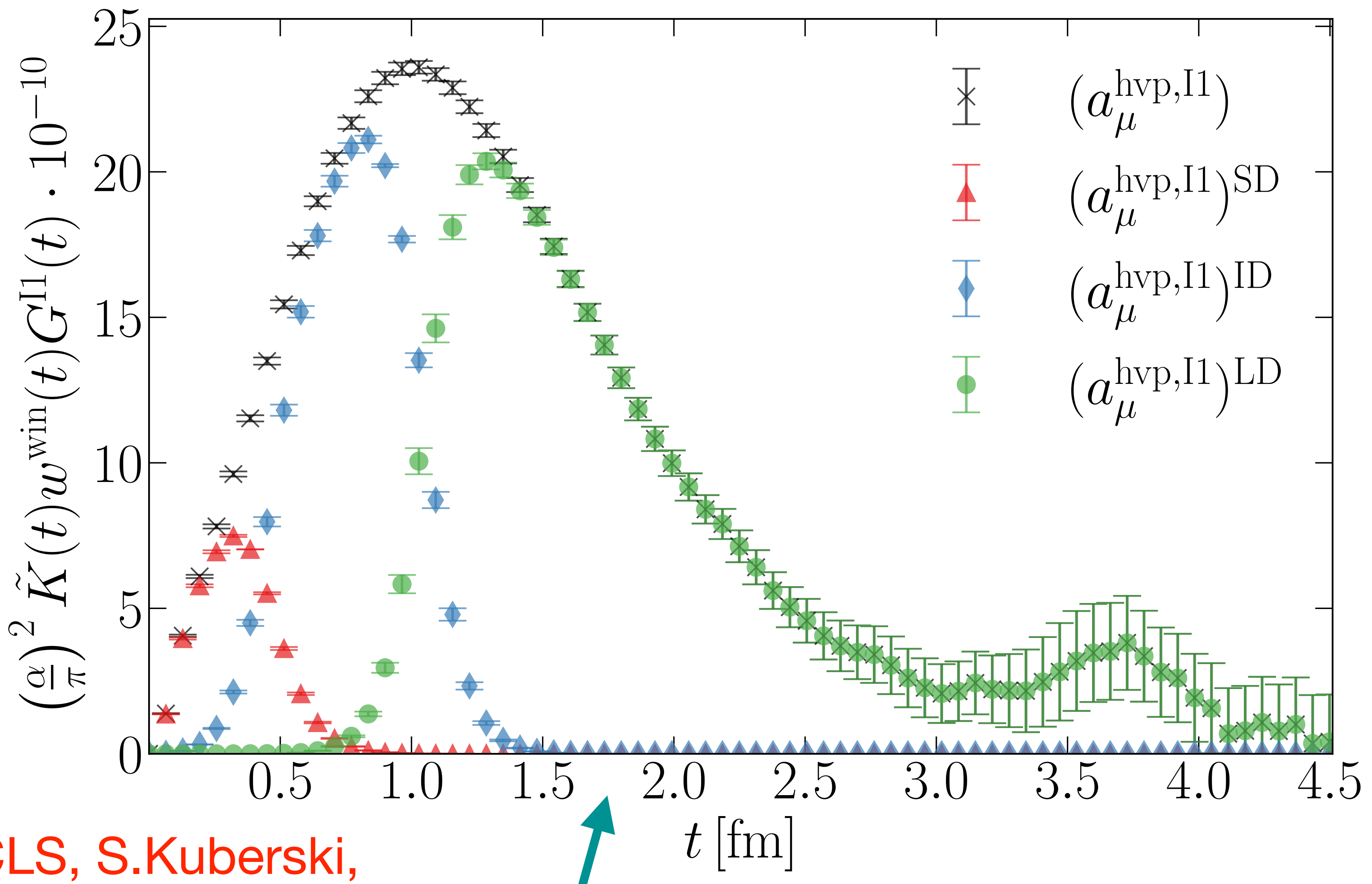
Key issue is growth of stat. noise at large t values.

Other issues: FV corrn., systs. from IB and disc. , scale setting

See S.Kuberski, POSLAT2023, 125

Lattice HVP - ‘window’ observables

$$\theta(t, t_1, \Delta t) = \frac{1}{2}\left[1 - \tanh\left(\frac{t - t_1}{\Delta t}\right)\right]$$

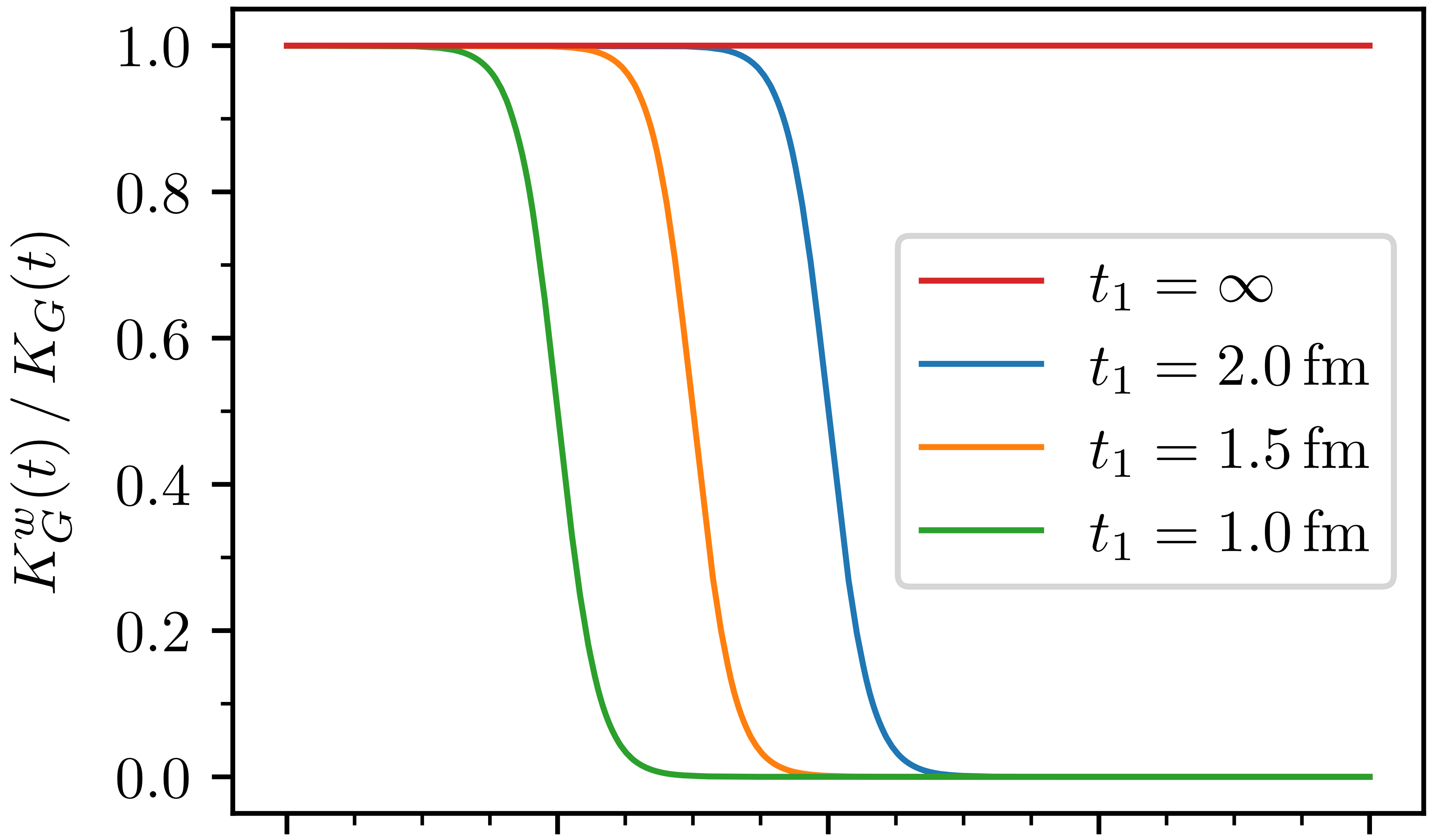


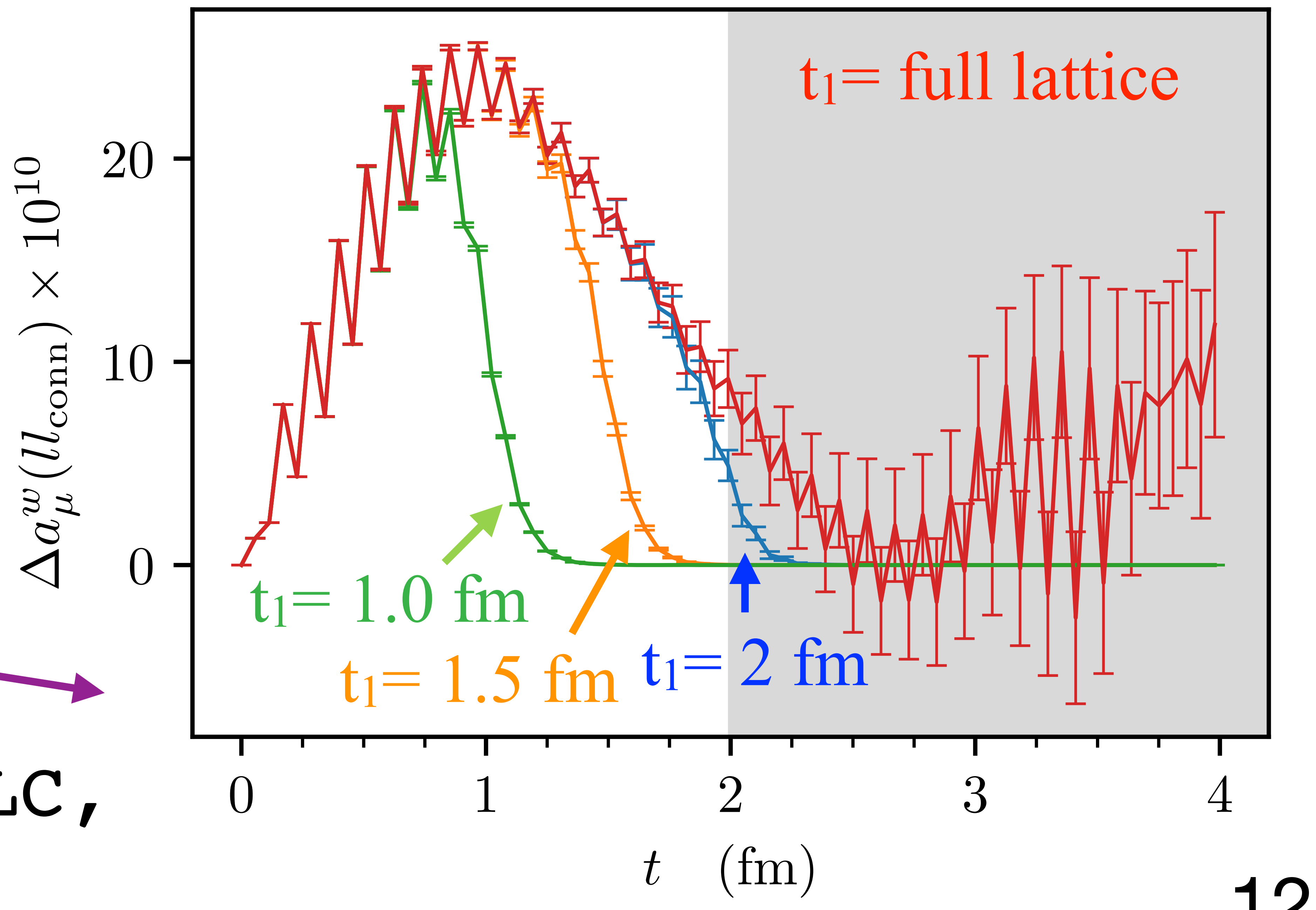
CLS, S.Kuberski, POSLAT2023, 125

Short-distance (SD), intermediate distance (ID) and long-distance (LD)

Bernecker+Meyer, 1107.4388; RBC/UKQCD, 1801.07224

Use rounded window functions to divide time region. Cutting out large t values allows accurate comparison of lattice results.



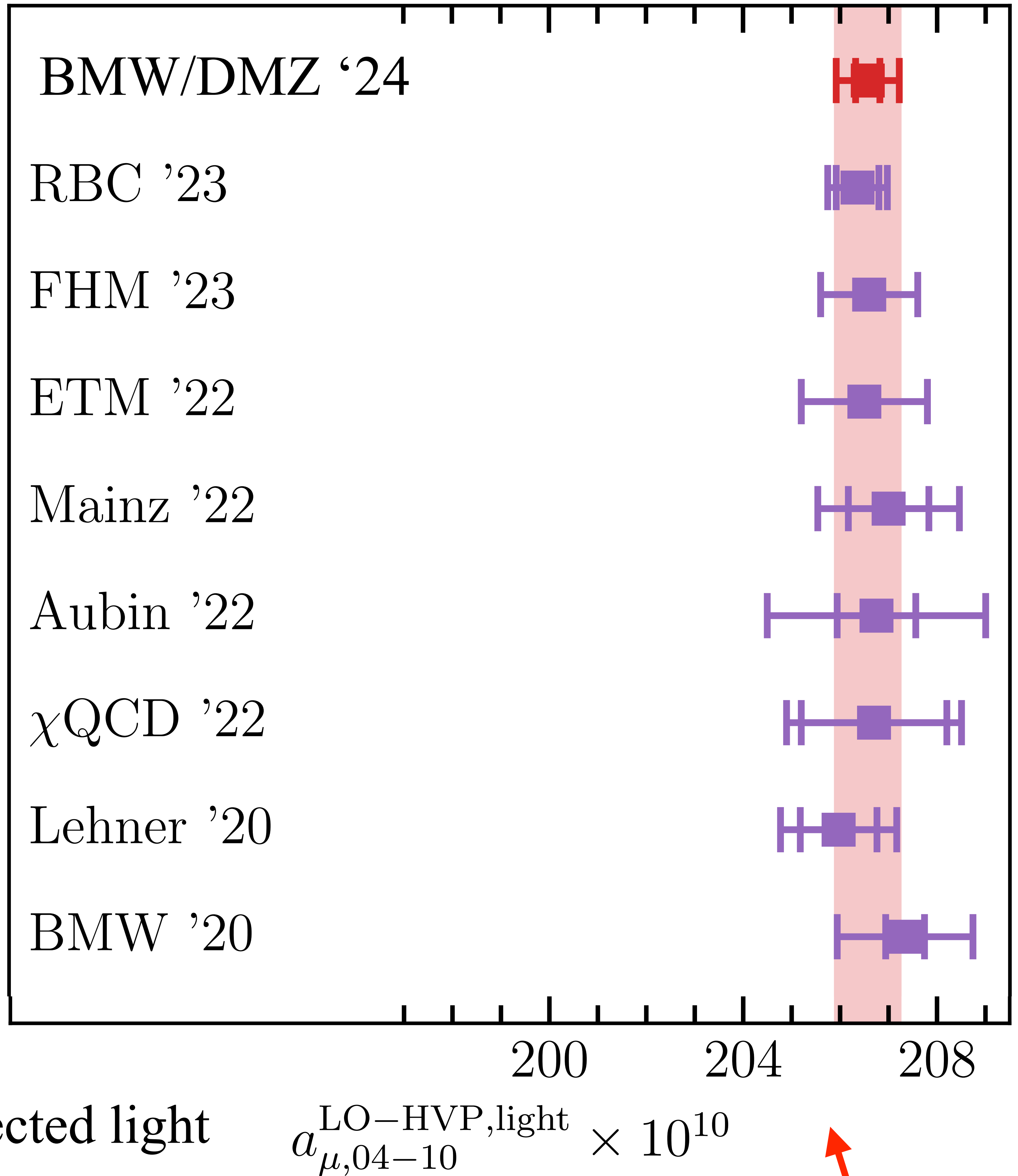


One-sided windows with variable size, t_1

Fermilab/HPQCD/MILC, 2207.04765

Other windows are available …

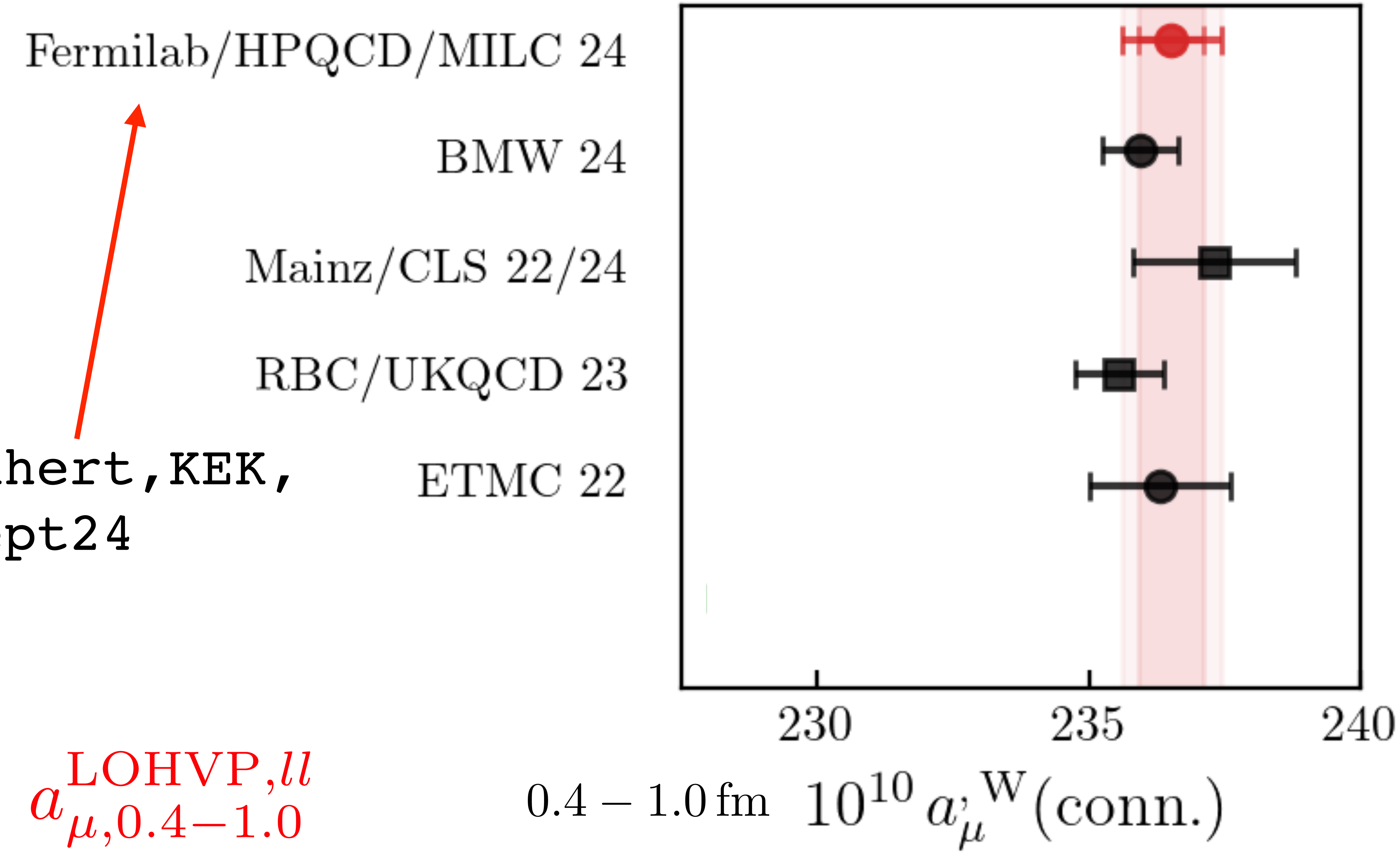
Lattice results for the 0.4 - 1.0 fm (Δt=0.15fm) intermediate window



Connected light only

BMW/DMZ24, 2407.10913

Excellent agreement between many different lattice analyses! Uncertainty $\lesssim 1\%$



Lahert, KEK, Sept24

$a_{\mu,0.4-1.0}^{\rm LOHVP,\mathit{ll}}$

Lattice WA w.o. BMW/DMZ'24 = 206.52(80) (100% correlated systematics)

BMW/DMZ'24 = 206.57(65) (0.3% uncertainty)

E. Neil, Theory initiative meeting, April 2024.

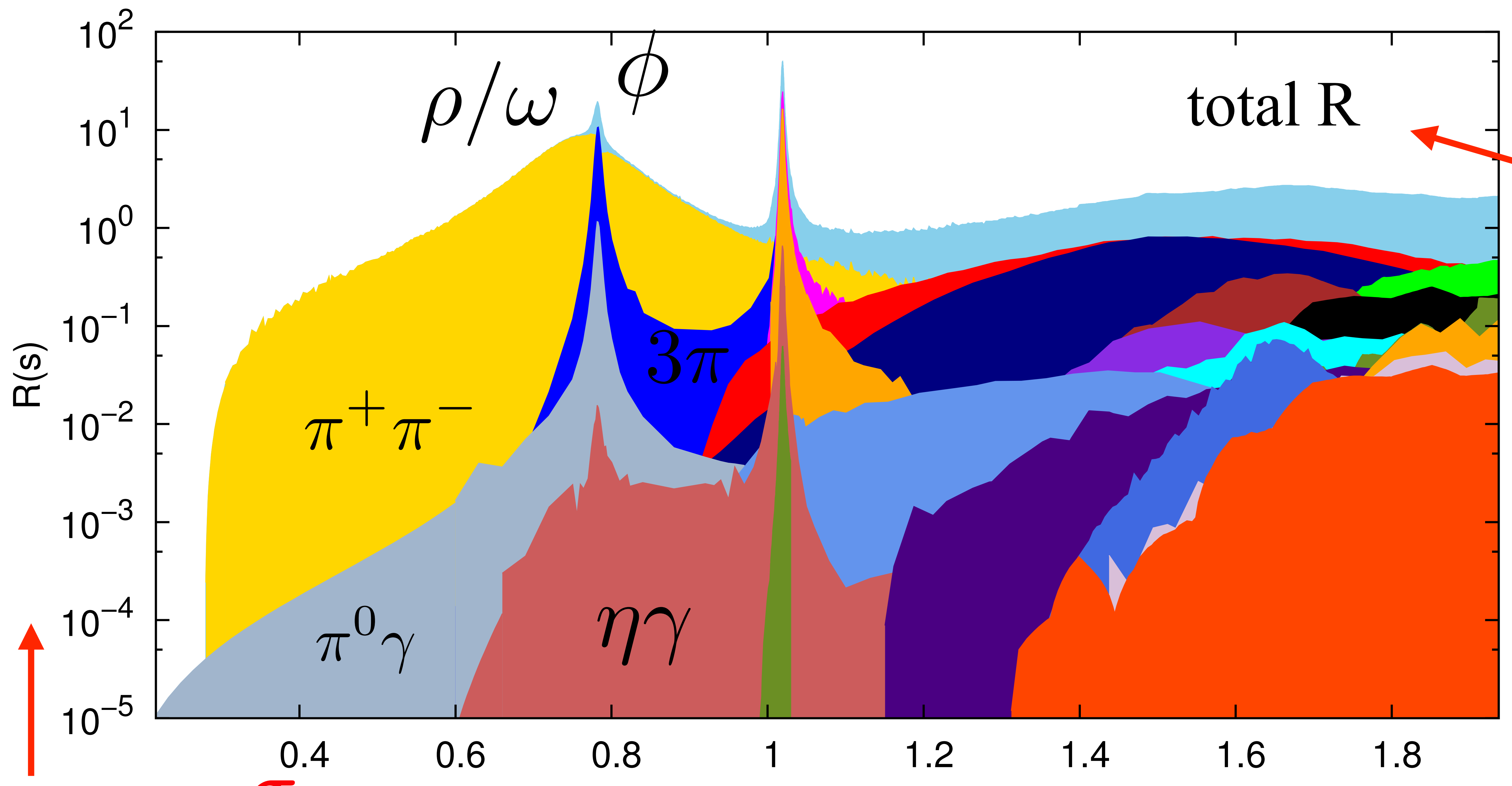
Data-driven HVP

Keshavarzi et al,1911.00367
A. Keshavarzi, LAT2023

$$a_\mu^{\rm LOHVP} = \frac{1}{4\pi^3}\int_{s_0}^{\infty} ds\,\sigma^0_{{\rm had},\gamma}(s) K(s)$$

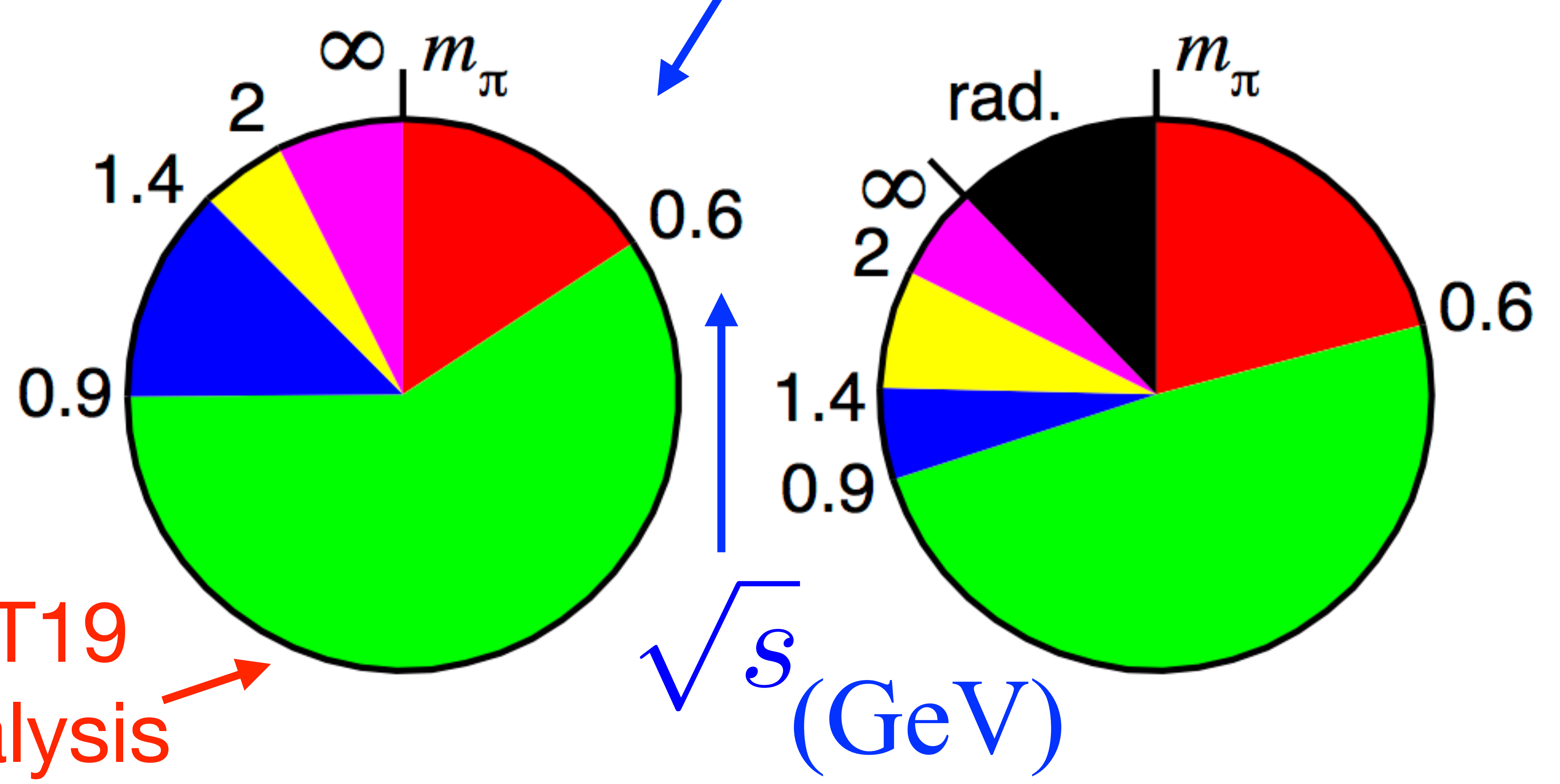
K(s) emphasises low s region

bare cross-section, e⁺e⁻ →hadrons, final-state radiation included



$R = \frac{\sigma}{\sigma_{\rm pt}}$

KNT19 analysis



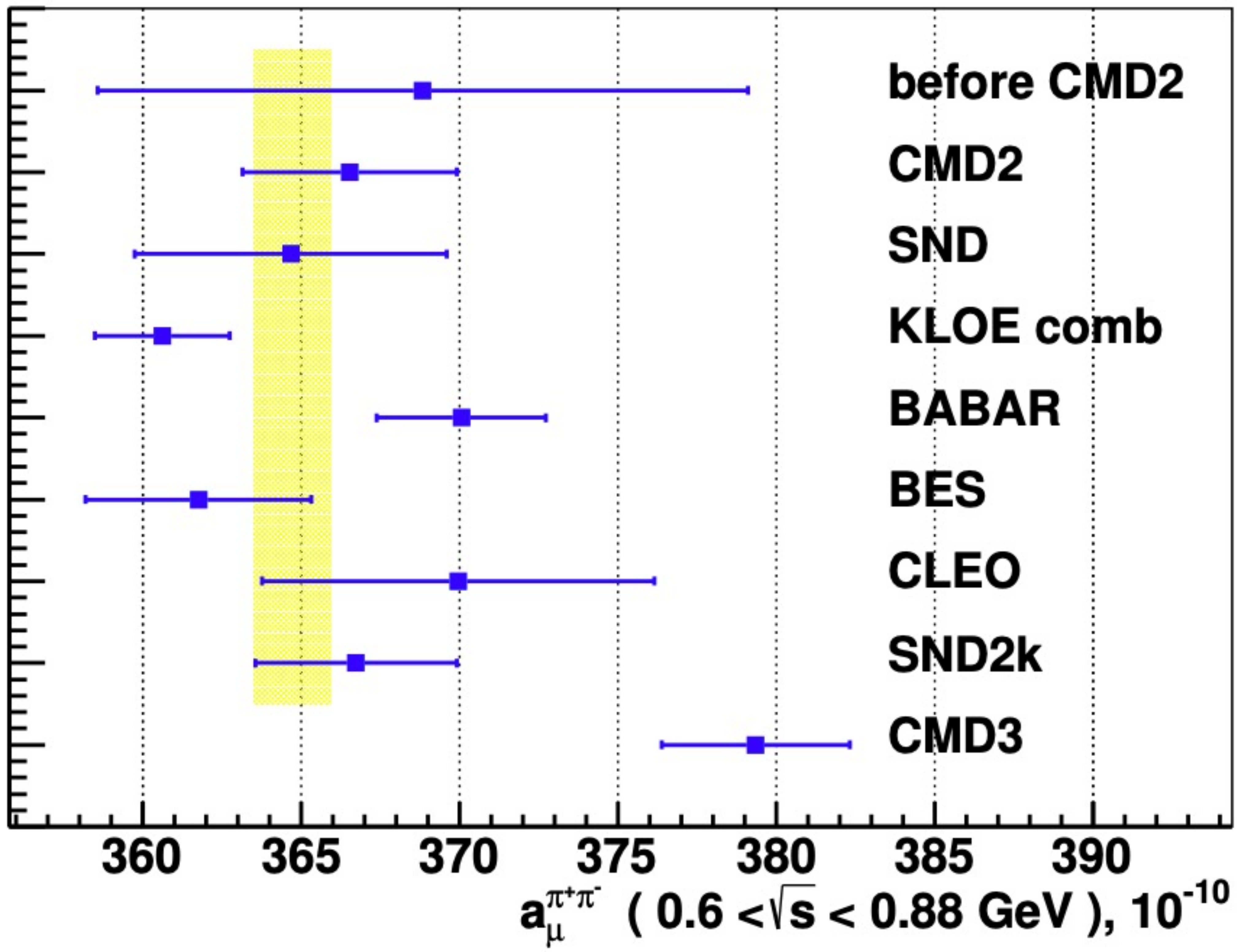
Data: exclusive final states below 2 GeV, inclusive above.

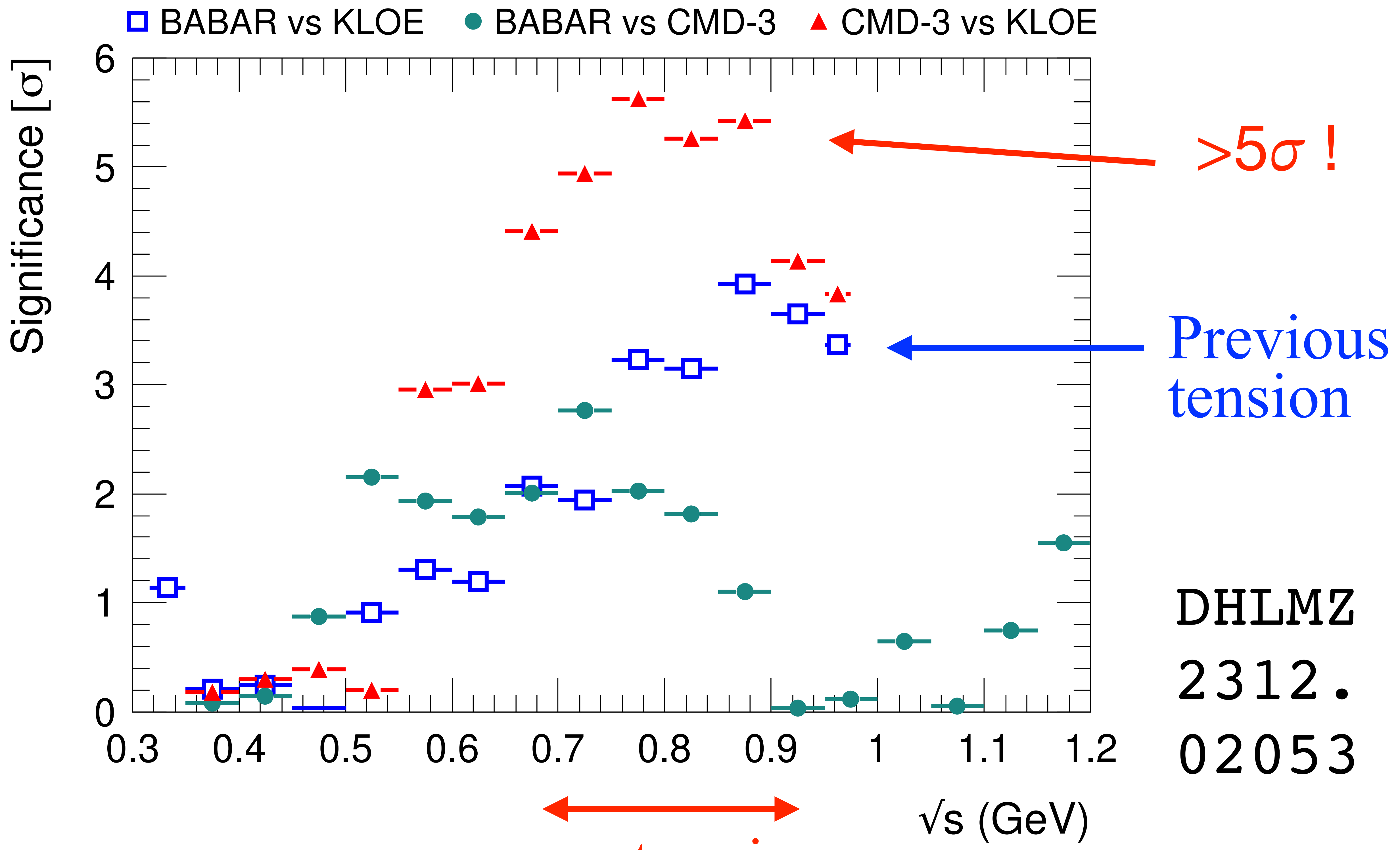
Radiative return (BaBar/KLOE) and direct scan (CMD3) experiments

Tension between experiments now a major issue

Issues with data for data-driven HVP

1) CMD3@VEPP2000, Novosibirsk, energy scan up to 1.2 GeV. New results for $e^+e^- \to \pi^+\pi^-$: 2302.08834, now published. Cross-section higher than previous expts.





DHLMZ 2312.02053

Using ONLY CMD3 $\pi\pi$ data in 0.3-1.2 GeV region would push up data-driven LOHVP by 22(5)x10^{-10} and remove SM/Muon g-2 difference

A. Keshavarzi, LAT2023

BUT how to average sensibly over experiments? Just increase uncertainty (a lot) ?

FUTURE: new BaBar results (2x stats), 2025; new KLOE results (7xstats), 2026/7; new SND/BES/Belle

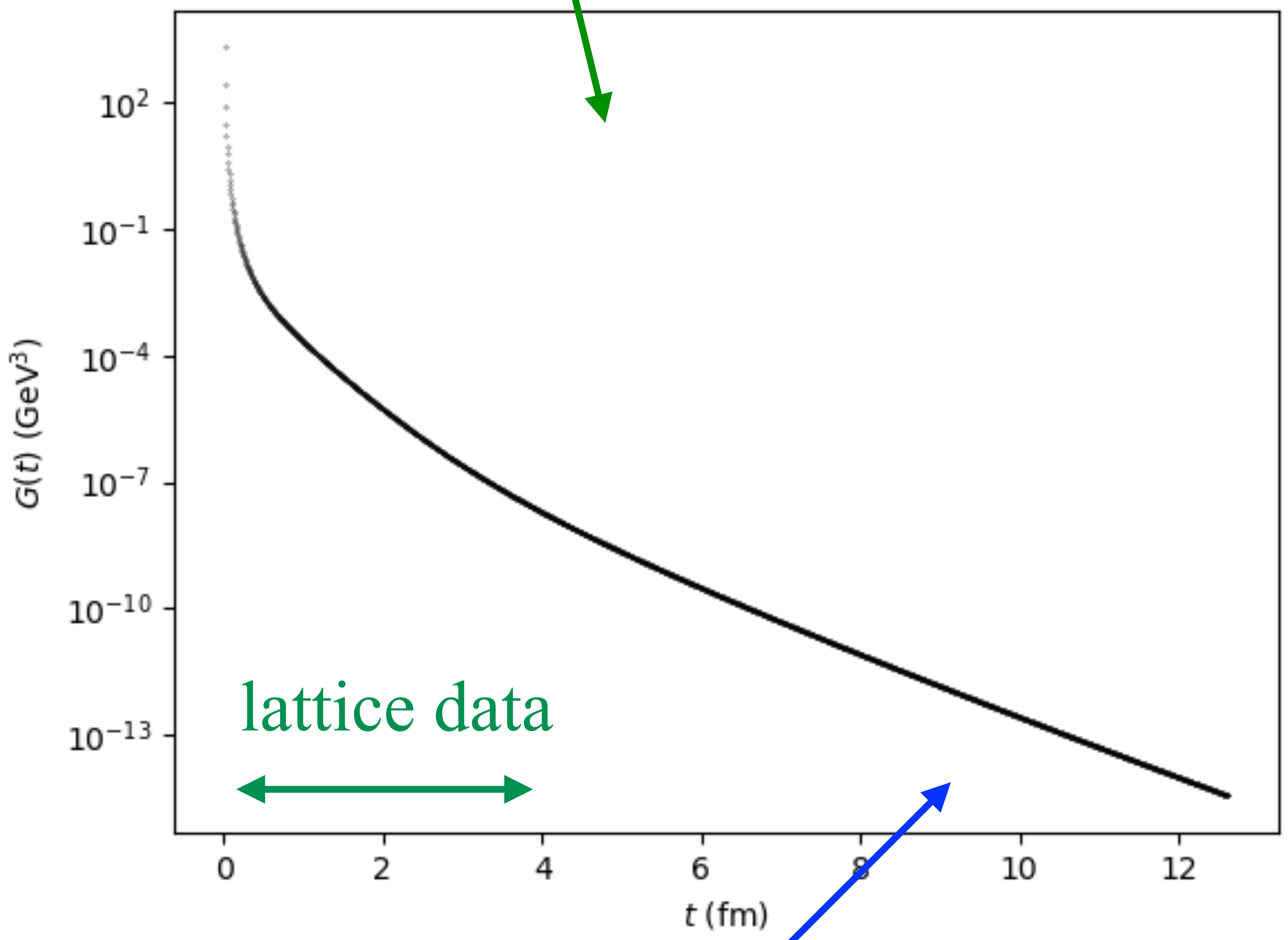
Comparing data-driven and lattice HVP results

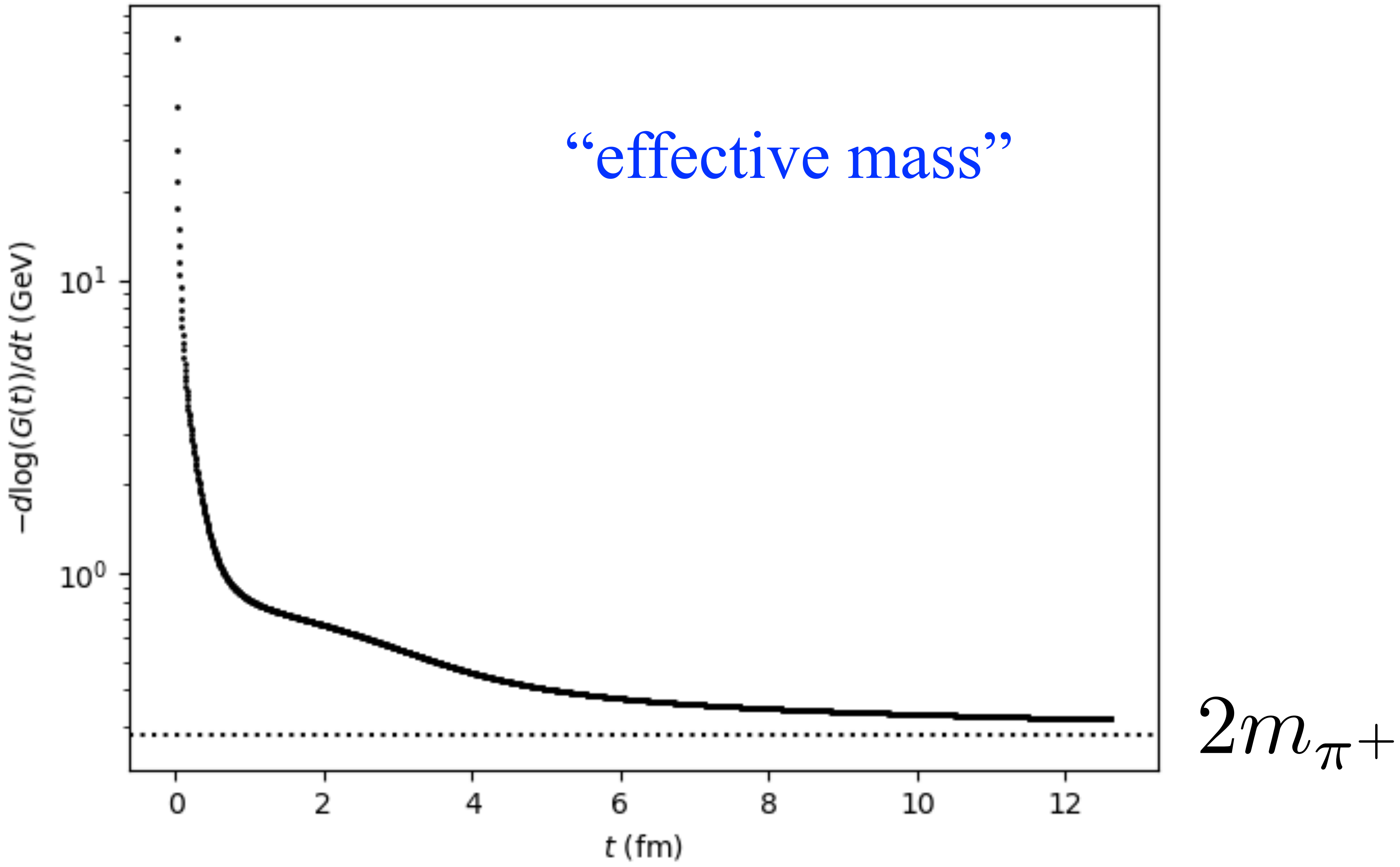
Can convert R(s) data into G(t)

$$G(t) = \frac{1}{12\pi^2} \int_0^\infty dE\, E^2 R(E^2) e^{-E|t|}$$

Using publicly available KNT19 R(s) data

Bernecker+Meyer, 1107.4388





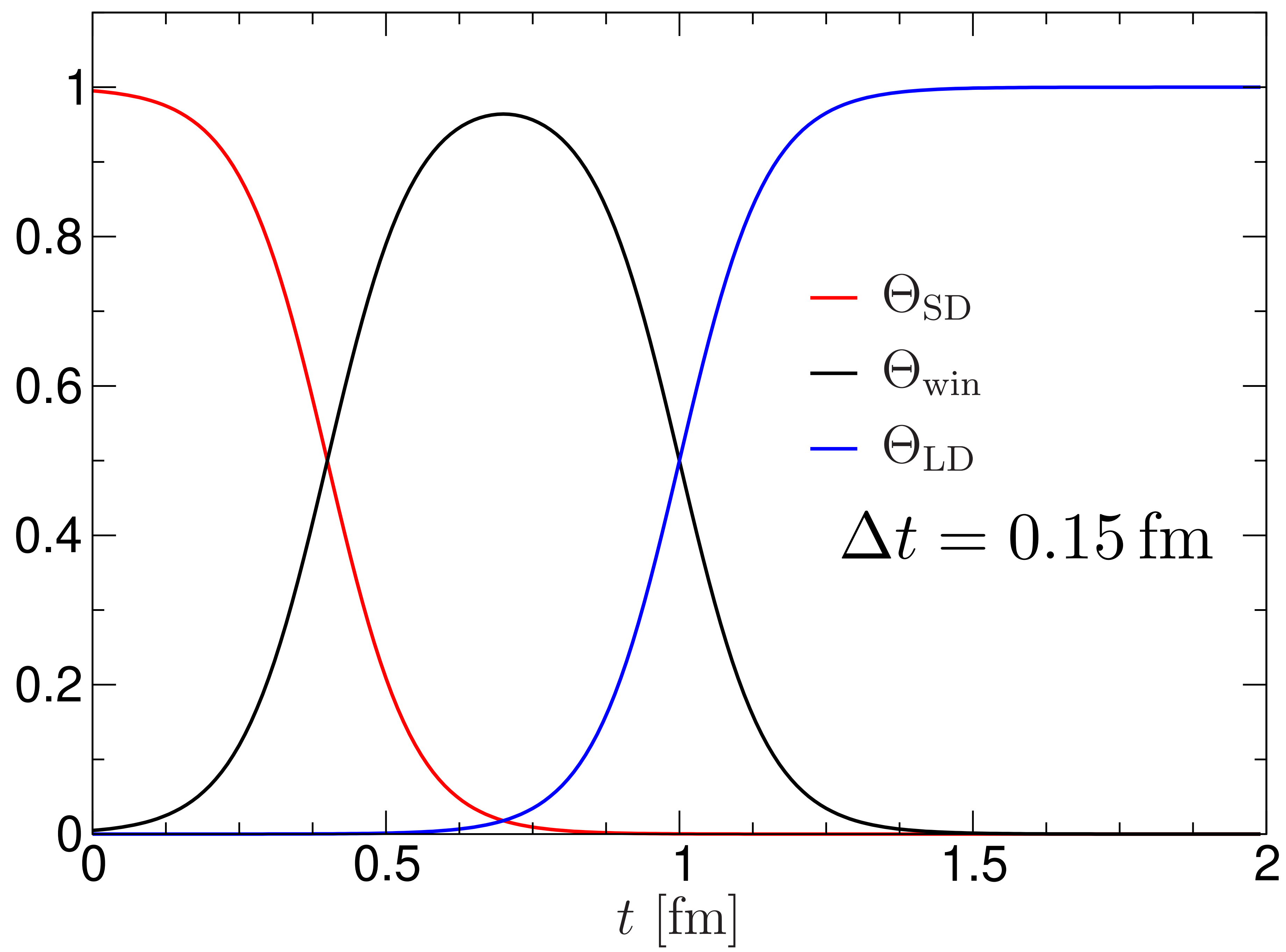
G(t) includes all flavours + disconnected+QED etc

Allows direct comparison of time-windowed values

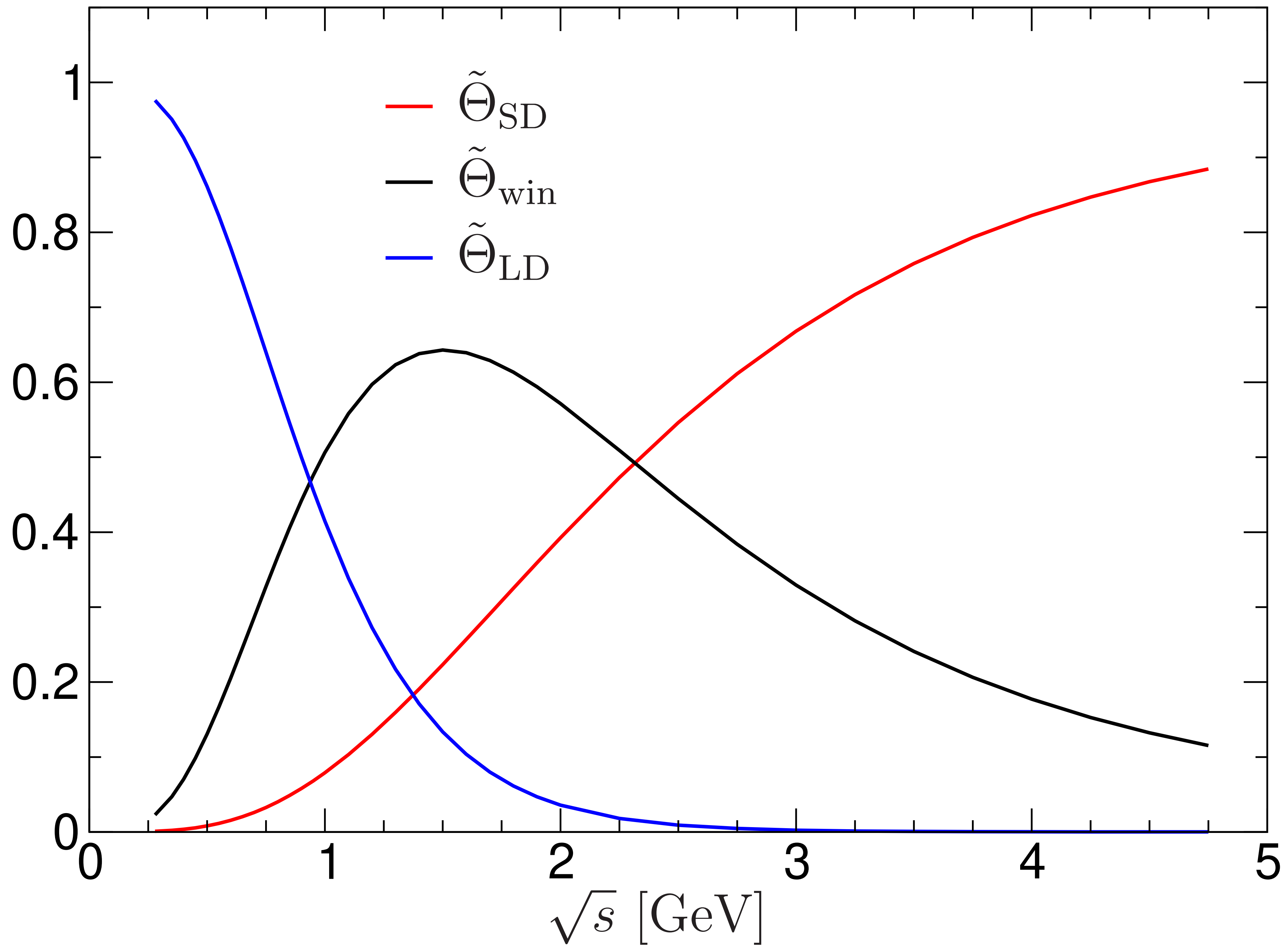
Time-windowed results are physical - disagreement in any window is a problem

Comparing data-driven and lattice HVP results

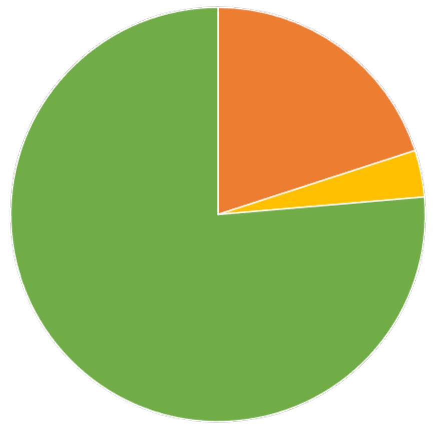
Colangelo et al
2205.12963



Mapping of window effects

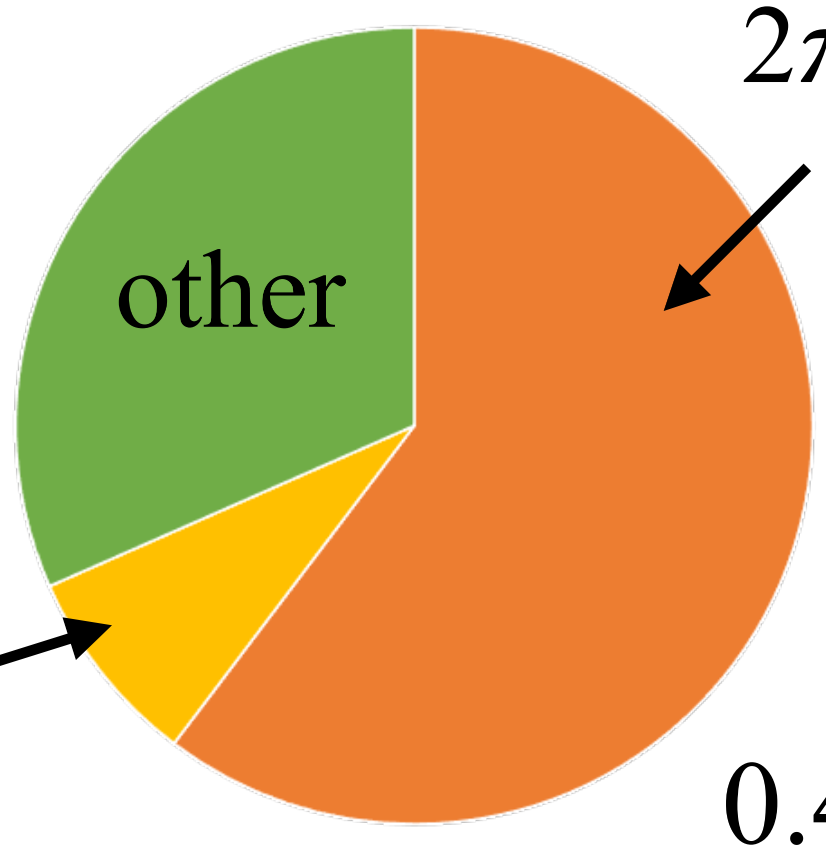


short-distance (SD)



0-0.4 fm

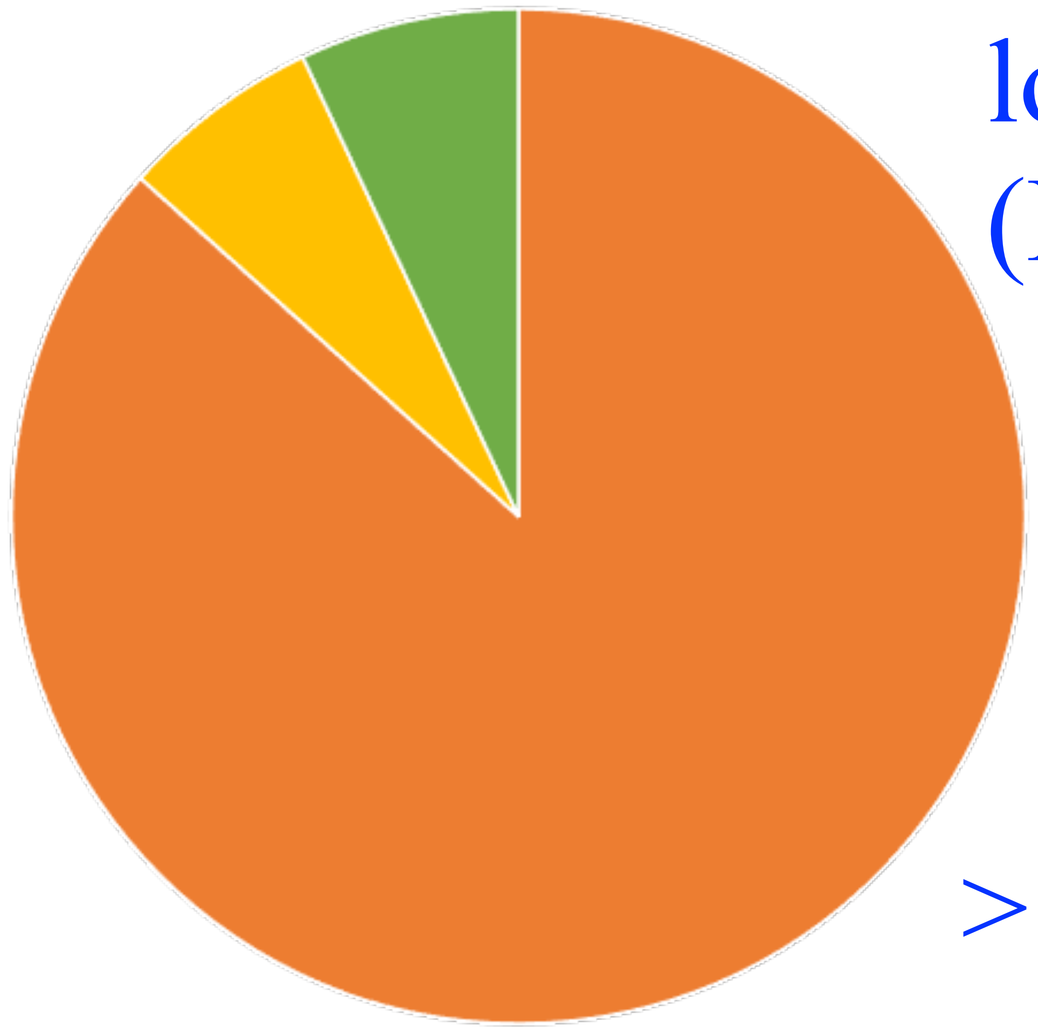
intermediate-distance



2π< 1 GeV

3π< 1.8 GeV

0.4-1.0 fm



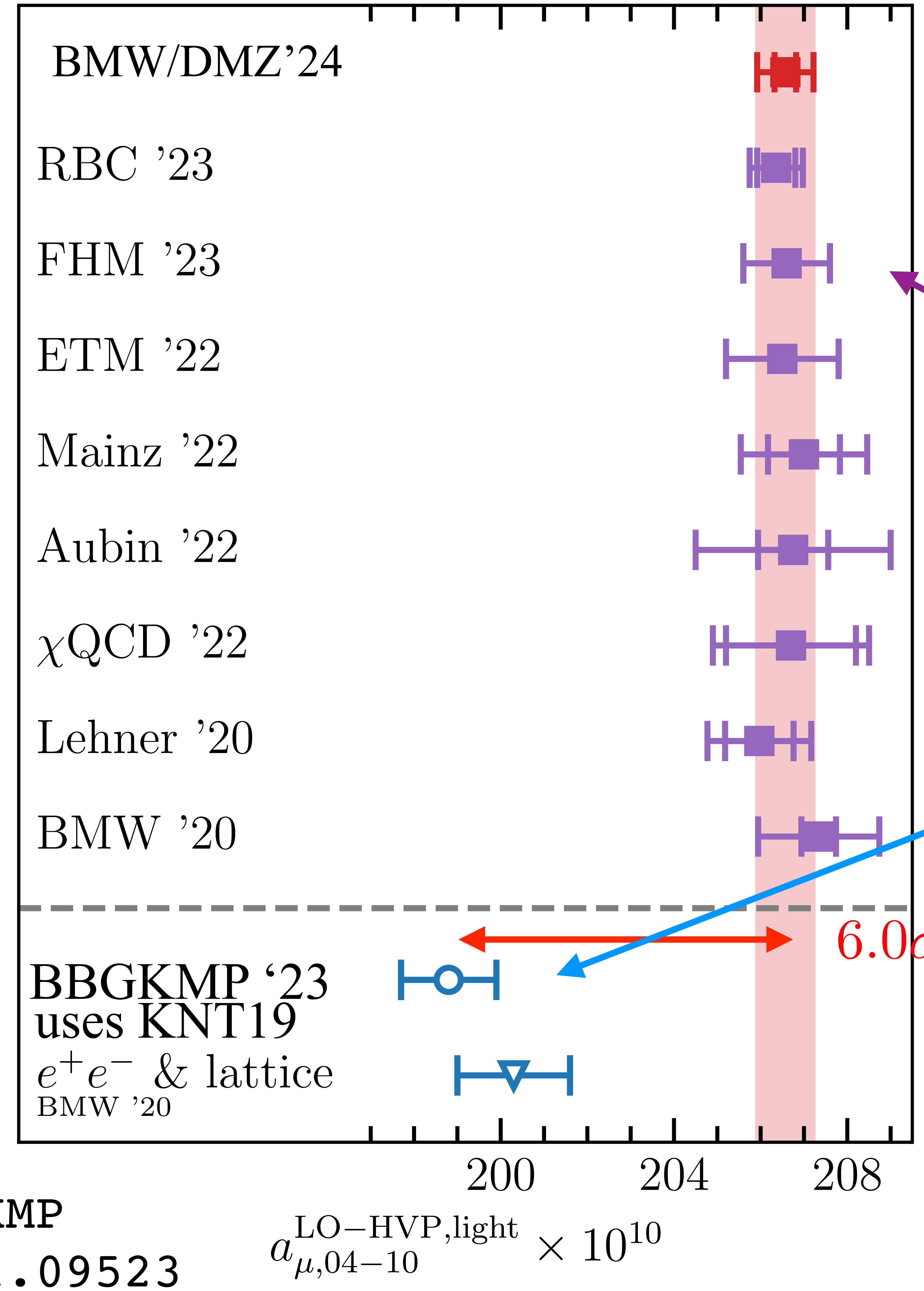
long-distance (LD)

Data-driven contributions

>1.0 fm

Comparing data-driven and lattice HVP results

Intermediate ‘window’ 0.4-1.0fm (Δt=0.15fm)



BBGKMP 2311.09523

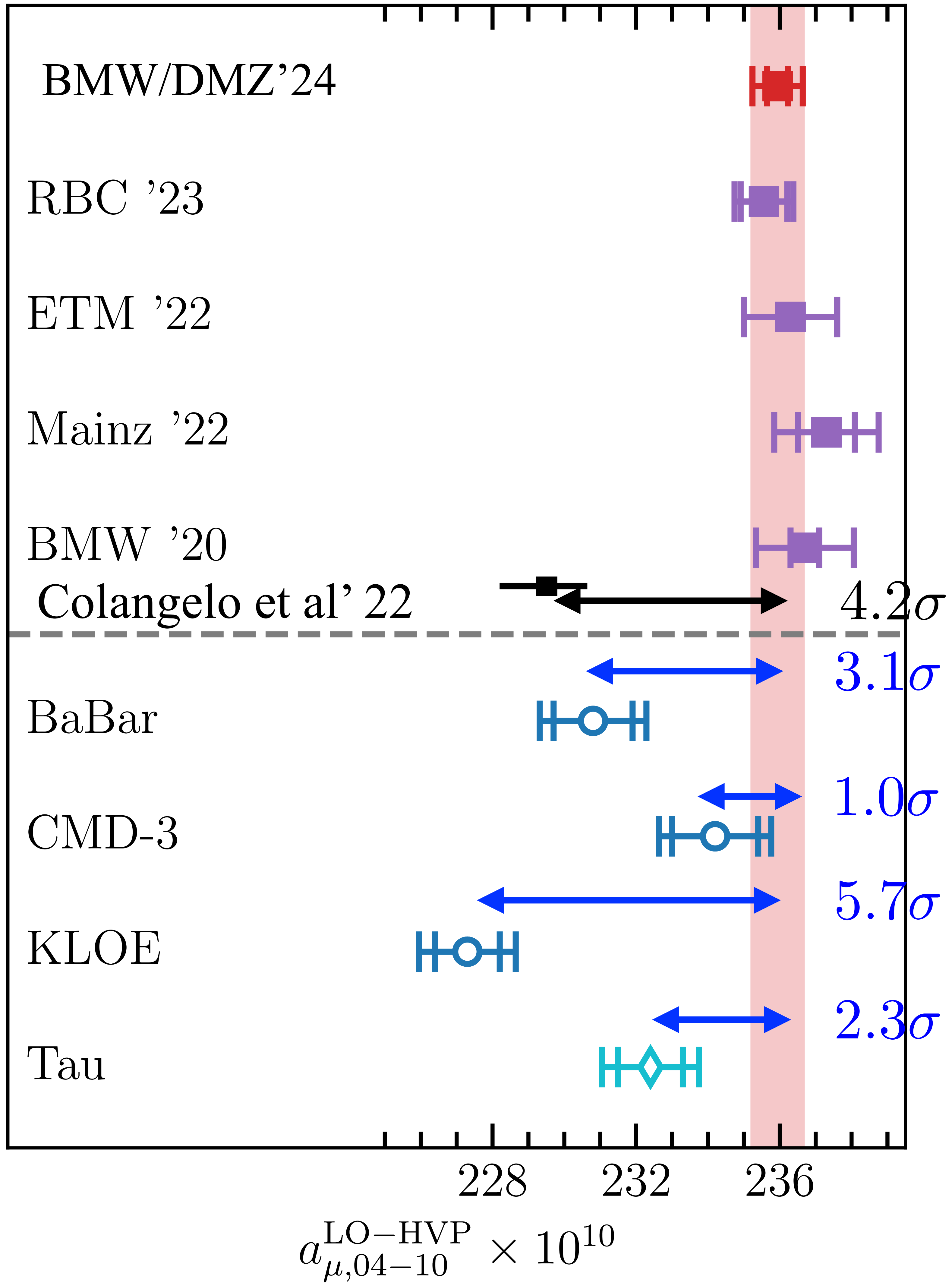
Light-conn.-only result: must remove other pieces from data-driven numbers.
Use isospin analysis to isolate I=1

6.0σ $= 7.7 \times 10^{-10} = 3.7\%$

Full HVP in window - compare directly to data-driven results (varying 2π input).

e+e− av. before CMD-3

BMW/DMZ 2407.10913

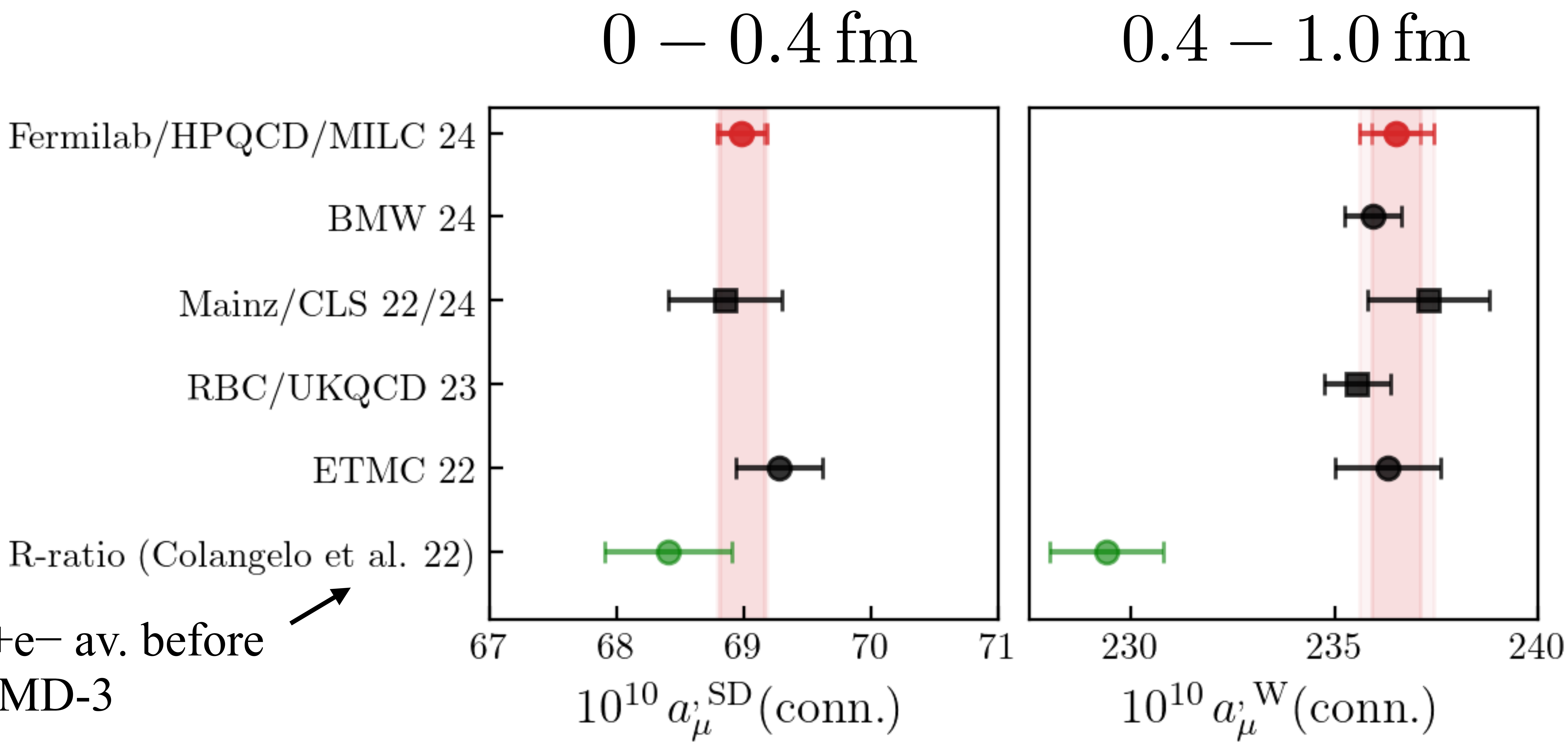


Conclude: Lattice disagrees with av. results from pre-CMD3 e^+e^-

Comparing data-driven and lattice HVP results

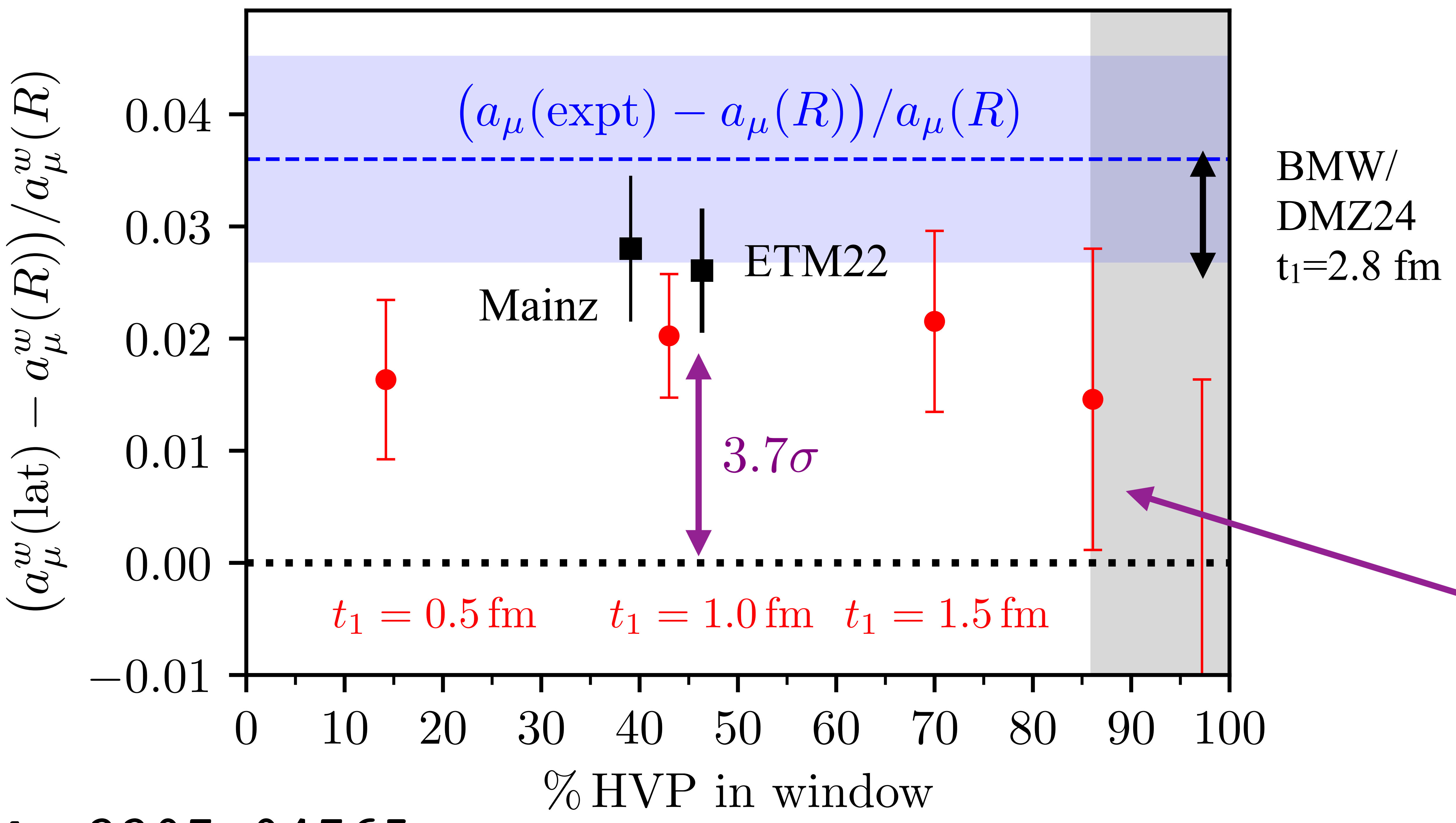
Short and intermediate windows (Δt=0.15fm)

Lahert,KEK, Sept24



e+e− av. before CMD-3

Comparing data-driven and lattice HVP results



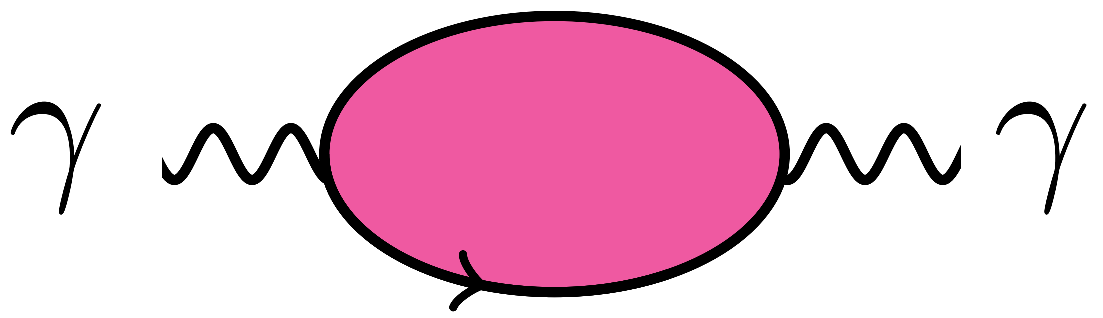
FHM, 2207.04765

One-sided window, 0 - t_1

Full HVP in window - compare directly to data-driven results (KNT19).

t_1=1.0 fm (43% HVP) = SD+ID. Lattice agreement on 2-3% difference with KNT19.

Lattice stat. errors large for $t_1 \geq 2$ fm for this (2019) data



Overall conclusion from windows comparisons:

Lattice QCD values higher than pre-CMD3 e^+e^- results at large-time/low s, i.e. where 2π tensions now seen.

See also analyses of hadronic contribution to running of α. Lattice differences with pre-CMD3 e^+e^- seen at low Q^2. (washed out by M_Z, so no impact on EW fits)

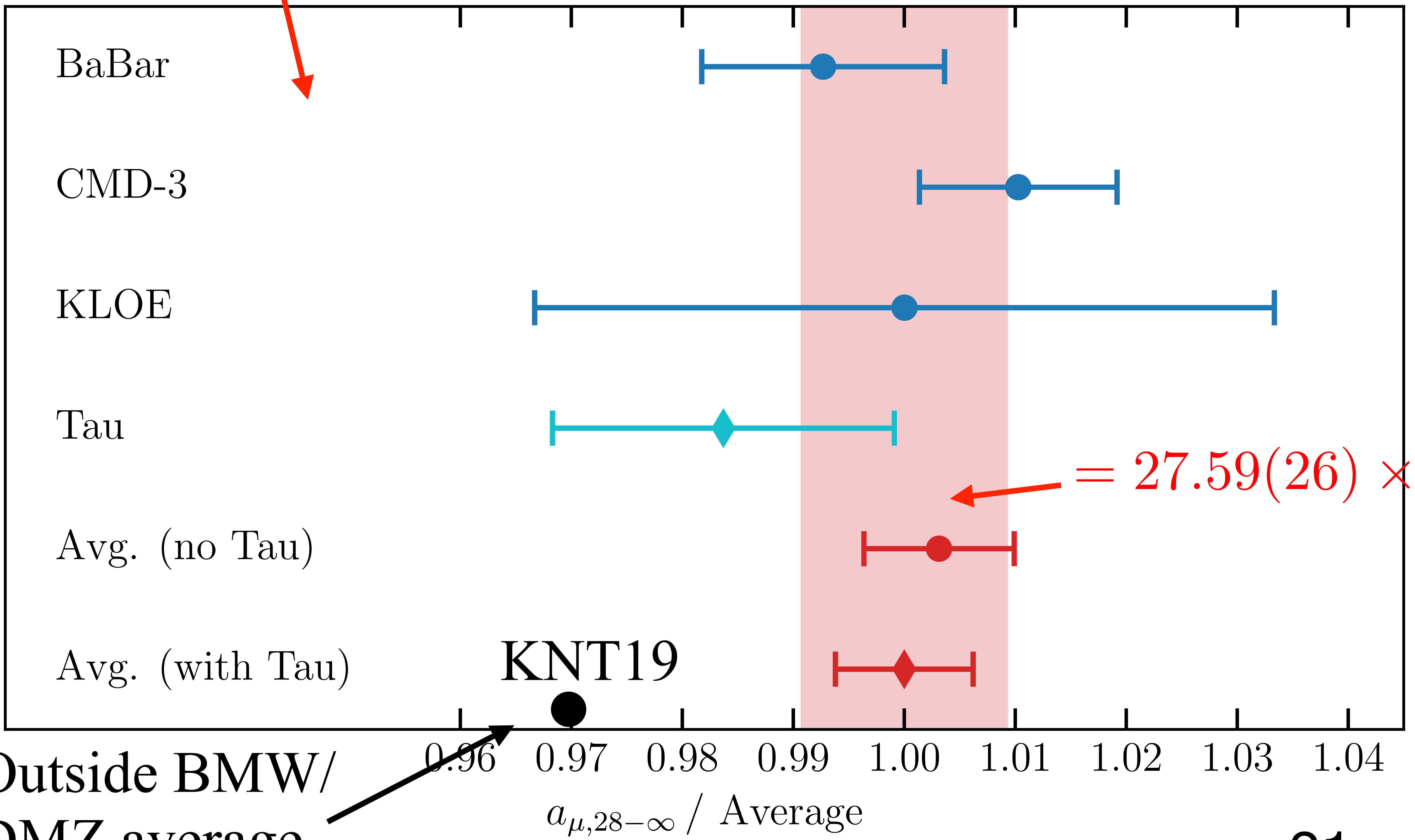
BMW,2002.12347, Mainz, 2203.08676

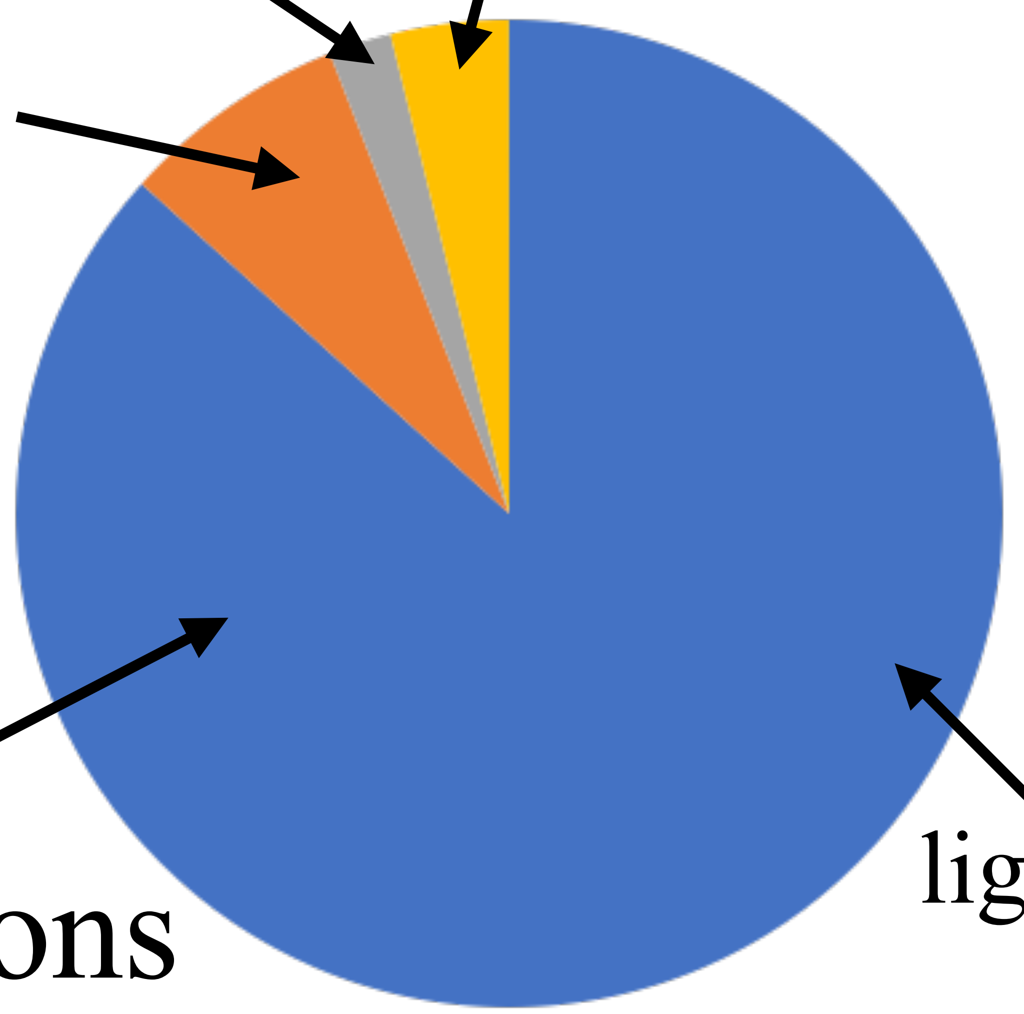
BMW/DMZ24 strategy for full HVP

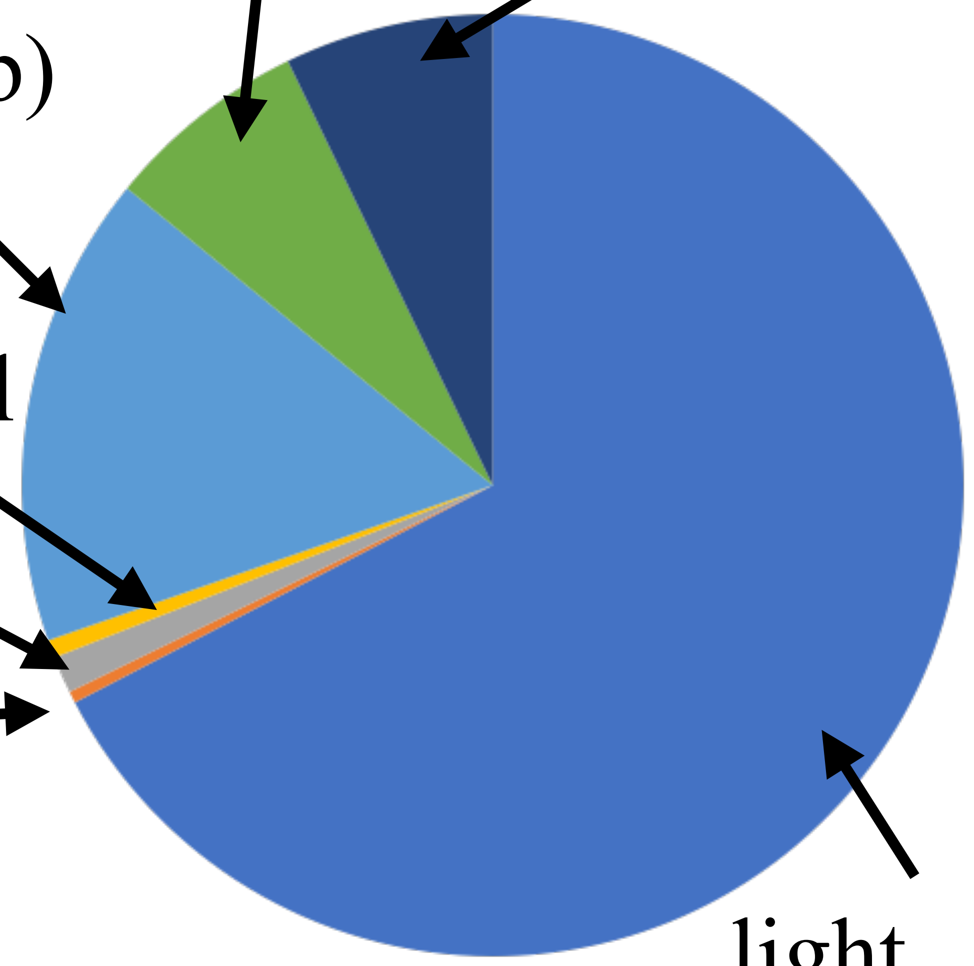
Take tmax=2.8fm for lattice, add 2.8-∞ from data-driven results.

CLAIM: reasonable experimental agreement on 2.8-∞ since >50% is from region BELOW ρ peak. (See slide 14)

Contribution of tail to HVP and its error is small







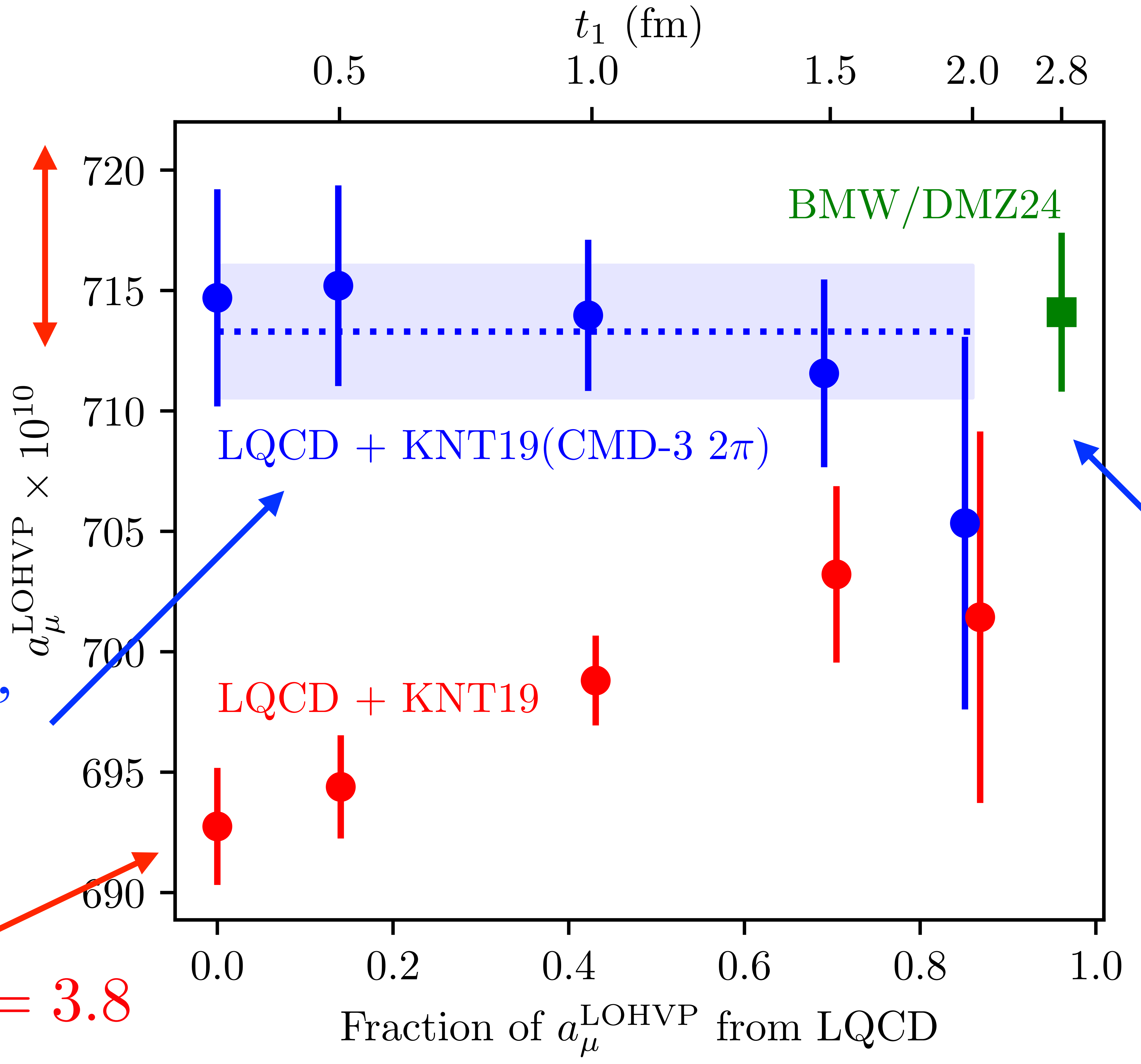
Pragmatic hybrid strategy for further full HVP results

GPL+CD et al,
Thanks to A. Keshavarzi

No new physics in muon g-2

Fit to a constant, χ^2/dof =0.6

$\chi^2/\mathrm{dof} = 3.8$



Use LQCD in one-sided time window up to t_1 .
Add in data-driven result for t_1 to ∞.

Totals should agree for different t_1

- test of validity of data-driven (and LQCD)
- choose smallest error or fit to a constant including correlations

Using 2019 FHM LQCD results for one-sided windows (2207.04765):

- totals are flat in t_1 for CMD3 2π
- total w. CMD-3 agrees with BMW/DMZ '24 for all values of t_1
- newer lattice data have much better uncertainties for $t_1 \gtrsim$ 2fm

Smaller t_1 : reduces lattice stat. and finite vol. error but increases input from data-driven tail

Hybrid strategy best to optimise uncertainty on total HVP?

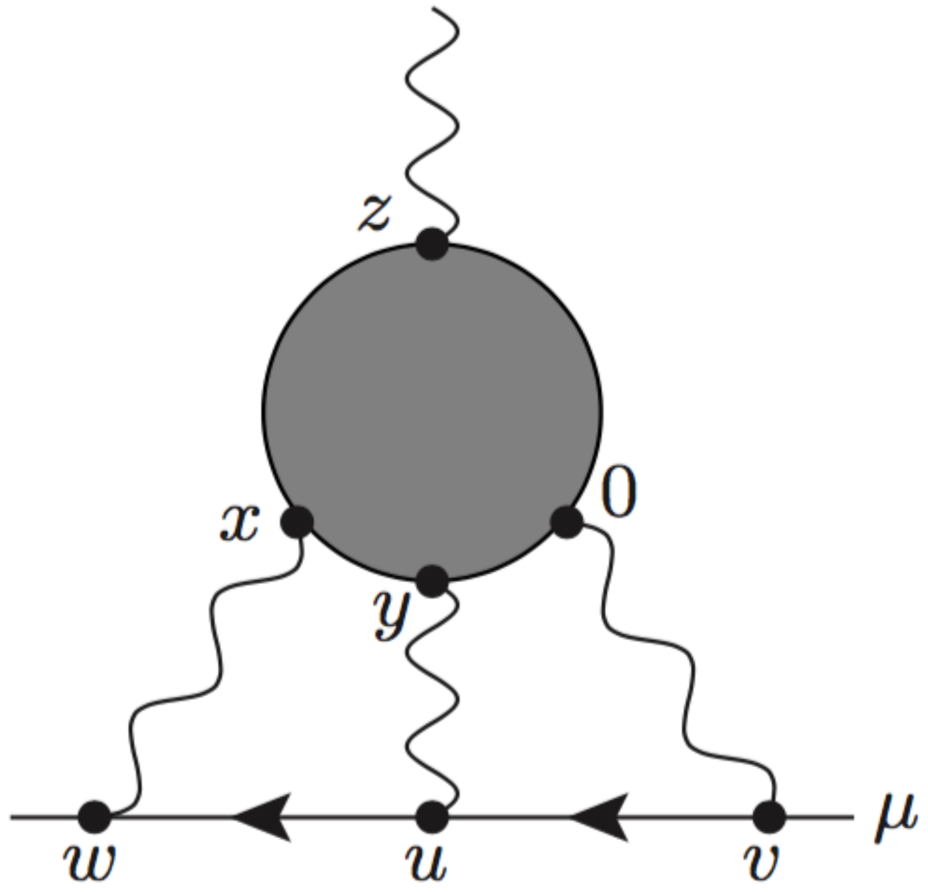
Larger t_1 : CMD3/KNT19 tension falls: <0.3% total HVP for $t_1 \geq 2.5$ fm

HLbL contribution

Theory white paper 2020 :
LO, phenomenology

$$a_\mu^{\mathrm{HLbL}} = 9.2 \pm 1.9 \times 10^{-10}$$

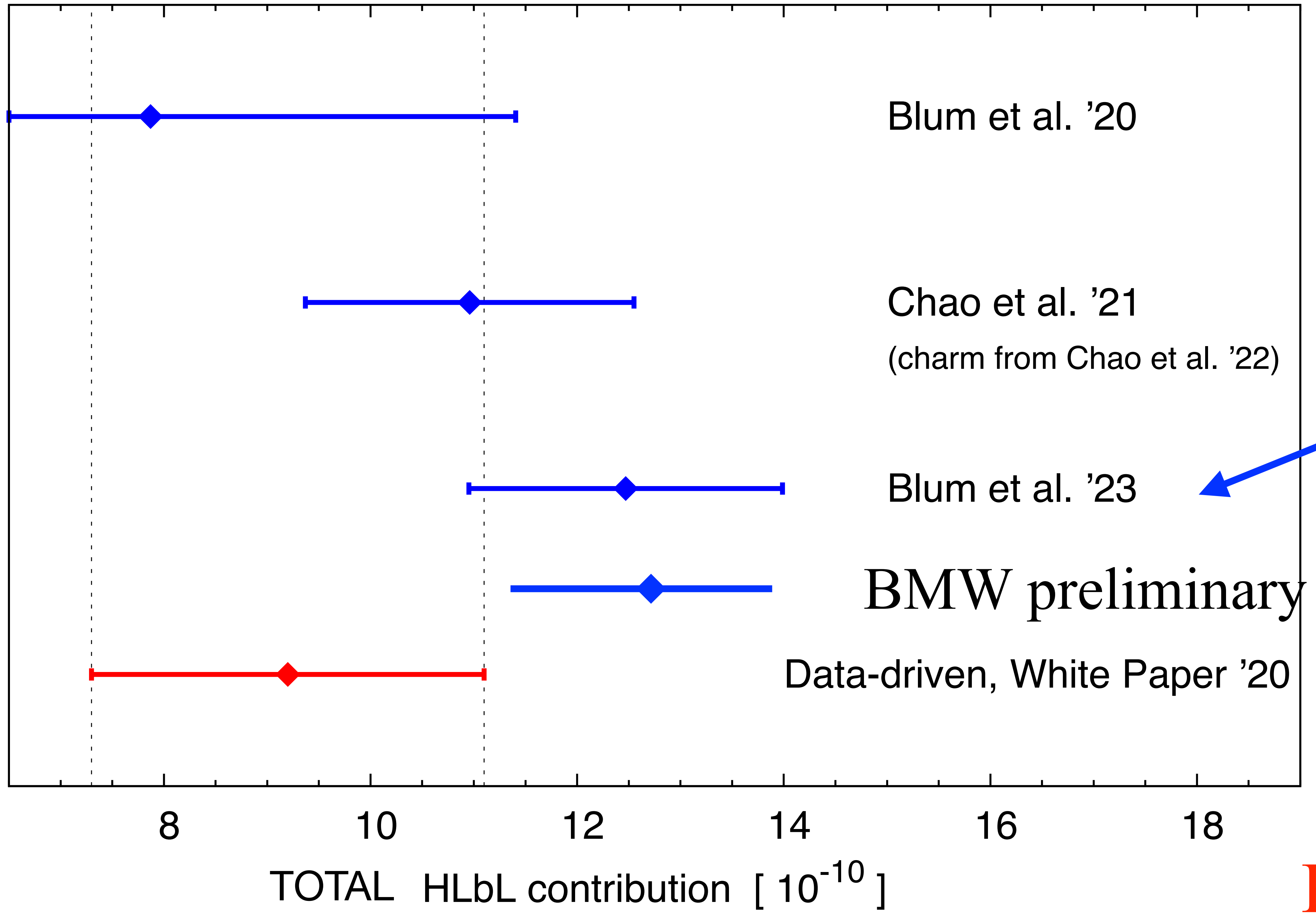
aim for 10% uncertainty



Method 1 : direct lattice calculation

Mainz, 2104.02632
+charm piece,
2204.08844

RBC/UKQCD, 2304.04423



Work at a finer lattice spacing (0.08fm) ongoing

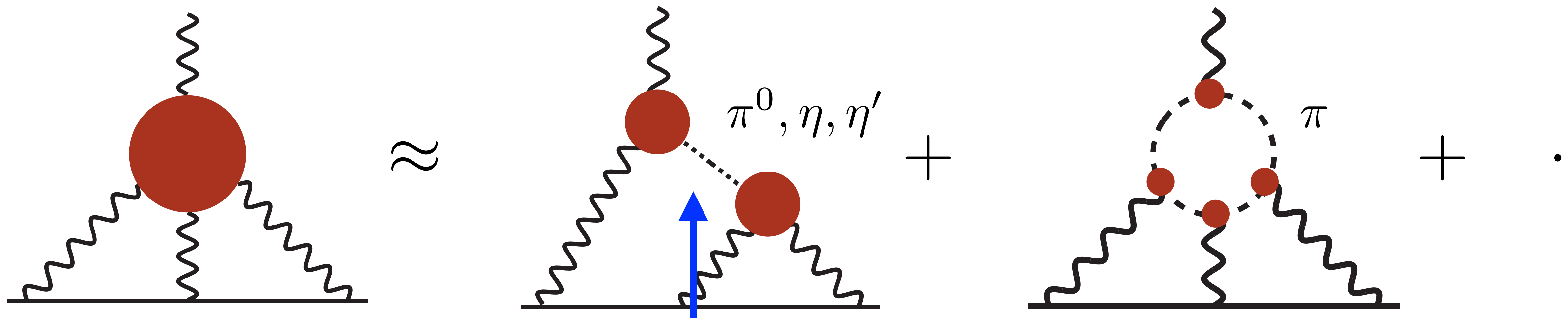
ETM calculation underway, physical light quarks, a=0.08fm.

Results from different groups agree.
Uncertainty ~10-13%.
Slightly (~3x10^{-10}) higher than WP20

H.Meyer , Theory initiative meeting, April 2024;
A.Gerardin, Lattice2023

HLbL contribution

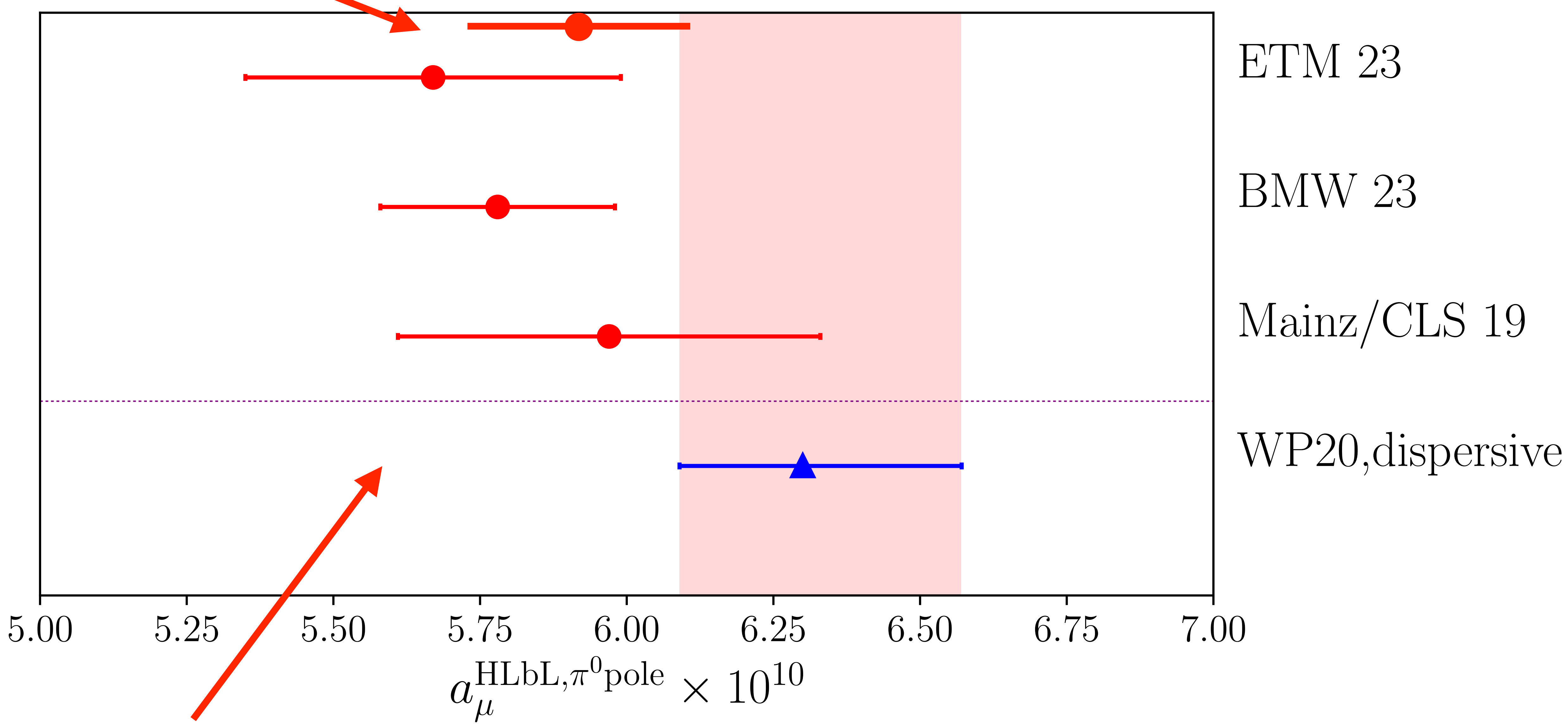
Method 2 : dispersive approach with lattice QCD input



Pseudoscalar transition form factor

$\mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)$

RBC/UKQCD preliminary



Calculate PVV 3-point function and take weighted sum over time-insertions of one V to fix γ energy

Details: A.Gerardin, Lattice2023

PS poles dominate - other contributions $\sim \pm 1.5 \times 10^{-10}$ tend to cancel (WP20)

Good lattice agreement, using 4 different quark actions

BMW 23 add η, η'

$$a_\mu^{\mathrm{HLbL,ps-poles}} = 8.51(52) \times 10^{-10}$$

$$= 9.38(40) \times 10^{-10} \quad \text{(WP20)}$$

NOT total HLbL

Lattice is 2σ lower than WP20 for η but the difference is small: 0.5×10^{-10}

ETM, 2308.12548,
BMW,2305.04570,
Mainz, 1903.09471

CONCLUDE : HLbL looking good, lattice providing critical input

Conclusions

There is almost certainly less new physics in muon g-2 than previously hoped, and perhaps none.

Lots still to understand in $e^+e^- \to$ hadrons data, tensions between expts. and with τ. Lattice evidence stacks up in favour of CMD3

Opportunity for lattice to finalise HVP results in next few years and provide SM result (uncertainty needed ~0.5%).

Requires multiple results from different groups using blinded analyses (underway).

This could include making use of data-driven results (even with tensions) for the long-time tail, since quickest route to numbers with reasonable uncertainties.

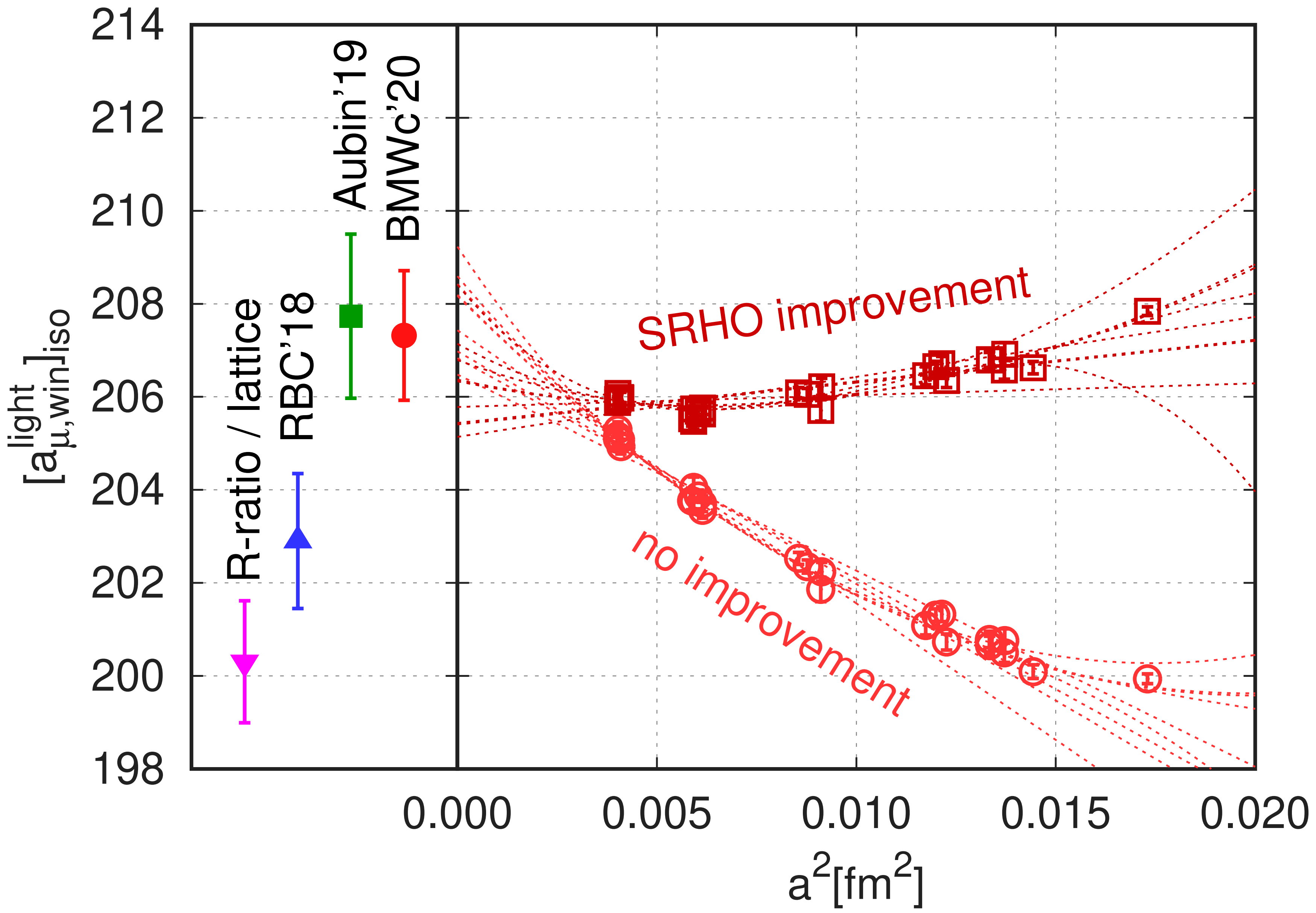
Progress on HLbL contribution also important and continuing.

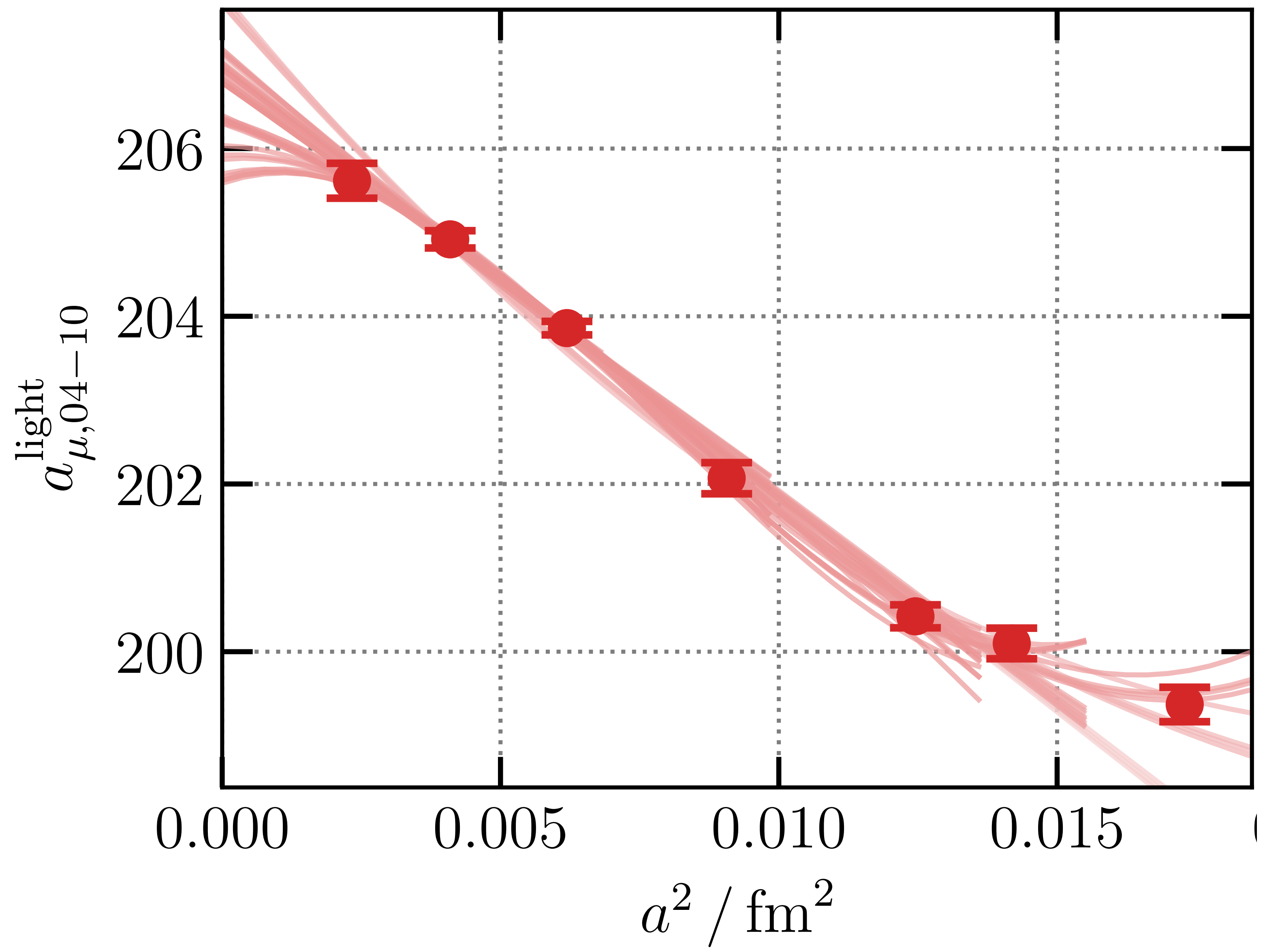
Timescales: New theory white paper, end 2024; FINAL muon g-2 result 2025, further experimental info. (e^+e^-, J-PARC, MuonE) later in 2020s, early 2030s.

Spares

BMW/DMZ ’24 and BMW20

Divide time region for light-q-conn into several windows: 0-0.4, 0.4-0.6, 0.6-1.2, 1.2-2.8. Correlated fit to last 3 allows different fit forms in different regions, lowers uncertainty.

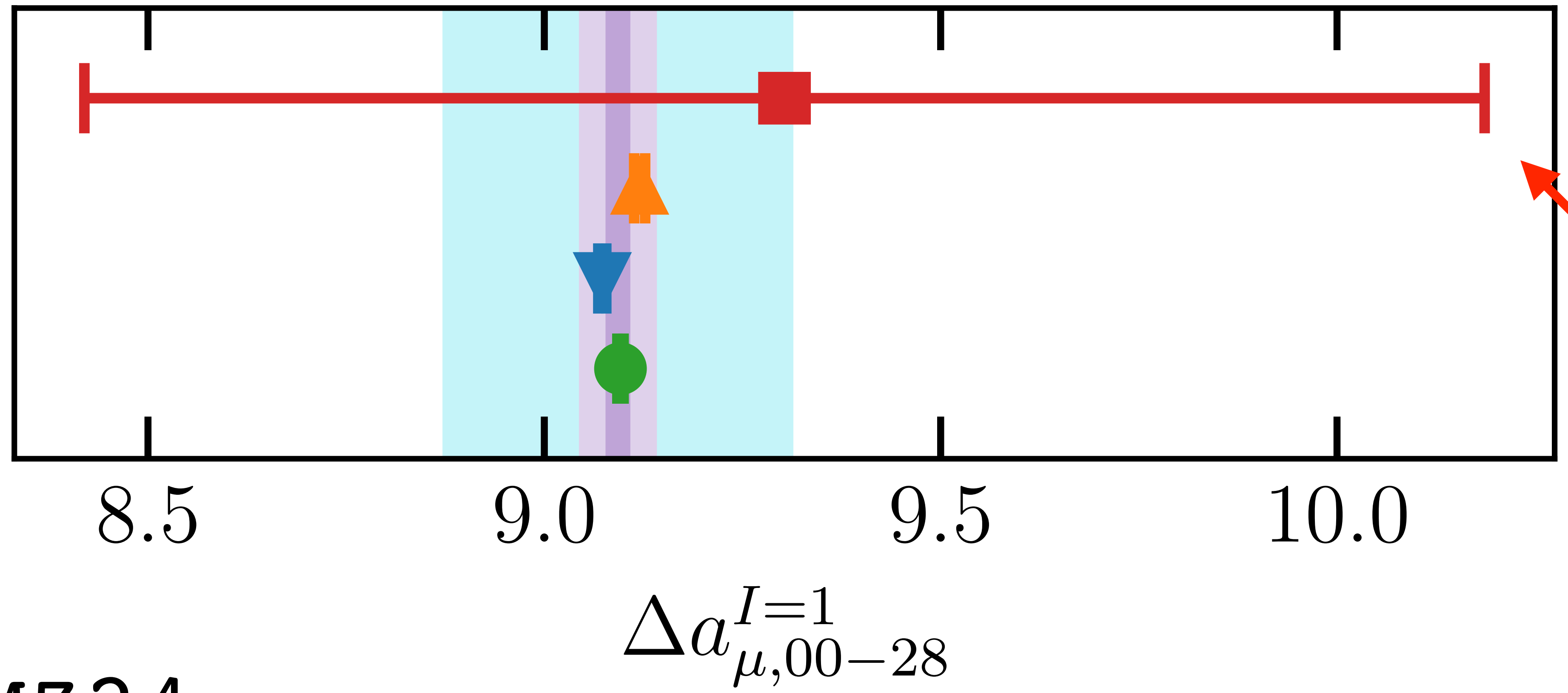




BMW20, 2002.12347

BMW/DMZ24, 2407.10913

BMW/DMZ ’24 and BMW20



BMW/DMZ24, 2407.10913

Finite-volume correction from L=6.8fm to ∞

BMW/DMZ ’24 have correction 9.31(88) for 0-2.8fm window. Test versus models using data-driven input.

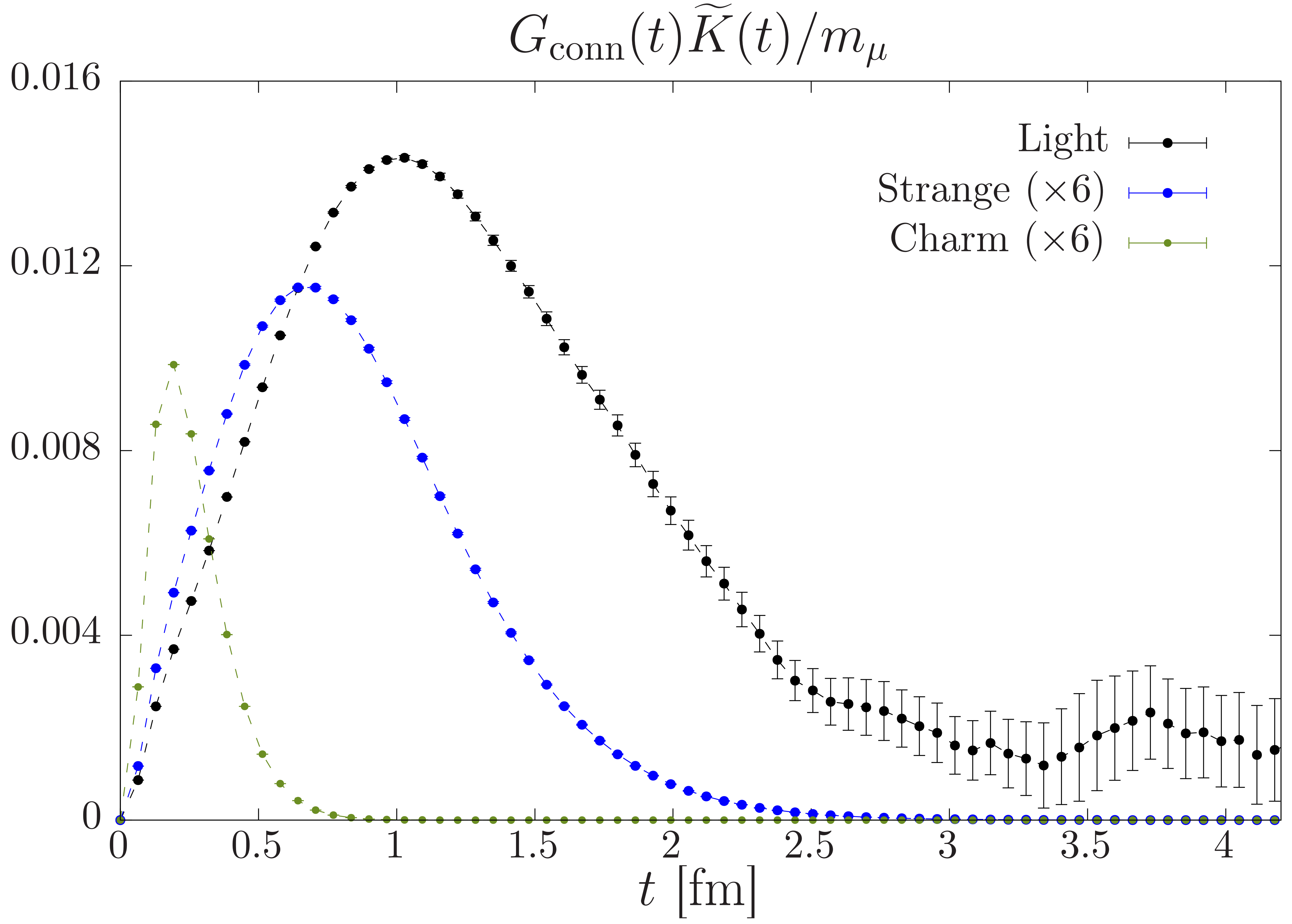
BMW ’20 have correction 18.7(2.5) for full calculation .

Difference between BMW/DMZ ’24 and BMW20 for the total HVP:

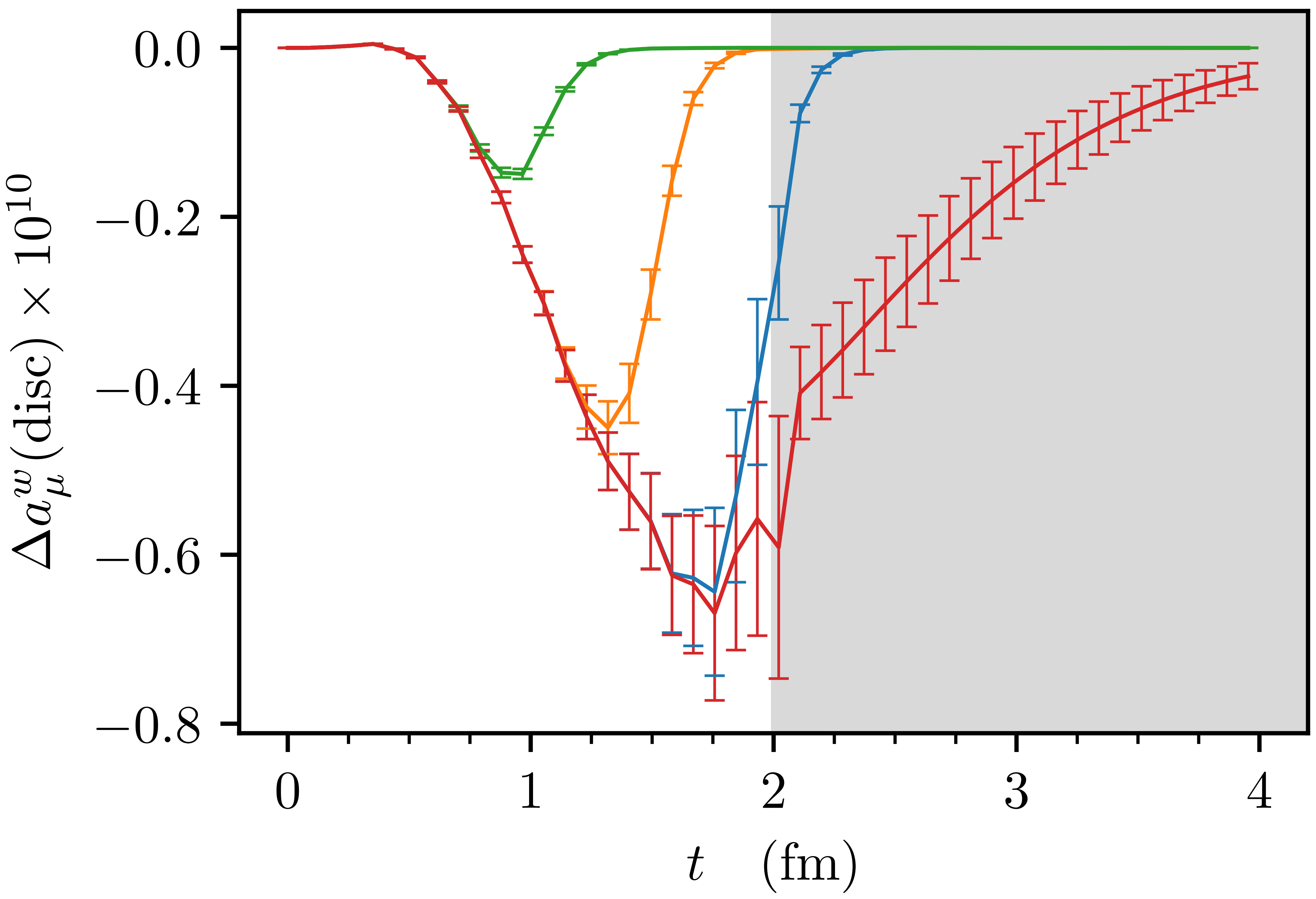
$$a_\mu^{\mathrm{LOHVP,BMW/DMZ24}} - a_\mu^{\mathrm{LOHVP,BMW20}} = 6.5(5.5) \times 10^{-10}$$

i.e. 1.2σ

Different flavour lattice correlators



Gerardin,
Lattice2023

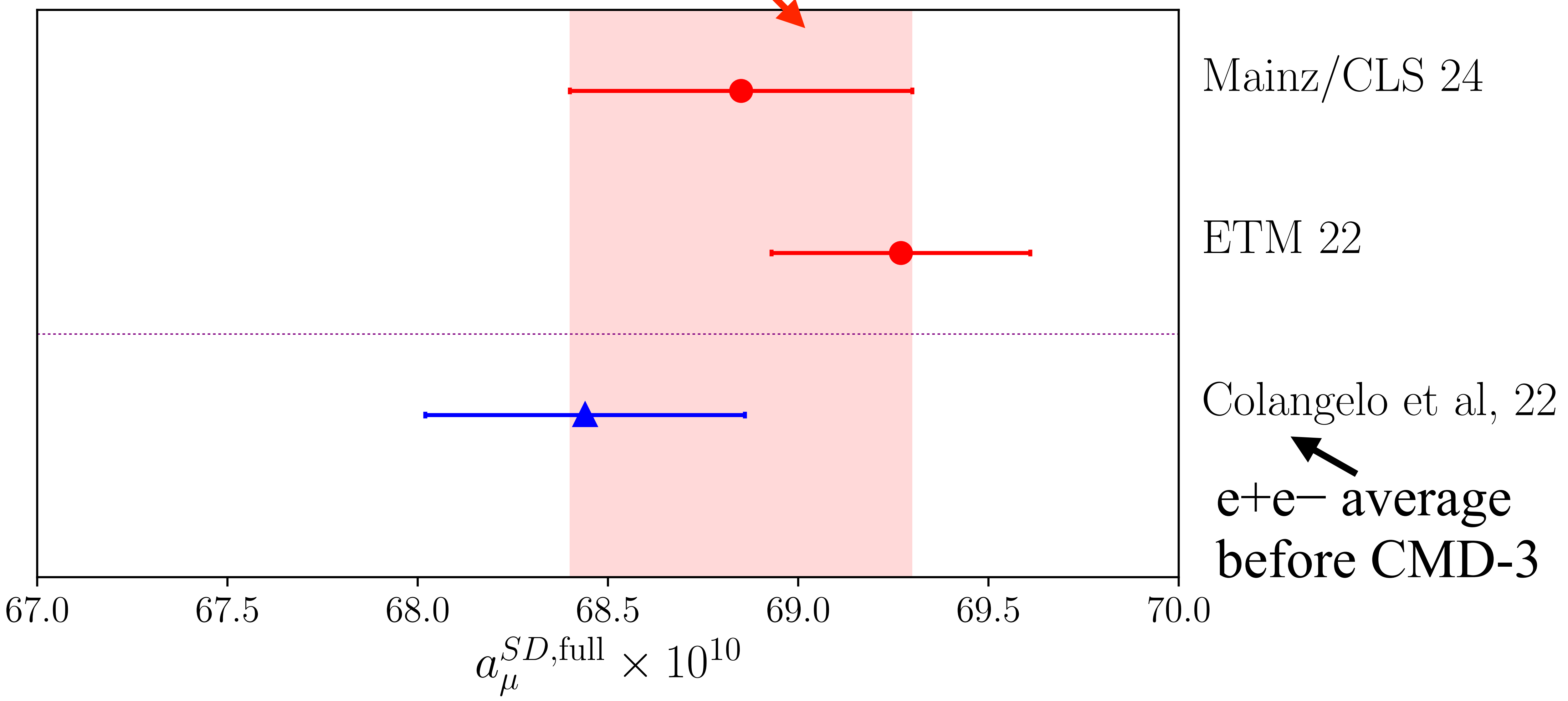


FHM, 2207.04765

Comparing data-driven and lattice HVP results

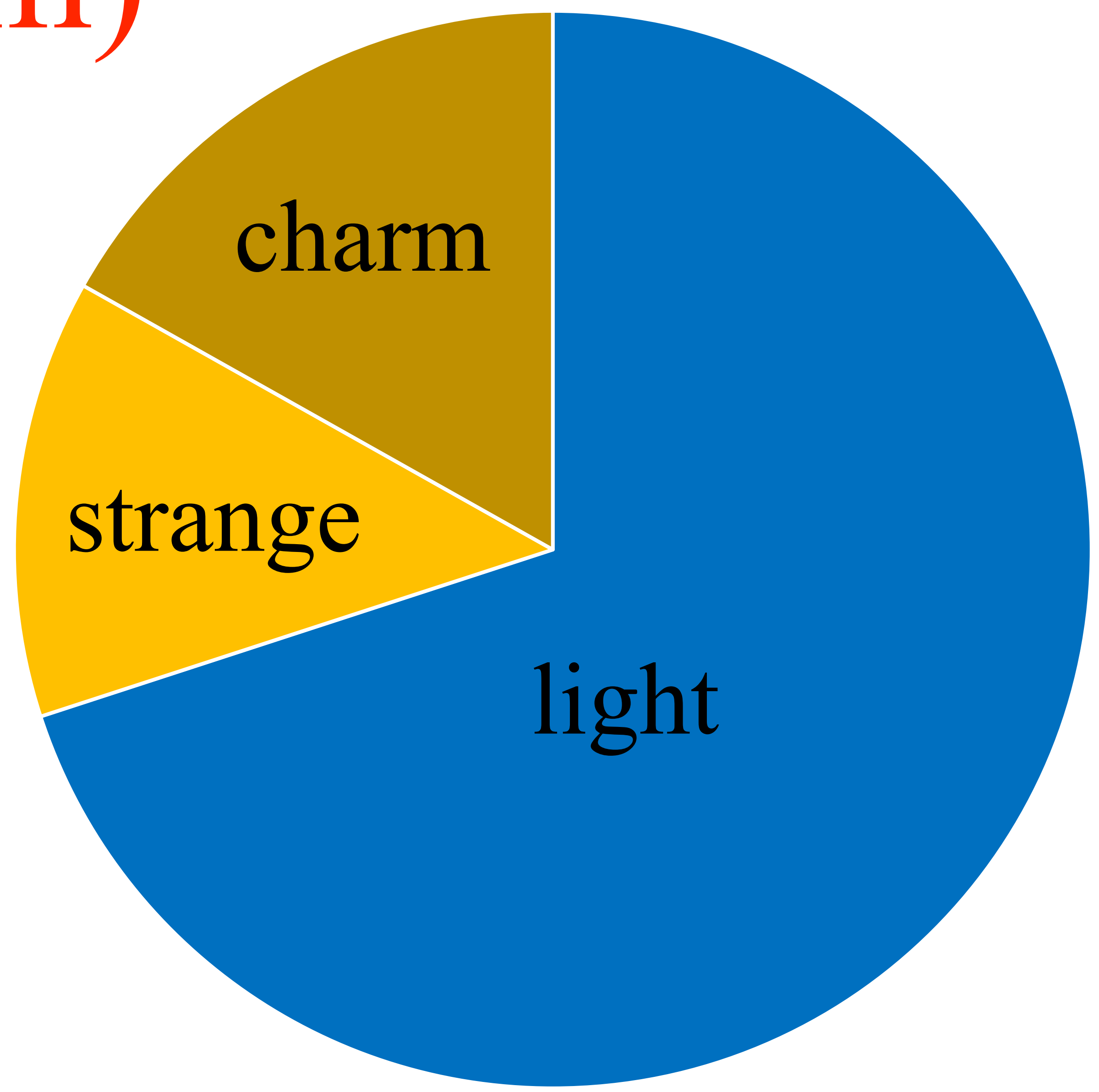
Short-distance 'window' 0.0 - 0.4 fm (Δt=0.15fm)

Full HVP in window - compare directly to data-driven results



e+e− average before CMD-3

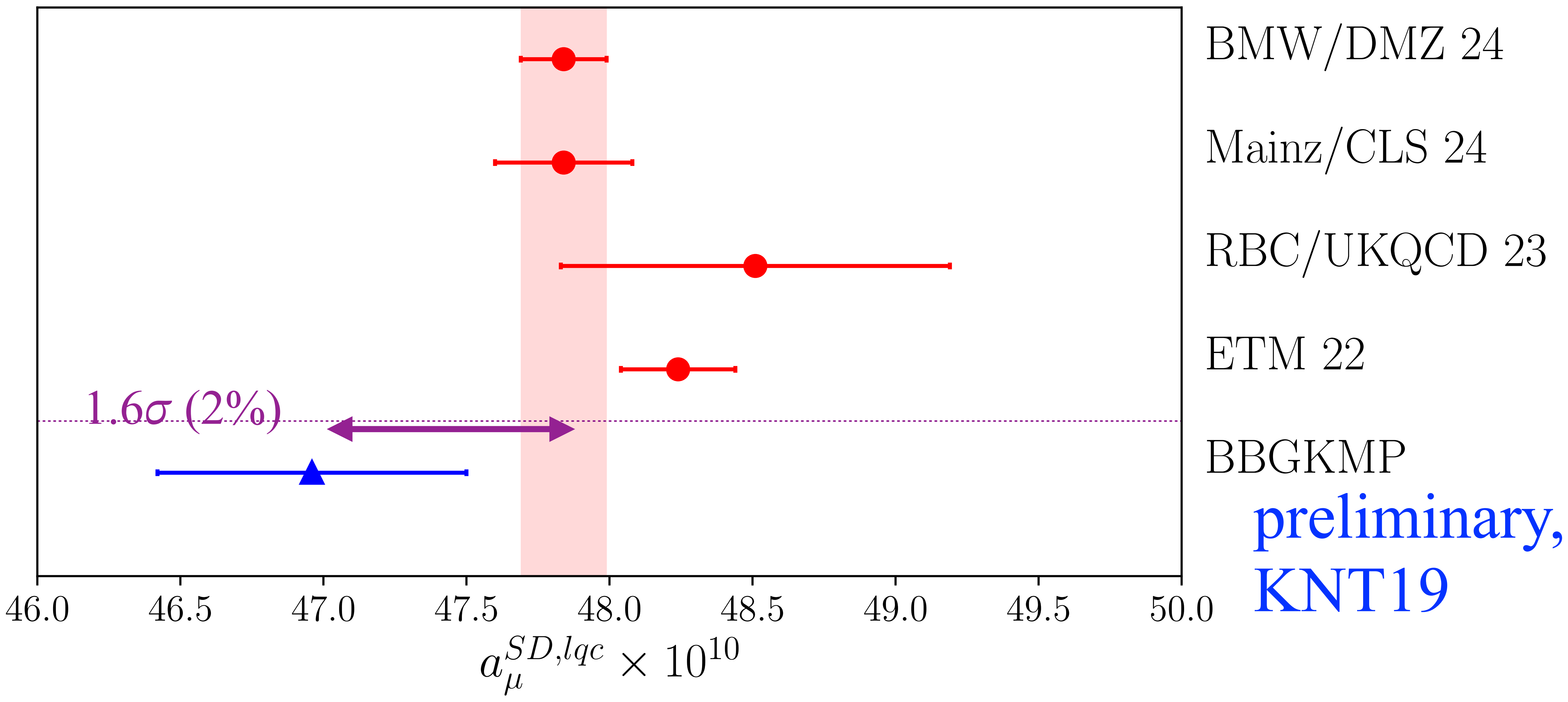
Flavour contributions to window



pre-CMD3 data-driven is in reasonable agreement at these short times (2π still contributes 20% here).

Comparing data-driven and lattice HVP results

Short-distance 'window' 0.0 - 0.4 fm (Δt=0.15fm)



preliminary, KNT19

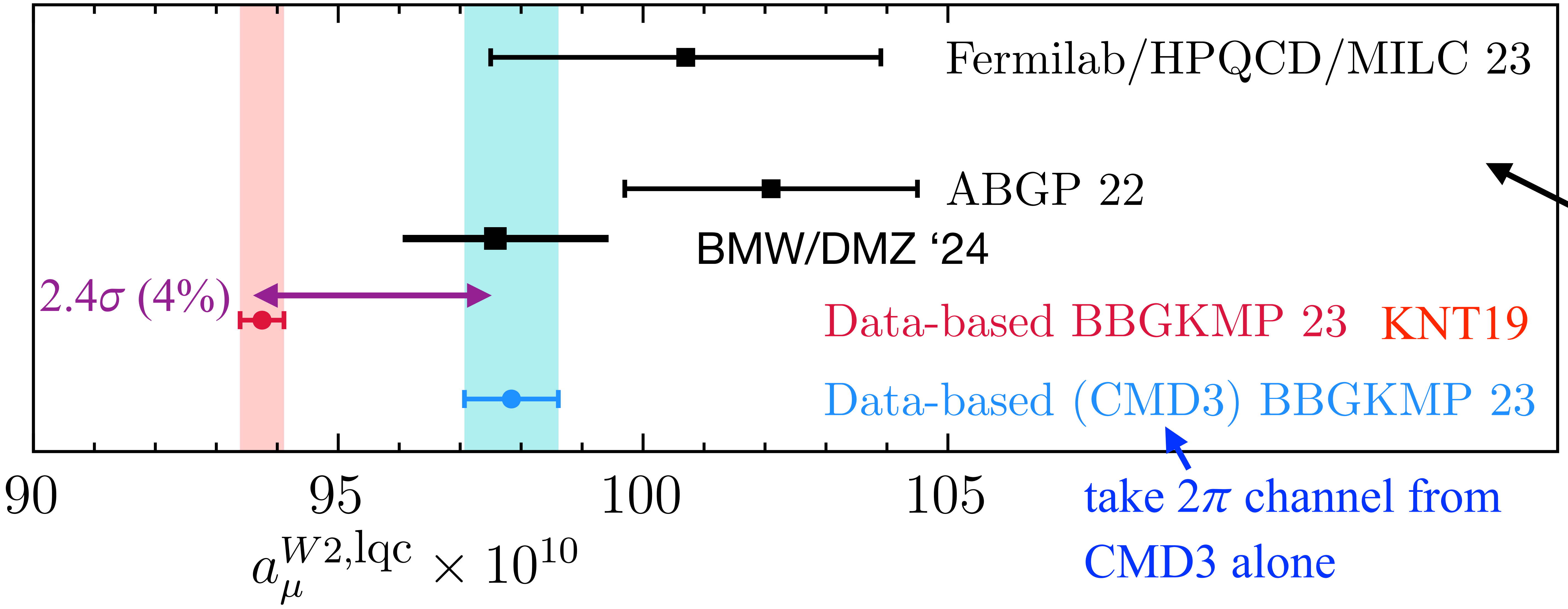
Light-quark-connected only. Lattice agreement good (errors ~0.5%).

pre-CMD3 data-driven result is a little lower than the lattice but not significantly (2π still contributes 20% here so CMD3 would push it up ~1%).

Comparing data-driven and lattice HVP results

A 'window' at larger times 1.5 - 1.9 fm (Δt=0.15fm) (W2)

BBGKMP 2311.09523



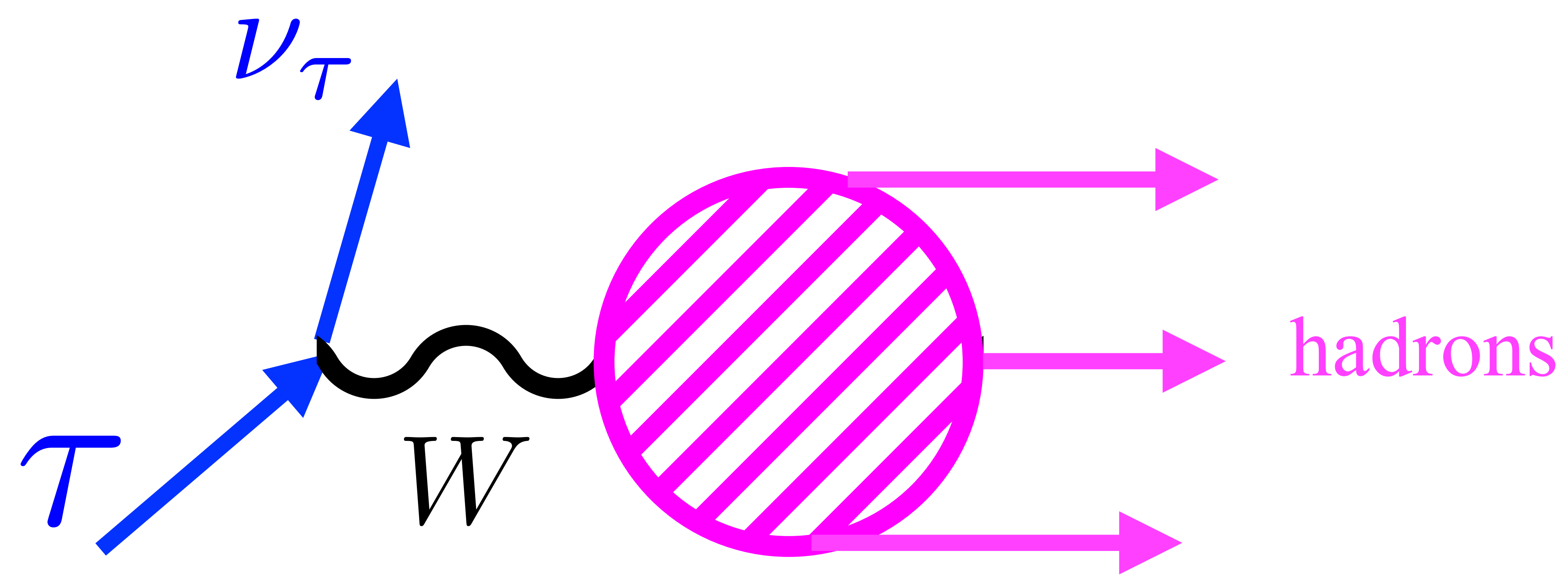
take 2π channel from CMD3 alone

Aubin et al '22, 2204.12256

Statistically noisier but better control of finite-vol, pion-mass corrections.

Lattice results agree. Some tension between lattice and pre-CMD3 e^+e^-. Lattice stat. errors too high (~2%) for this small window to be clear ?

Issues with data for data-driven HVP



2) Inclusion of LEP data for τ hadronic decay

Can select states (even number of pions) corresponding to vector current $\bar{u}\gamma_\mu d$

Measure spectral function, v1(s) : distribution in s=(mass hadrons)^2

ALEPH: hep-ex/0506072

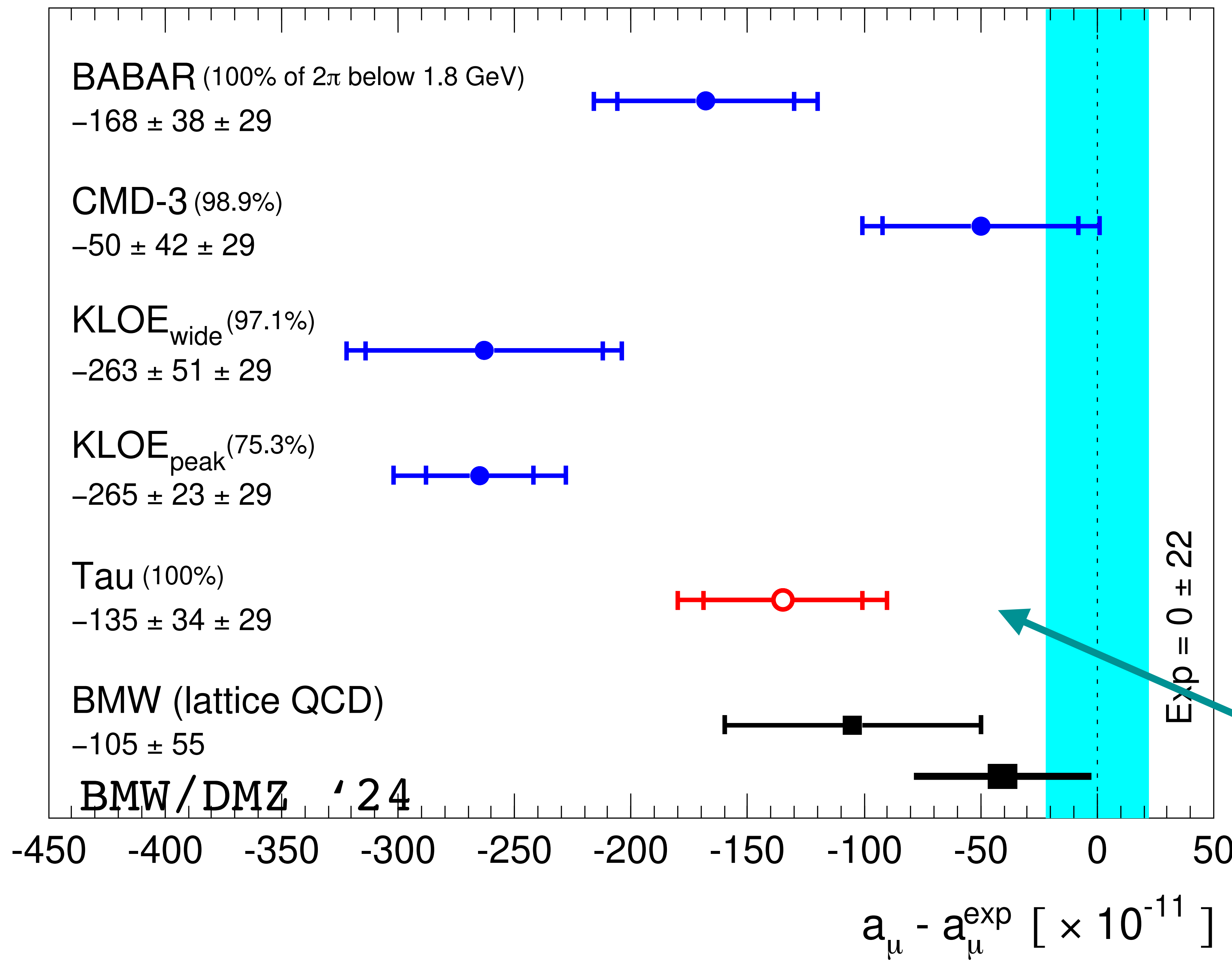
$$\sigma^{I=1}(e^+e^- \to X^0) = \frac{4\pi\alpha^2}{s} v_{1,X^-}$$ exact isospin limit

key modes: $\pi^+\pi^-$ ↑ $\pi^0\pi^-$ ↑

Need correction for IB = ρ-ω mixing in e+e− +EM, FSR

Opportunity for lattice?

M. Bruno talk Mon.



τ result for 2π (up to 1.8 GeV) in HVP agrees well with BaBar, higher than KLOE.

DHLMZ 2312.02053

3) BaBar study of initial-state radiation (2308.05233) suggests issues with PHOKHARA Monte Carlo. May affect KLOE and BES radiative return experiments. Further study needed.

MuONe experiment @ CERN

Strong Liverpool involvement

R.Pilato, TI workshop, Sept. 2023

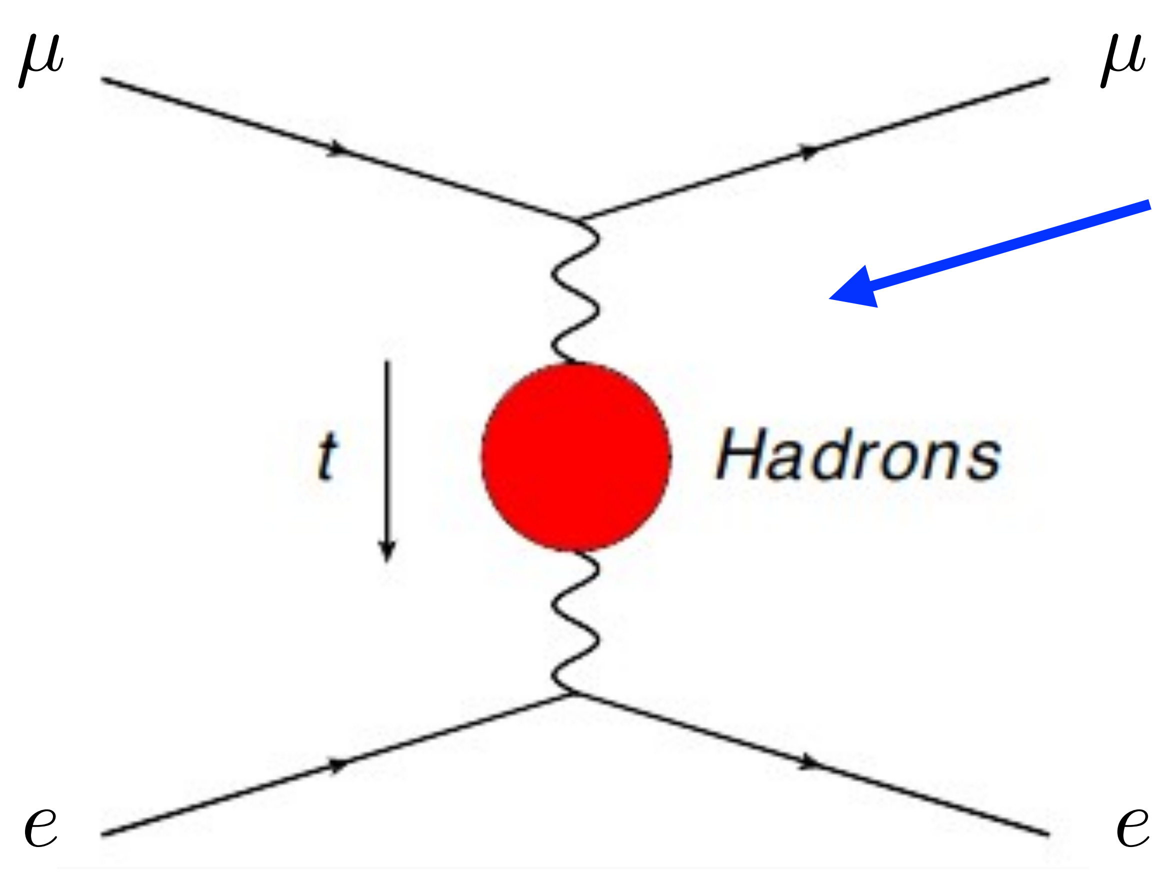
Measure hadronic contribution to running of α from μ scattering from atomic electrons.

$$a_\mu^{\mathrm{LOHVP}} = \frac{\alpha_0}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\mathrm{had}}(t(x))$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

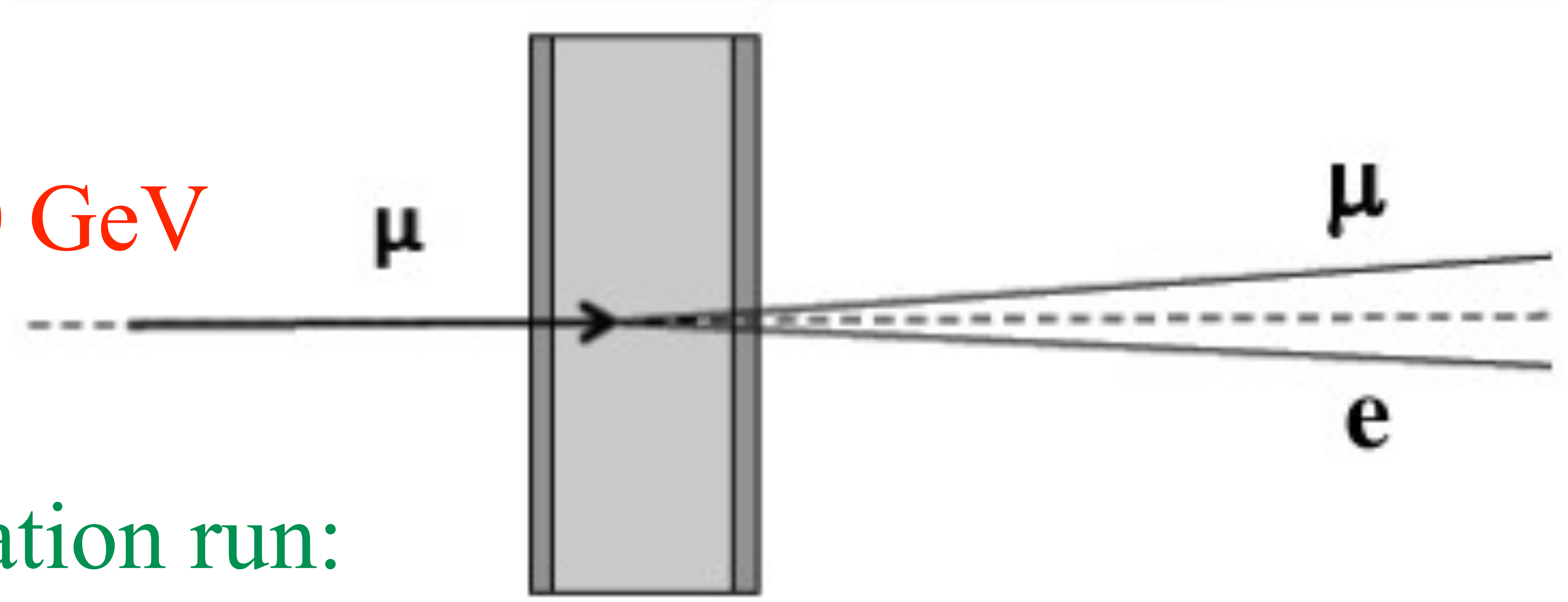
$$-0.153\,\mathrm{GeV}^2 < t < 0$$

gives 88% of a_μ^{LOHVP}



$\Delta\alpha = \Delta\alpha_{\mathrm{lep}} + \Delta\alpha_{\mathrm{had}} + \Delta\alpha_{\mathrm{top}}$

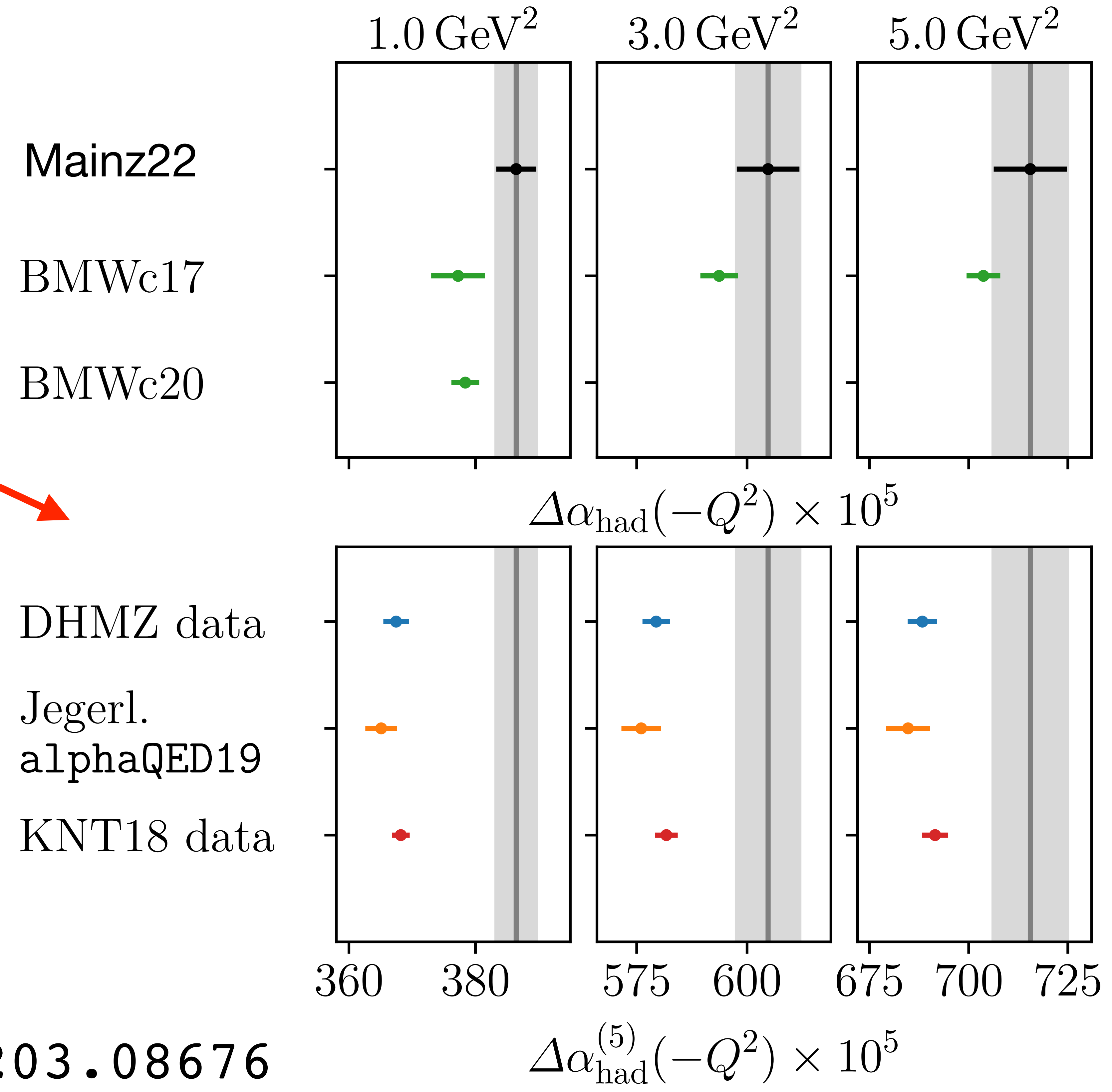
160 GeV



Be

validation run: Sept. 2023

Tensions in $\Delta\alpha_{\mathrm{had}}$ between lattice and pre-CMD3 e^+e^- at small t.



Schedule:

- 2-4 month test run 2025 with ~1/4 apparatus - measure to few %
- Full apparatus to be installed after Long shutdown 3 (2026-2029) .
- 3 years running will achieve 0.3% stat. + 0.3% syst.
- theory work also needed

we need to have lattice HVP finalised before then

Mainz, 2203.08676