Heavy Black Hole Effective Theory

LepageFest October 14-15, 2024

Poul H. Damgaard
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Classical Gravity and Quantum Field Theory

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- Then there was Iwazaki (1971)

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Quantum Theory of Gravitation vs. Classical Theory*)

——*Fourth-Order Potential*——

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Research Institute for Fundamental Physics, Kyoto University, Kyoto

(Received May 18, 1971)

The perihelion-motion of Mercury depends on the fourth-order potential in quantum field theory; it is a "Lamb shift". In spite of the unrenormalizability of the theory, we have extracted a finite and physically meaningful quantity, a fourth-order potential, from fourth-order graphs. We have also discussed briefly renormalization of the Newtonian potential in the fourth-order perturbation.

Iwasaki got (almost) everything right

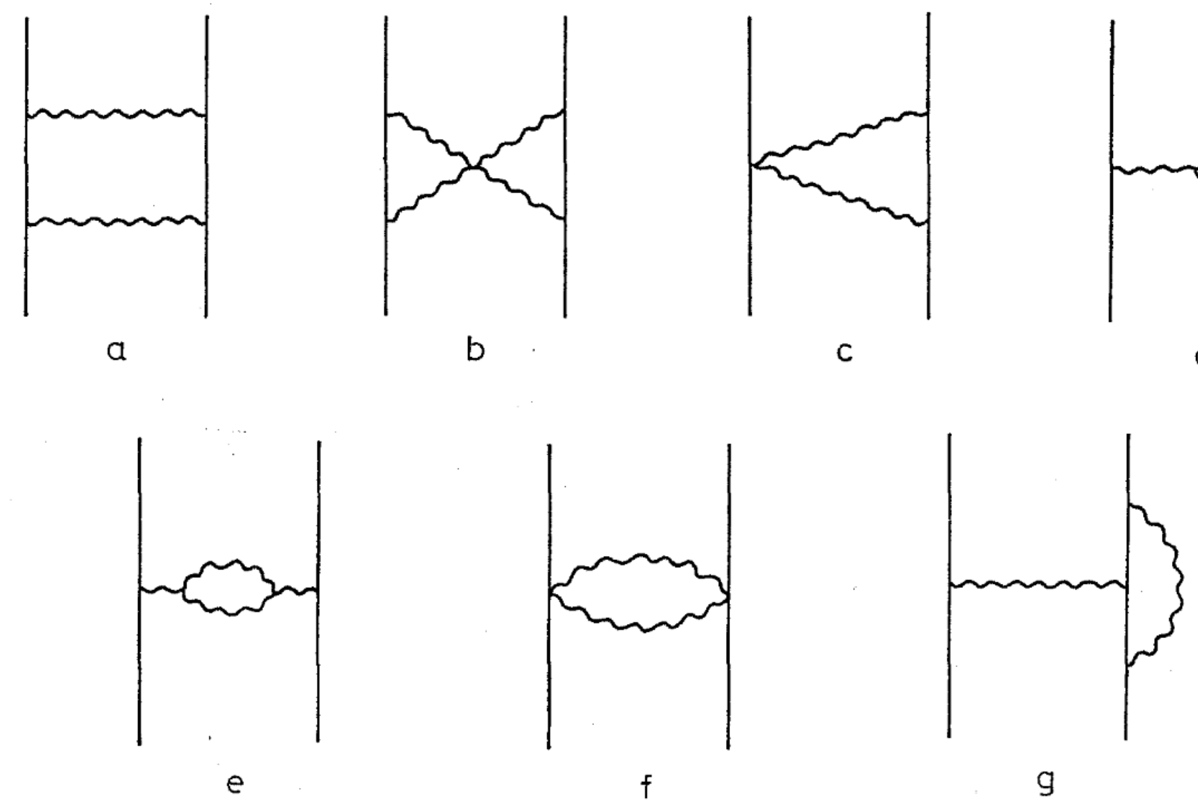

Fig. 1. Some of the fourth-order diagrams. The wavy line represents the graviton and the solid line matter.

EFFECTIVE LAGRANGIANS FOR BOUND STATE PROBLEMS IN QED, QCD, AND OTHER FIELD THEORIES

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and

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Received 13 August 1985

A renormalization group strategy for the study of bound states in field theory is developed Our analysis is completely different from conventional analyses, based upon the Bethe–Salpeter equation, and it is far simpler This is illustrated in state-of-the-art calculations for the ground state splittings in muonium and positronium

Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium

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(Received 10 August 1994)

A rigorous QCD analysis of the inclusive annihilation decay rates of heavy-quarkonium states is presented. The effective-field-theory framework of nonrelativistic QCD is used to separate the short-distance scale of annihilation, which is set by the heavy-quark mass M, from the longer-distance scales associated with quarkonium structure. The annihilation decay rates are expressed in terms of nonperturbative matrix elements of four-fermion operators in nonrelativistic QCD, with coefficients that can be computed using perturbation theory in the coupling constant $\alpha_s(M)$. The matrix elements are organized into a hierarchy according to their scaling with v, the typical velocity of the heavy quark. An analogous factorization formalism is developed for the production cross sections of heavy quarkonium in processes involving momentum transfers of order M or larger. The factorization formulas are applied to the annihilation decay rates and production cross sections of S-wave states at next-to-leading order in v^2 and P-wave states at leading order in v^2.

PACS number(s): 13.25.Gv, 12.38.Bx, 12.39.Hg, 13.40.Hq

$$
\begin{aligned}
L_{\mathrm{eff}} = &-\tfrac{1}{2}(E^2 - B^2) + \psi_{\mathrm{e}}^{\dagger}(\mathrm{i}\partial_t - e\phi + \mathbf{D}^2/2m)\psi_{\mathrm{e}} \\
&+ \psi_{\mathrm{e}}^{\dagger}\,[c_1\mathbf{D}^4/8m^3 + c_2(e/2m)\boldsymbol{\sigma}\cdot\boldsymbol{B} \\
&+ c_3(e/8m^2)\,\nabla\cdot\boldsymbol{E} + c_4(e/8m^2)\,\{\mathrm{i}\mathbf{D}\cdot\boldsymbol{E}\times\boldsymbol{\sigma}\}]\,\psi_{\mathrm{e}} \\
&+ \psi_{\mathrm{e}}^{\dagger}\,[d_1(e/8m^3)\,\{\mathbf{D}^2,\boldsymbol{\sigma}\cdot\boldsymbol{B}\}]\,\psi_{\mathrm{e}} \\
&- (d_2/m_{\mathrm{e}}m_{\mu})(\psi_{\mathrm{e}}^{\dagger}\boldsymbol{\sigma}\psi_{\mathrm{e}})\cdot(\psi_{\mu}^{\dagger}\boldsymbol{\sigma}\psi_{\mu}) + \ldots, \qquad (1)
\end{aligned}
$$

Applying the same ideas to gravity

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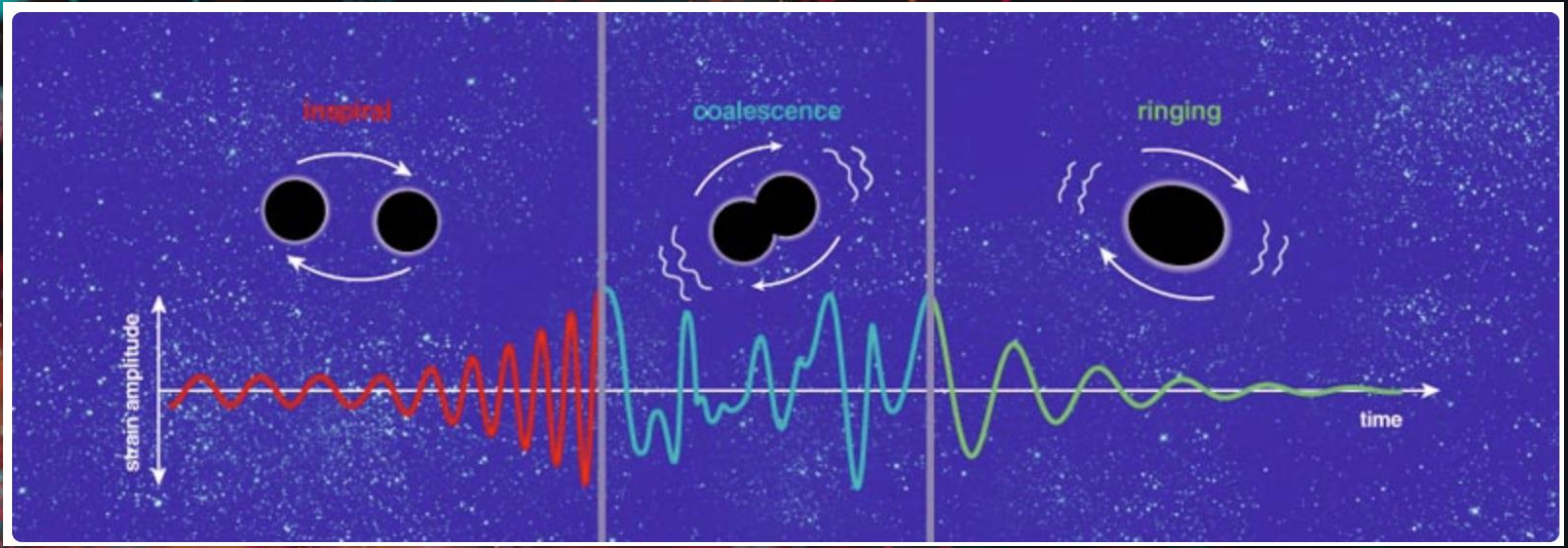

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$$\omega_\mu{}^{ab} = -\frac{\kappa}{4}\partial^b h_\mu{}^a - \frac{\kappa^2}{16}h^{\rho b}\partial_\mu h^a{}_\rho + \frac{\kappa^2}{8}h^{\rho b}\partial_\rho h_\mu{}^a - \frac{\kappa^2}{8}h^{\rho b}\partial^a h_\mu{}^\rho - (a \leftrightarrow b).$$

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$$\mathcal{L}^{s=1/2}_{\text{HBET}} = \sqrt{-g}\bar{Q}\left[ie^\mu{}_a\gamma^a D_\mu + mv_\mu v^a(e^\mu{}_a - \delta^\mu_a)\right]Q \tag{4.6a}$$
$$+ \frac{\sqrt{-g}}{2m}\bar{Q}\left[ie^\mu{}_a\gamma^a D_\mu + mv_\mu\gamma^a(e^\mu{}_a - \delta^\mu_a)\right]\sum_{n=0}^{\infty}G_n[h]\frac{F[h]^n}{m^n}\left[ie^\rho{}_c\gamma^c D_\rho + mv_\rho\gamma^c(e^\rho{}_c - \delta^\rho_c)\right]Q,$$

Classical Physics from the S-matrix

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$$\hat{S} \;=\; \exp\left[\frac{i}{\hbar}\hat{N}\right]$$

Classical Physics from the S-matrix

$$\hat{T} = G_N \hat{T}_0 + G_N^{3/2} \hat{T}_0^{\text{rad}} + G_N^2 \hat{T}_1 + G_N^{5/2} \hat{T}_1^{\text{rad}} + G_N^3 \hat{T}_2 + \ldots$$

$$\hat{N} = G_N \hat{N}_0 + G_N^{3/2} \hat{N}_0^{\text{rad}} + G_N^2 \hat{N}_1 + G_N^{5/2} \hat{N}_1^{\text{rad}} + G_N^3 \hat{N}_2 + \ldots$$

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$$\begin{aligned}
\hat{N}_0 &= \hat{T}_0 \\
\hat{N}_0^{\text{rad}} &= \hat{T}_0^{\text{rad}} \\
\hat{N}_1 &= \hat{T}_1 - \frac{i}{2\hbar} \hat{T}_0^2 \\
\hat{N}_1^{\text{rad}} &= \hat{T}_1^{\text{rad}} - \frac{i}{2\hbar} (\hat{T}_0 \hat{T}_0^{\text{rad}} + \hat{T}_0^{\text{rad}} \hat{T}_0) \\
\hat{N}_2 &= \hat{T}_2 - \frac{i}{2\hbar} (\hat{T}_0^{\text{rad}})^2 - \frac{i}{2\hbar} (\hat{T}_0 \hat{T}_1 + \hat{T}_1 \hat{T}_0) - \frac{1}{3\hbar^2} \hat{T}_0^3
\end{aligned}$$

Classical Physics from the S-matrix

$$\Delta \tilde{P}_1{}^{\nu}(\gamma, b)|_{\text{cons}} = p_\infty \frac{b^\nu}{|b|} \sin\left(\frac{\partial \tilde{N}(\gamma, J)}{\partial J}\right) + p_\infty^2 L^\nu \left(\cos\left(- \frac{\partial \tilde{N}(\gamma, J)}{\partial J}\right) - 1\right).$$

$$\chi = -\frac{\partial \tilde{N}(\gamma, J)}{\partial J} = -\frac{1}{p_\infty}\frac{\partial \tilde{N}(\gamma, b)}{\partial b},$$

Looking forward

Looking forward

The Bound-State Problem

Reduction of the Bethe-Salpeter equation to an equivalent Schrödinger equation, with applications

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(Received 10 February 1978)

We propose a new relativistic two-body formalism which reduces to a nonrelativistic Schrödinger theory for a single effective particle. The formalism is equal in rigor to that of Bethe and Salpeter, and considerably simpler to apply. We illustrate its use by computing $O(\alpha^6)$ terms in the ground-state splitting of muonium and positronium involving infinite Coulomb exchange.