TECHNISCHE UNIVERSITÄT MÜNCHEN

Nonrelativistic Multiscale Systems with Effective Field Theories

Advanced Grant

EFT-XYZ

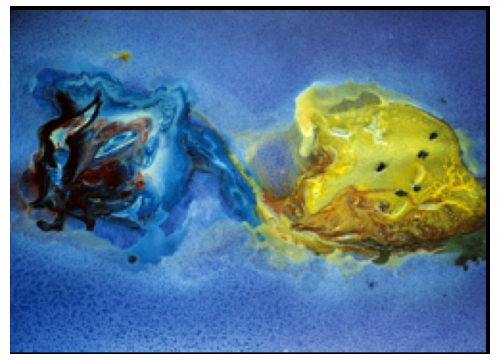
Nora Brambilla

T30f

TUMQCD LATTICE Collaboration

STRONG 2020

TUM-IAS Focus Group Physics with Effective Field Theories



Quark Confinement and the Hadron Spectrum since 1994

MCQST

MDSI

Munich Data Science Institute

ORIGINS Excellence Cluster

QWG

a

homage

to

Peter

Lepage

The future of heavy quarks is the future of physics

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Nonrelativistic (NR) bound states lie at the core of quantum physics spanning particle to nuclear physics, and condensed matter to astrophysics

They are at the origin of several past and contemporary revolutions.

They are multiscale systems, which is an opportunity for the physics but a challenge for a QFT description

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Focus of the talk

We introduce a nonrelativistic effective field theory (pNREFT) description that reinventes QM and allows precise calculations of bound state observables -

This framework is particularly suited to address strongly interacting systems

allows to use NR bound states to address contemporary challenges like:

- the exotics XYZ states and the nature of the strong force
- the in medium heavy pairs evolution with impact e.g. on the nuclear phase diagram (and dark matter properties, neutrino flavour oscillations...)

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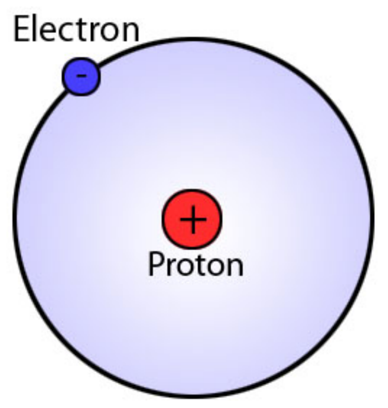
Novel tools to bridge perturbative methods with **lattice QCD** are key to this program, as well as the **combination** between **different EFTs**

Past Revolutions

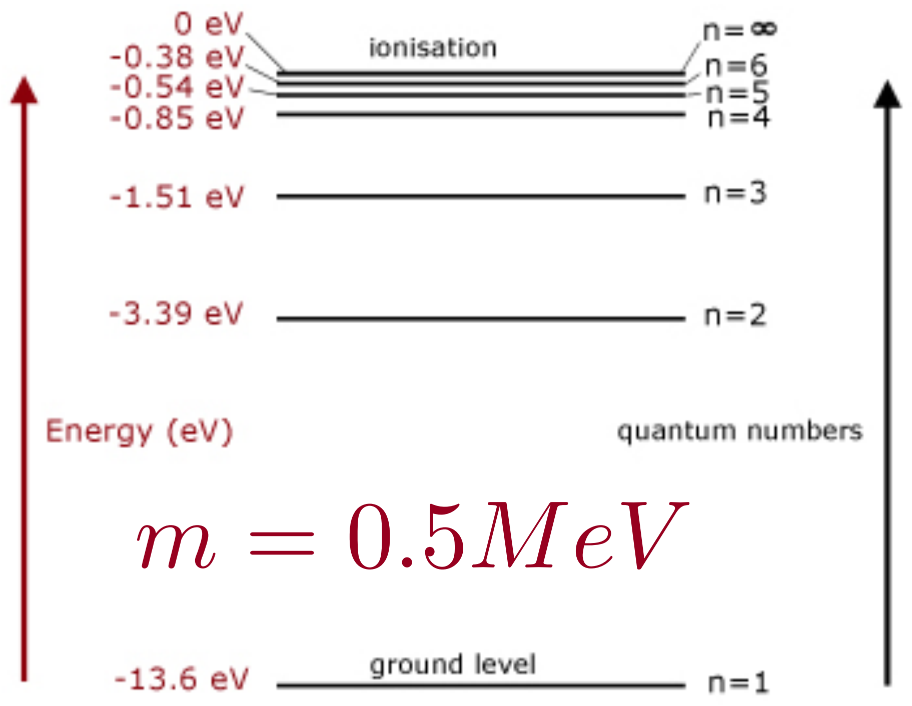
NR bound states accompanied the history of quantum theory from its beginning to the establishment of QFT

The prototype of NR system is the hydrogen atom and it is at the origin of the quantum revolution



$$v = \alpha = \frac{e^2}{4\pi}$$

fine structure constant



$$m = 0.5MeV$$

The relevant scales of the non-relativistic bound state dynamics are

- $E \sim \frac{\mathbf{p}^2}{2m} \sim V \sim mv^2,$
- $p \sim 1/r \sim mv;$

a crucial observation: if v(elocity)$\ll 1$, then $m \gg mv \gg mv^2$.

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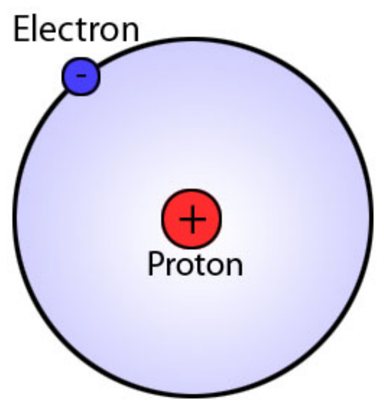

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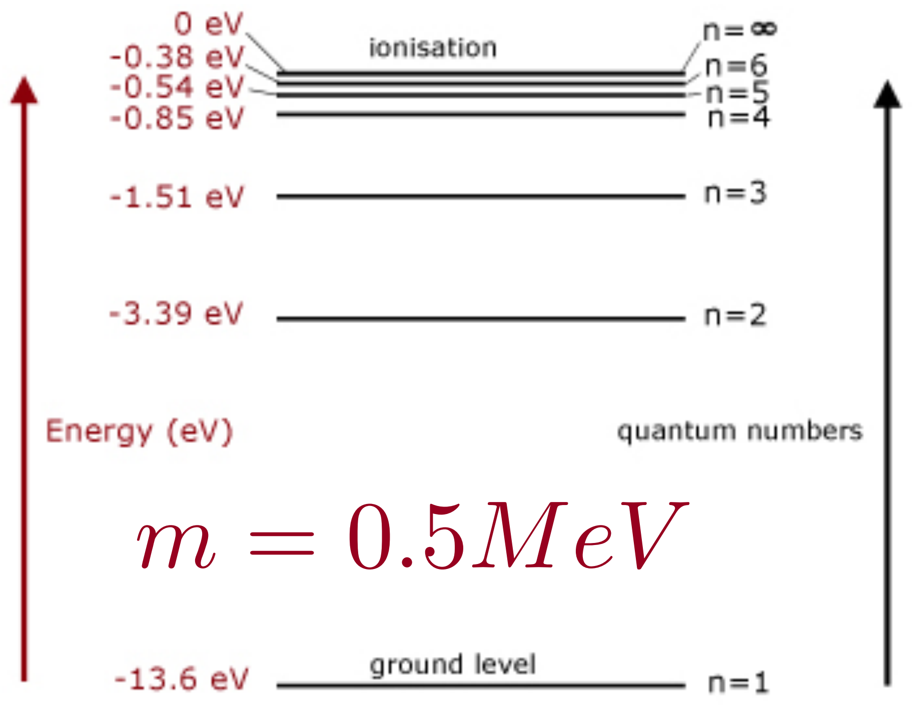

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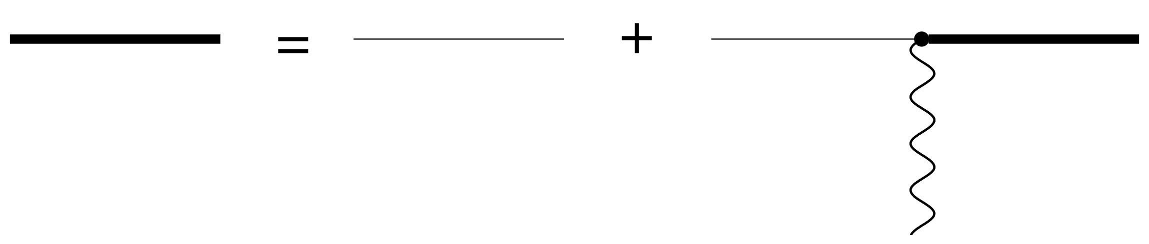
To explain the energy levels of hydrogen, from QM…

- 1926 Schrödinger equation: $\left(\frac{\mathbf{p}^2}{2m} + V\right)\phi = E\phi$

- 1928 Dirac equation: $(i\not{D} - m)\,\psi = 0$

$$\begin{cases} g^D = g_0^D + g_0^D(-ie\not{A})g^D \\ g_0^D = \frac{i}{\not{p} - m} \end{cases}$$



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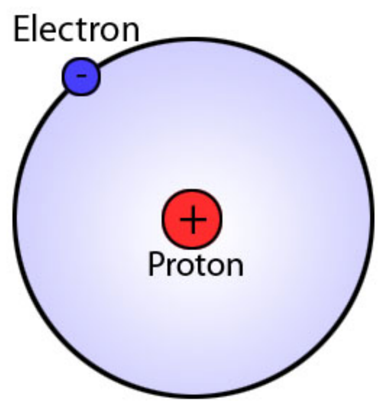

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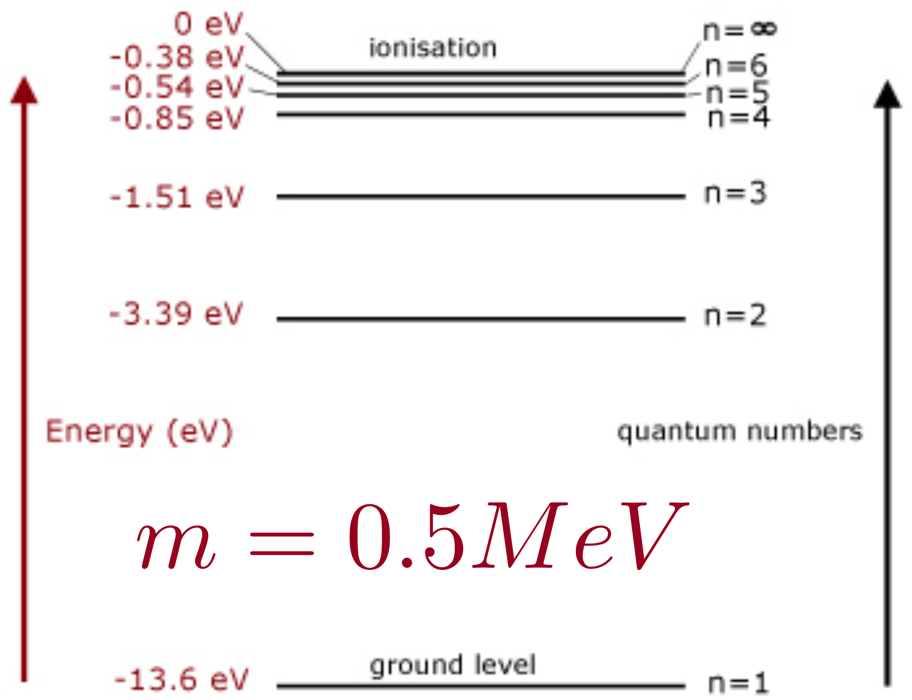

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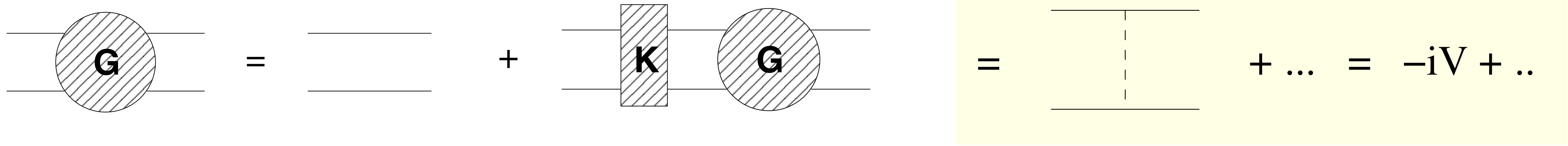
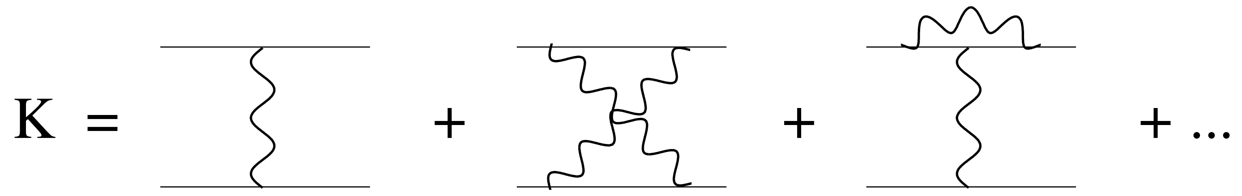
….to the relativistic quantum theory of bound states

- 1951 Bethe–Salpeter equation:

$$\begin{cases} G = G_0 + G_0 K G \\ G_0 = g_0^D \otimes g_0^D \end{cases}$$



All the complexity of the field theory is in the kernel



which only in the non-relativistic limit reduces to the Coulomb potential, but, in general, keeps entangled all bound-state scales.

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SLAC-PUB-1848
November 1976
(T/E)

$\mathcal{O}(\alpha)$ CORRECTIONS TO THE DECAY RATE OF ORTHOPOSITRONIUM*

William E. Caswell, G. Peter Lepage, and Jonathan Sapirstein

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

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- **cumbersome in perturbation theory :**

It shows the difficulty of the approach the fact that going from the calculation of the $m\alpha^5$ correction in the hyperfine splitting of the positronium ground state to the $m\alpha^6 \ln\alpha$ term took twenty-five years!

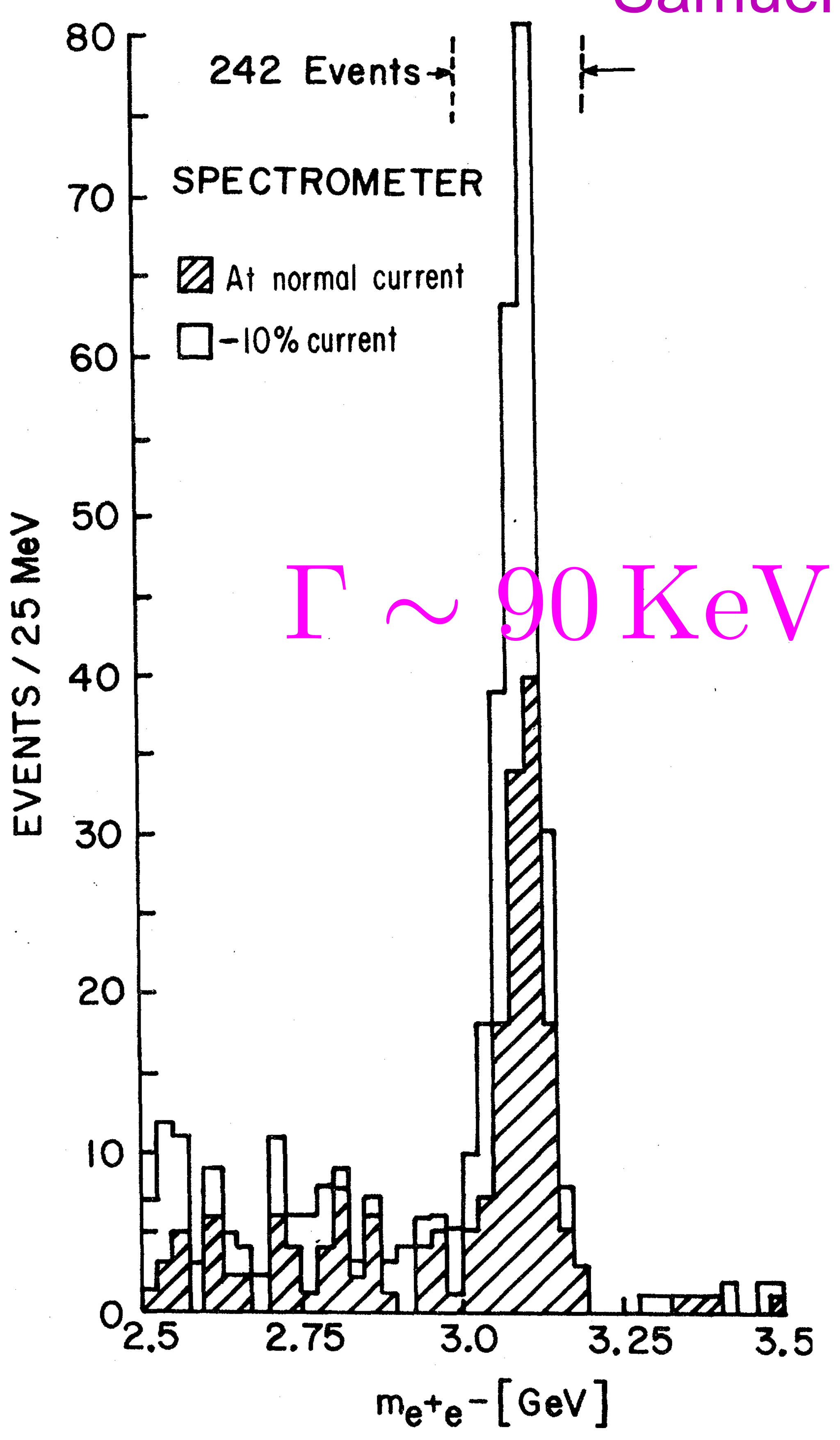
- Karplus Klein PR 87(52)848, Caswell Lepage PRA (20)(79)
  Bodwin Yennie PR 43(78)267

- **poorly suited to achieve factorization (important in QCD)**

Past Revolutions

The discovery of the J/psi (ccbar lowest state) is at the origin of the November revolution in 1974

Samuel Ting: It was like to stumble in a village where people were living 70000 years



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Aubert et al. BNL 74

- Discovery of the first quark of heavy type Q (m_c > Lambda_QCD)
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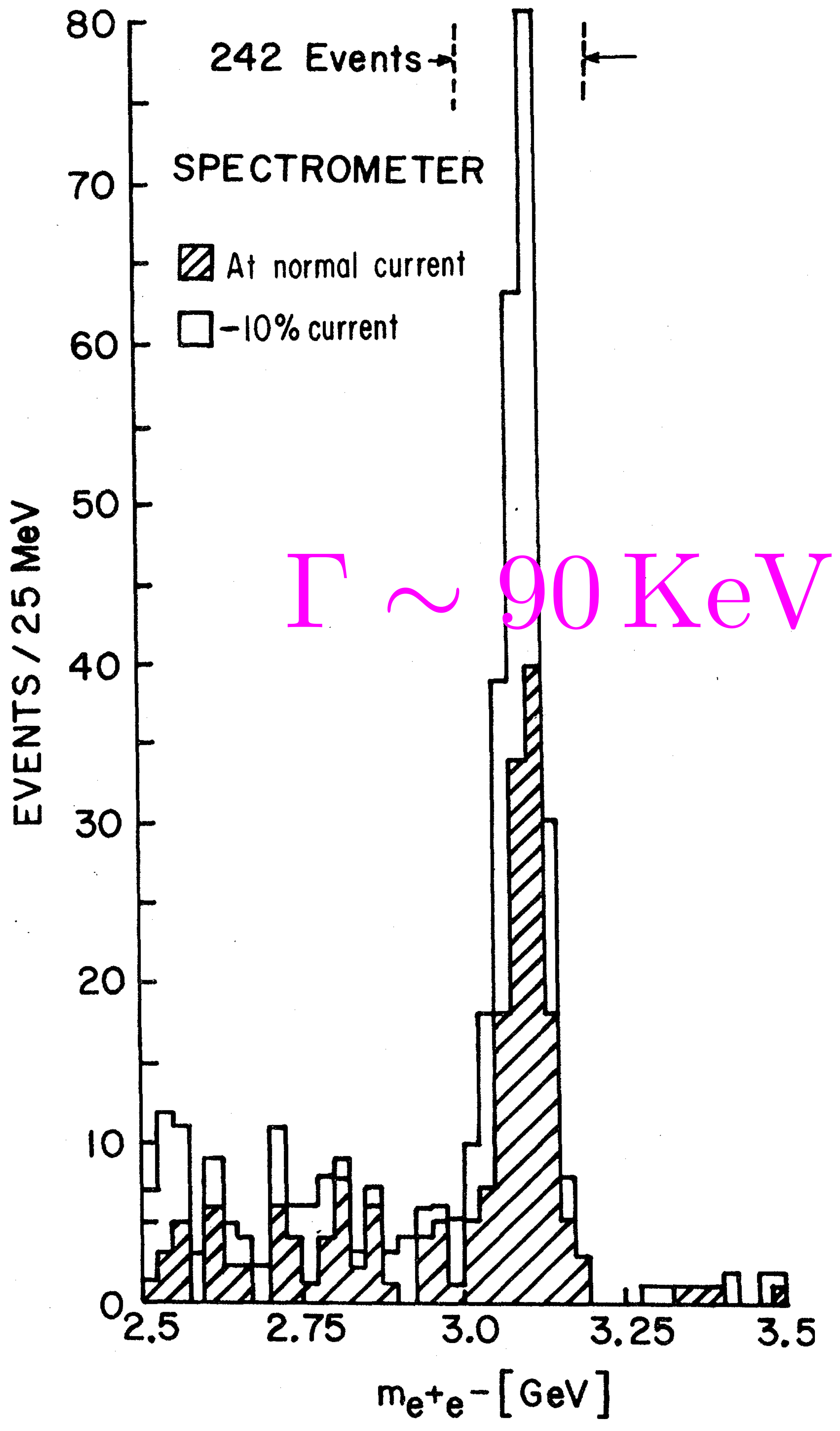

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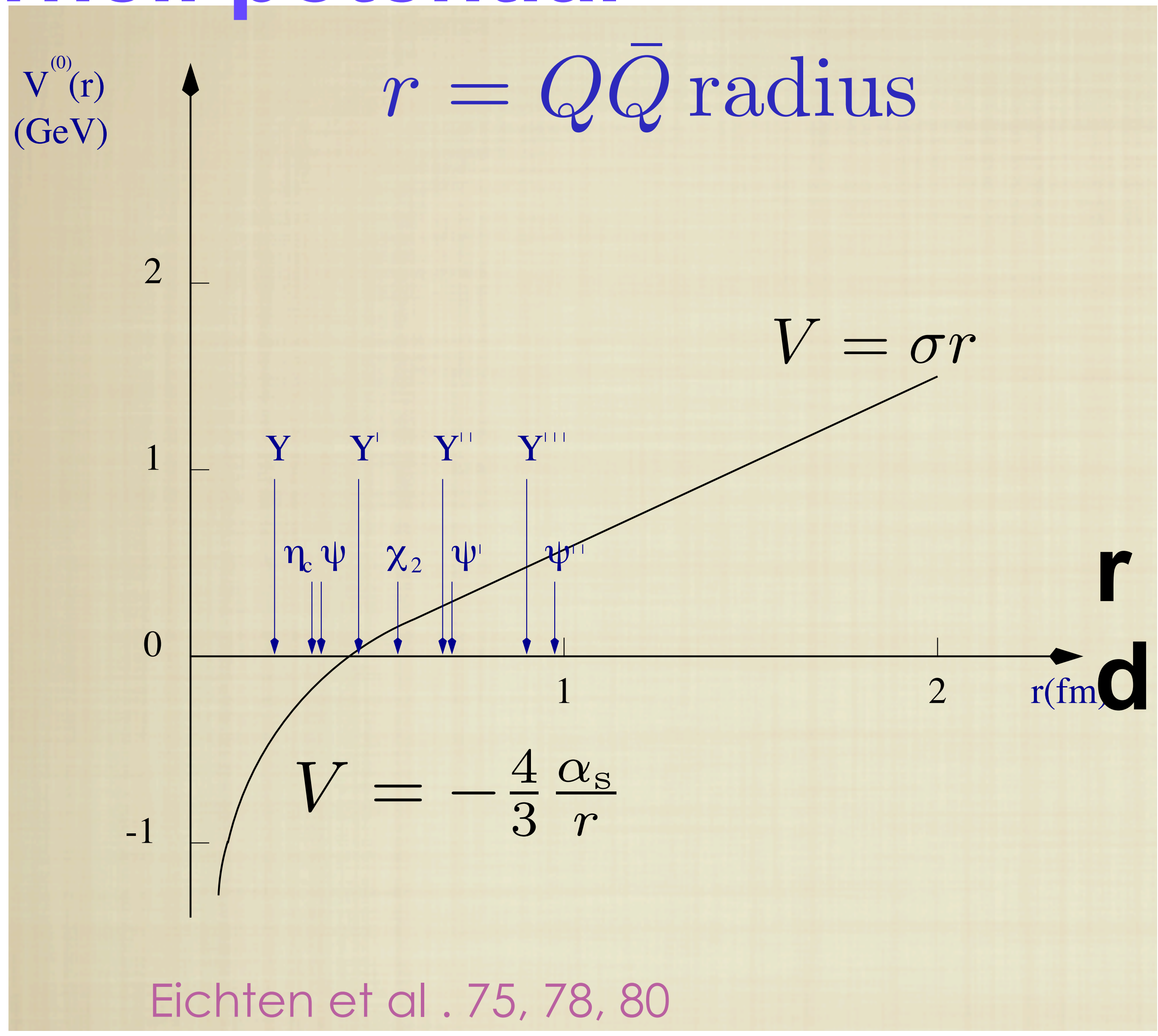
—>**narrow width and asymptotic freedom:** annihilation at large scale controlled by a small coupling constant $\alpha_s(2m_c) \ll 1$

—>**energy levels and confinement:** structure of levels controlled by a Cornell potential in a Schroedinger eq.

Cornell potential



r QQbar distance

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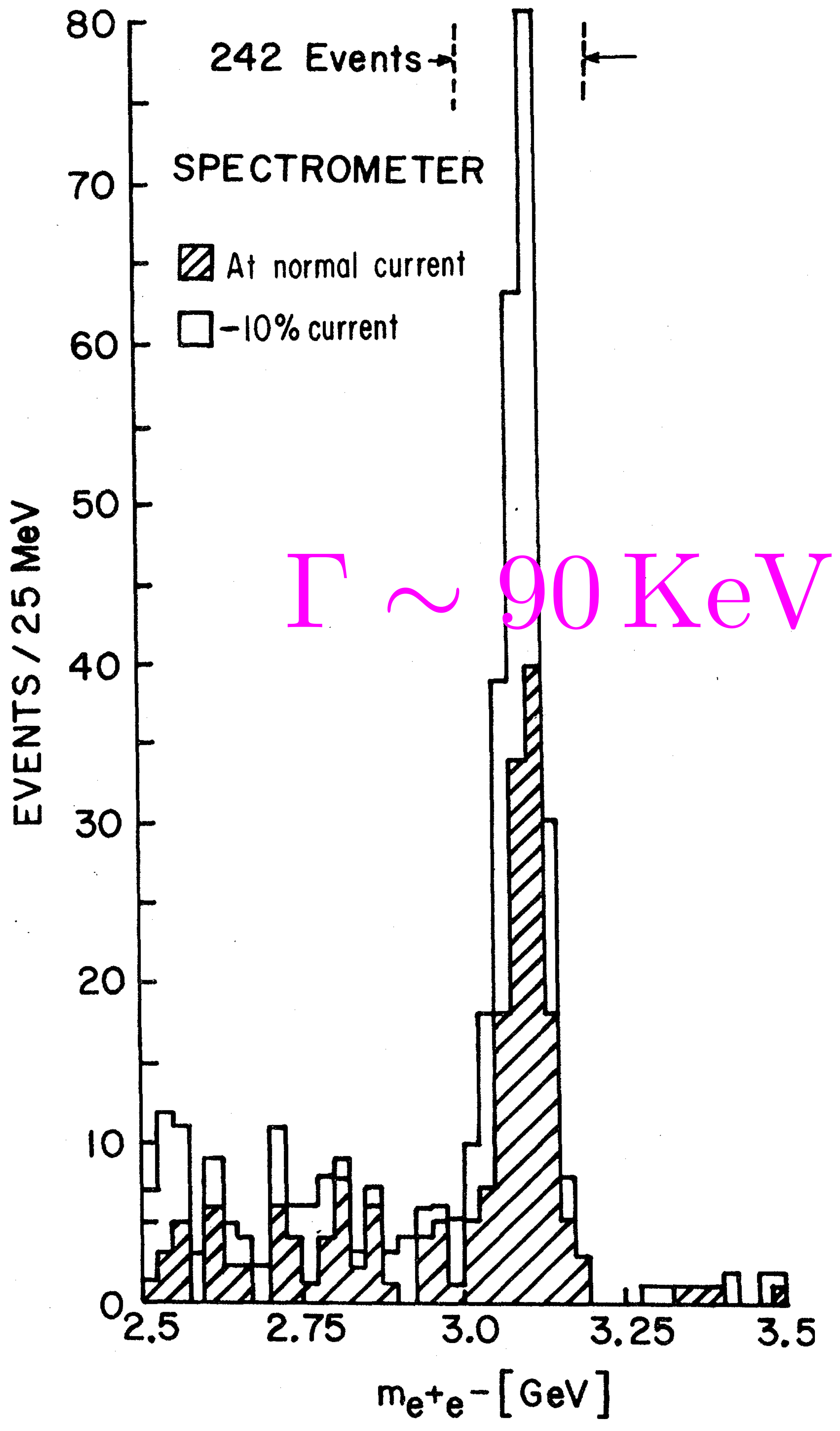

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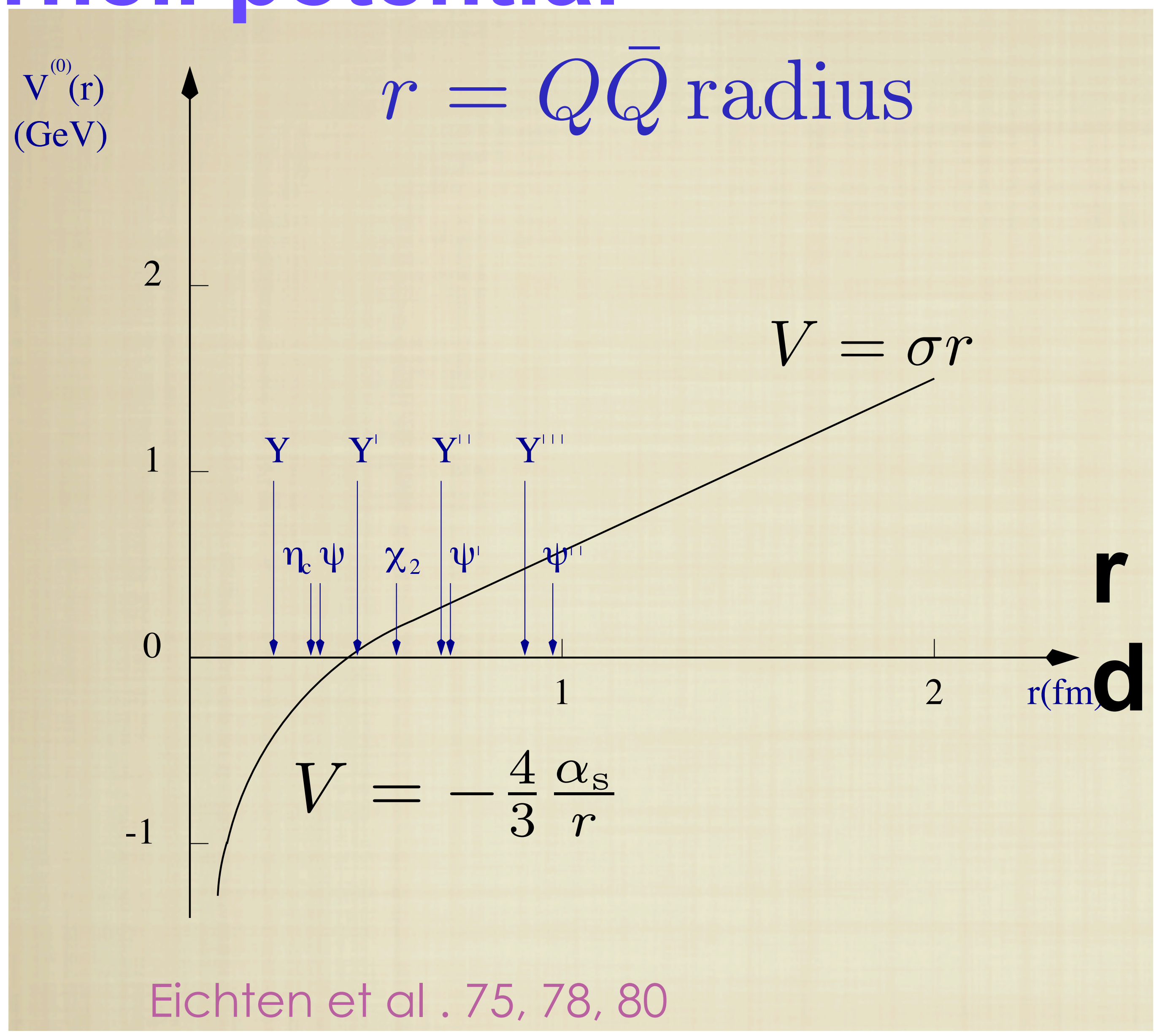
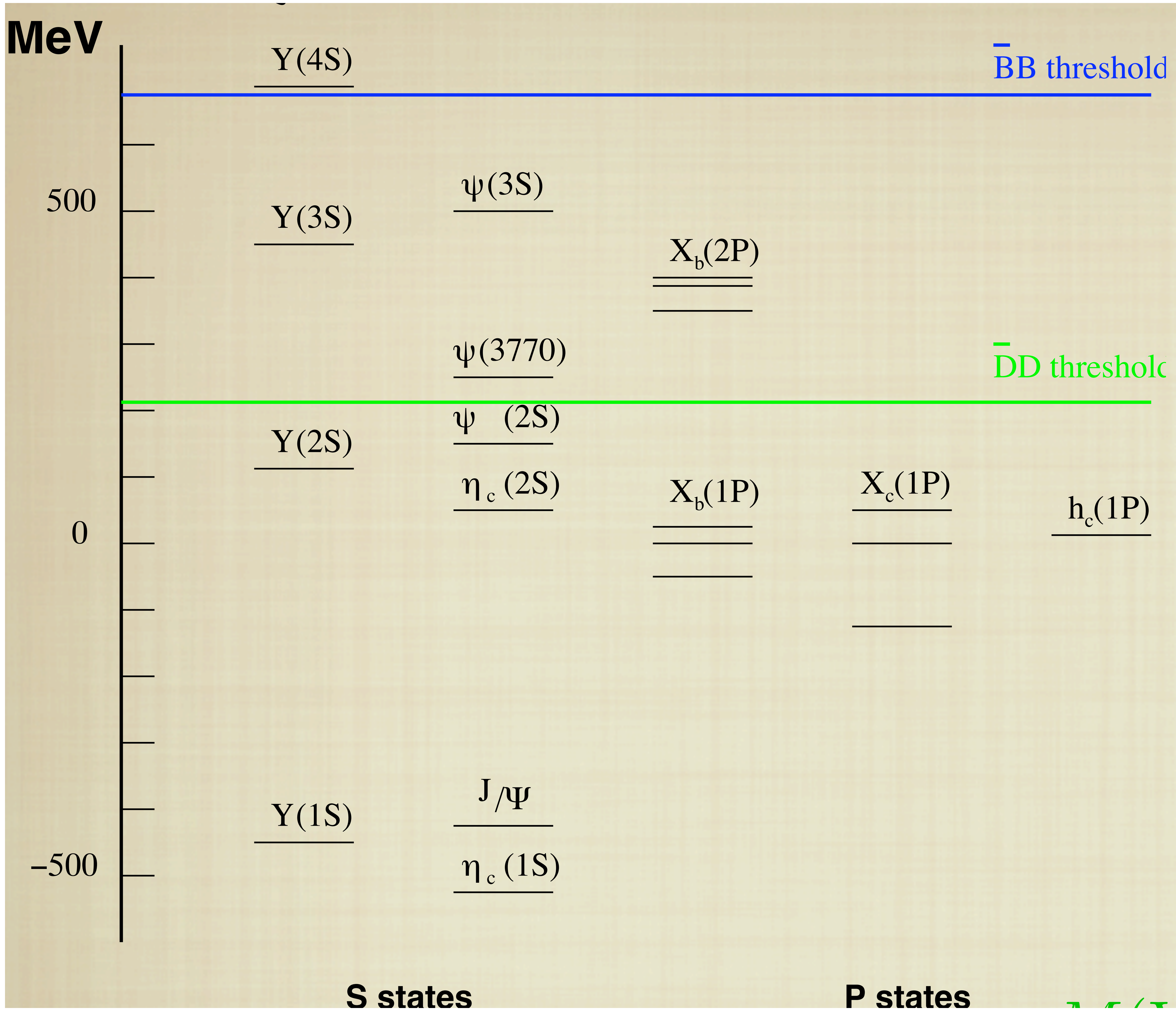


r QQbar distance

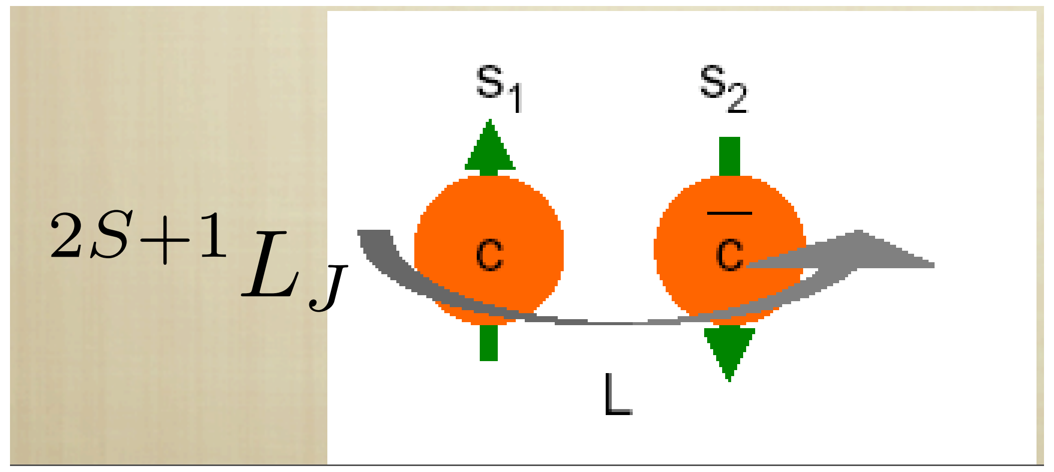
Heavy quarkonia are nonrelativistic systems:multiscale systems

Many scales: a challenge and a GREAT opportunity for the PHYSICS

Quarkonium scales



Levels normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$



$$M(\Upsilon(1S)) = 9.460\,\mathrm{GeV}$$
$$M(J/\psi) = 3.097\,\mathrm{GeV}$$

The mass scale is perturbative

$$m_Q \gg \Lambda_{\mathrm{QCD}}$$

$$m_b \simeq 5\,\mathrm{GeV};\, m_c \simeq 1.5\,\mathrm{GeV}$$

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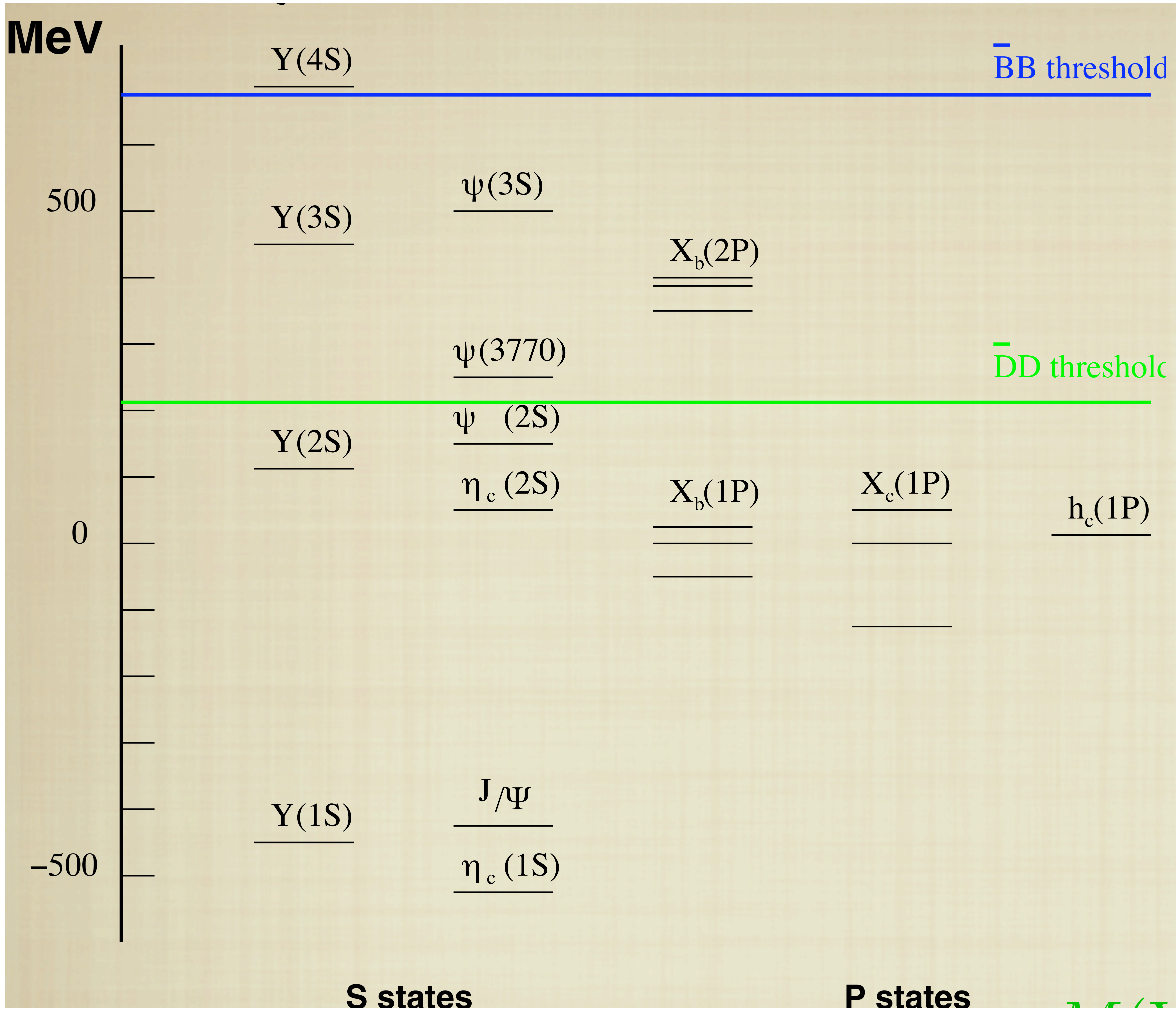

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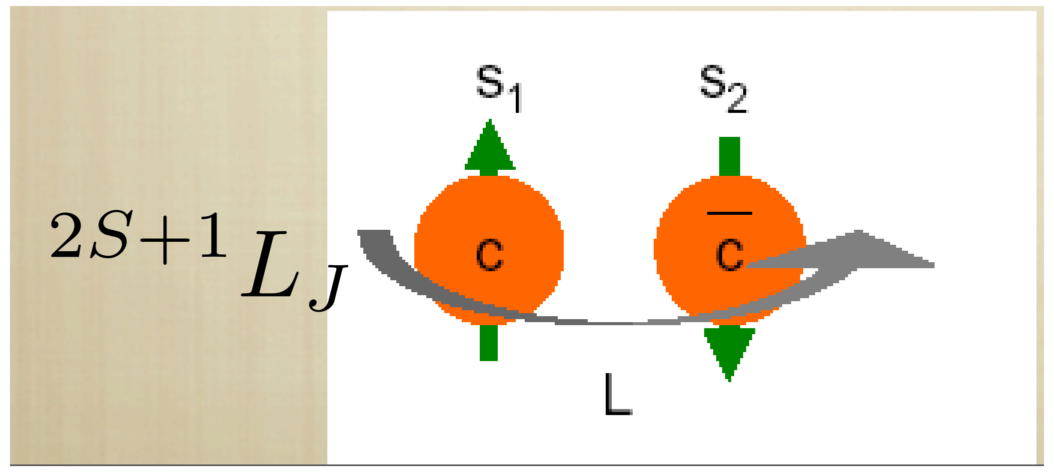

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$$\Delta E \sim mv^2, \Delta_{fs}E \sim mv^4$$

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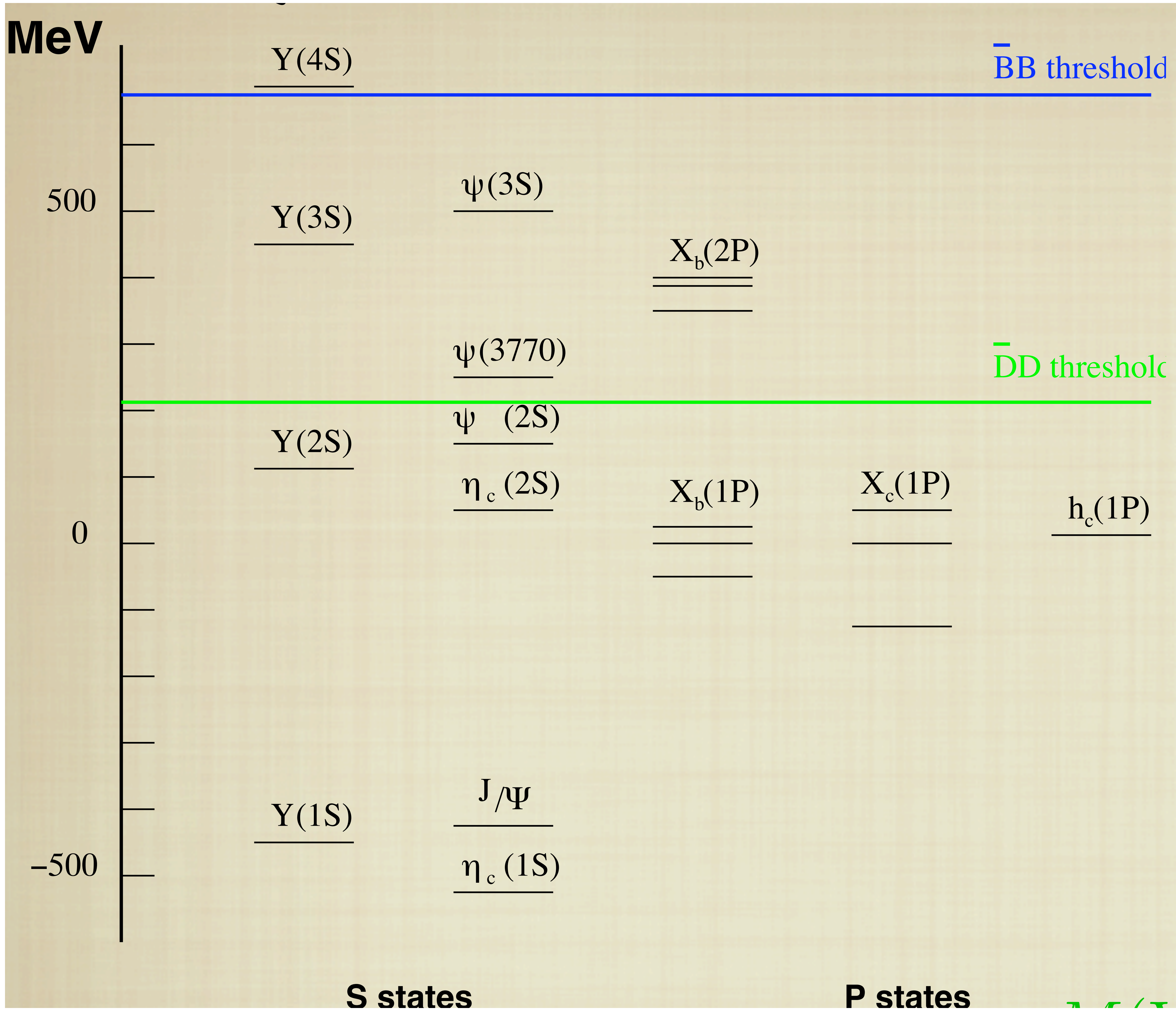

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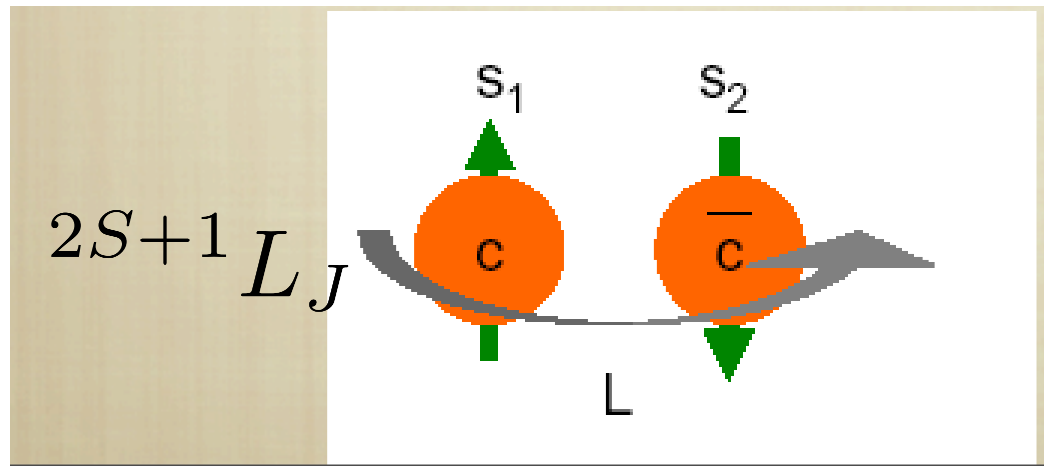

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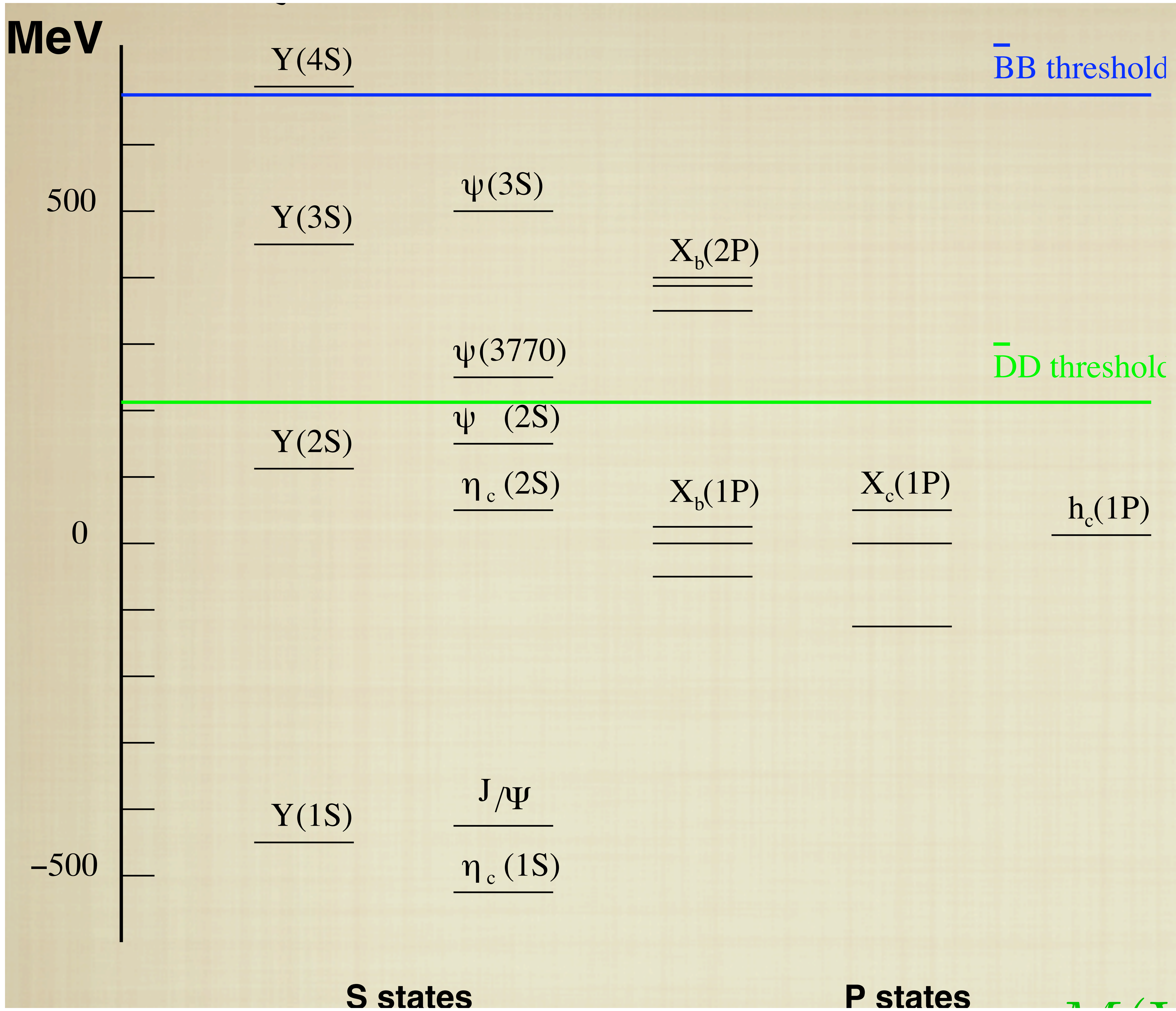

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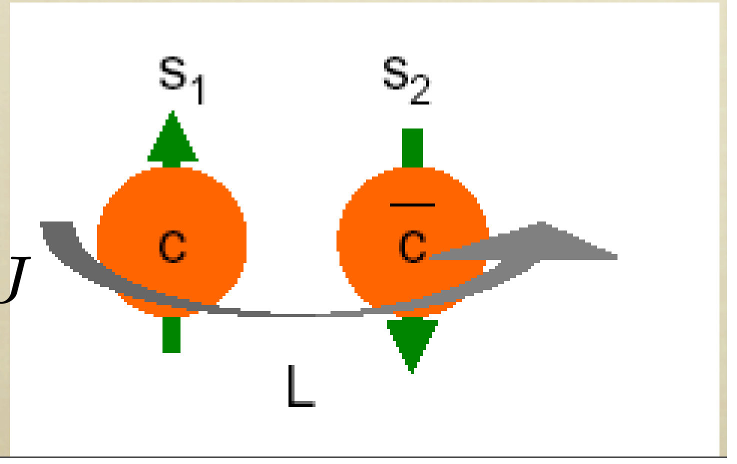
$^{2S+1}L_J$



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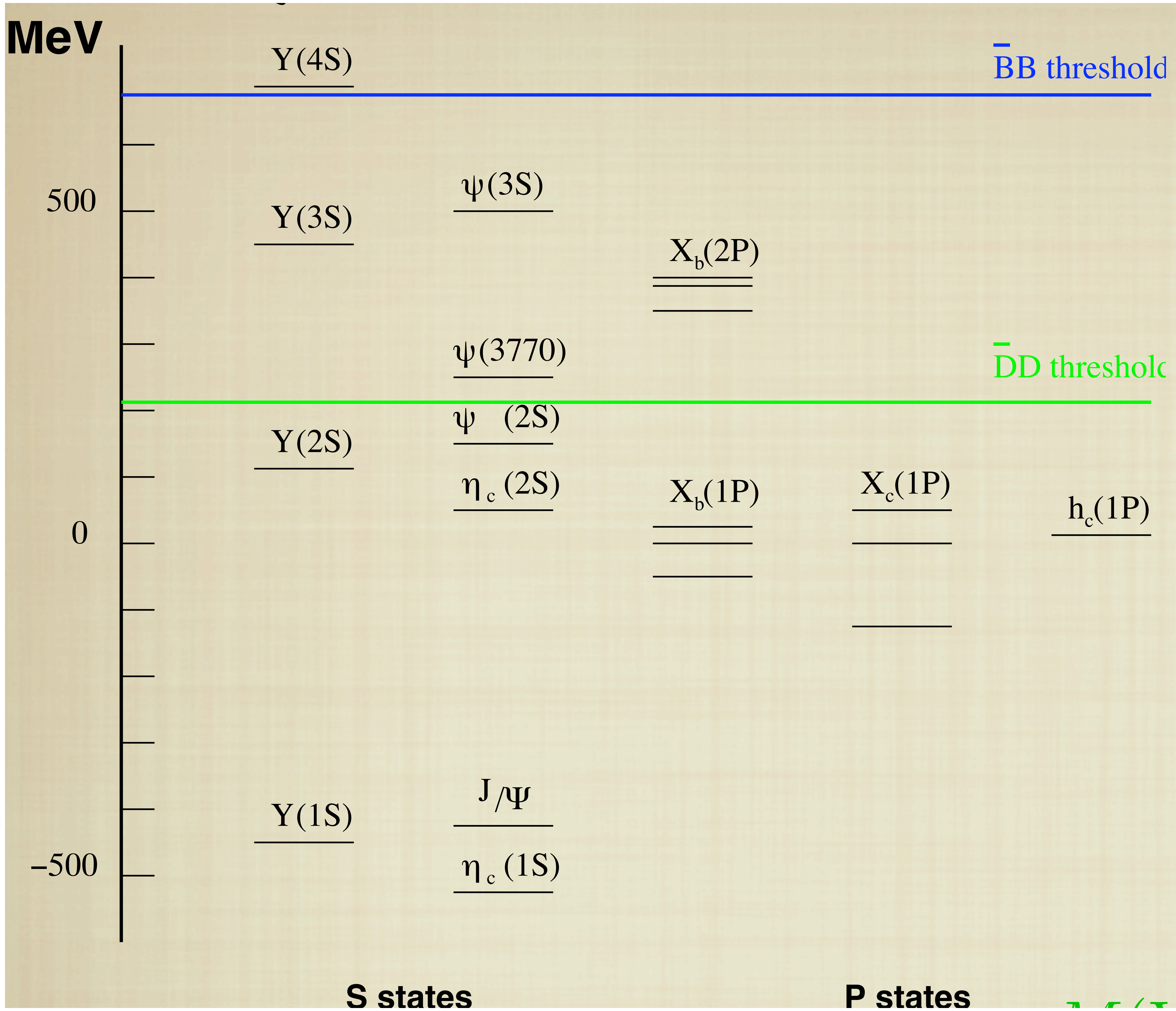

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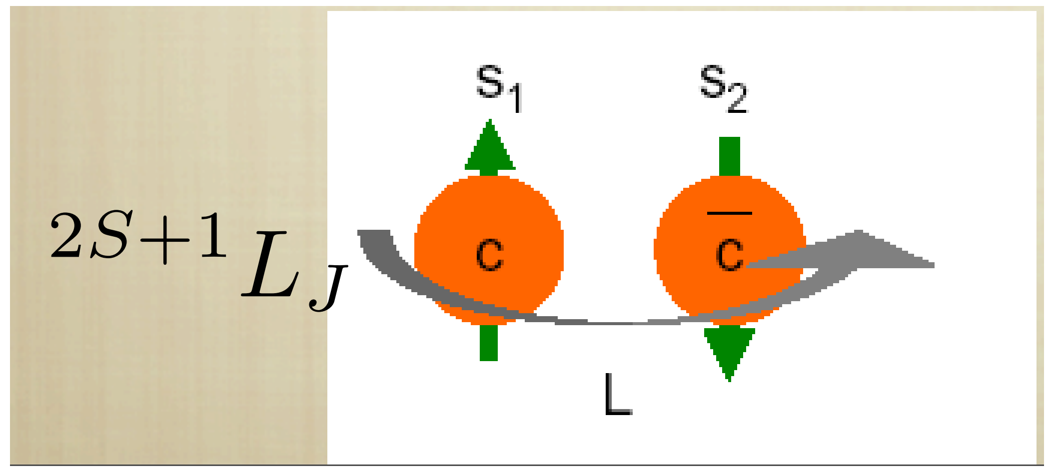

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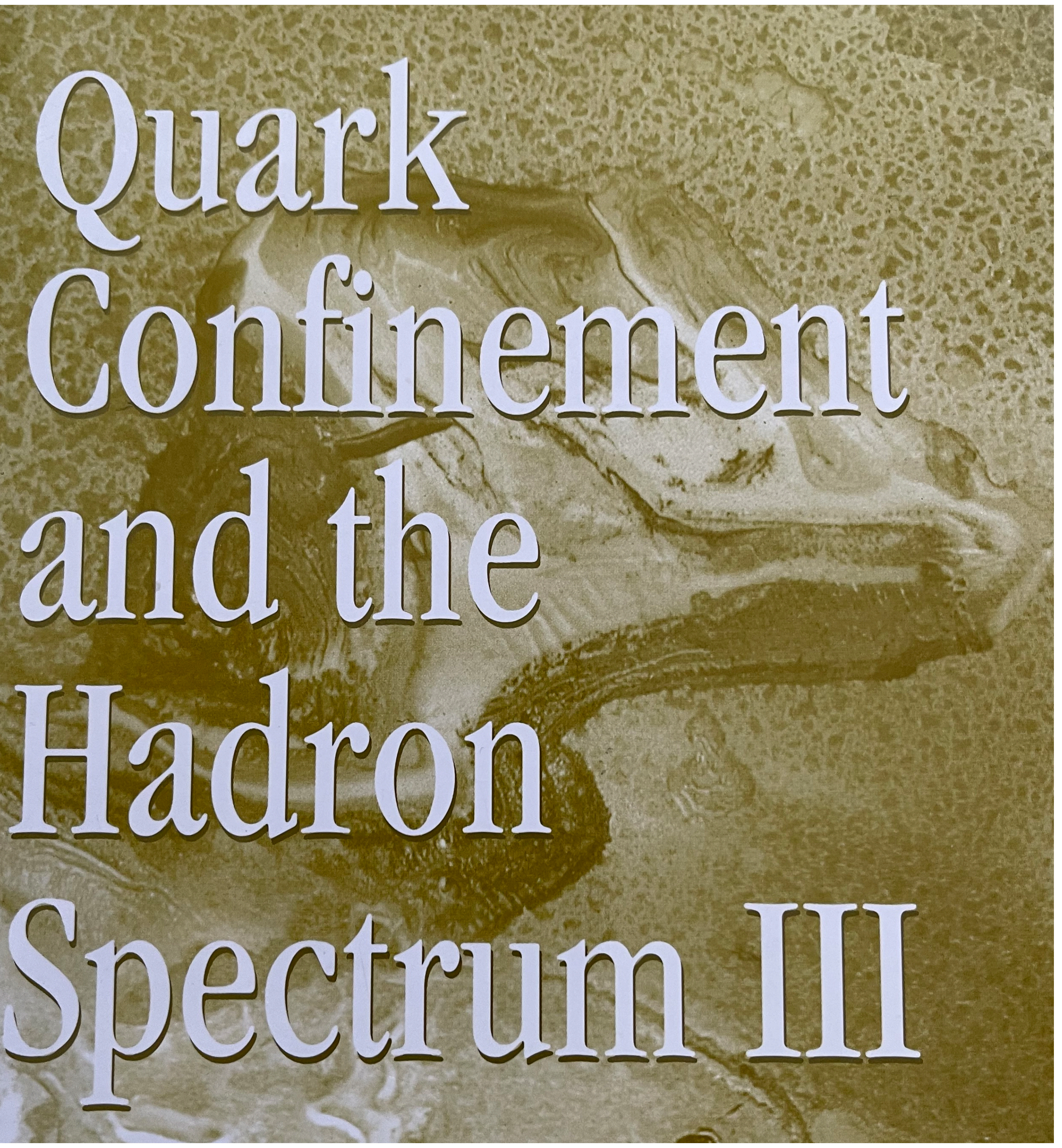

JLAB 1998

giving a talk on

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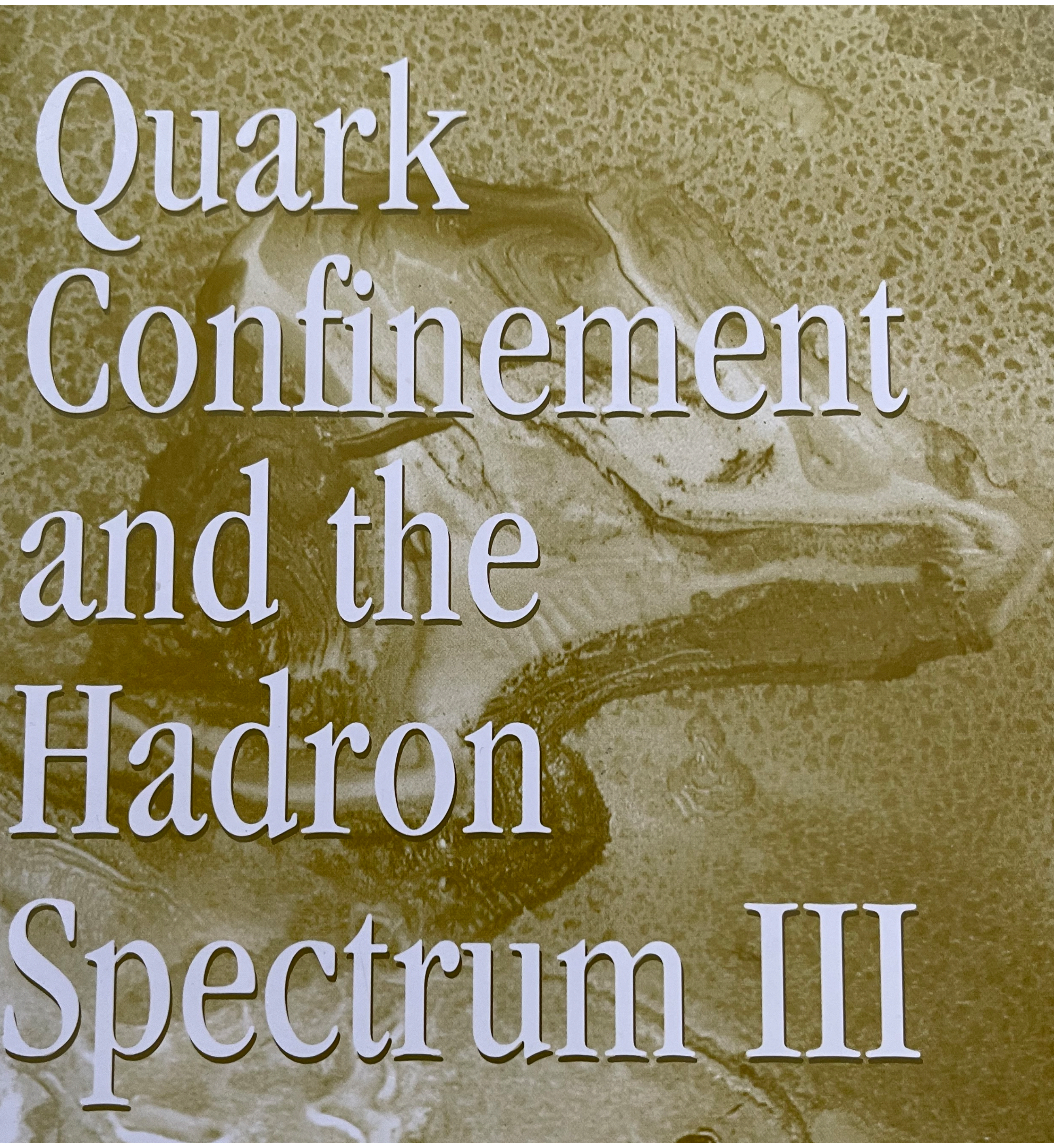

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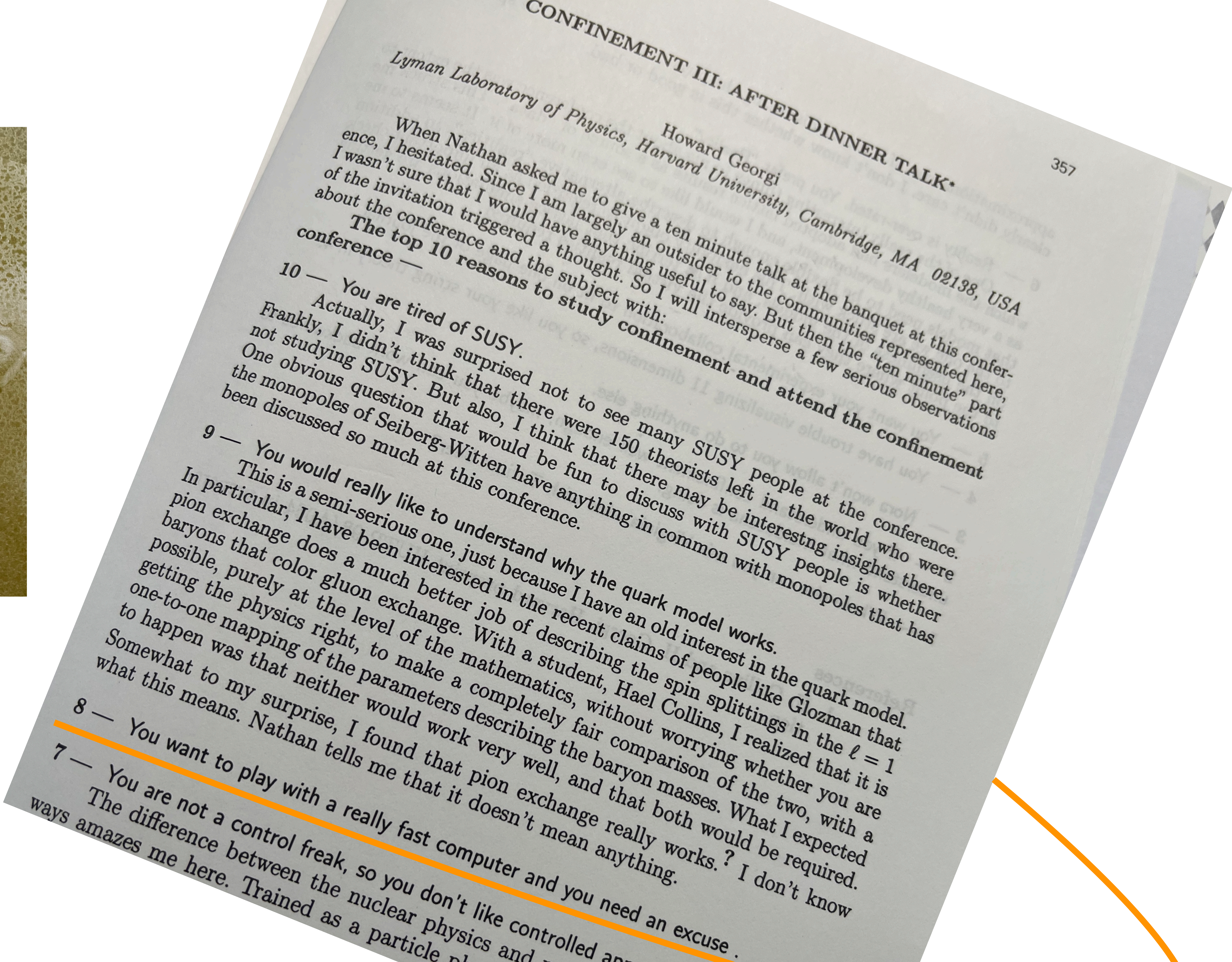
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CONFINEMENT III: AFTER DINNER TALK*

Howard Georgi

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

When Nathan asked me to give a ten minute talk at the banquet at this conference, I hesitated. Since I am largely an outsider to the communities represented here, I wasn't sure that I would have anything useful to say. But then the "ten minute" part of the invitation triggered a thought. So I will intersperse a few serious observations about the conference and the subject with:

The top 10 reasons to study confinement and attend the confinement conference —

10 — You are tired of SUSY.

Actually, I was surprised not to see many SUSY people at the conference. Frankly, I didn't think that there were 150 theorists left in the world who were not studying SUSY. But also, I think that there may be interestng insights there. One obvious question that would be fun to discuss with SUSY people is whether the monopoles of Seiberg-Witten have anything in common with monopoles that has been discussed so much at this conference.

9 — **You would really like to understand why the quark model works.**

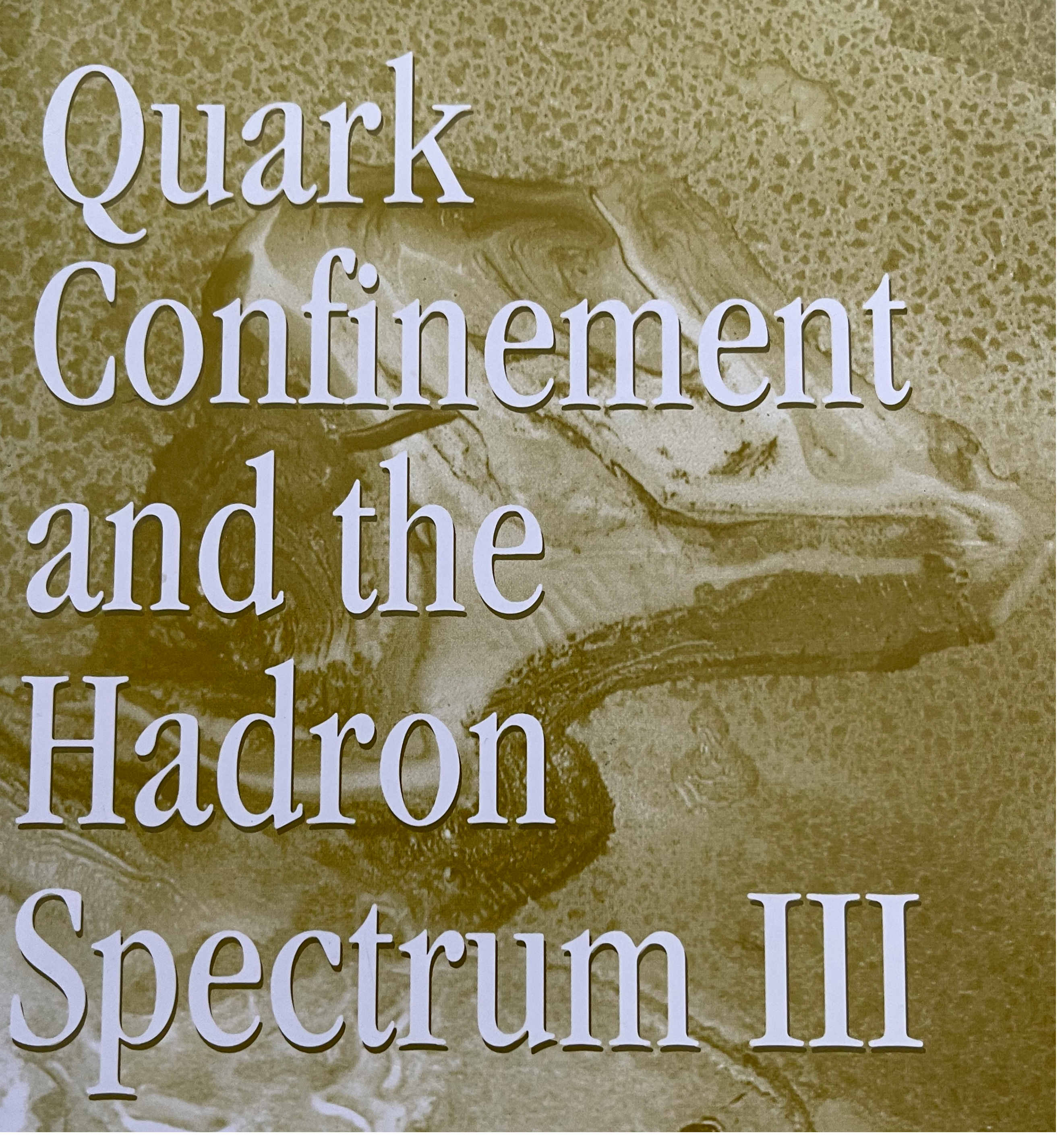
This is a semi-serious one, just because I have an old interest in the quark model. In particular, I have been interested in the recent claims of people like Glozman that pion exchange does a much better job of describing the spin splittings in the $\ell = 1$ baryons that color gluon exchange. With a student, Hael Collins, I realized that it is possible, purely at the level of the mathematics, without worrying whether you are getting the physics right, to make a completely fair comparison of the two, with a one-to-one mapping of the parameters describing the baryon masses. What I expected to happen was that neither would work very well, and that both would be required. Somewhat to my surprise, I found that pion exchange really works. ? I don't know what this means. Nathan tells me that it doesn't mean anything.

8 — You want to play with a really fast computer and you need an excuse .

7 — You are not a control freak, so you don't like controlled an

The difference between the nuclear physics and

ways amazes me here. Trained as a particle p



JLAB

1998

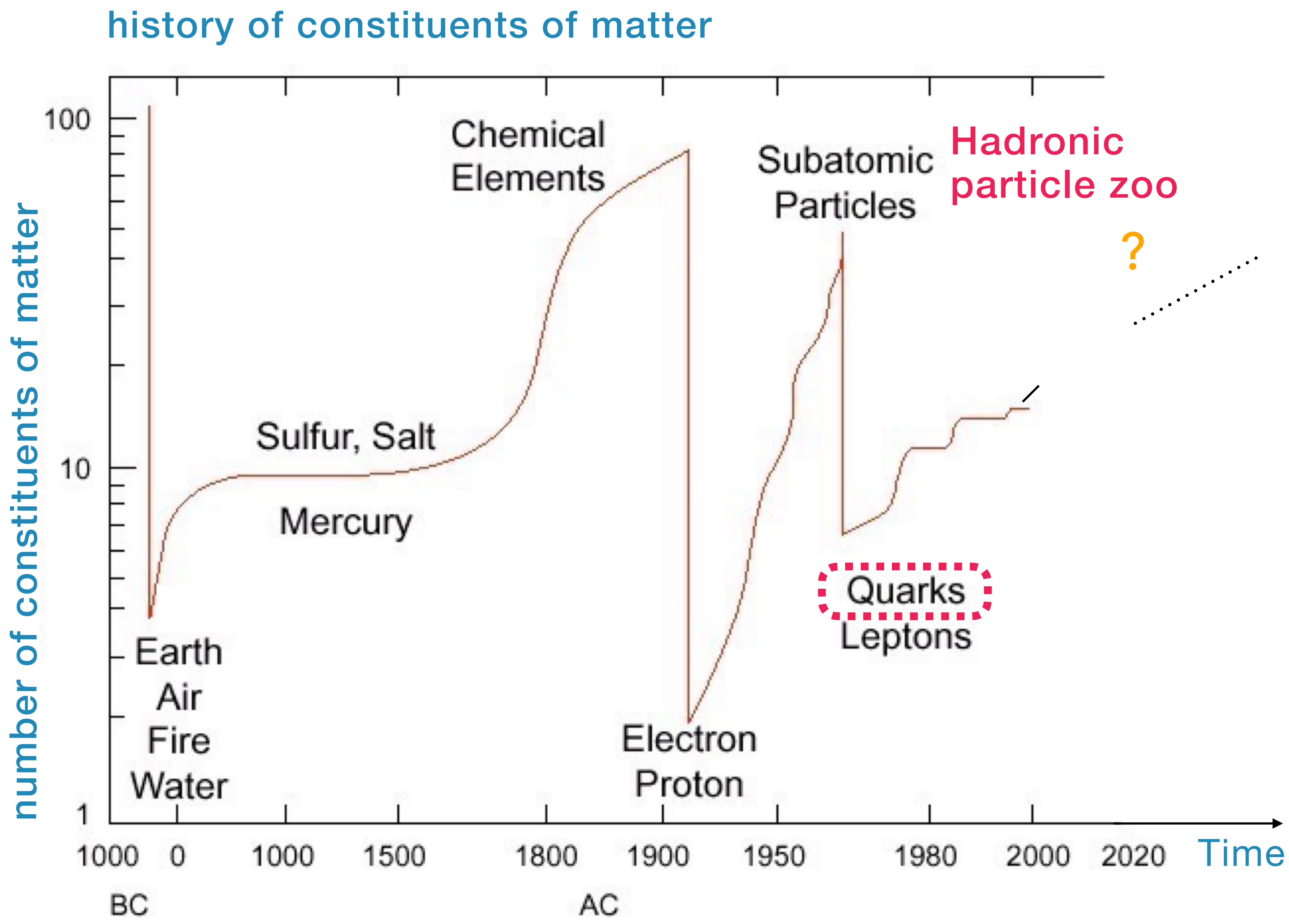
Talk of Peter

Lattice QCD on Small Computers

We demonstrate that lattice QCD calculations can be made 10^3–10^6 times faster by using very coarse lattices. To obtain accurate results, we replace the standard lattice actions by perturbatively-improved actions with tadpole-improved correction terms that remove the leading errors due to the lattice. To illustrate the power of this approach, we calculate the static-quark potential, and the charmonium spectrum and wavefunctions using a desktop computer. We obtain accurate results that are independent of the lattice spacing and agree well with experiment.

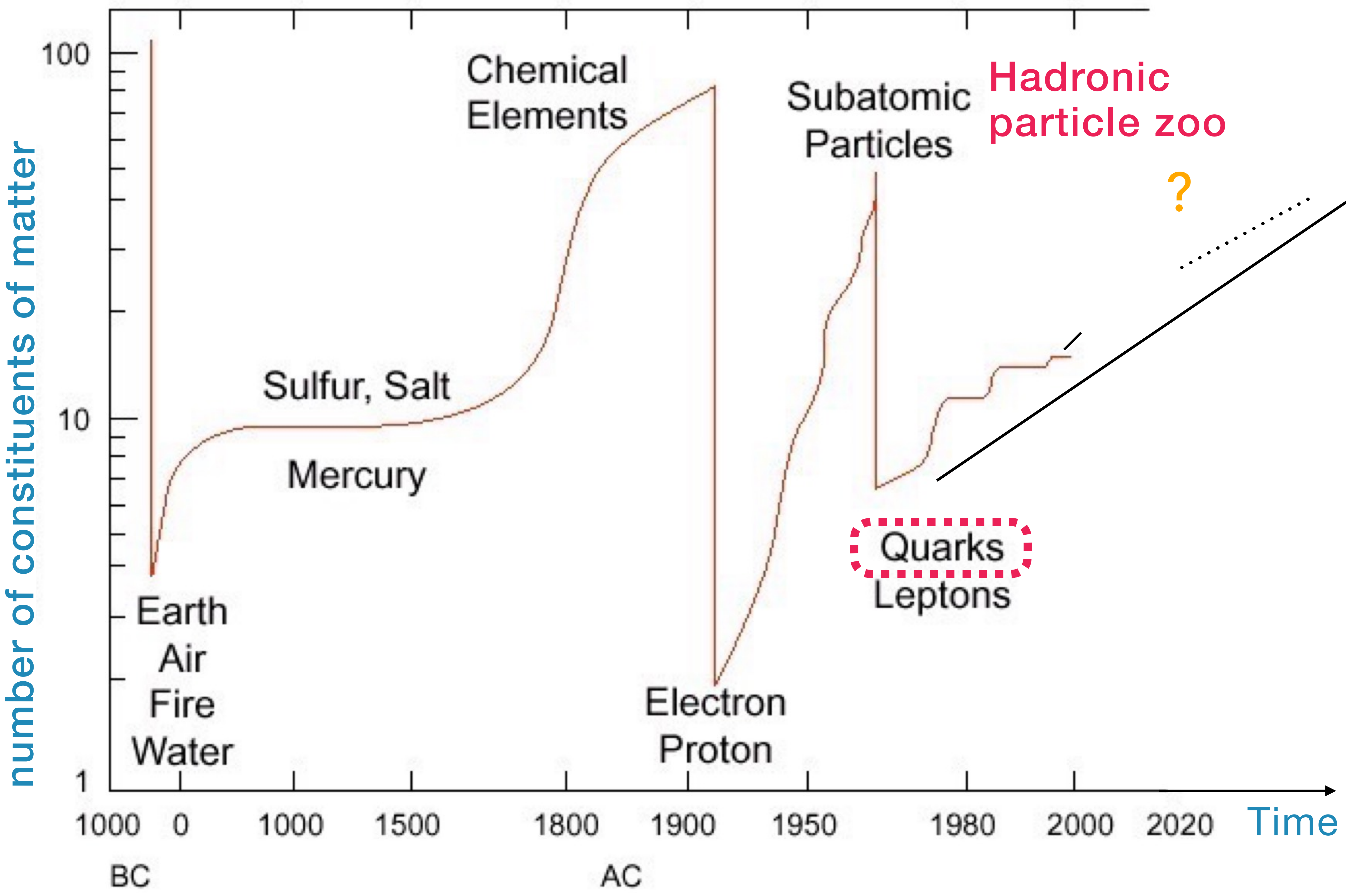
Today NR bound systems are at the center of new Revolutions

Constituents of matter and fundamental forces

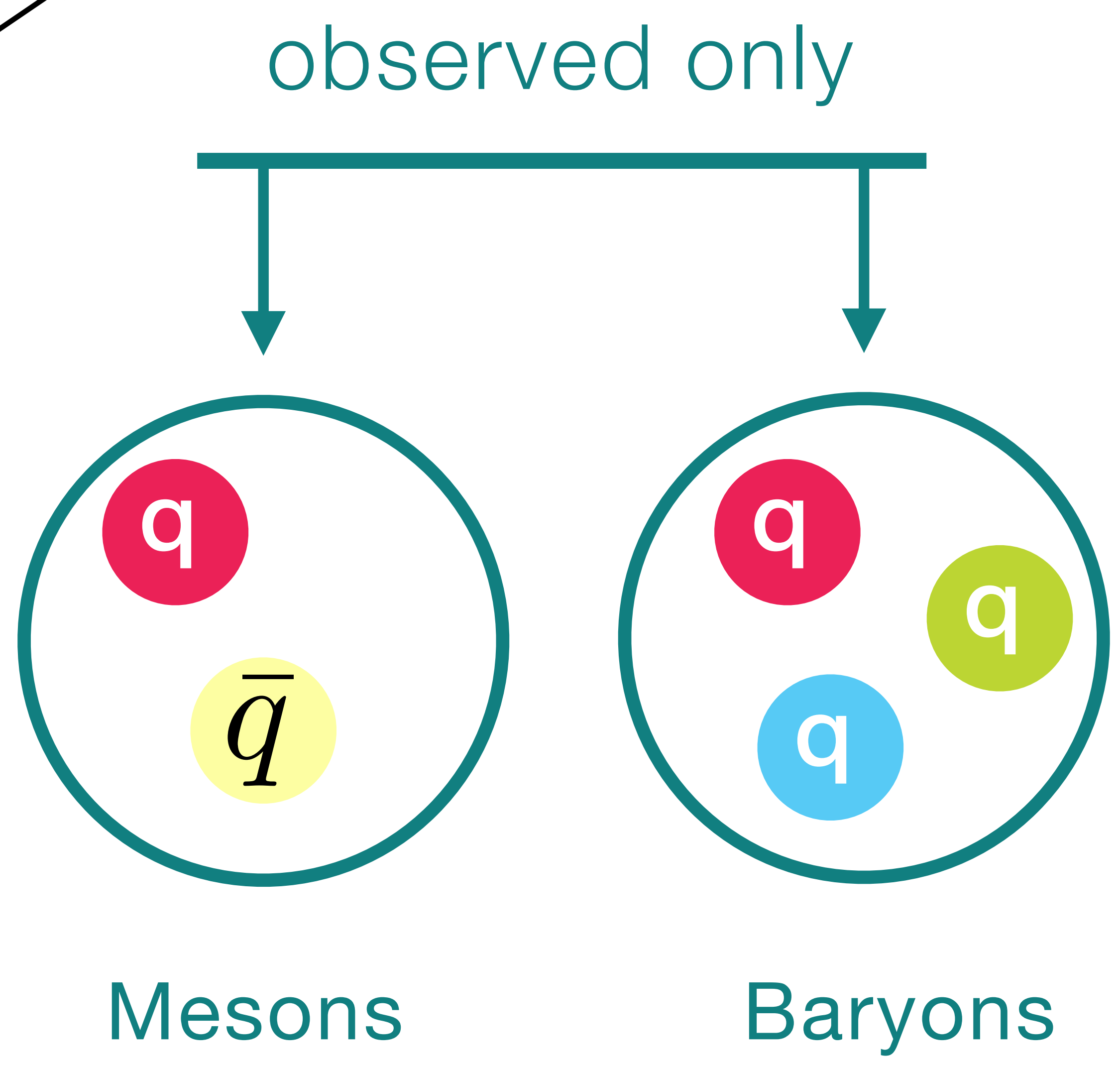


Constituents of matter and fundamental forces

history of constituents of matter



Quark Model 1964
Gell-Mann Zweig

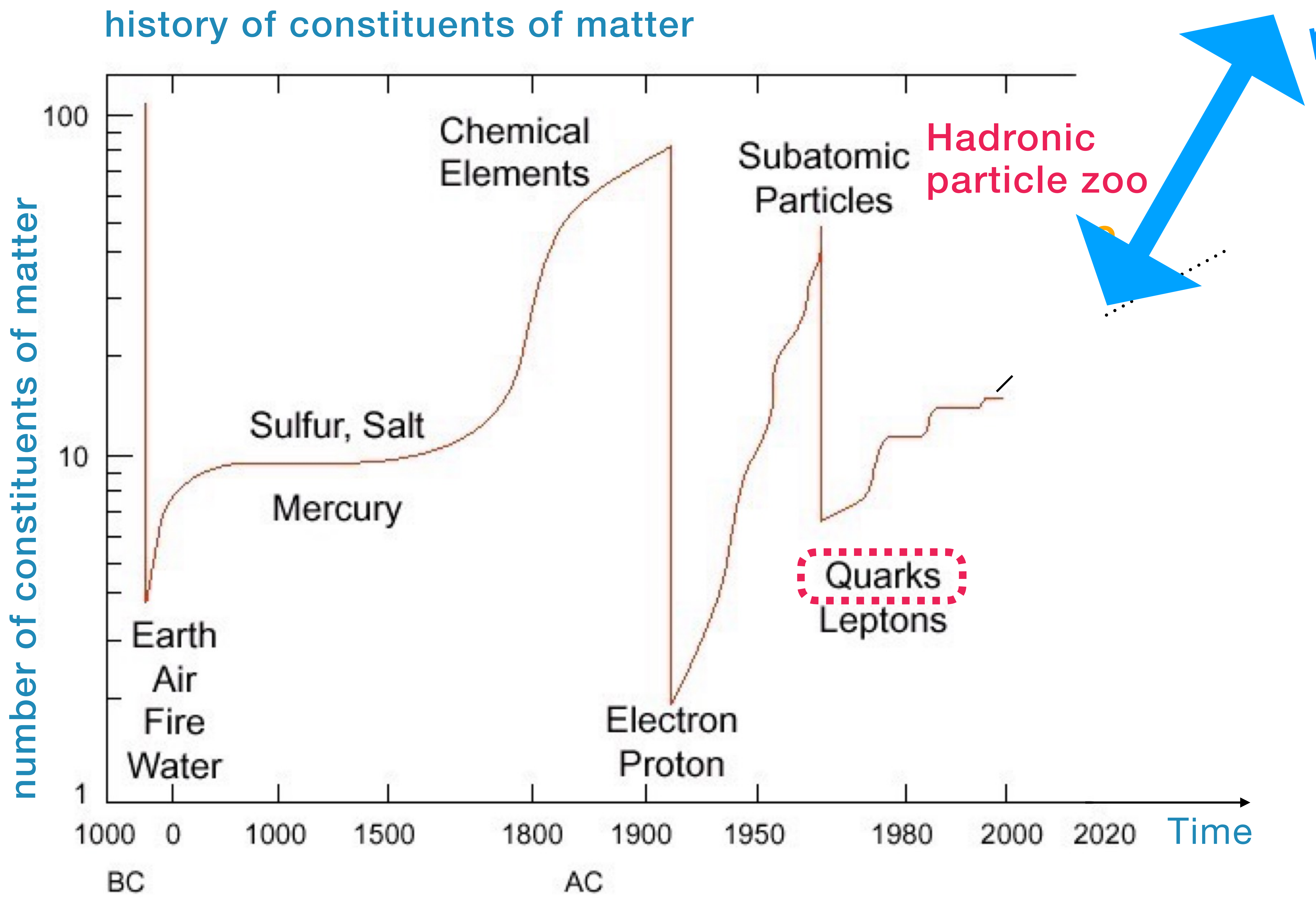


$qq\bar{q}\bar{q}\quad qqqq\bar{q}\ldots$

possible but not observed

Constituents of matter and fundamental forces

Beyond the standard model of particle physics



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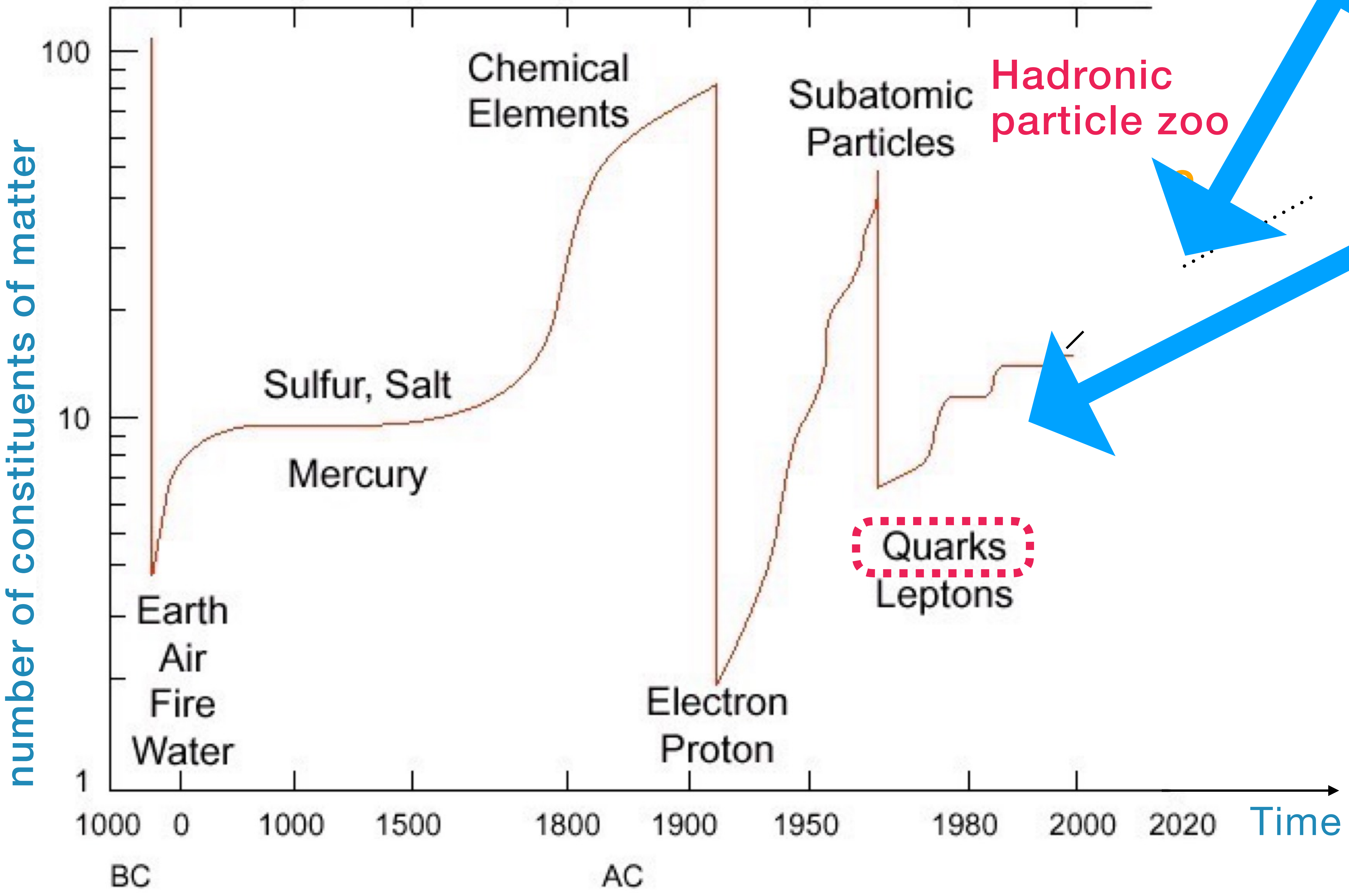


Beyond the standard quark model

With the XYZ exotic states discovery,

states observed in the sector with two heavy quarks

THE ECONOMIC TIMES

Scientists at CERN observe three "exotic" particles for first time

JARY 26, 2016 | 3 MIN READ

ysicists May Have Discovered a New "Tetraquark" Particle

Data from the DZero experiment shows evidence of a particle containing four different types of qu

HARD SCIENCE — JULY 16, 2022

Tetraquarks and pentaquarks: "Unnatural" forms of exotic matter have been found

Scientists have found three new examples of a very exotic form of matter made of quarks. They can yield insights into the early Universe.

WIRED

'Impossible' Particle Adds a Piece to the Strong Force Puzzle

The unexpected discovery of the double-charm tetraquark gives physicists fresh insight into the strongest of nature's fundamental forces.

CORRIERE DELLA SERA

Nuova straordinaria particella scoperta al Cern: il pentaquark

Consentirà di saperne di più sulla «forza forte» che tiene unite le particelle nel nucleo ... i e sui componenti della materia

INDIA TODAY

Mysterious 'X' particles that formed moments after the big bang found in Large Hadron Collider

BBC

Pentaquarks: scientists find new "exotic" configurations of quarks

Scientists have found new ways in which quarks, the tiniest particles known to humankind, group together.

Le Monde

of unknown nature discovered ... dron sector with tw

Les surprises du tétraquark, « collage » de particules élémentaires

La découverte d'une nouvelle particule à la structure particulièrement stable pourrait permettre aux chercheurs de vérifier leurs théories sur l'interaction forte.

ZEIT ONLINE

Cern-Forscher entdecken neues Teilchen

Die Physiker am Kernforschungszentrum in Genf haben die Existenz des Pentaquark-Teilchens nachgewiesen. Bislang war es nur in theoretischen ... beschrieben worden.

LHCb discovers longest-lived exotic matter yet

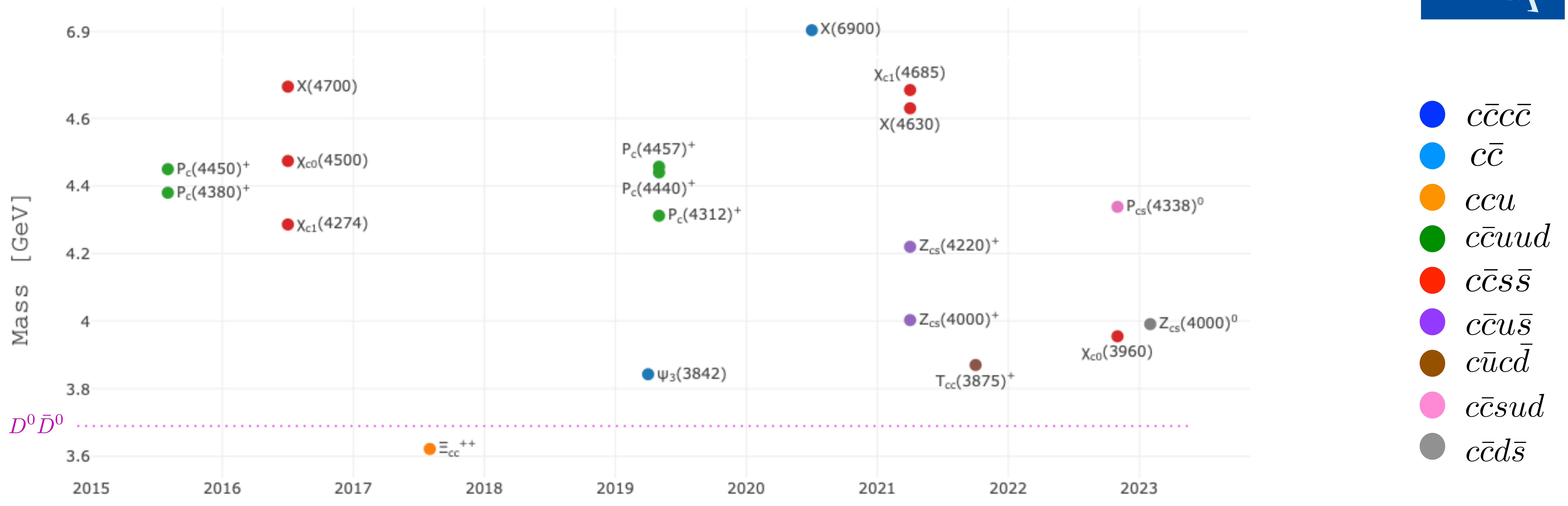
08/04/21 | By Sarah Charley

The newly discovered tetraquark provides a unique window into the interactions of the particles that make up atoms.

symmetry

The present revolutions

XYZ Revolution: Discoveries of Exotic States in the Sector With Two Heavy Quarks: Beyond the Standard Quark Model!



Date of arXiv submission

https://qwg.ph.nat.tum.de/exoticshub/

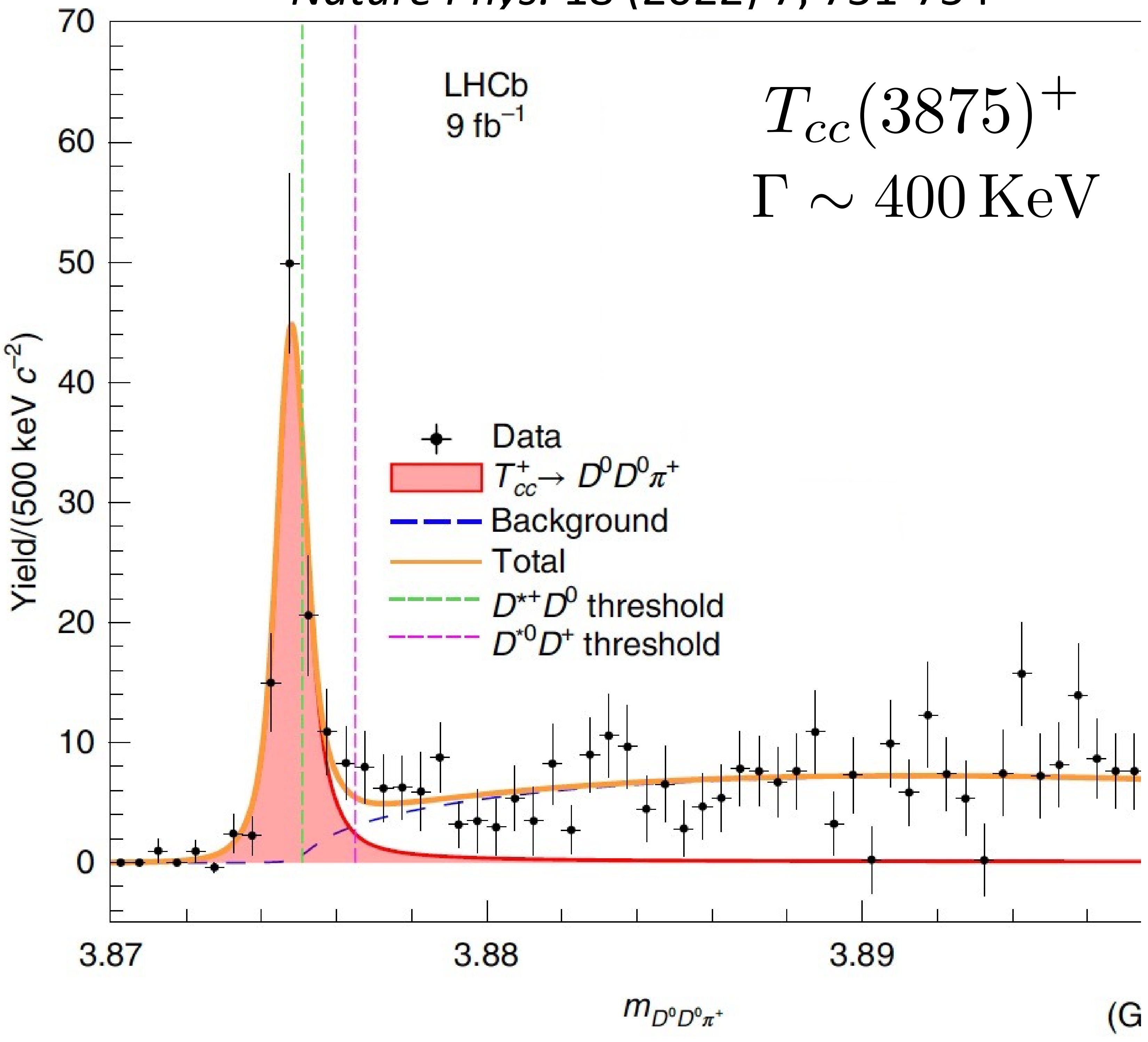
INTERPLAY AMONG MANY EXPERIMENTS:

UPCOMING EXPERIMENTS: STCF, Electron ion

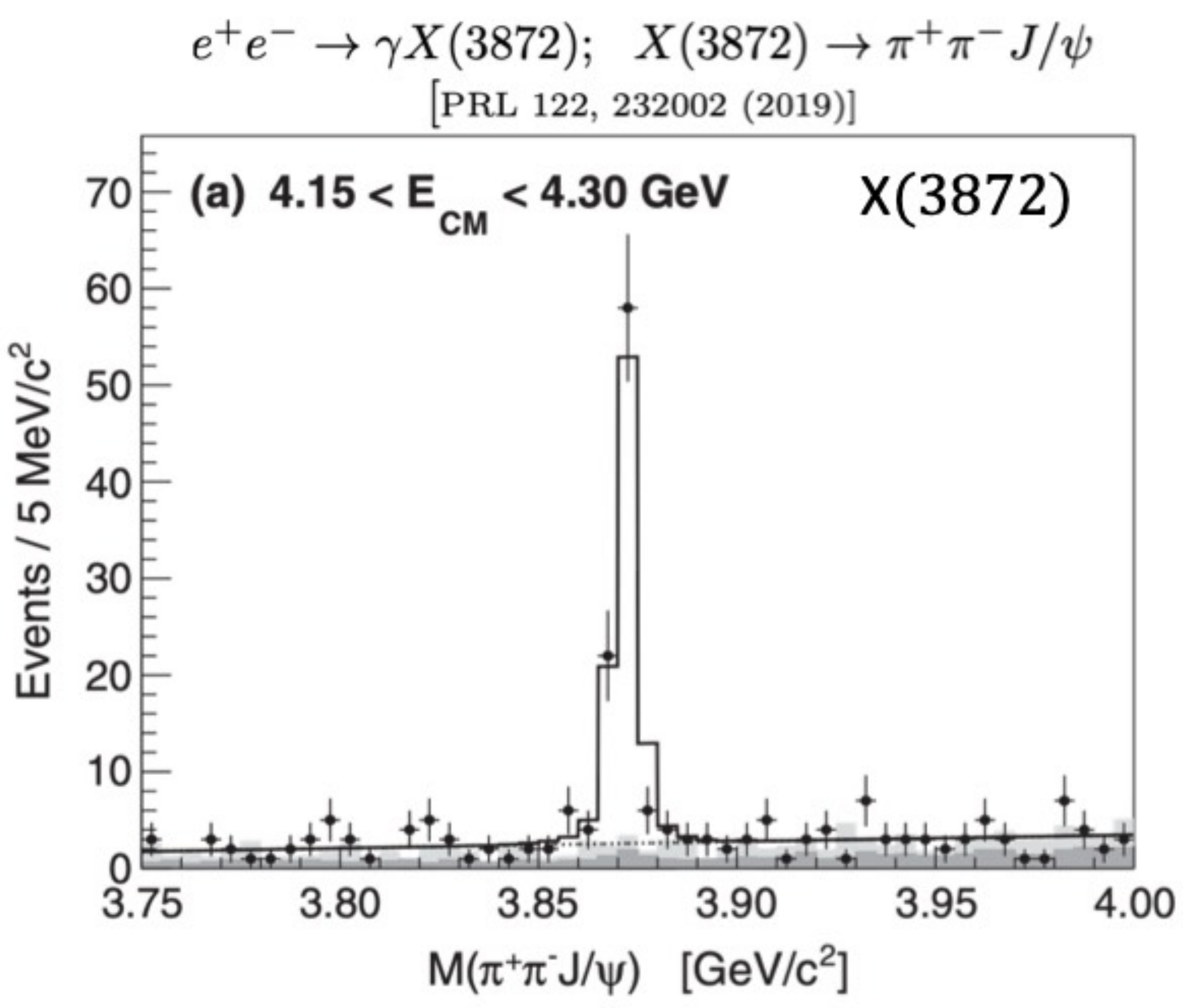
XYZ REVOLUTION: A New Spectroscopy Is Born!

Some surprisingly narrow states even if above/at strong decay thresholds

Nature Phys. 18 (2022) 7, 751-754



$$M_{T_{cc}(3875)^+} - (M_{D^{*+}} + M_{D^0}) = -0.27 \pm 0.06 MeV$$



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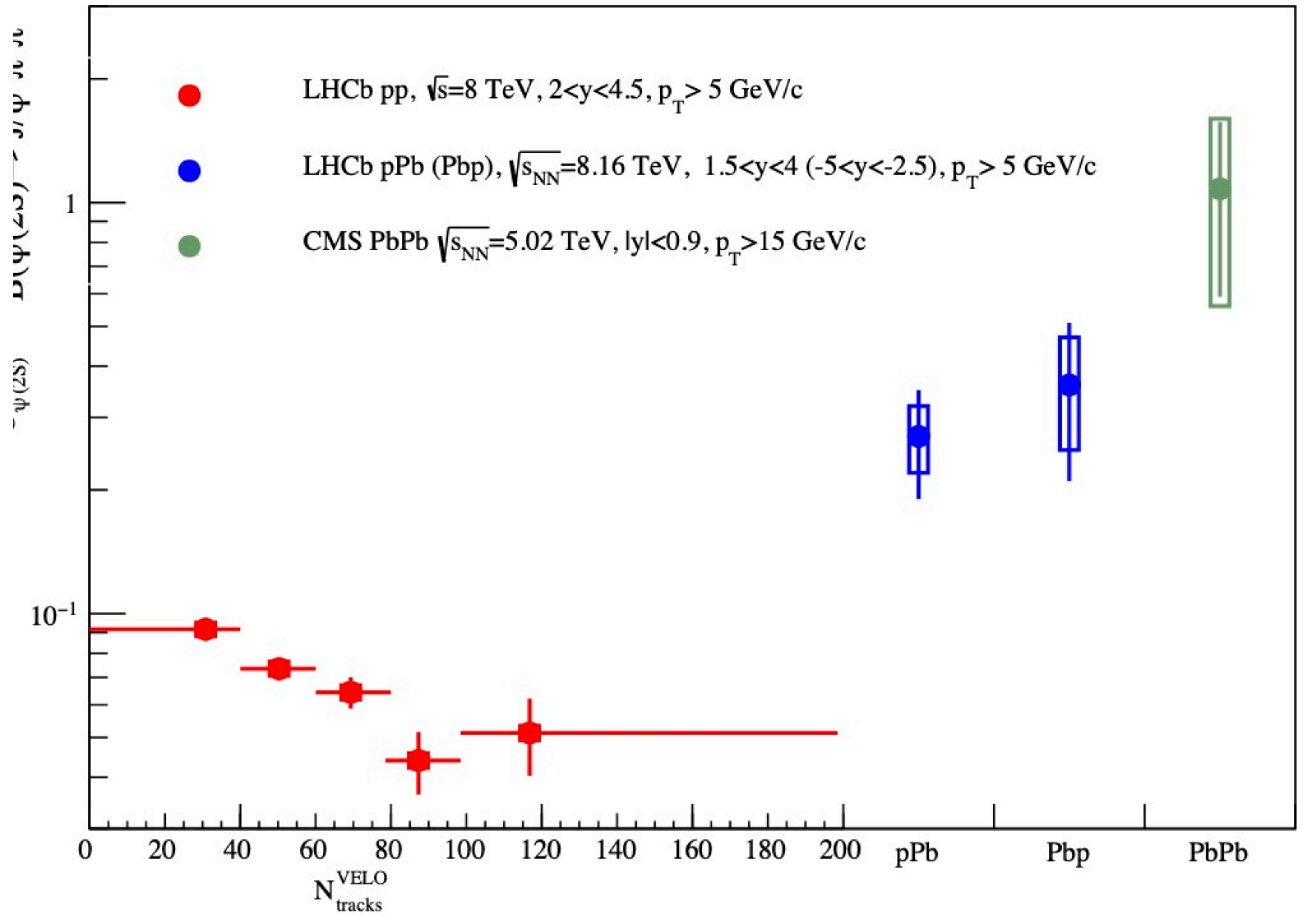
Produced in heavy ions where the deconfined strongly coupled QCD medium (Quark Gluon Plasma-QGP) is formed



New perspectives for XYZ studies!

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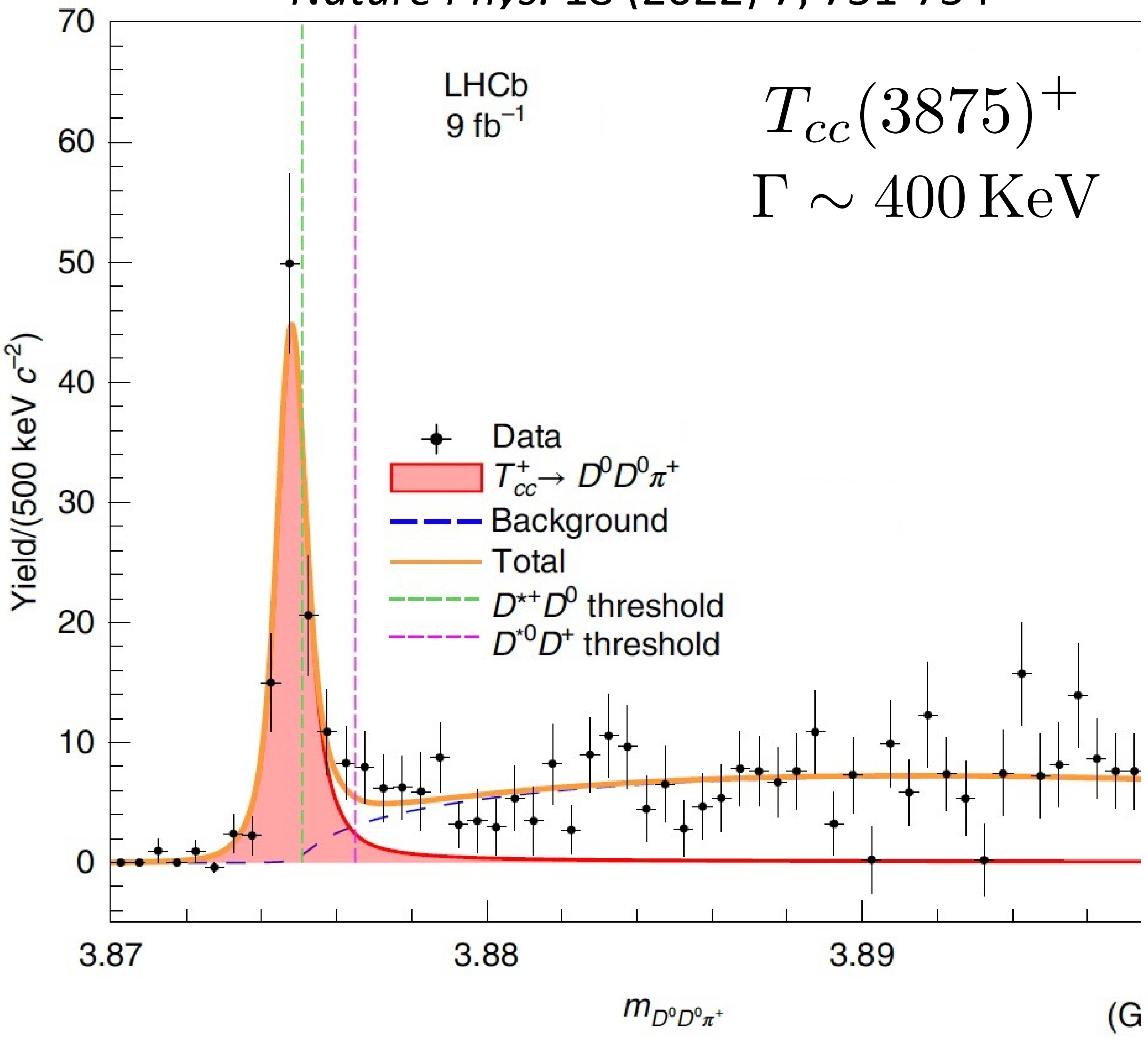

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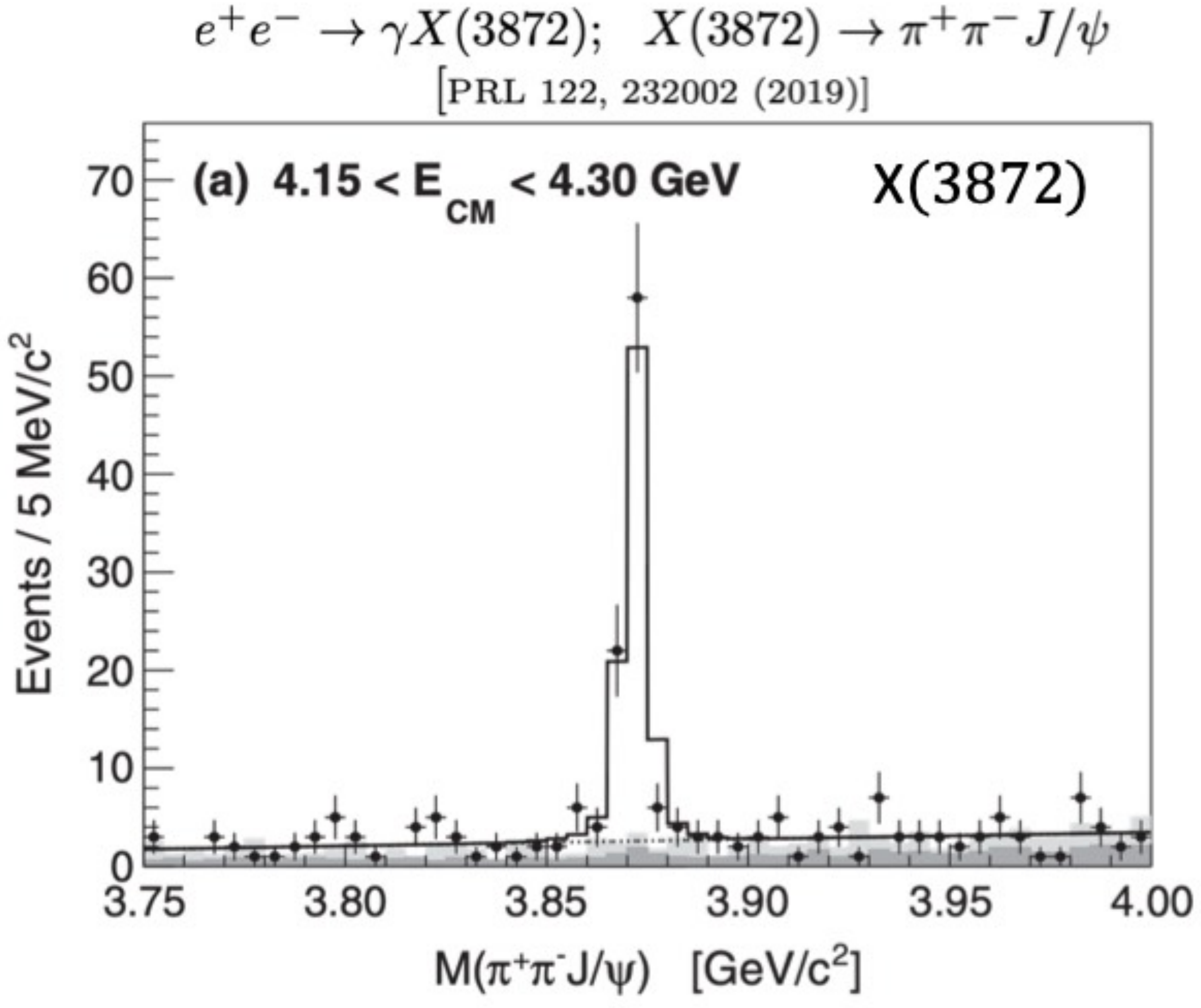

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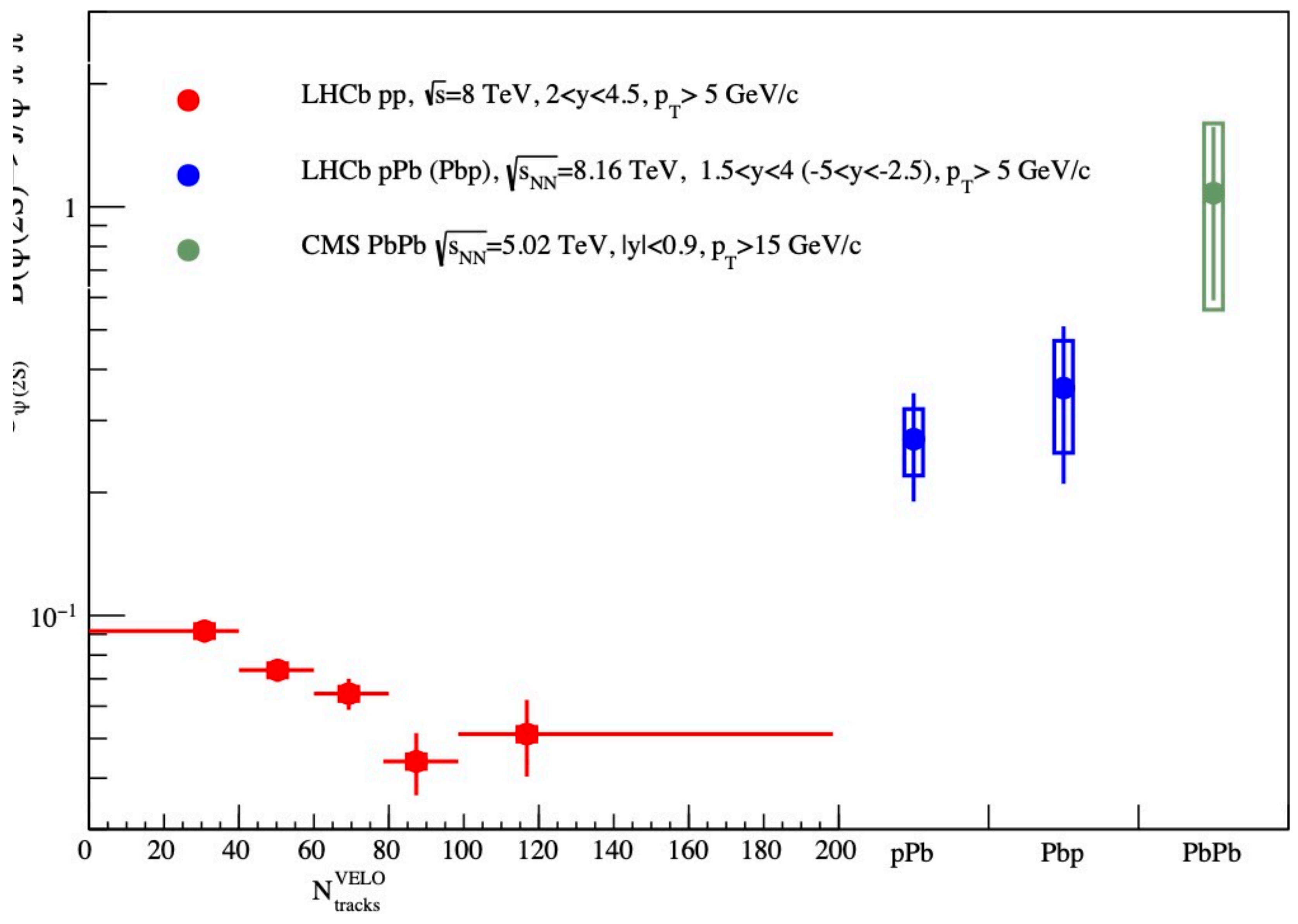

New perspectives for XYZ studies!

XYZs not merely composite particles, have unique properties
Novel strongly correlated exotics systems: we need flexible tools to address spectra, production, medium propagation in QCD ! Challenge from the Nonperturbative binding

The present revolutions: nuclear matter phase diagram

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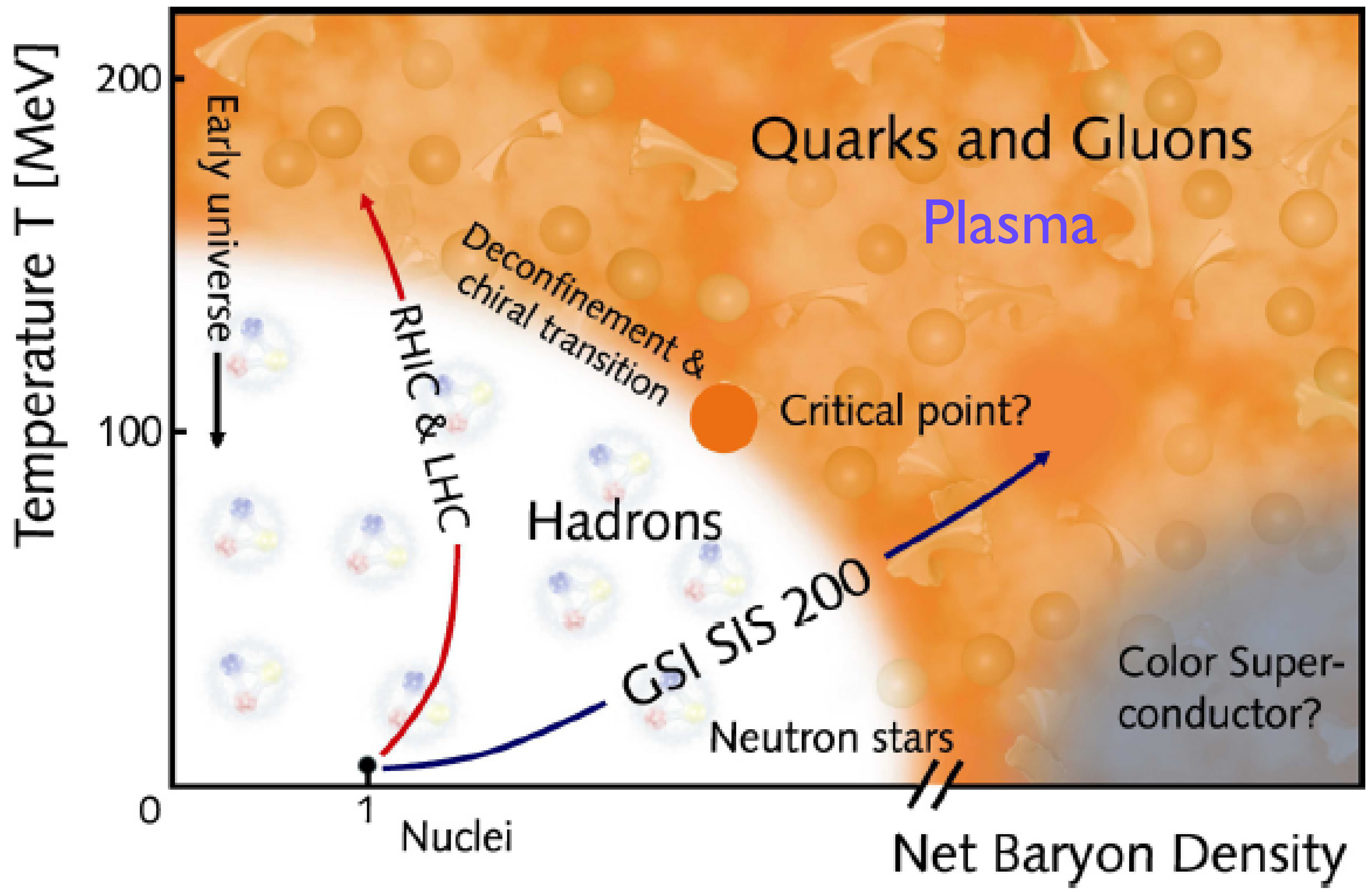

Nuclear Matter

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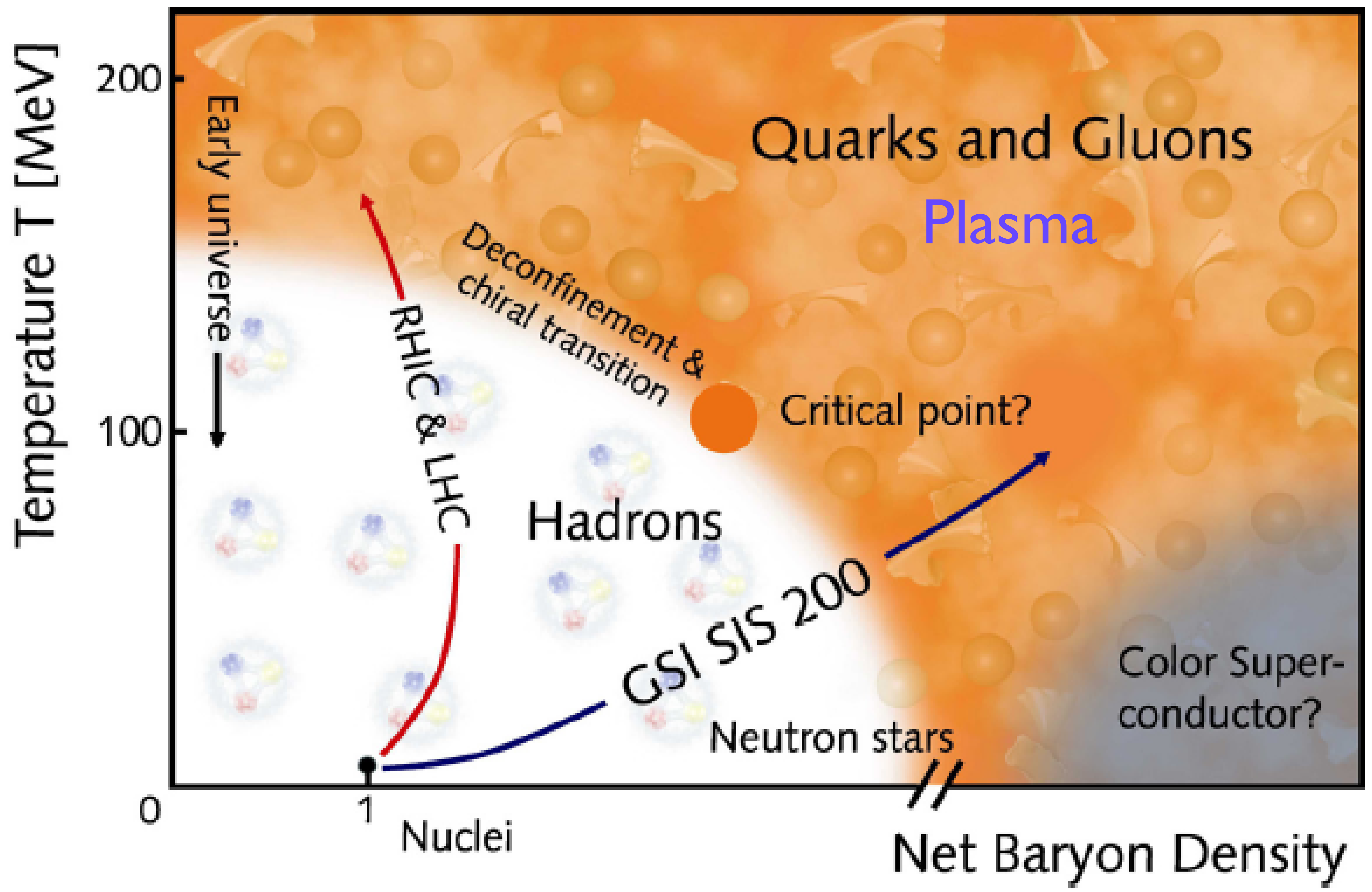

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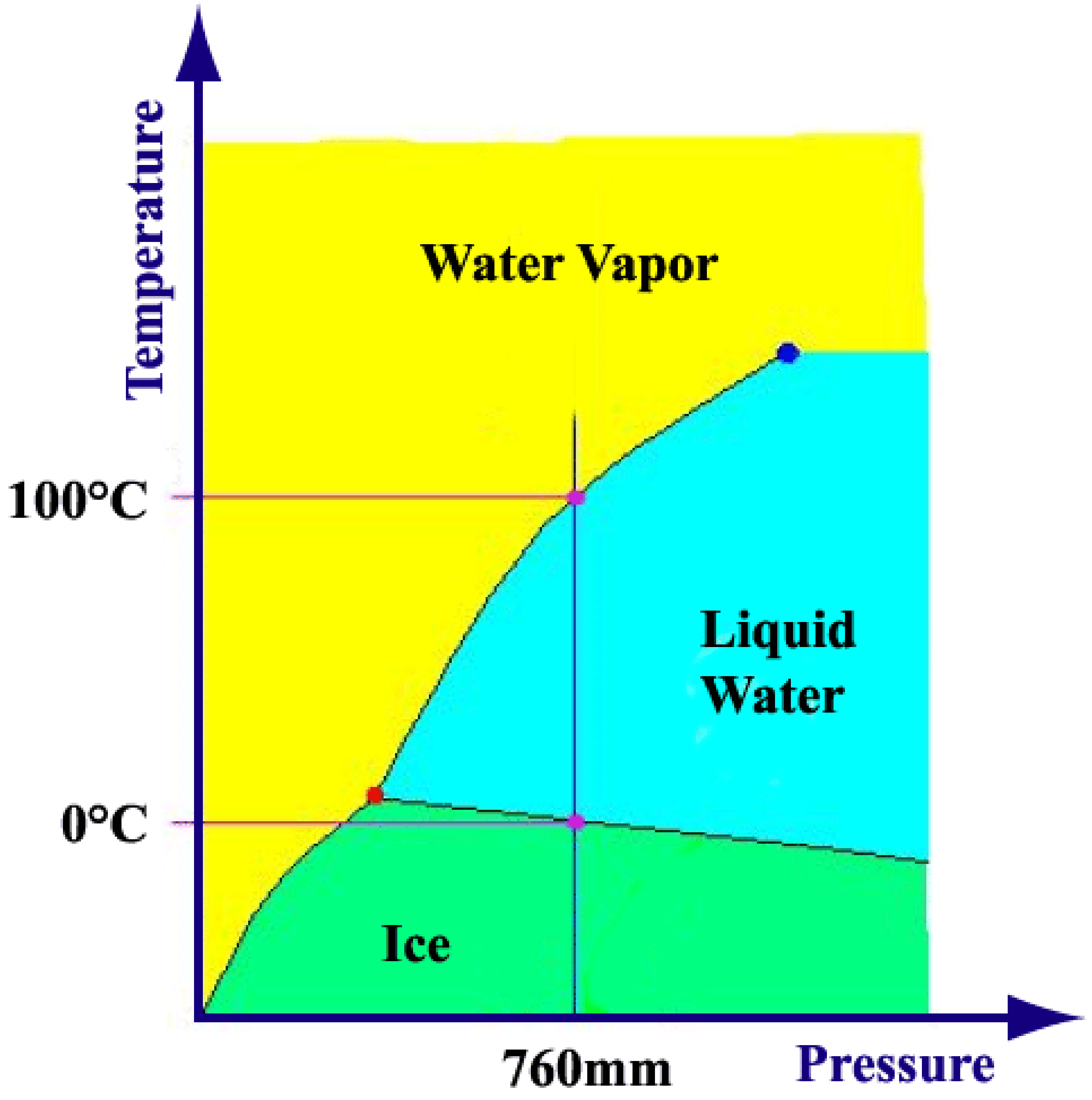

Water

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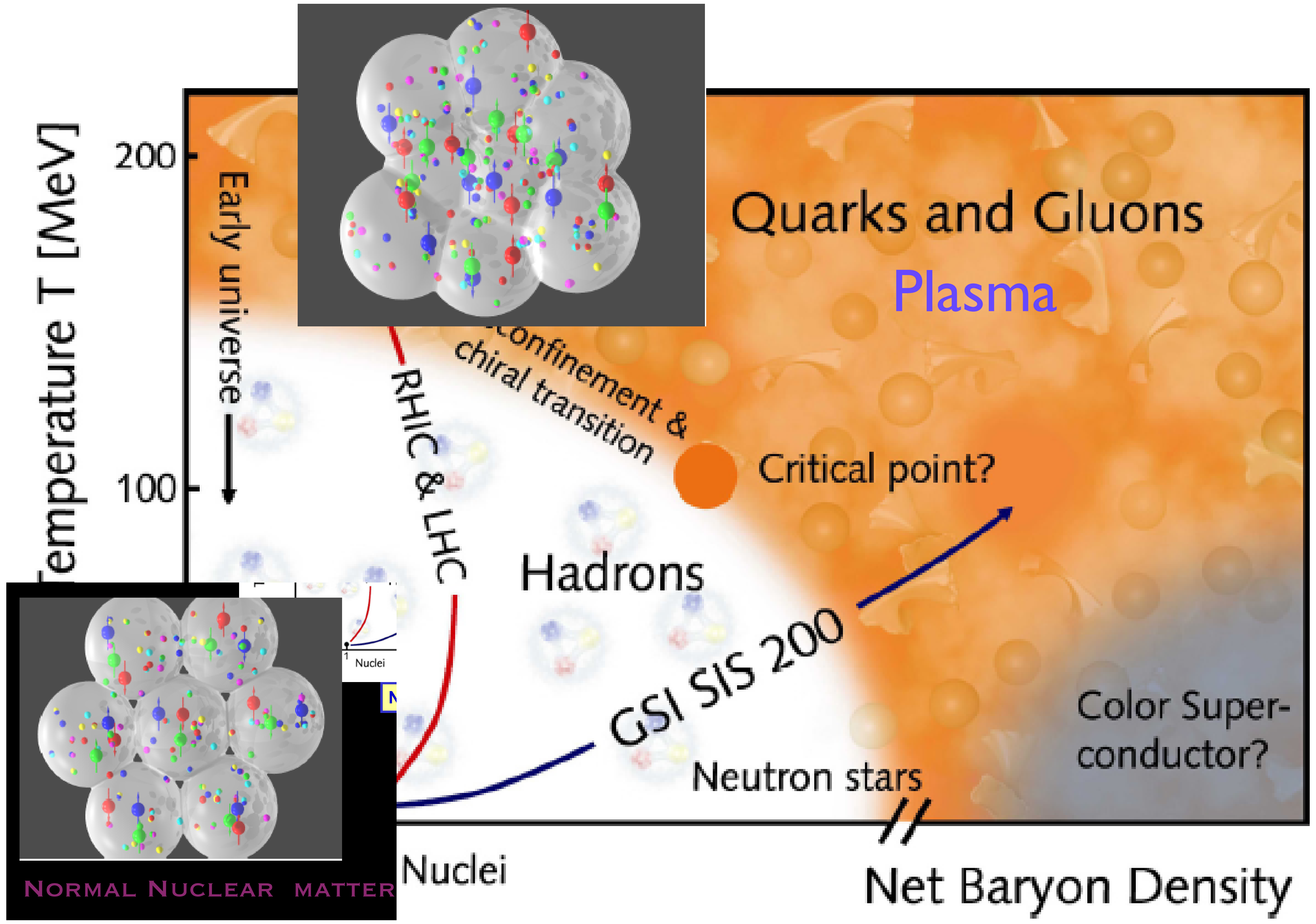

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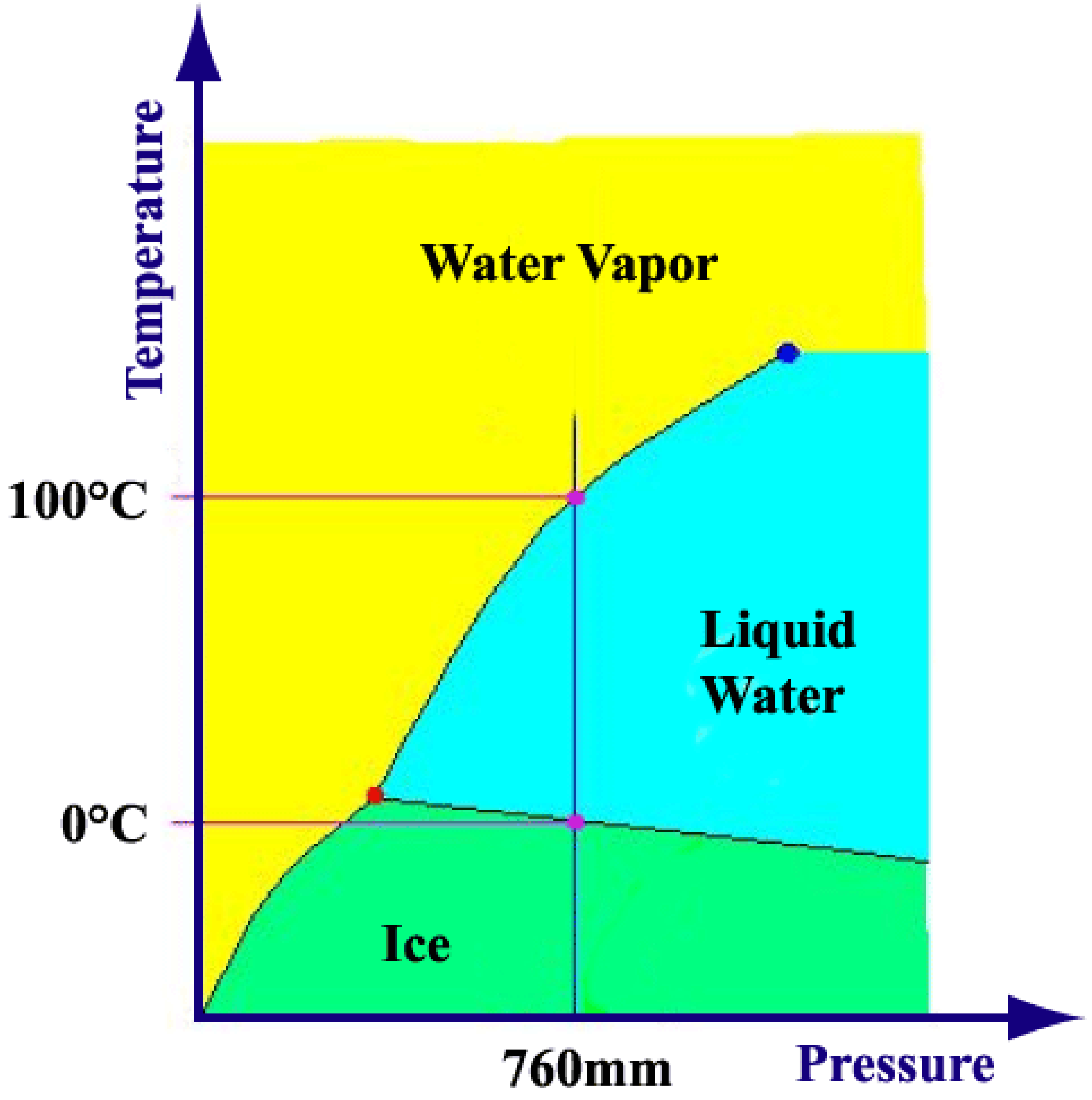

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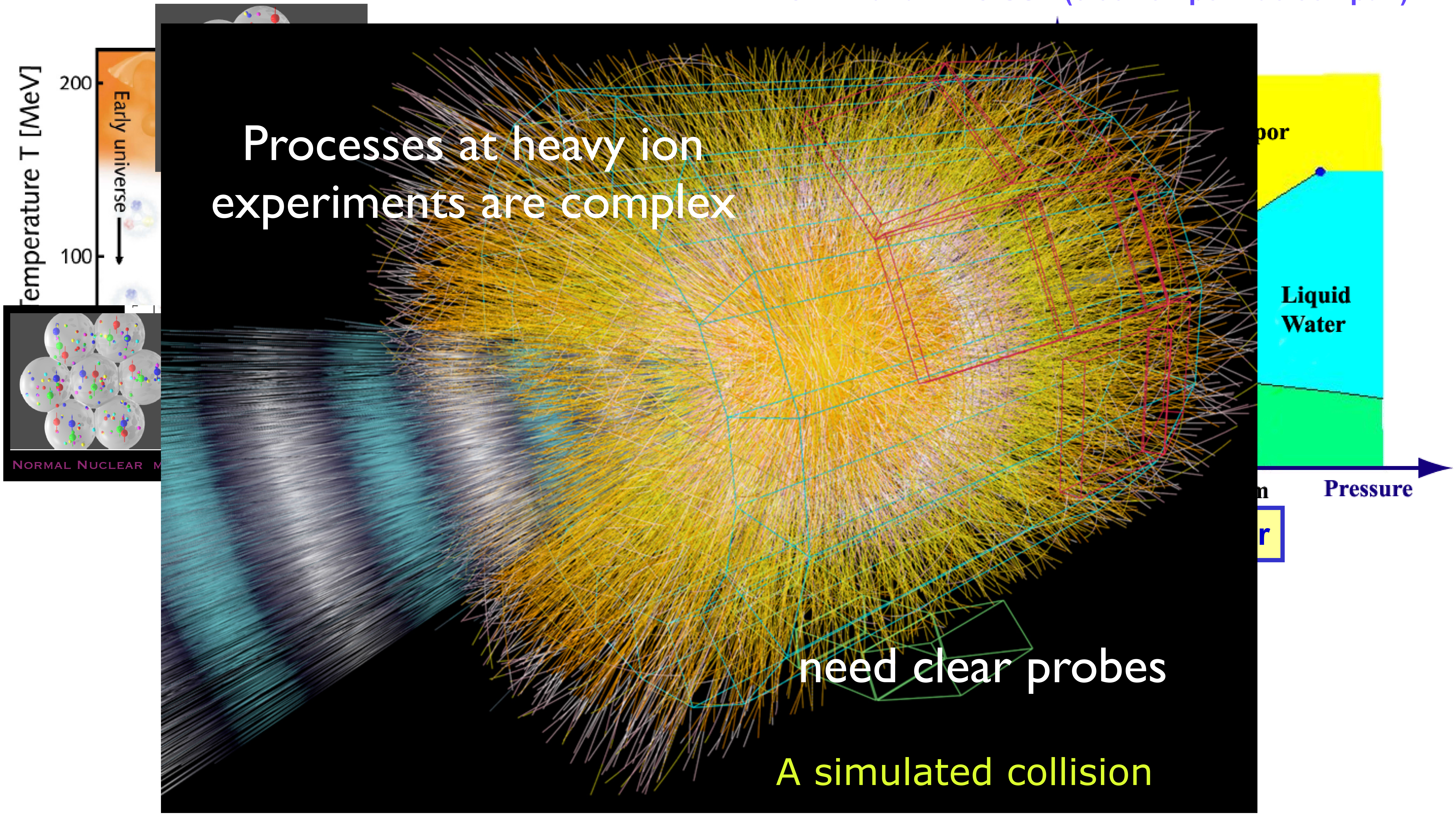
The present revolutions: nuclear matter phase diagram

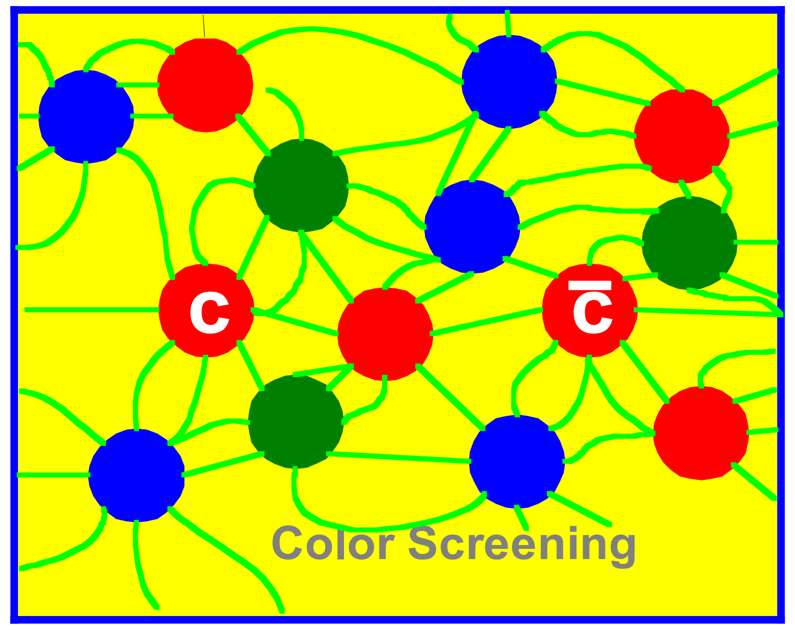
investigated in heavy ions collision at the LHC at CERN and RHIC USA (5.36 TeV per nucleon pair)



The present revolutions: nuclear matter phase diagram

Quarkonia are probe of QGP formation

Matsui Satz 1986 idea **of color screening in medium**



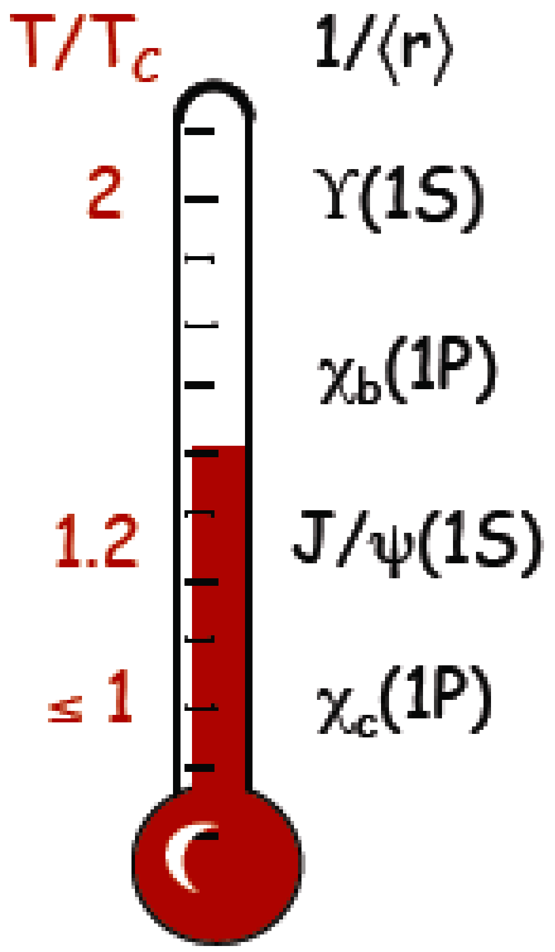
Debye charge screening

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$m_D \sim gT$$

$$r \sim \frac{1}{m_D}$$

→ Bound state dissolve



Sequential Melting at different Temperature

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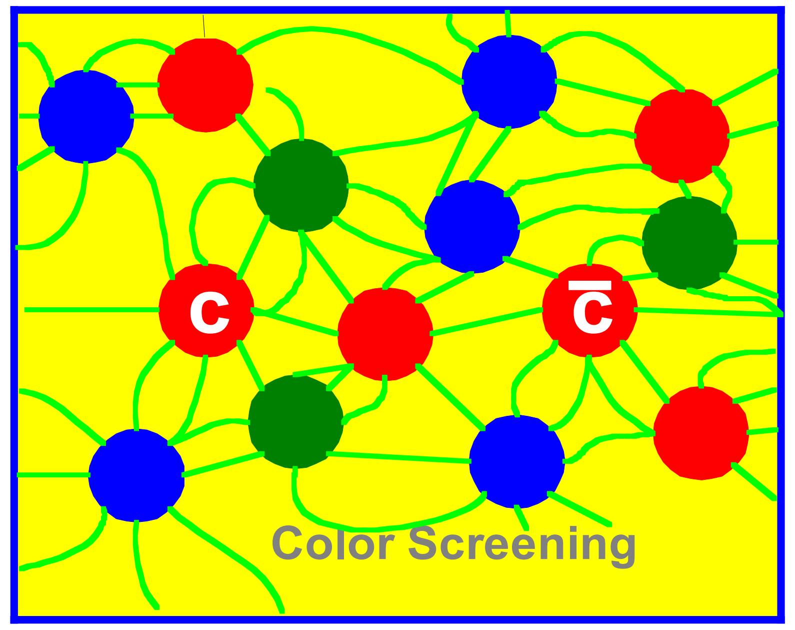

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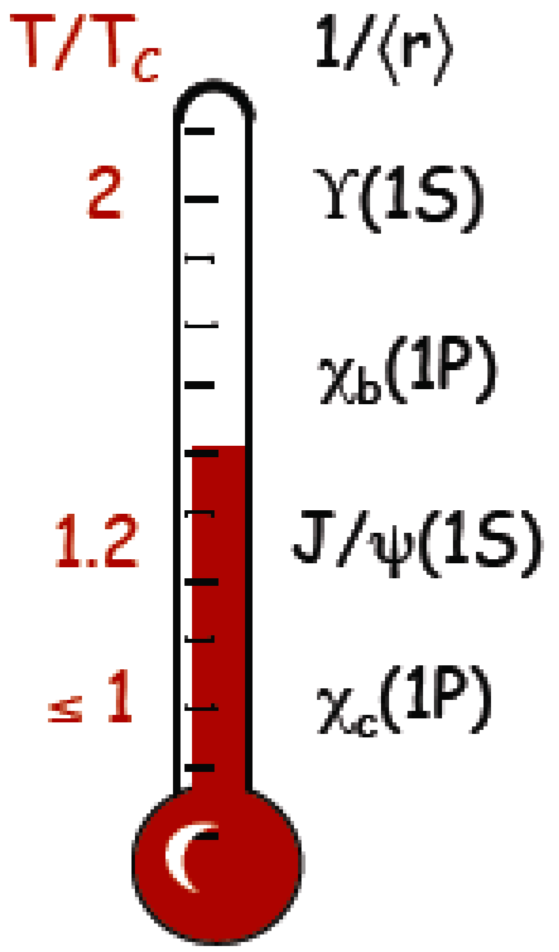

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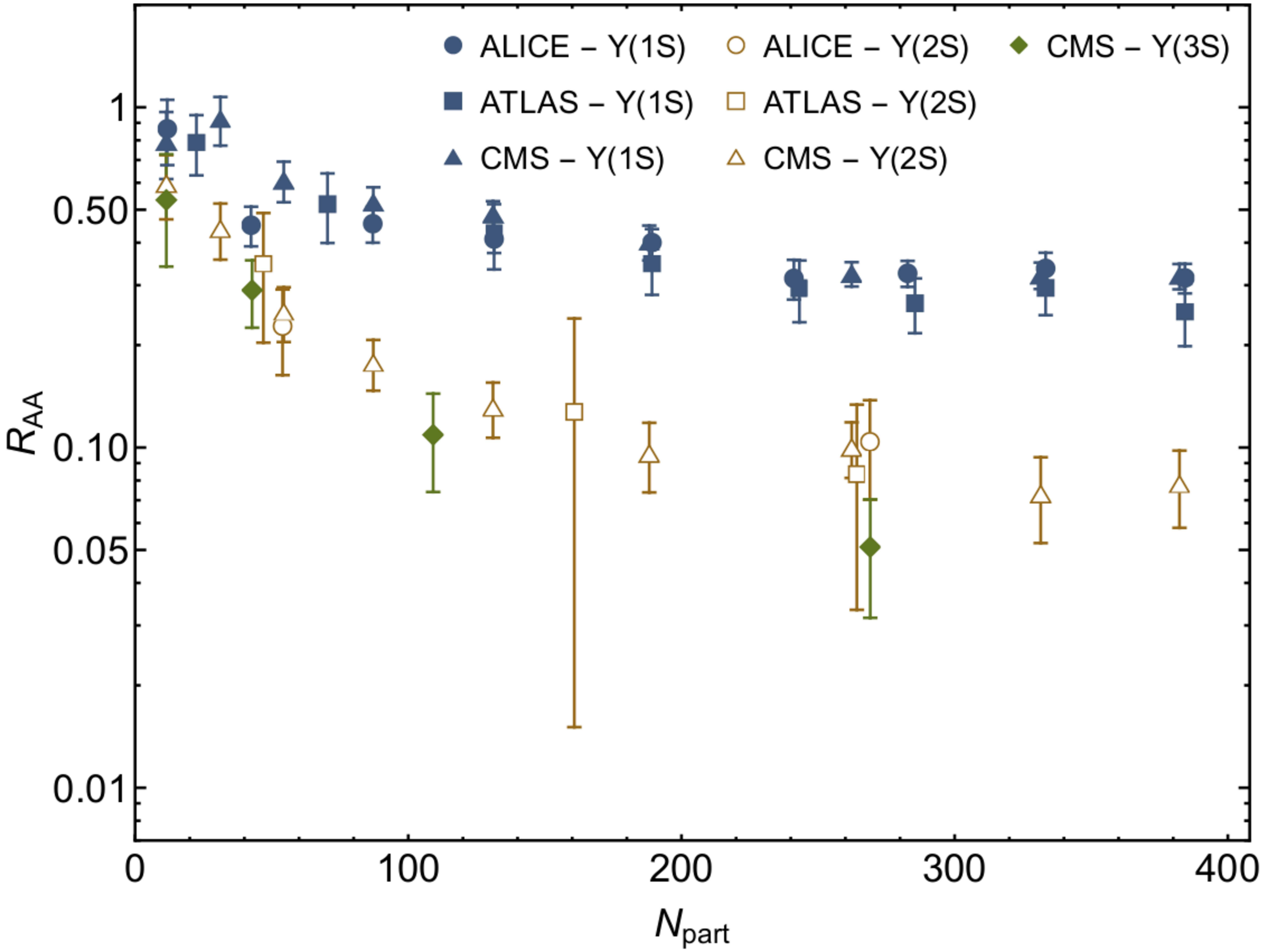
Experimental measurements:

R_{AA} is the nuclear modification factor = yield of quarkonium in PbPb / yield in pp.



CMS PLB 790 (2019) 270
ALICE PLB 822 (2021) 136579
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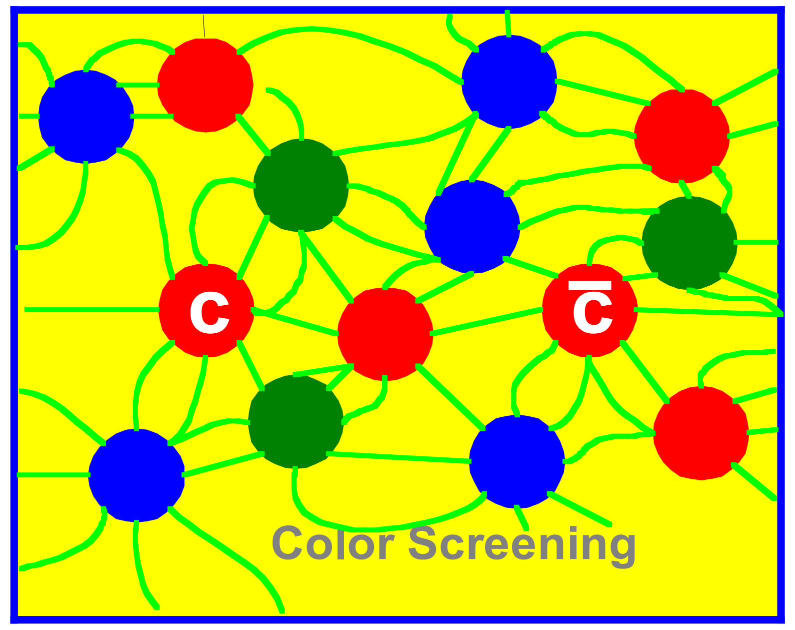

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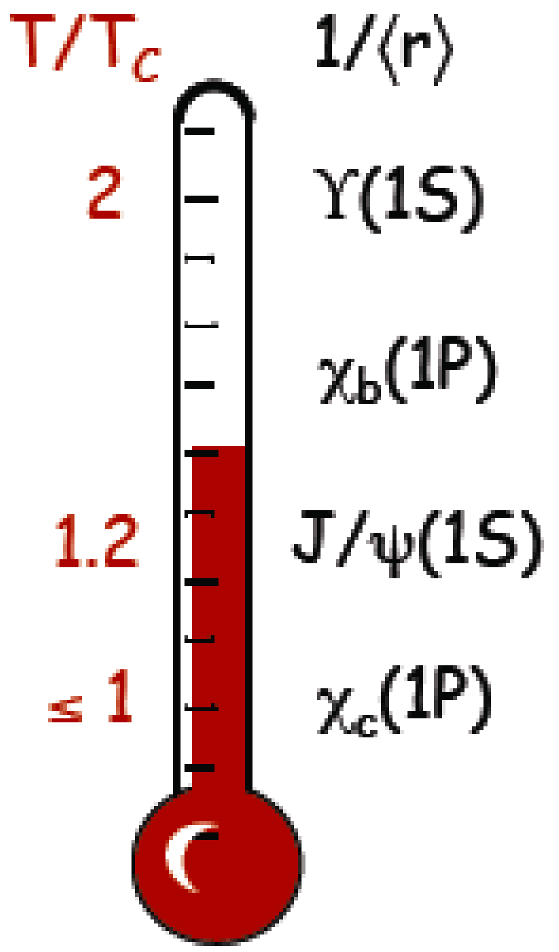

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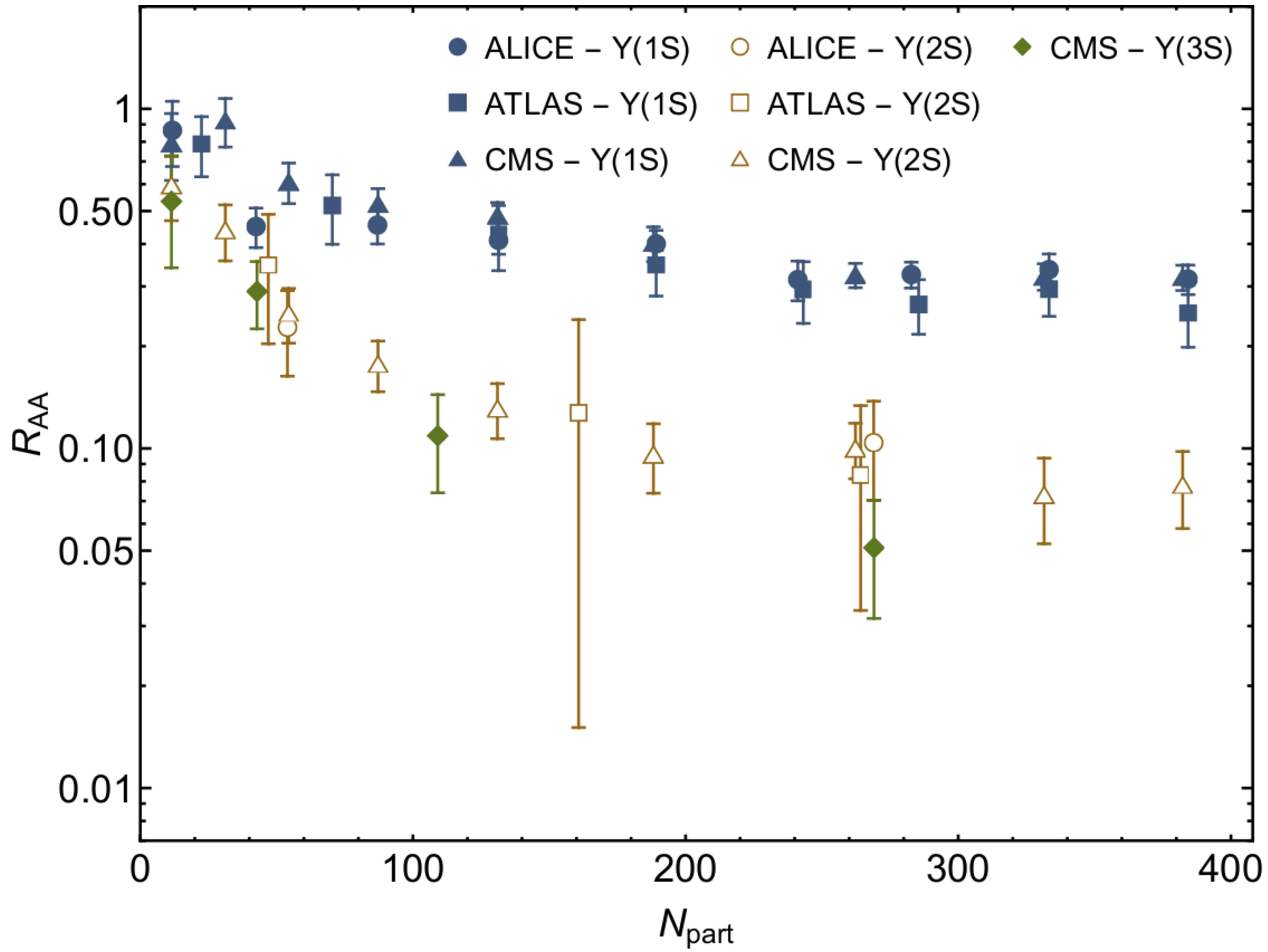

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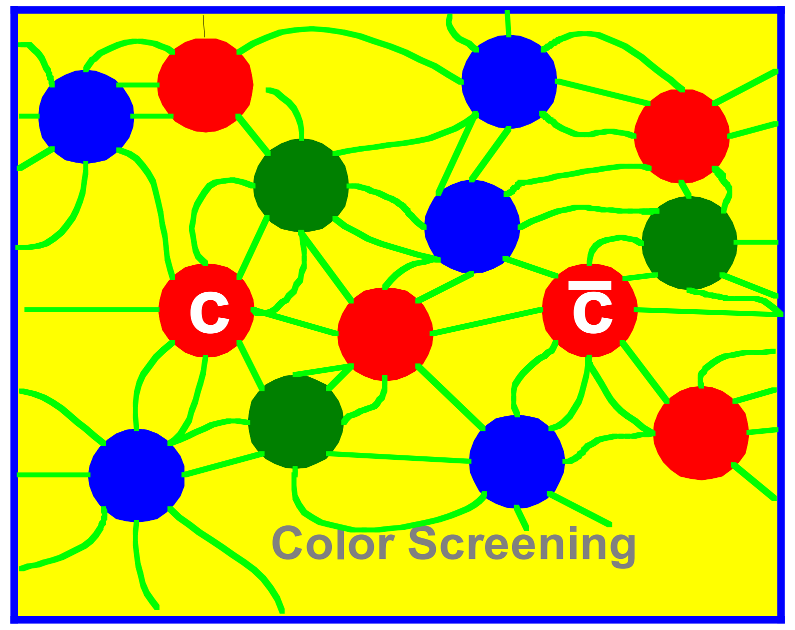

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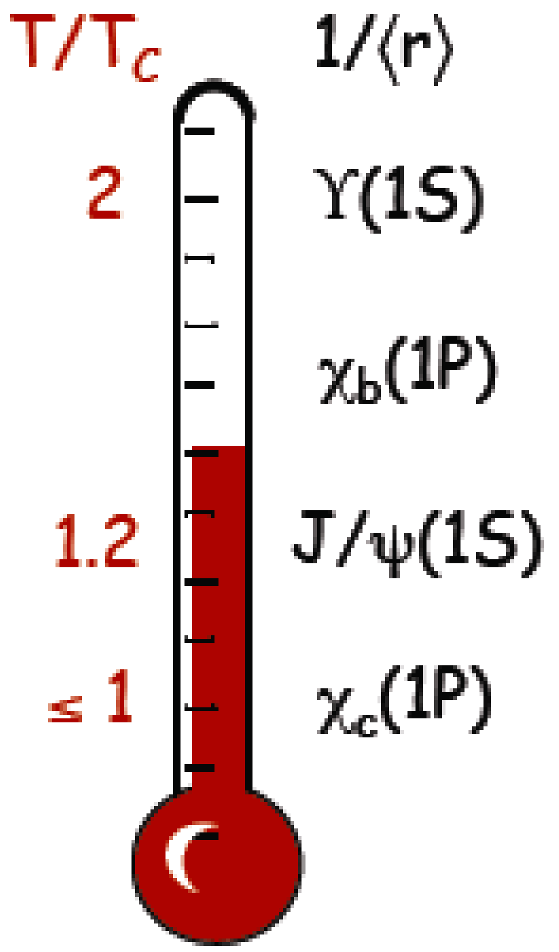

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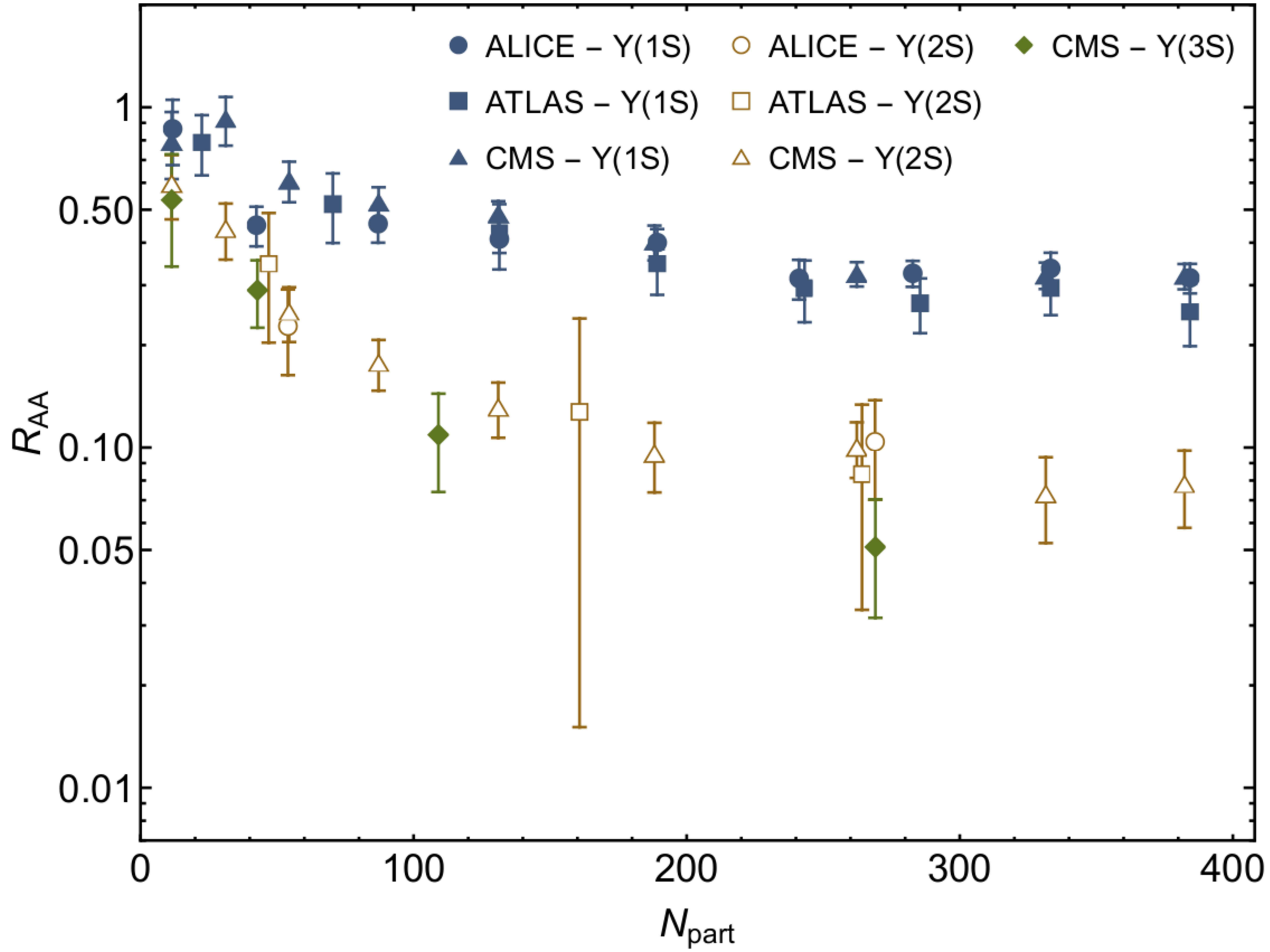

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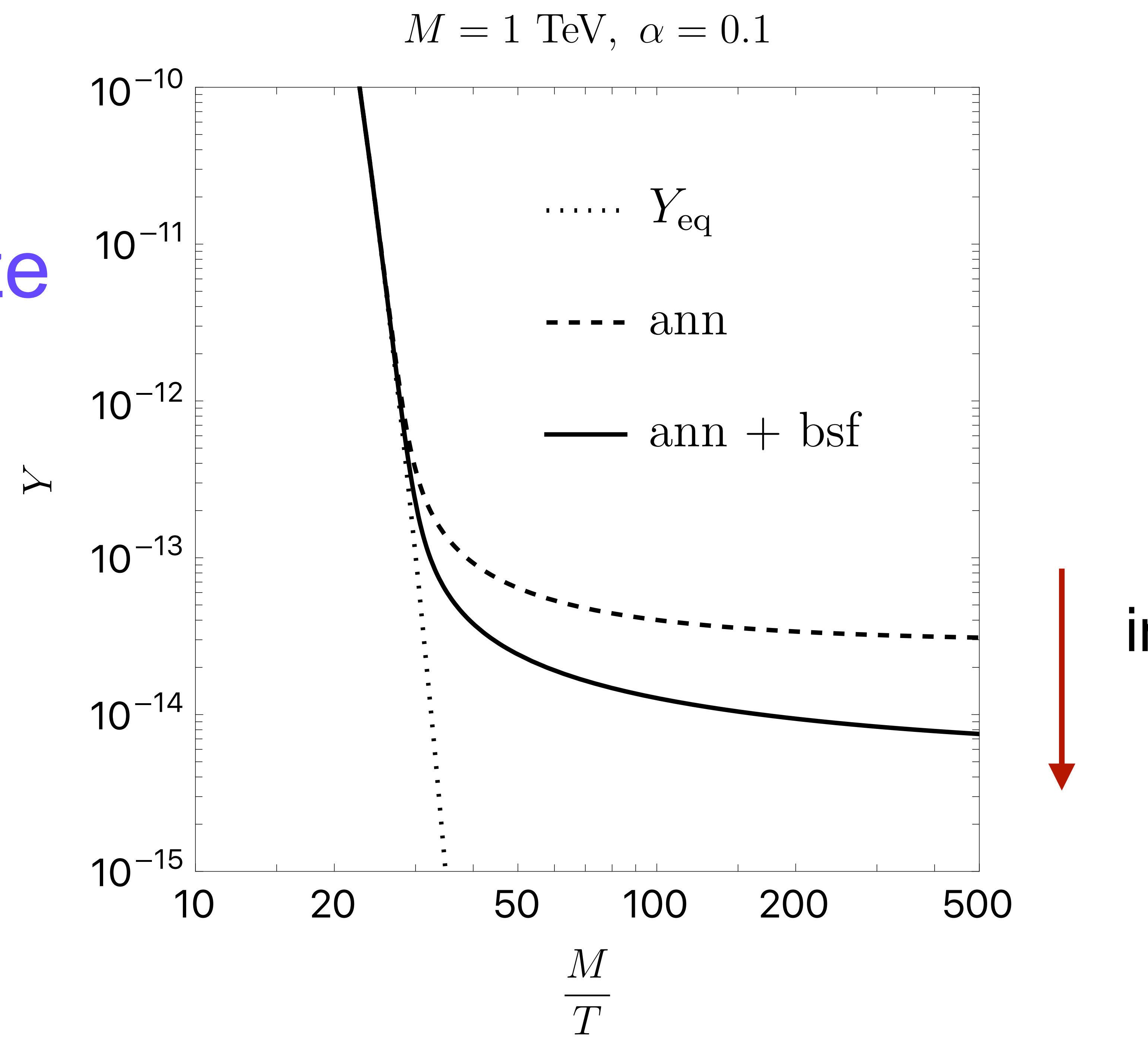
XYZ states are also produced and evolve in heavy ion collisions

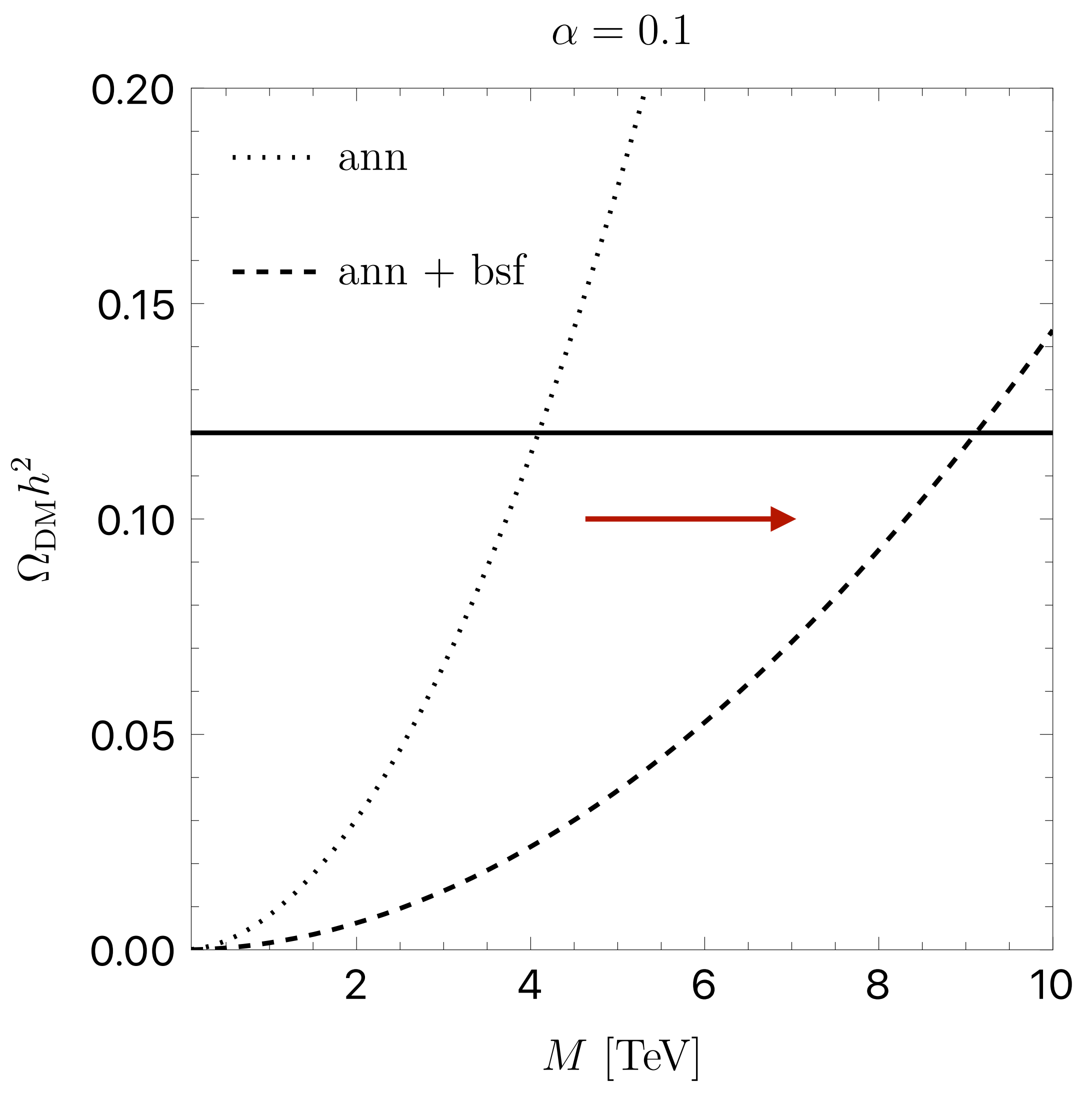
The present revolutions: dark matter evolution in early universe or neutrino evolution (fast flavour conversion)in supernova

A similar description can be applied to other NR system evolving in medium: e.g. **heavy dark matter pairs evolving in the early universe**: in order to predict the cosmological abundance of dark matter an estimation of particle rates in an expanding thermal environment is needed. Bound state effects at finite T may have large impact on the result

Freeze Out



- Early universe ($T \gtrsim M$): Heavy DM in thermal equilibrium with dark medium
- Expanding universe ($T \lesssim M$): T cools down $\rightarrow$ detailed balance lost
- Evolution equation: $(\partial_t + 3H)n = -\frac{1}{2}\langle\sigma_{\rm eff} v_{\rm rel}\rangle(n^2 - n_{\rm eq}^2)$
- Accurate prediction of DM relic density requires precise determination of the relevant interaction rates in expanding thermal environment
- Observed DM relic abundance implies heavy DM: $\Omega_{\rm DM}h^2 \sim 3\times 10^{11}\frac{M}{\rm TeV}Y_0 \quad \rightarrow \quad M \sim {\rm TeV}$
- During and after chemical freeze-out, DM is non-relativistic: $H \sim \langle\sigma_{\rm eff} v_{\rm rel}\rangle n_{\rm eq} \quad \rightarrow \quad T \sim M/25$



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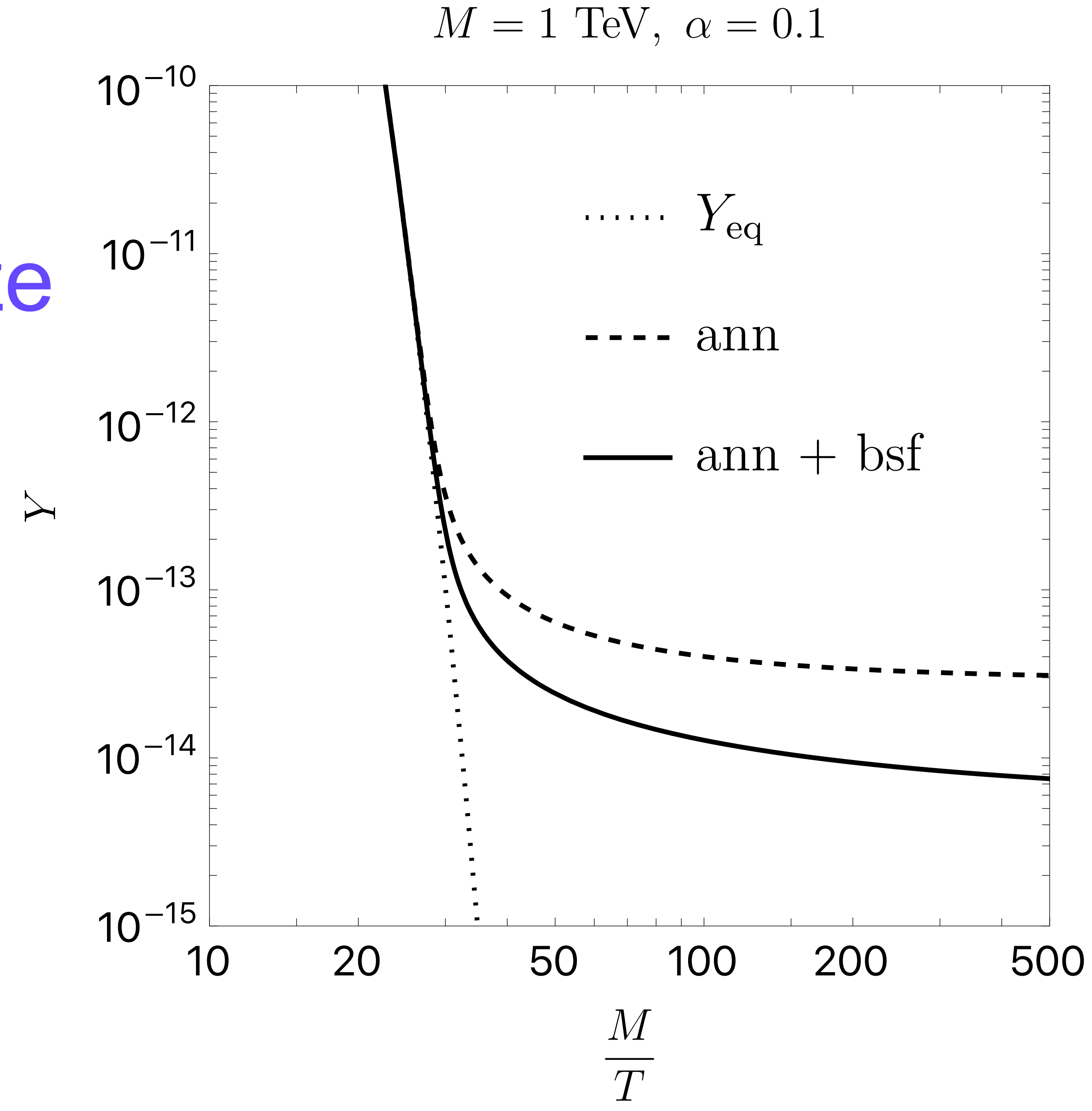
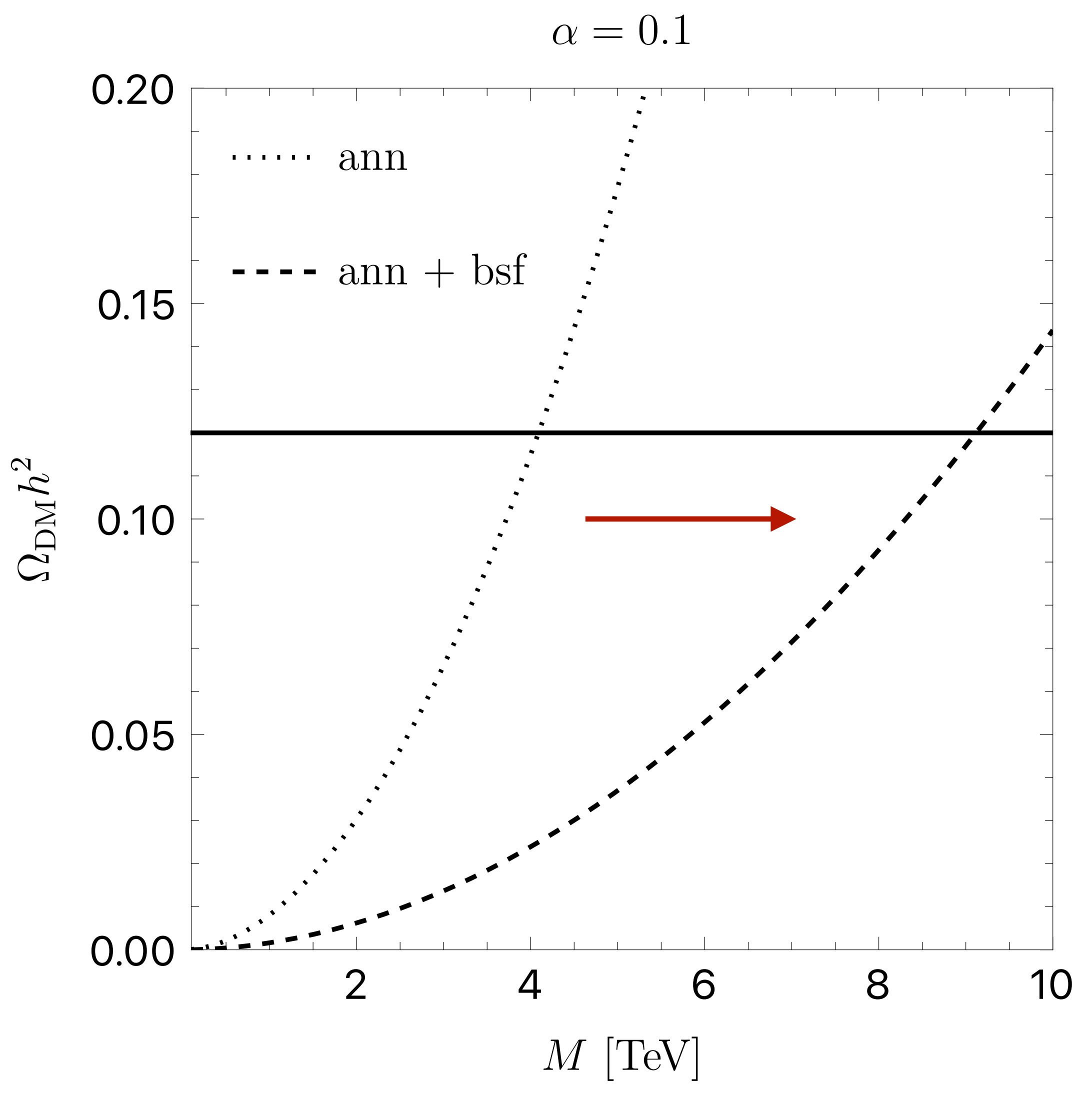




->obtain the quantum nonequilibrium evolution of DM pairs in the early universe Including dissipation, decoherence and recombination

QQbar systems (and NR bound systems) are important tools to address significant problems at the frontier of particle physics

To this aim they should be addresses in QFT, QFT at finite T , finite mu

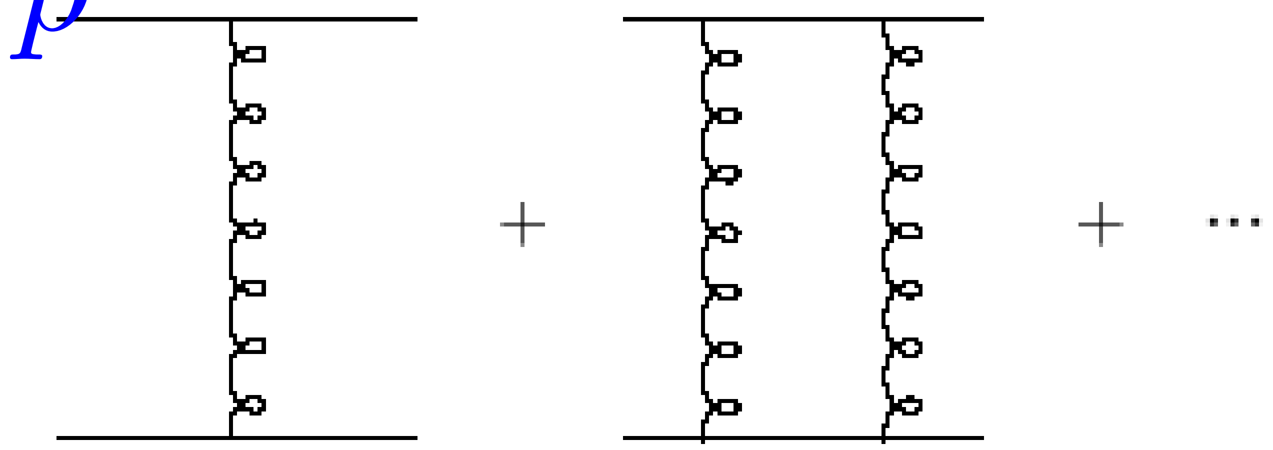
However...

For quarkonium to become a probe of strong interactions, it should be treated in QCD :a very hard problem

Close to the bound state $\alpha_s \sim v$

Q $\quad p$

$p \sim m\alpha_s$



$\bar{Q}$

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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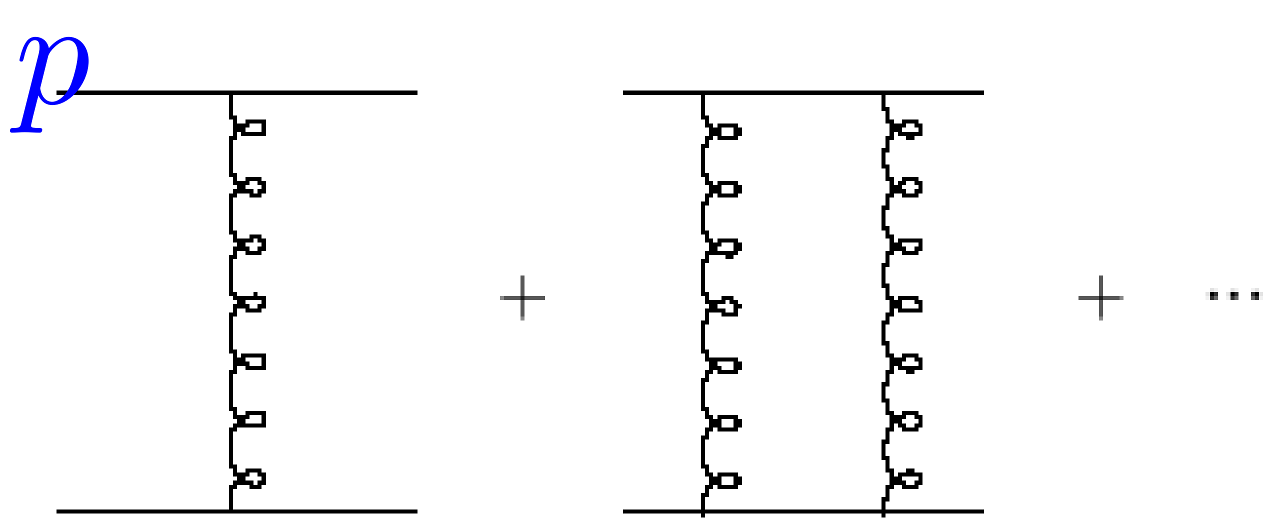

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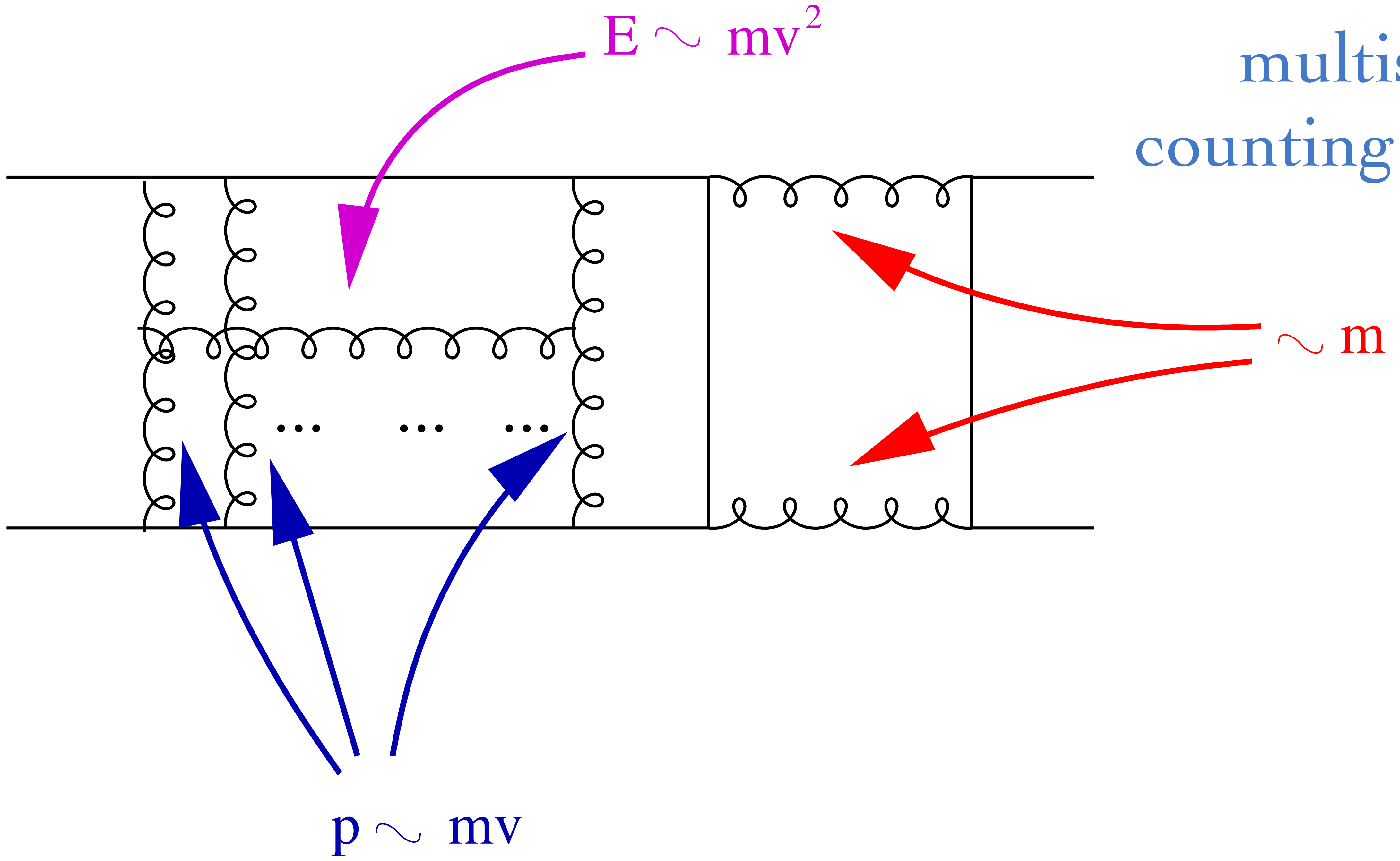

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

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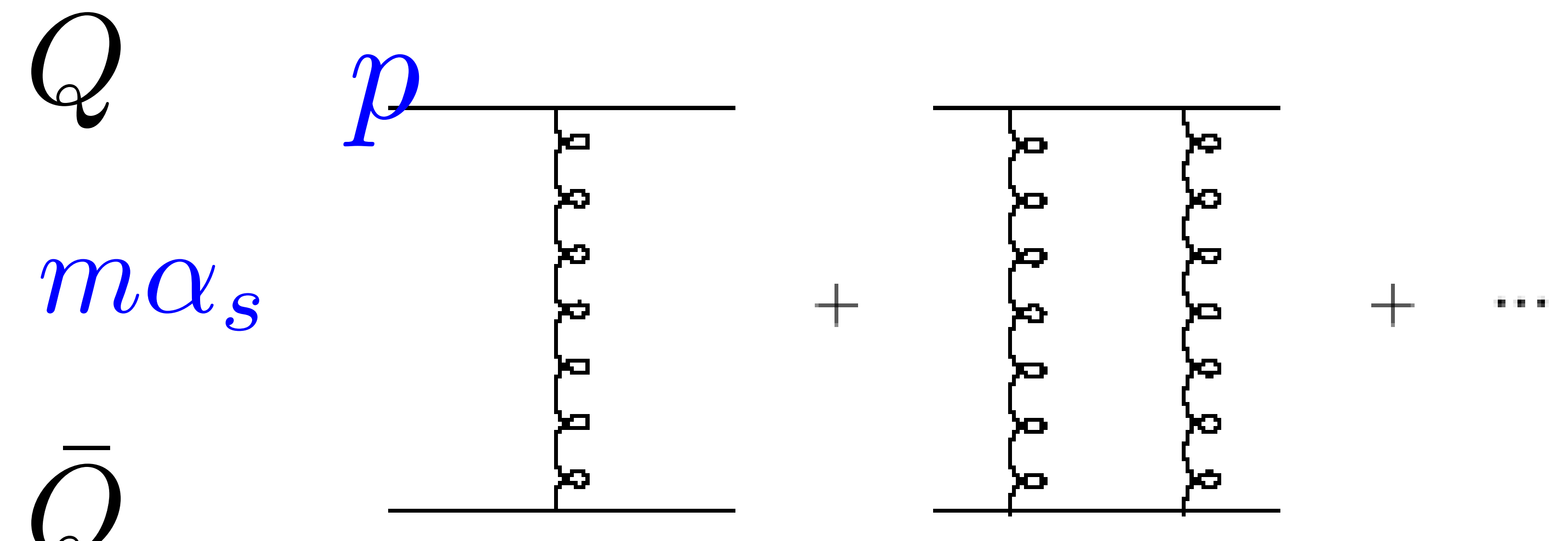

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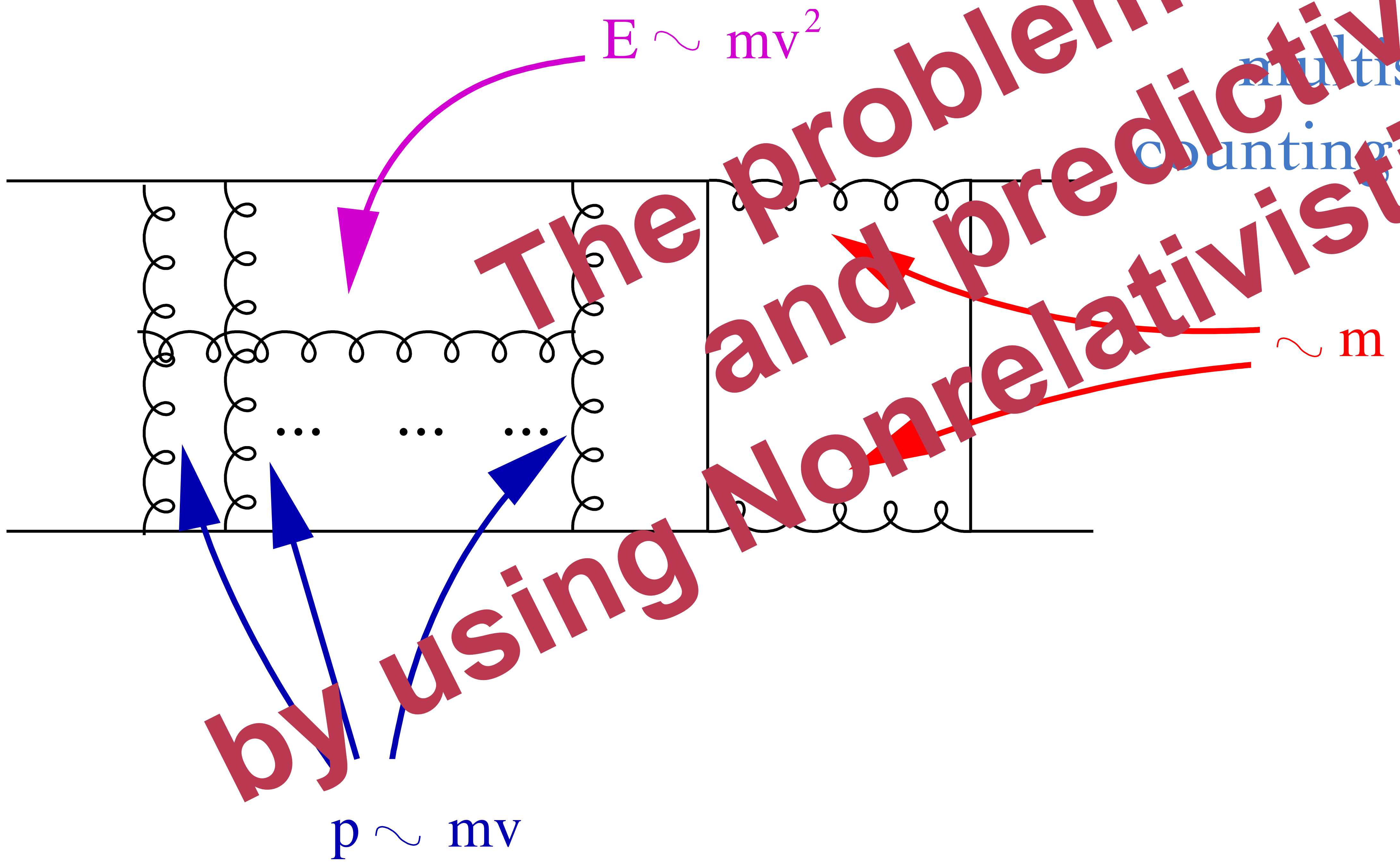

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The problem is greatly simplified and predictivity is achieved by using Nonrelativistic Effective Field Theories

Disentangling the bound state scales at the Lagrangian level in pNREFT has advantages

I. It facilitates higher order perturbative calculations

Relevant for: physics: Hydrogen atom (e.g. proton radius), positronium (e.g. width, hfs), muonium ttbar threshold production; Dark matter annihilation and production close to threshold; SUSY particles annihilation and production; QQbar, QQq and QQQ with small radius; extraction of SM parameters

II. In QCD (or in a strongly coupled theory) it factorizes automatically high energy contributions (perturbative) from low-energy (nonperturbative, thermal) ones

Relevant for: pionium and precision chiral dynamics; nucleon-nucleon systems; Quarkonium, Exotic X, Y, Z states, Quarkonium in hot QCD medium in heavy ion collisions; confinement and nonperturbative effects

III. It allows to integrate out hierarchically other scales using other EFTs (for example the temperature T using Hard Thermal Loop (HTL) EFT) and to apply lattice directly on the low energy factorized part

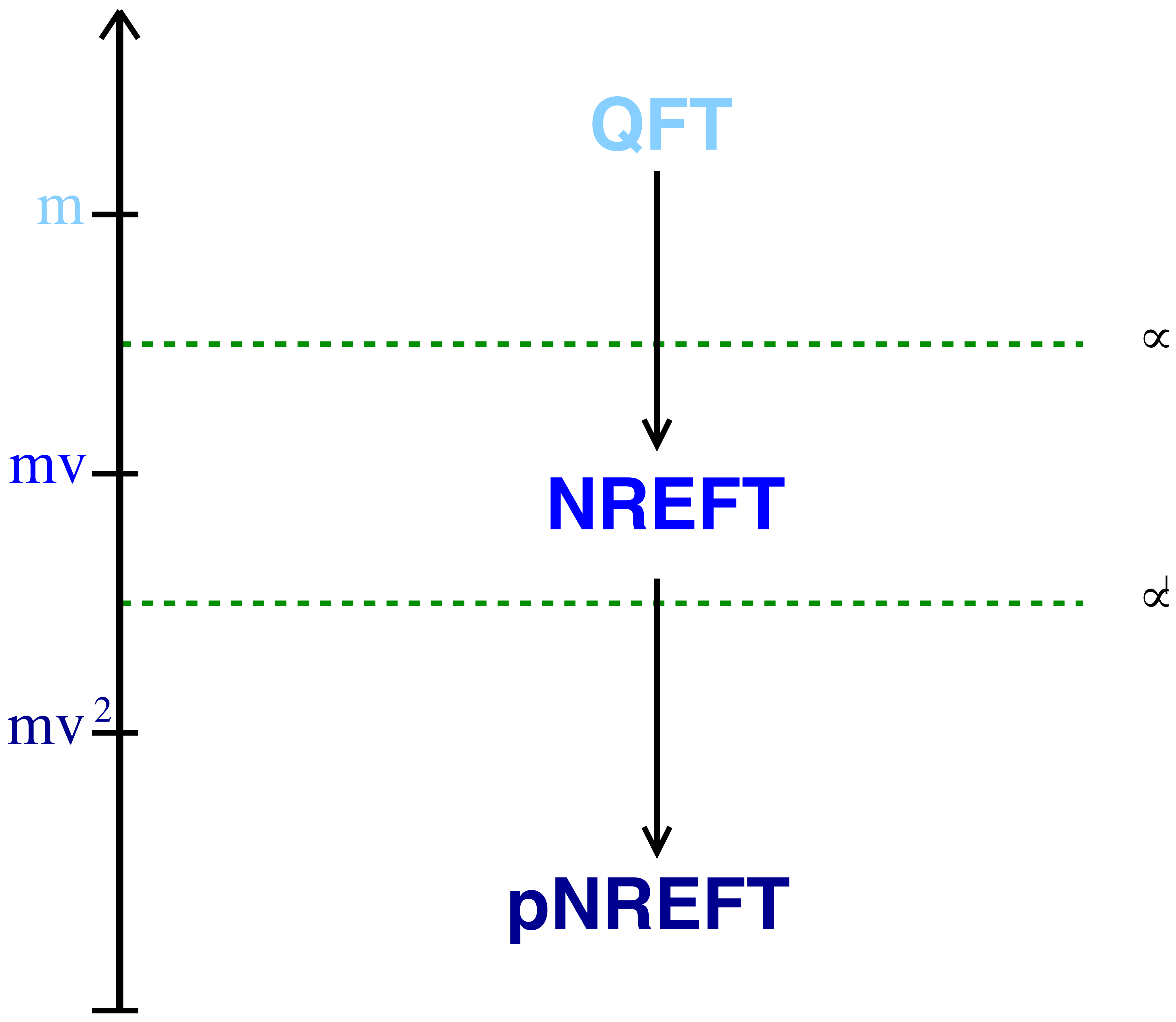
IV. It allows to define in QFT objects of great importance like potentials both in the perturbative and in the nonperturbative regime

V. More conceptually It provides a field theoretical foundation of the Schroedinger equation

Disentangling the bound state scales at the Lagrangian level has advantages : pNREFT

$$\mathcal{L}_{\rm pNREFT} = \int d^3 r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

separates the Schroedinger dynamics of the two particle field ϕ from the low energy dynamics encoded in $\Delta\mathcal{L}$



pNREFT is the lowest energy EFT that can be constructed for the NR bound system.

Notice: if QFT = QED, pNRQED gives a proper version of Quantum Mechanics

N B., A. Pineda, J. Soto, A.Vairo. Rev. Mod. Phys 77 (2005) 1423

The lowest dynamical energy and the corresponding pNREFT depend on the system in consideration, e.g. to describe Van der Waals interaction between bound states the lowest scale is lower than mv^2

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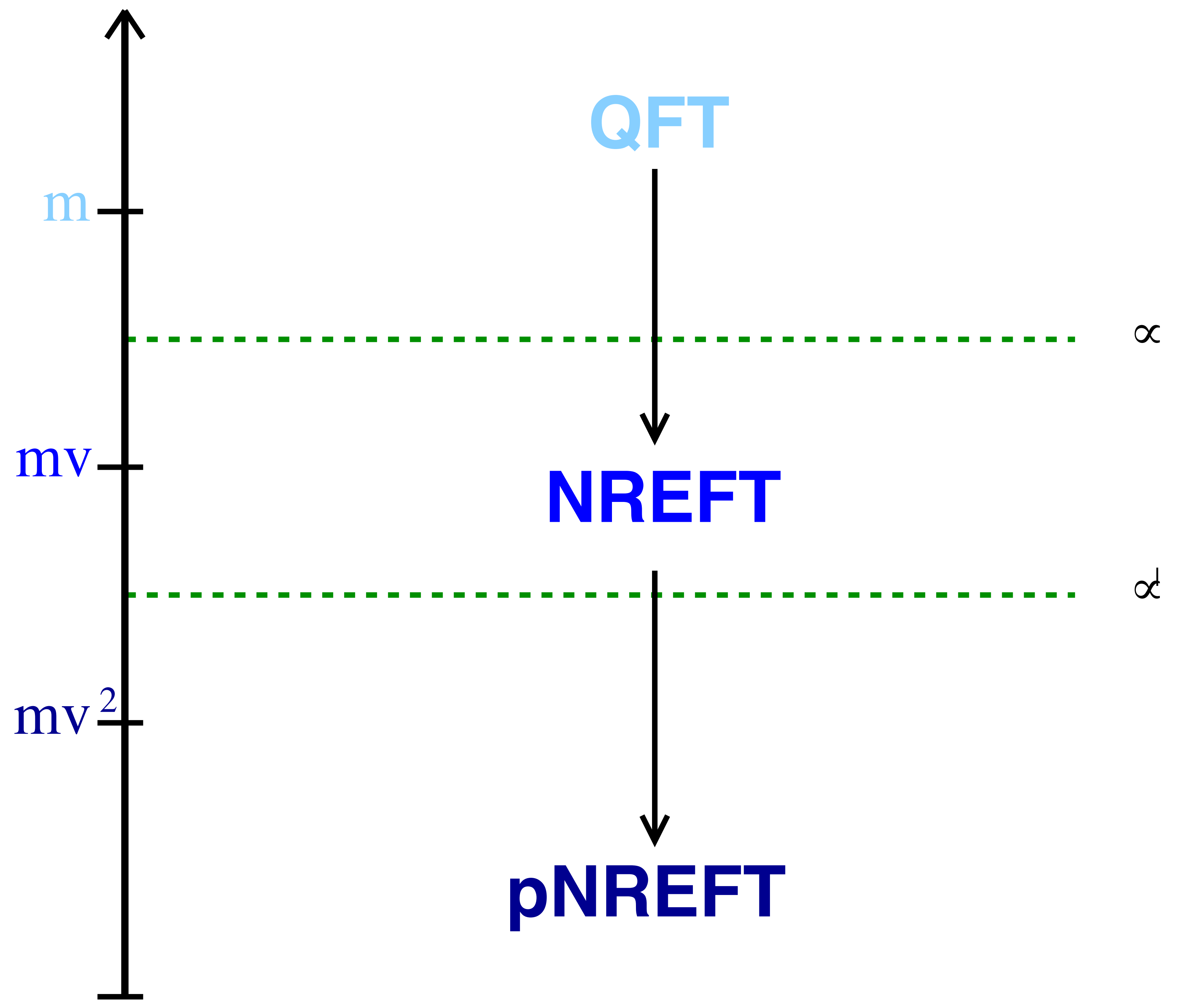

pNREFT is the lowest energy EFT that can be constructed for the NR bound system.

It implements the Schroedinger eq. as zero order problem, define the potentials at the level of the QFT, implements systematically retardation corrections (Lamb shift), it encodes Poincare' invariance, and it is equivalent at any given order of the expansion to the underlying QFT

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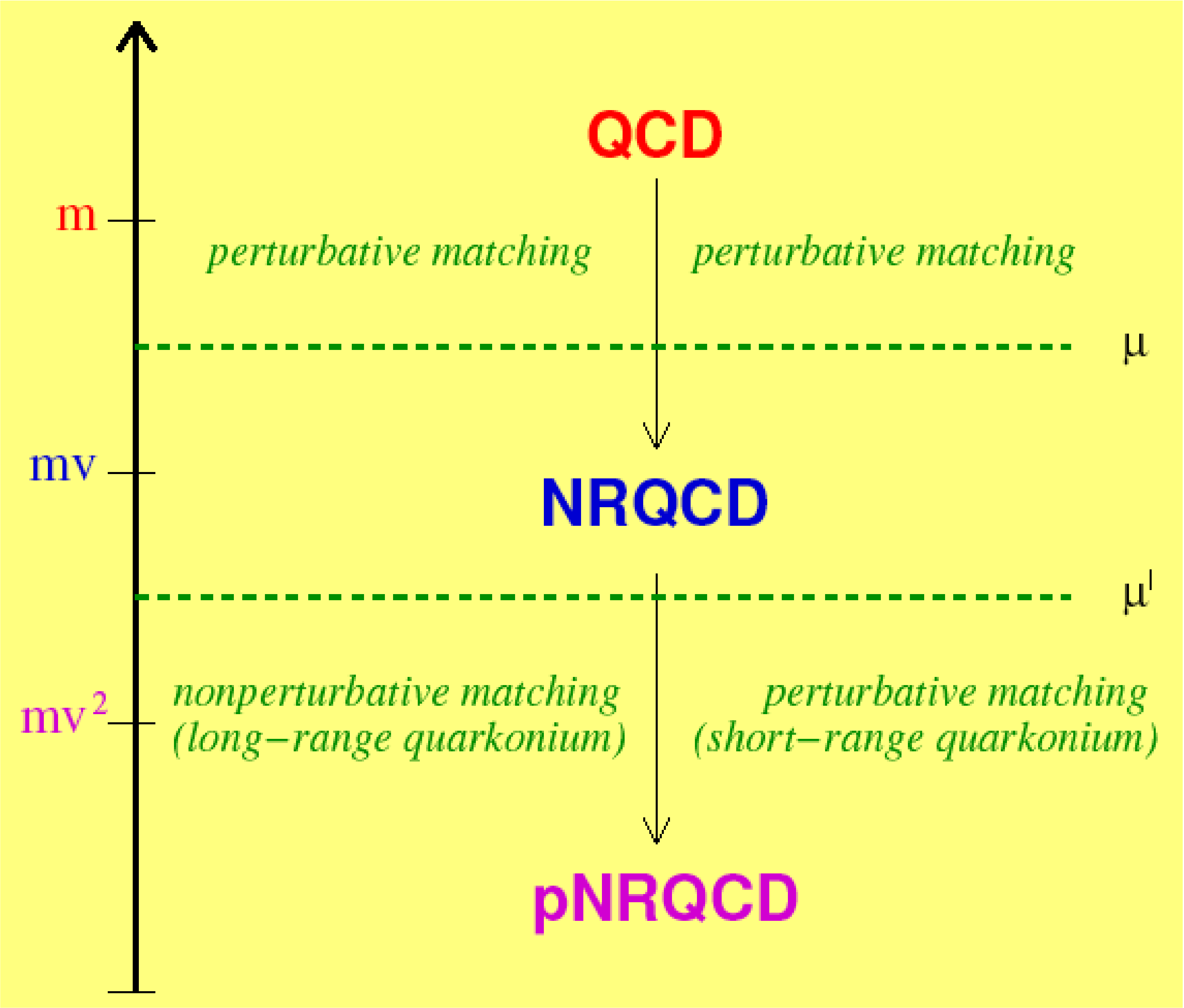


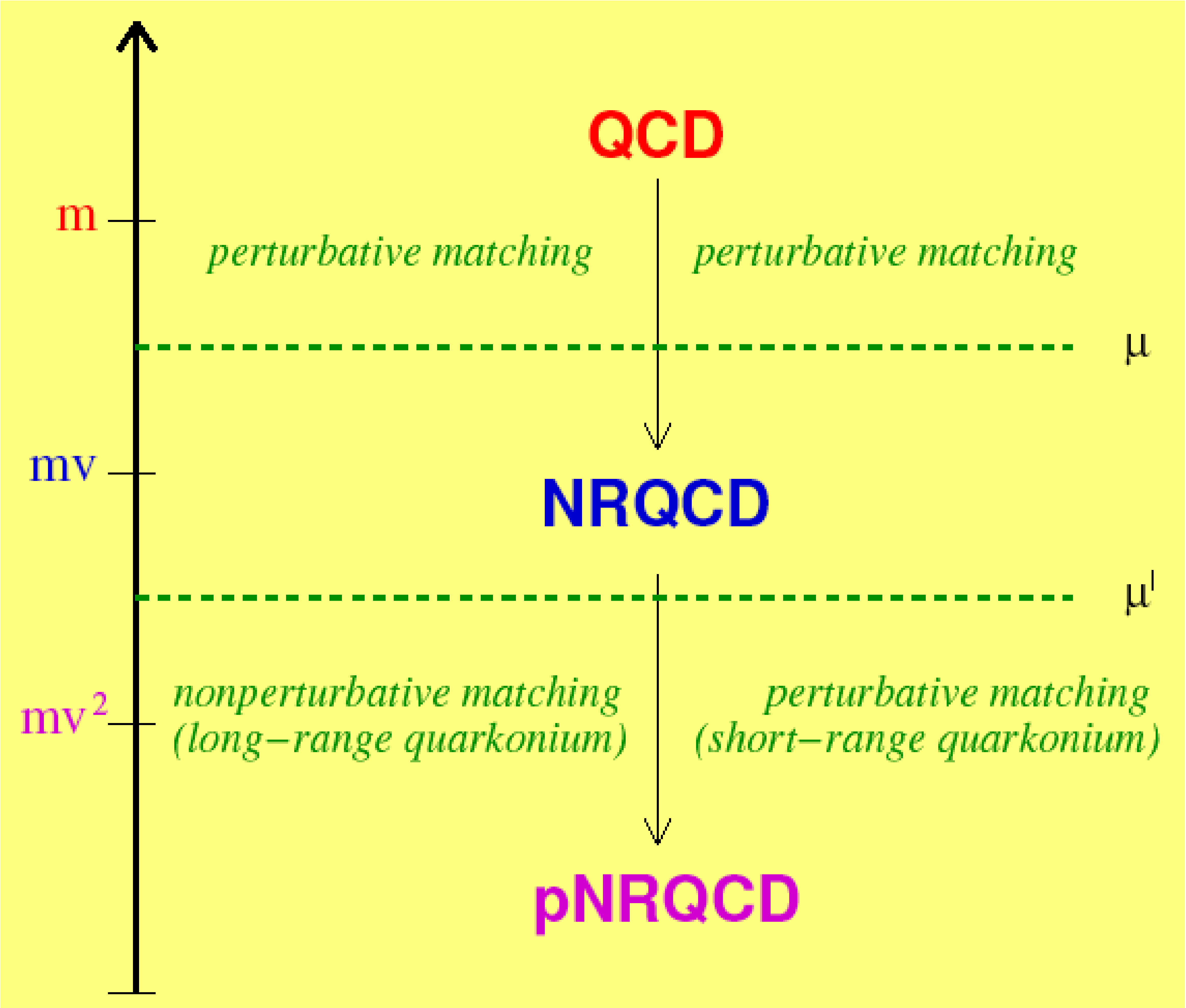
Color degrees of freedom
3X3=1+8
singlet and octet QQbar

Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

Quarkonium with NR EFT



$$\mathcal{L}_{\text{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

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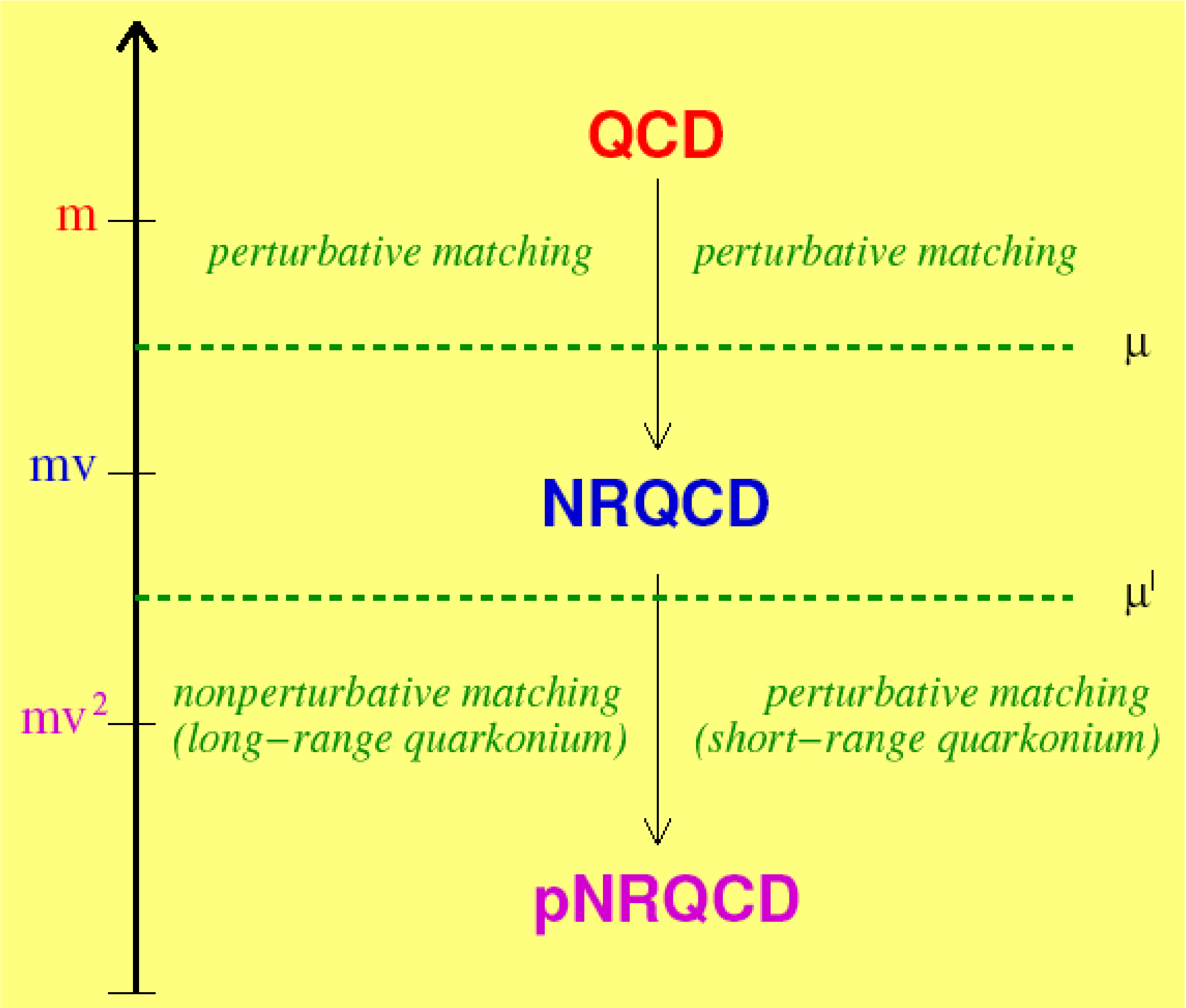

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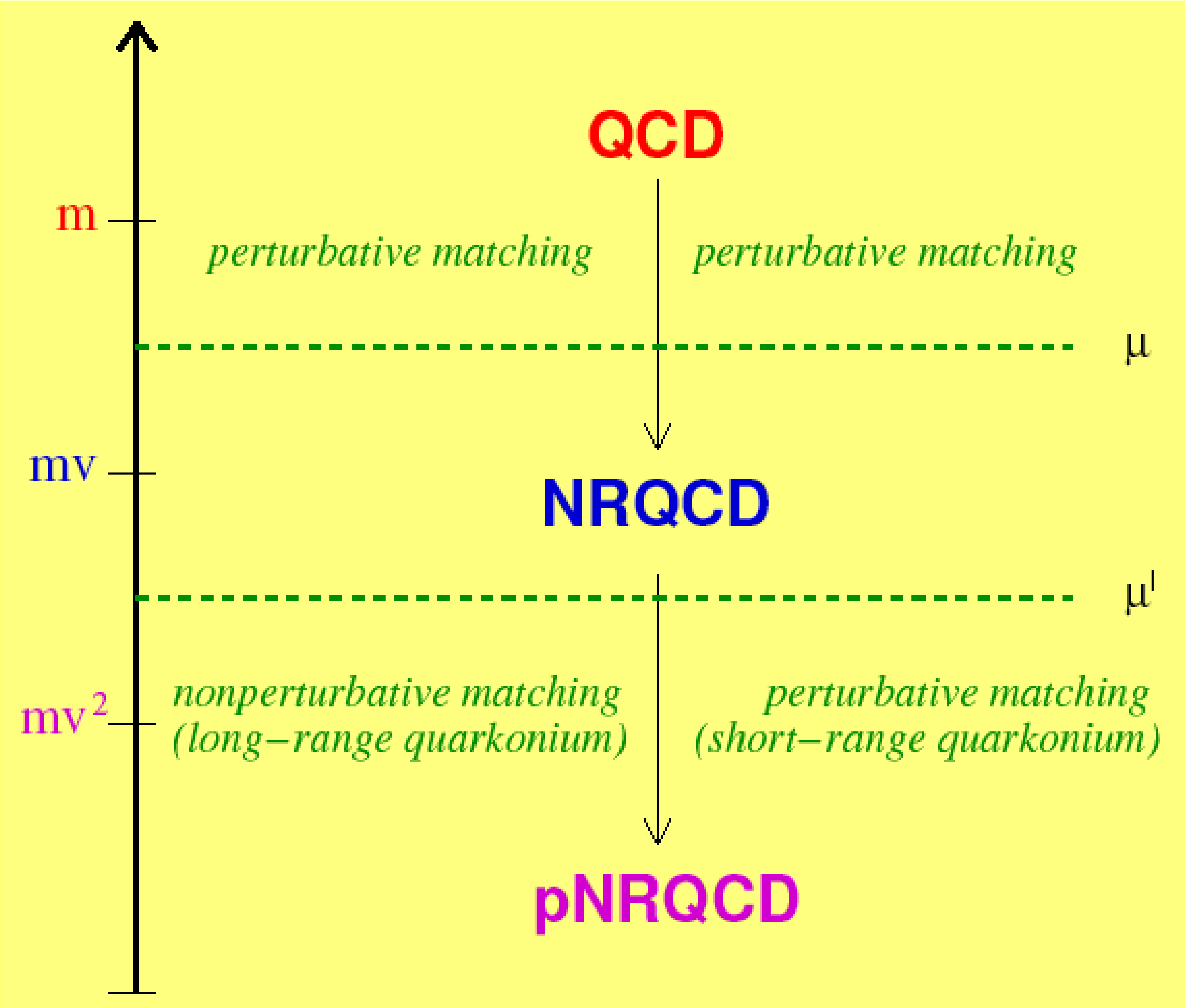

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$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

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momentum)

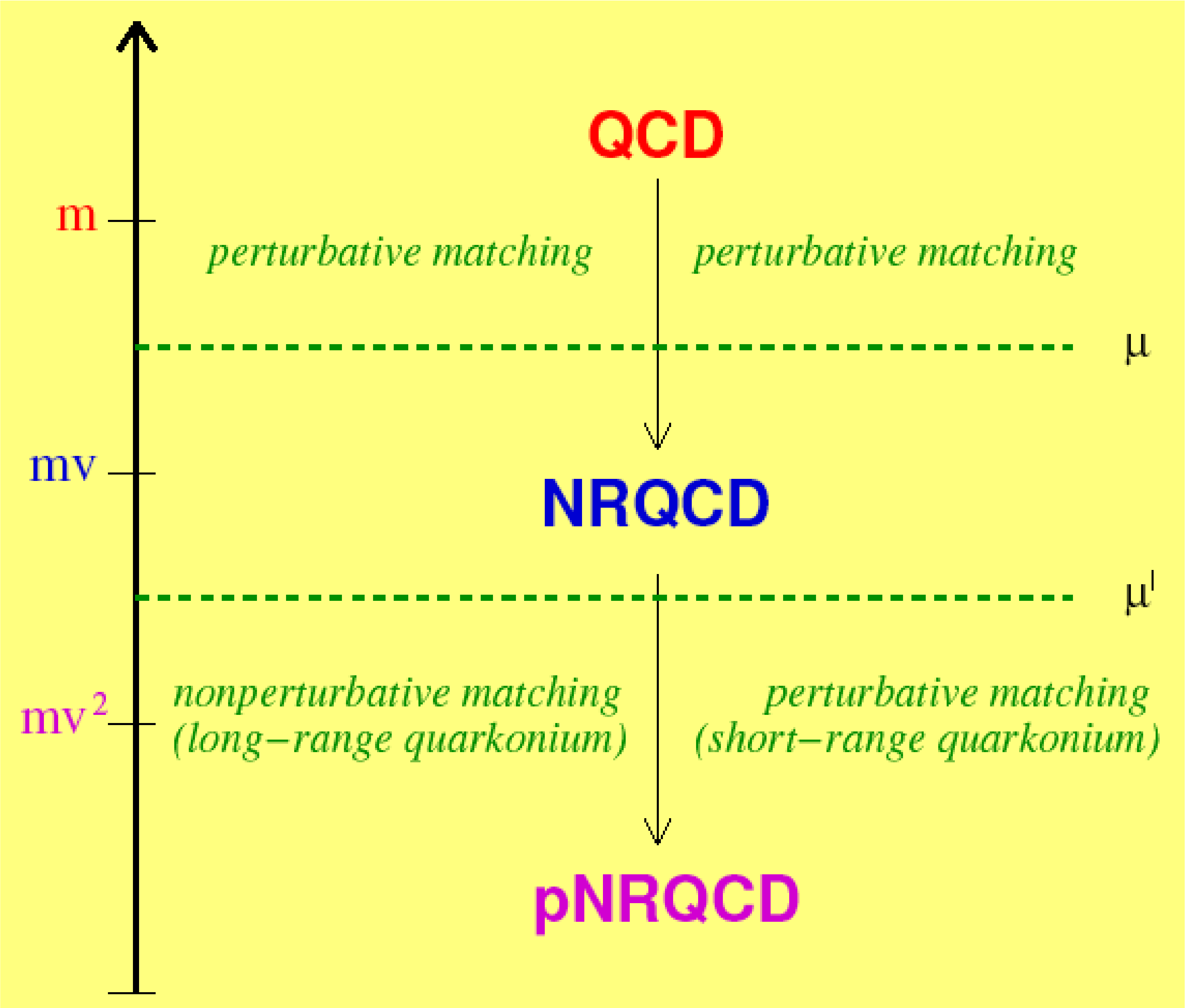
Ultrasoft
(binding energy)

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

Quarkonium with NR EFT

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative
momentum)

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

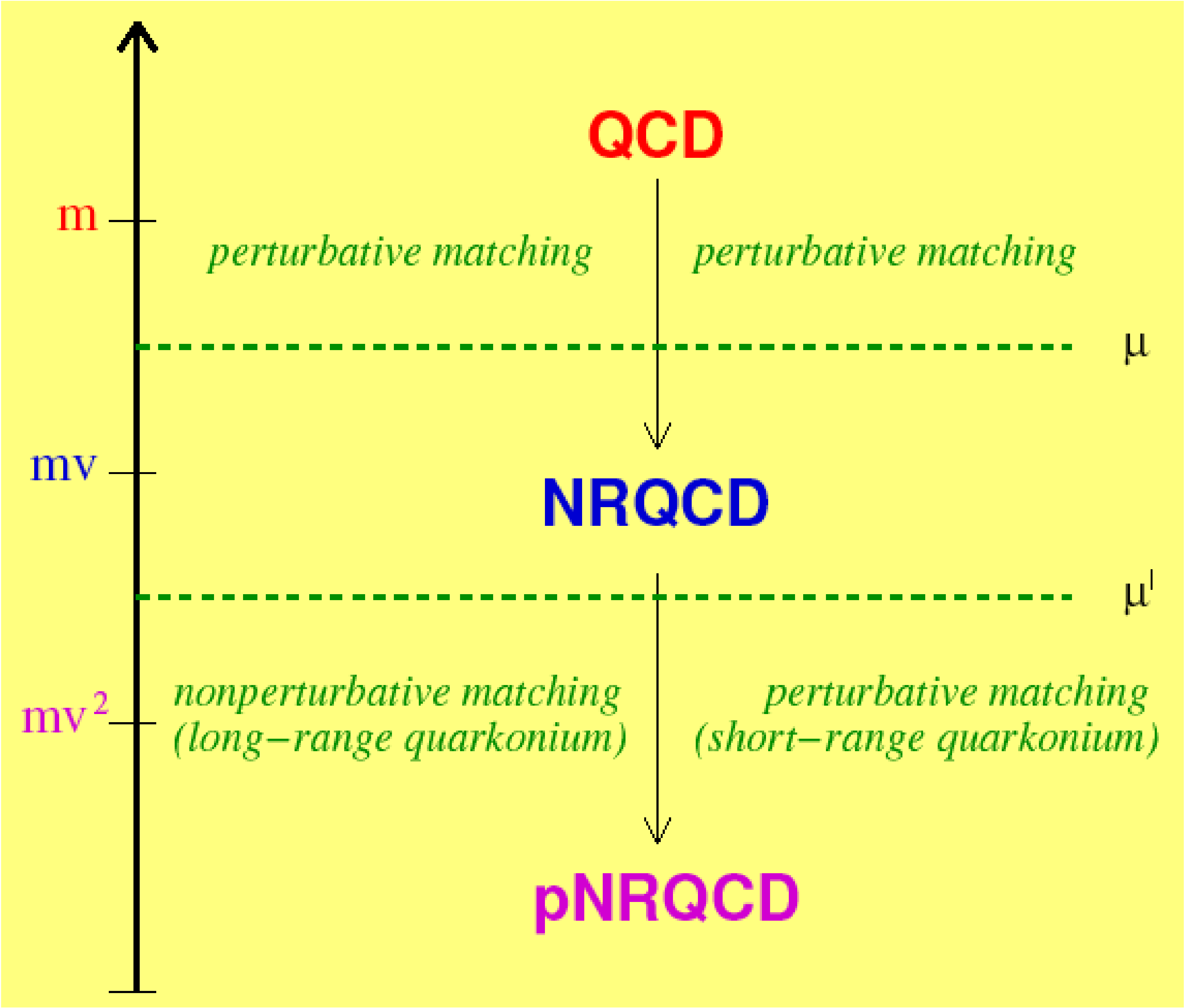
Ultrasoft
(binding energy)

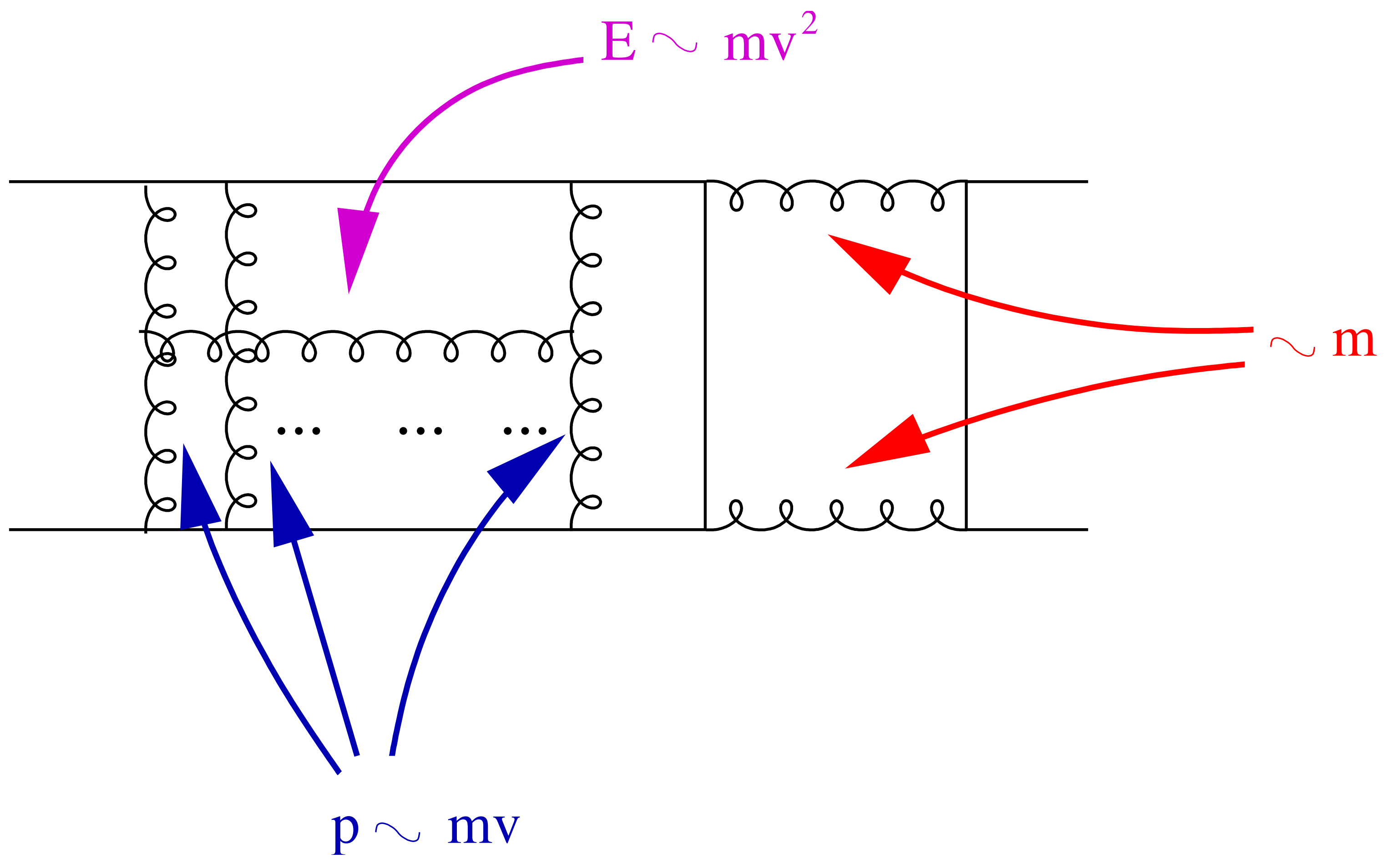
$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

QQbar systems with NR EFT: Non Relativistic QCD (NRQCD)

Caswell, Lepage 86, Lepage Thacker 88, Bodwin, Braaten, Lepage 95





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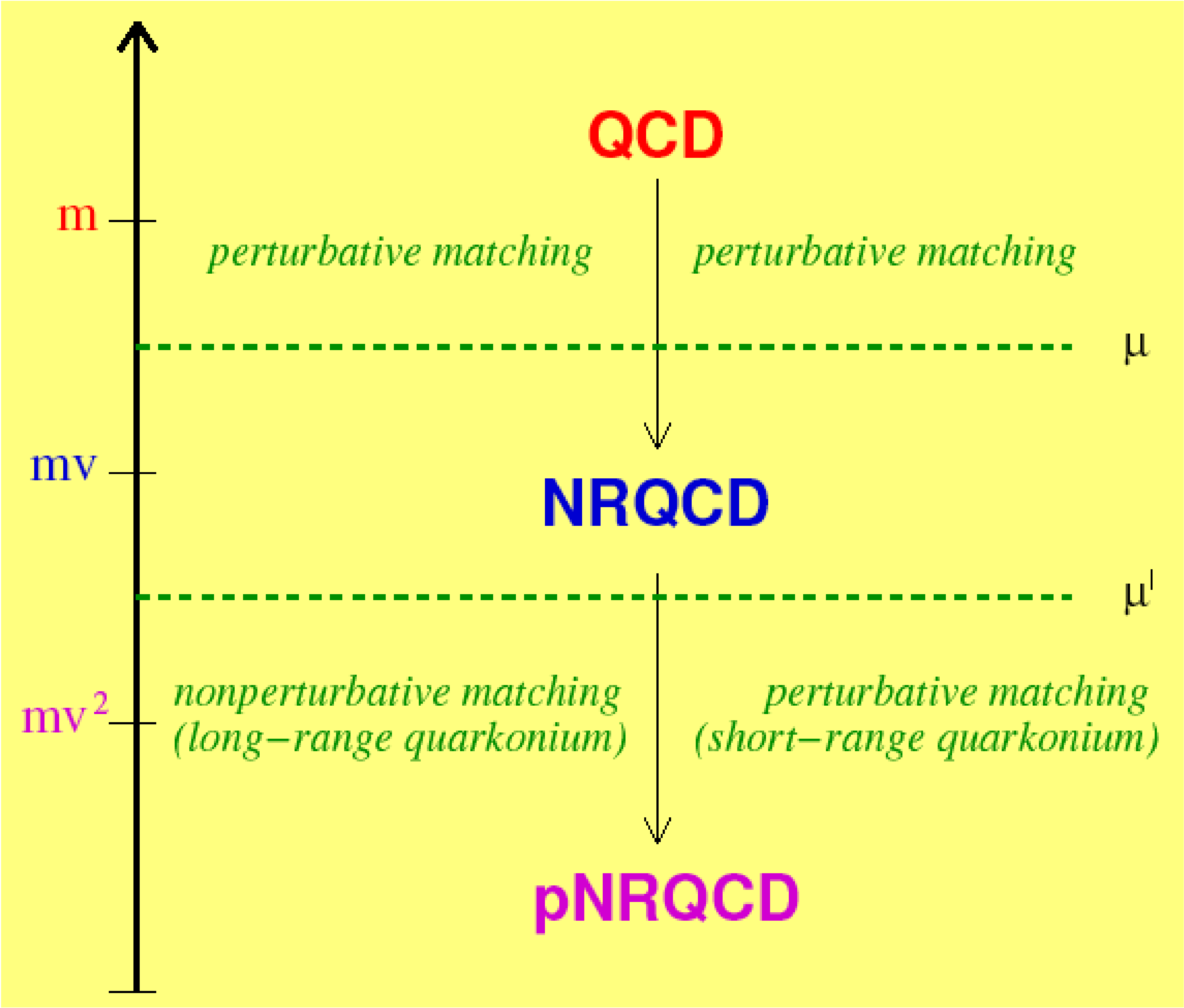

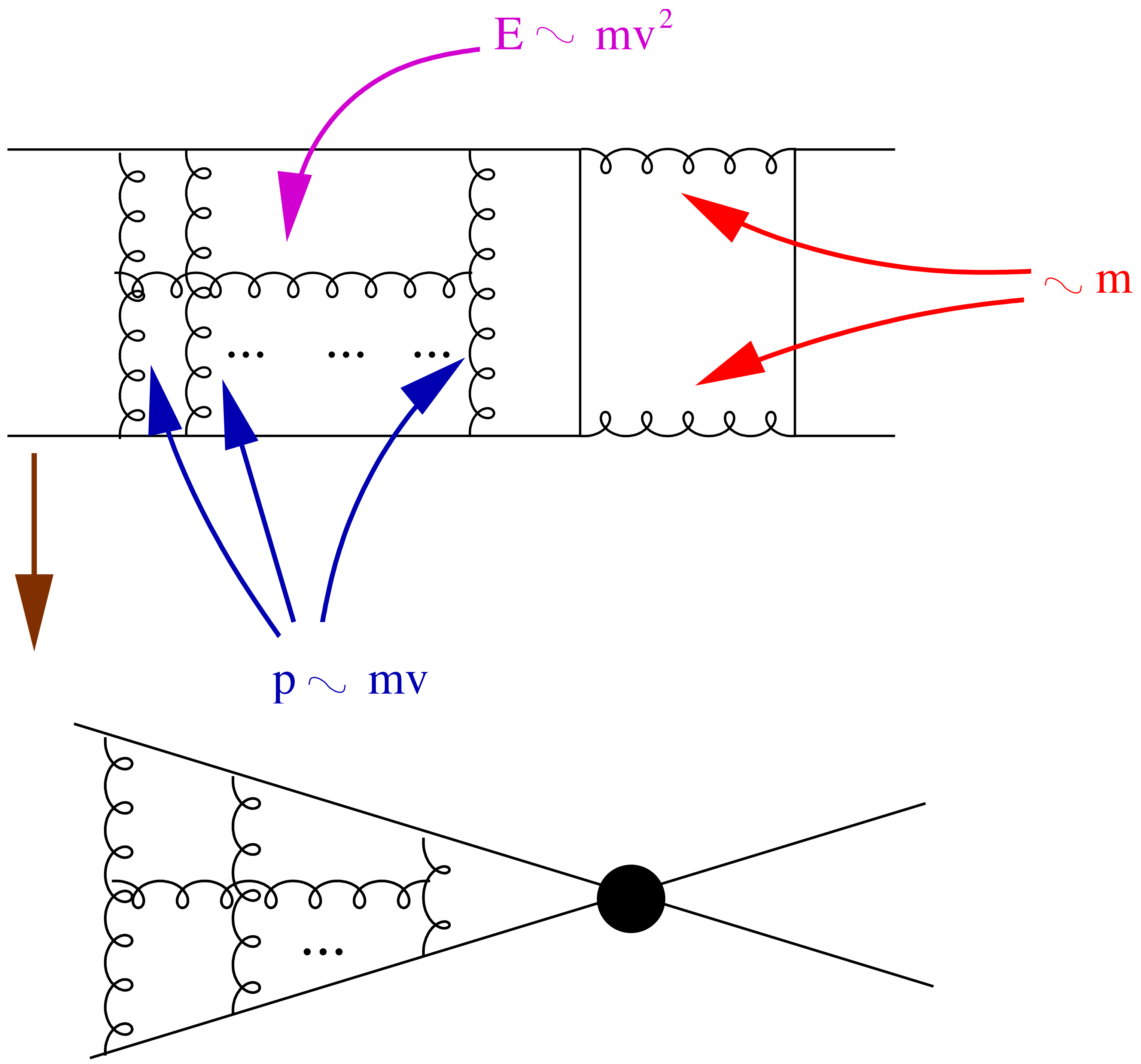

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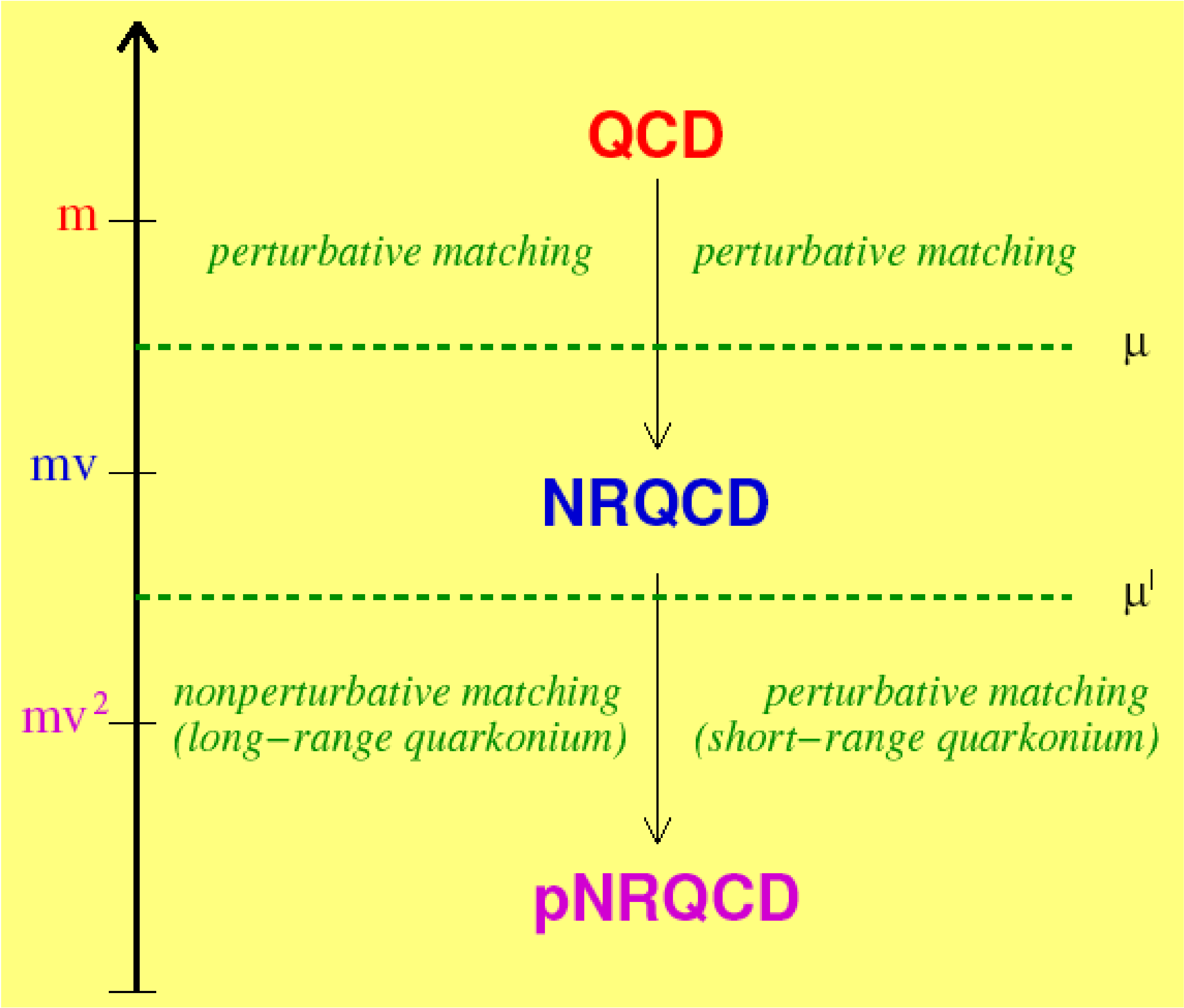
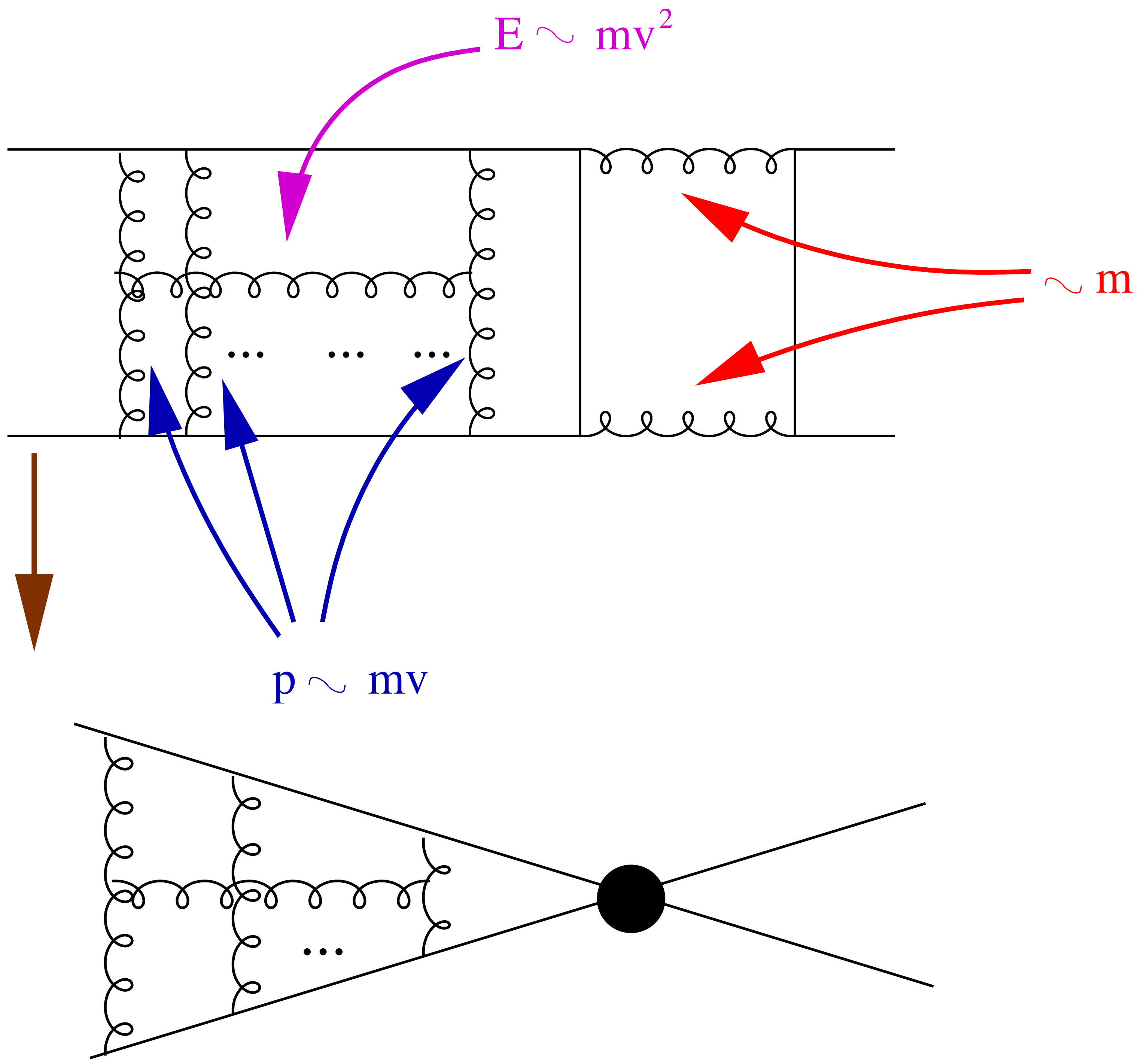


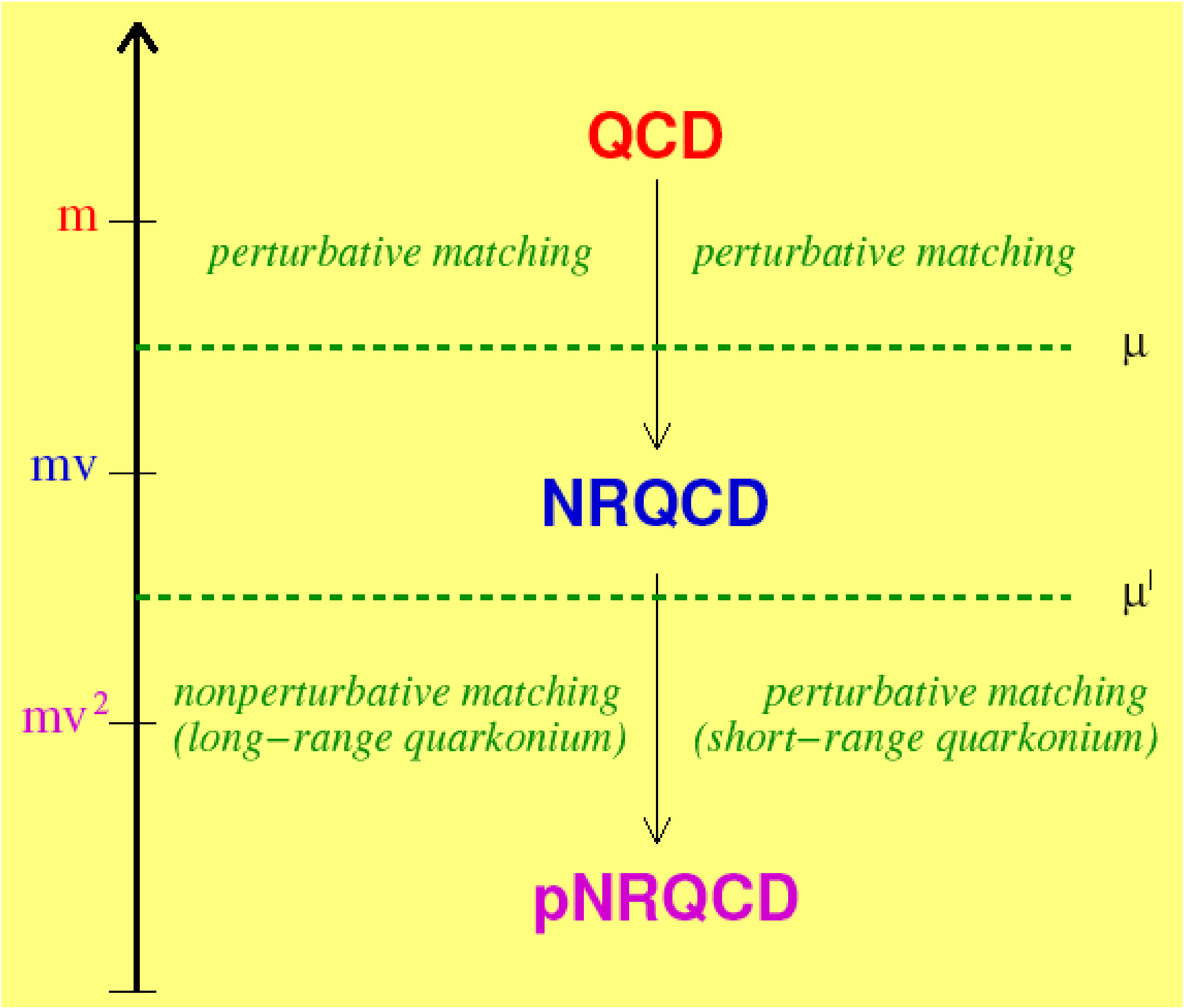


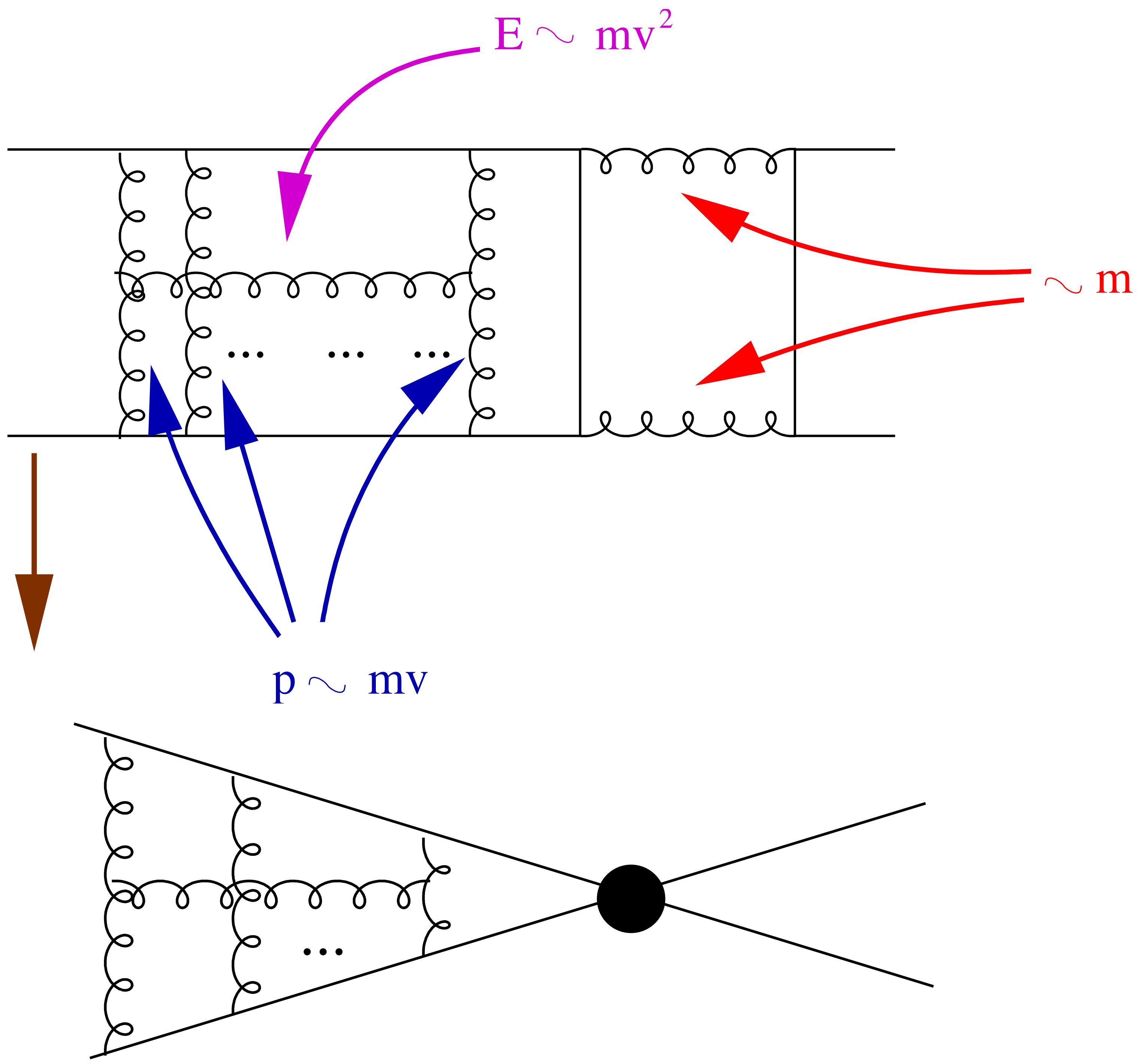
$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

Applications to productions,
Lattice NRQCD…
Still two scales entangled

Quarkonium with NR EFT: potential Non Relativistic QCD (pNRQCD)

Pineda Soto 97, N. B., Pineda, Soto, Vairo 98





For mv >>Lambda_QCD

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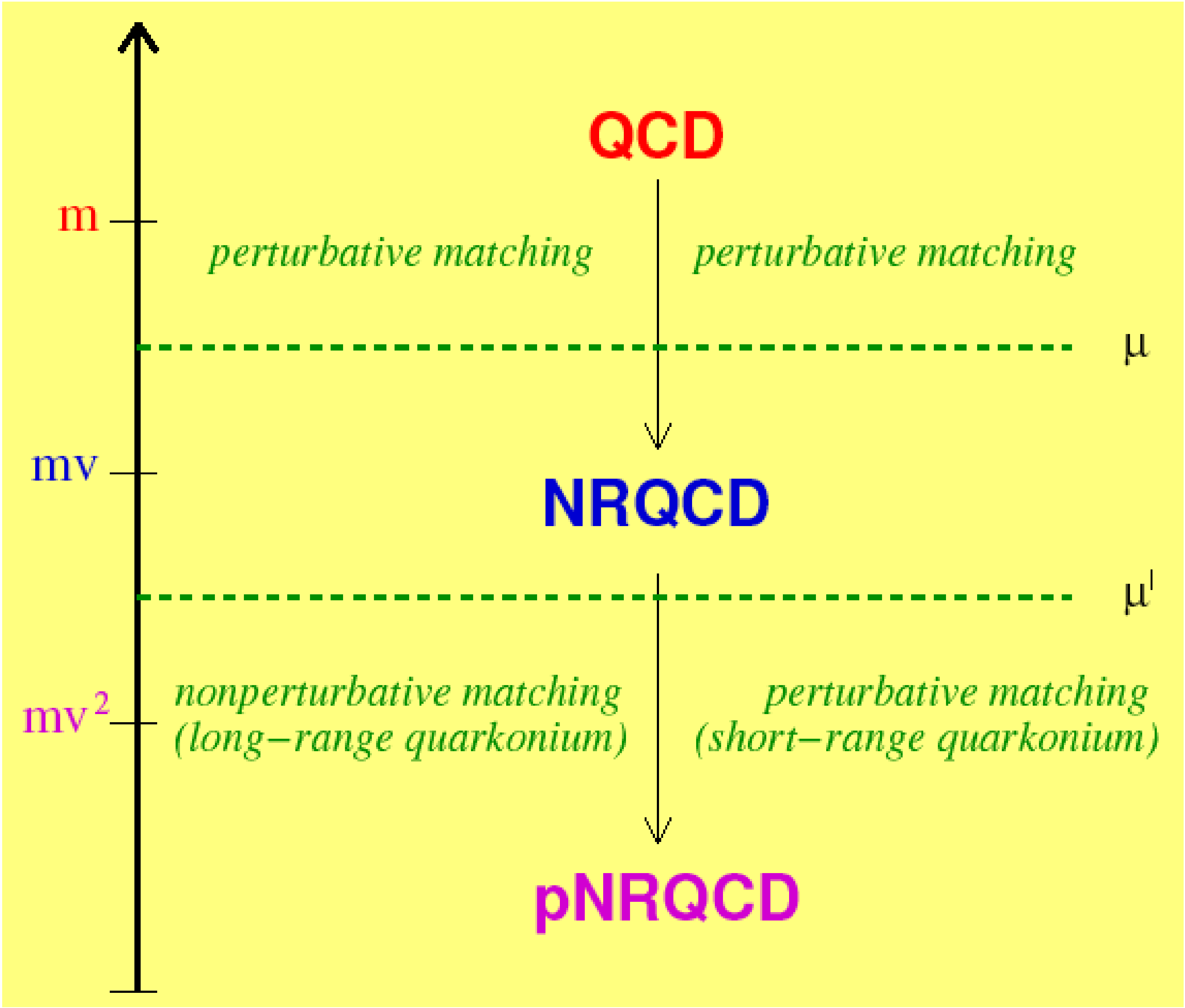

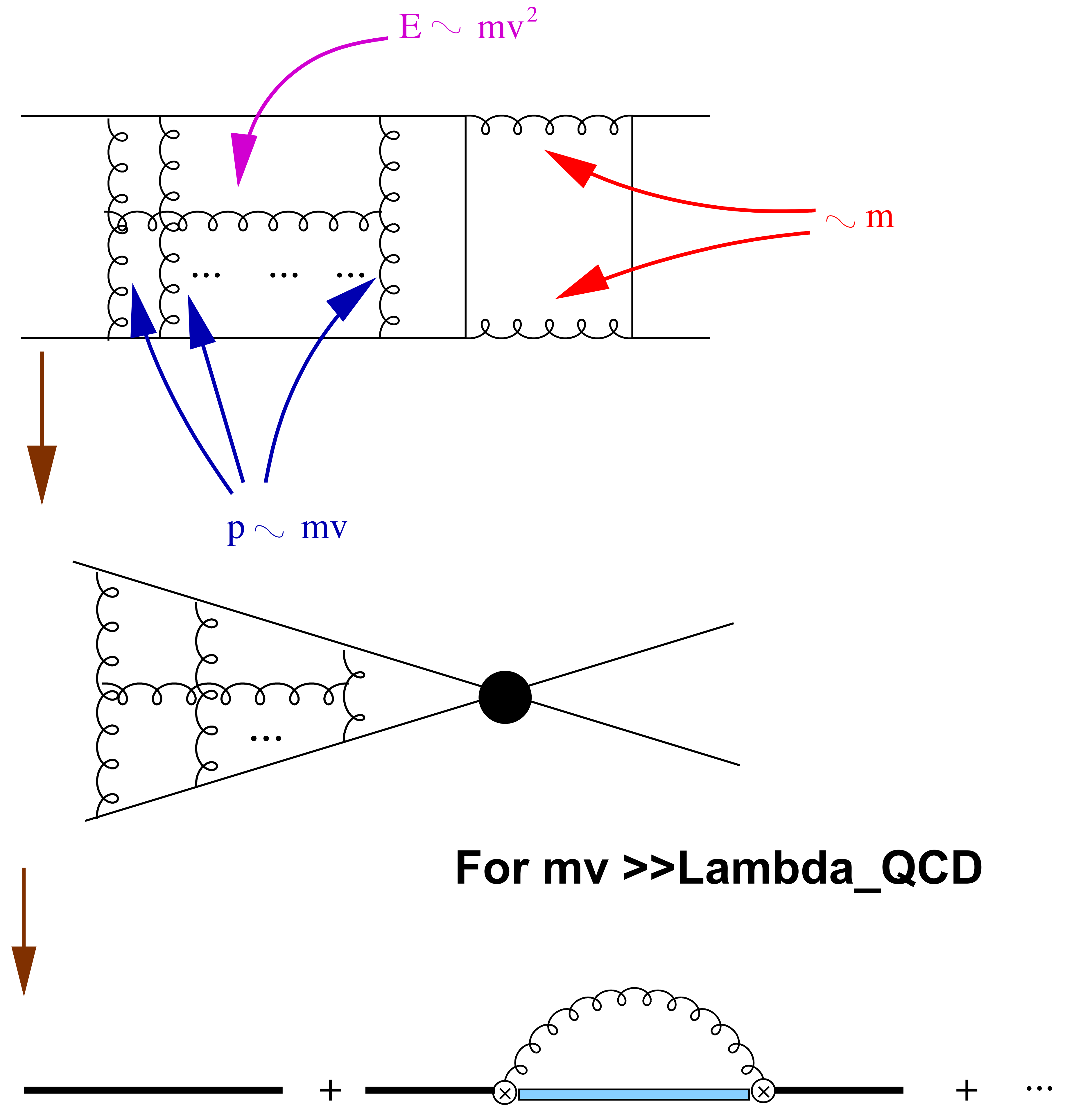

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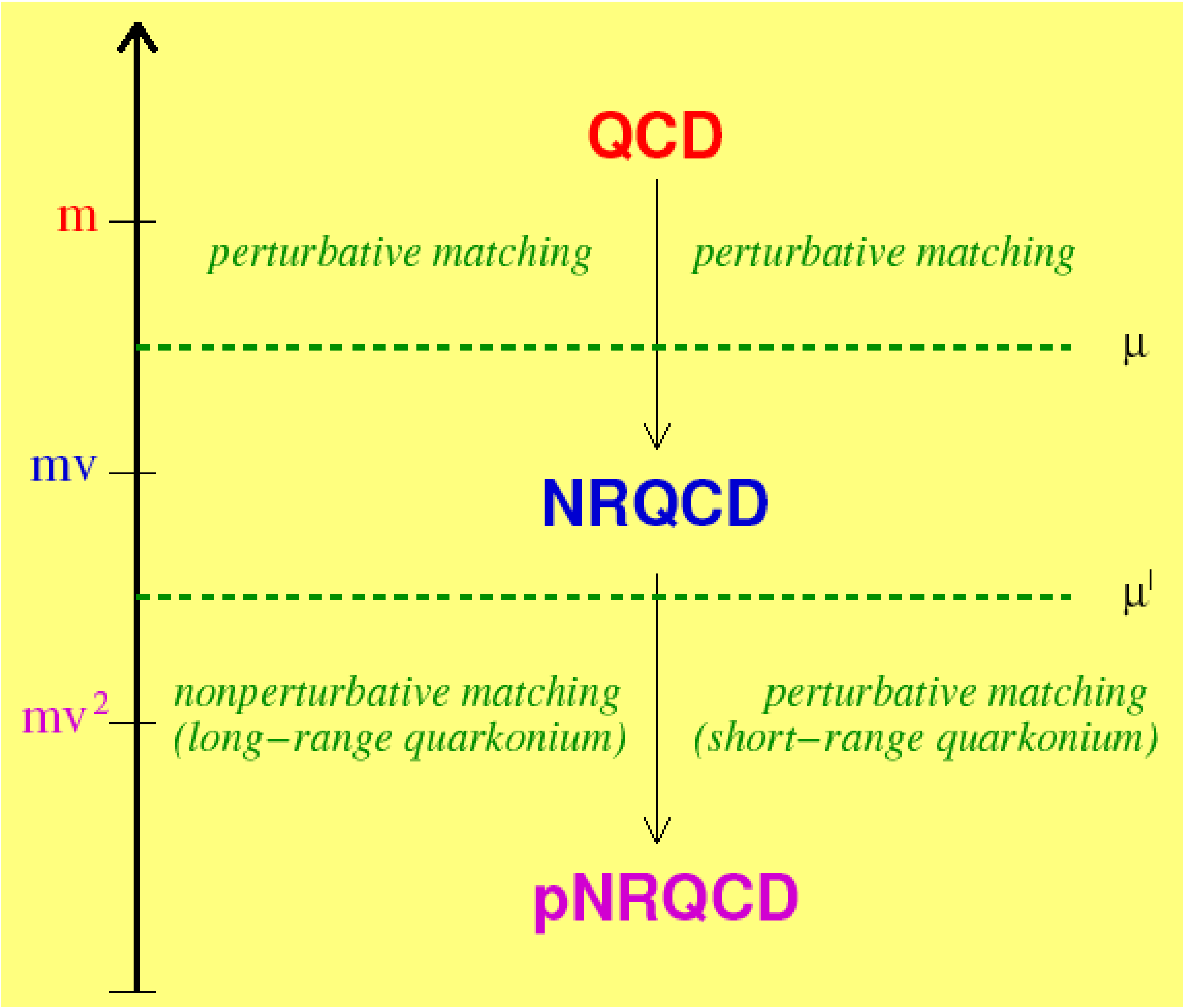

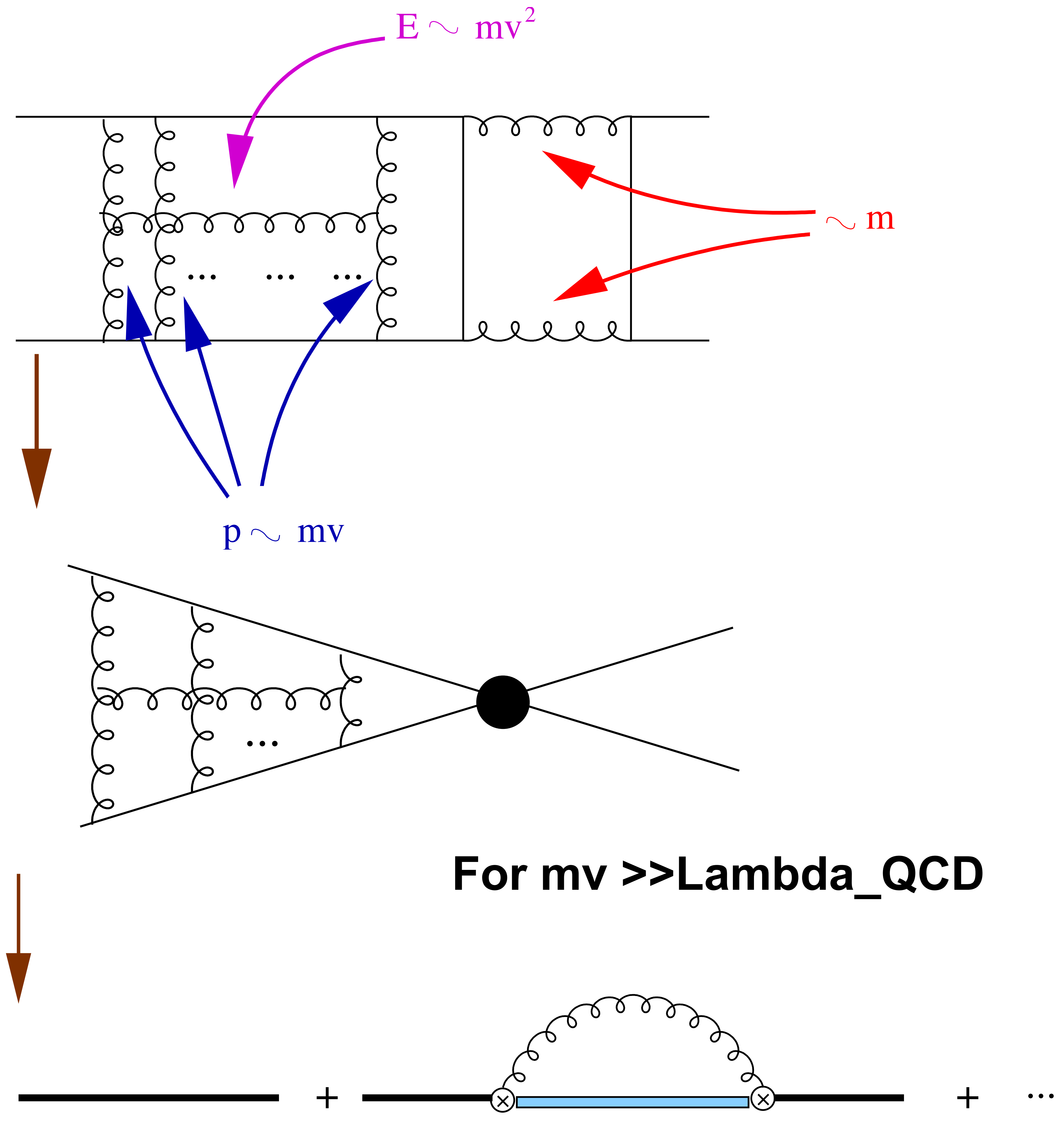

For mv >>Lambda_QCD

$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Quarkonium with NR EFT: potential Non Relativistic QCD (pNRQCD)

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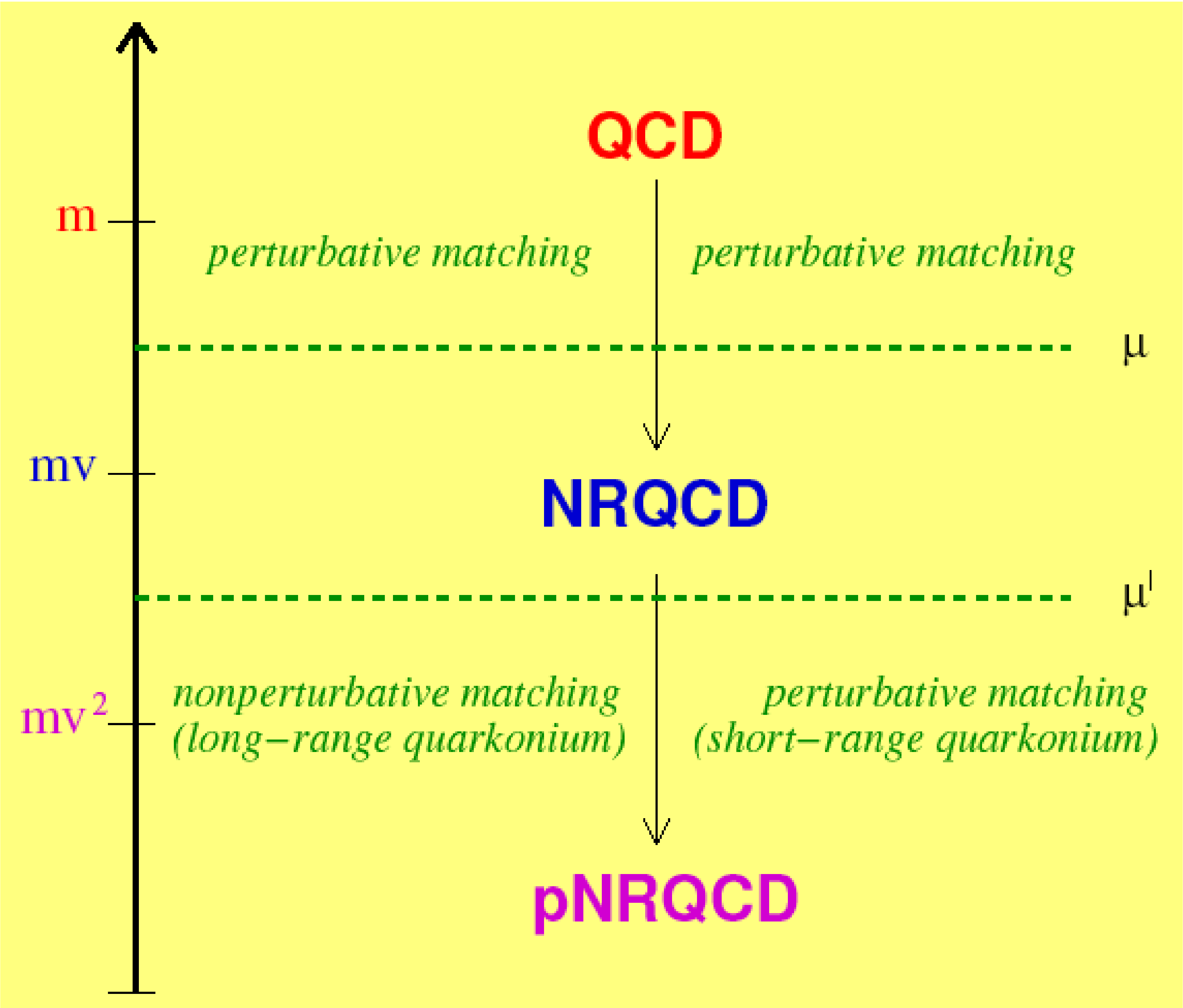

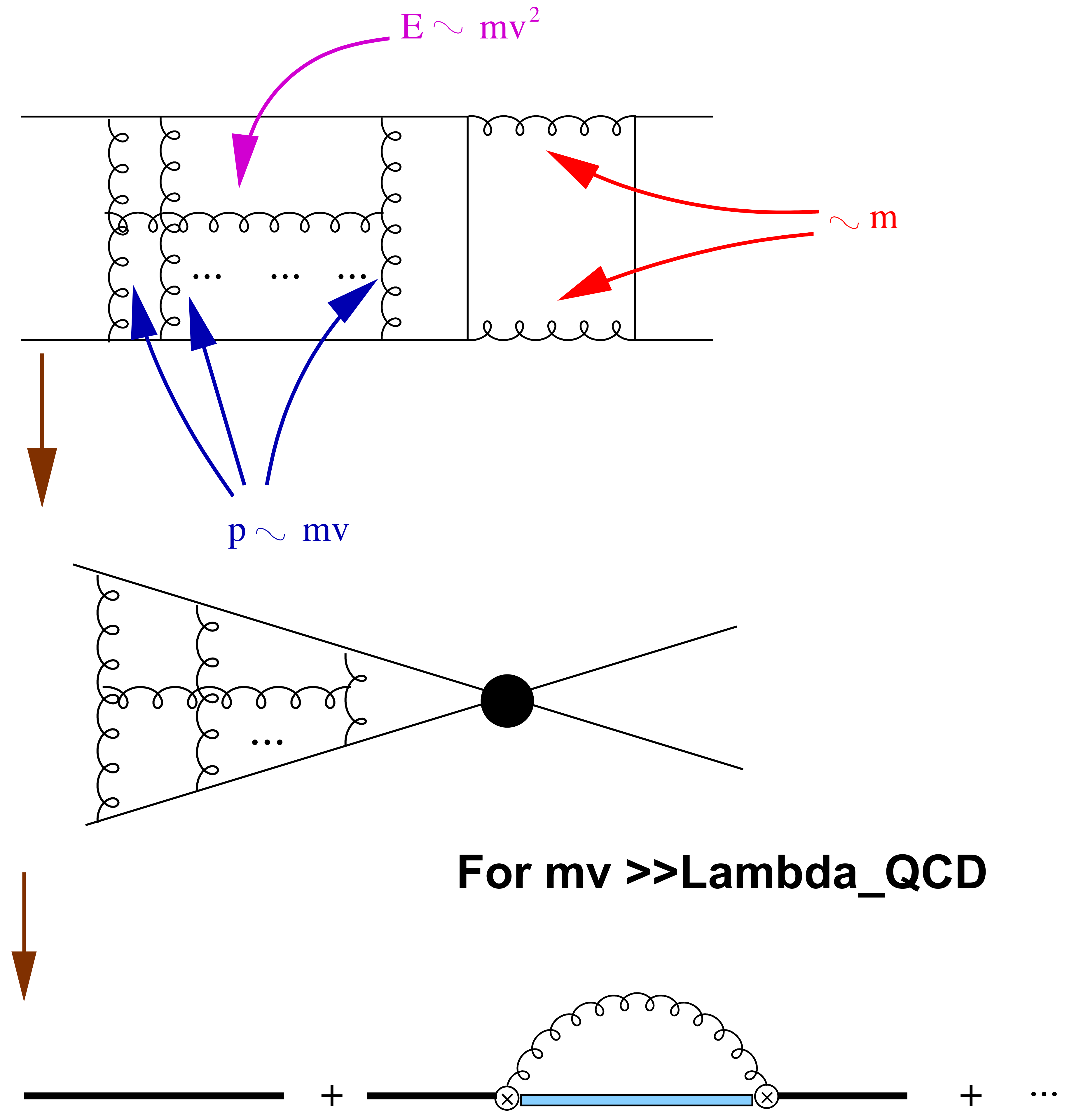

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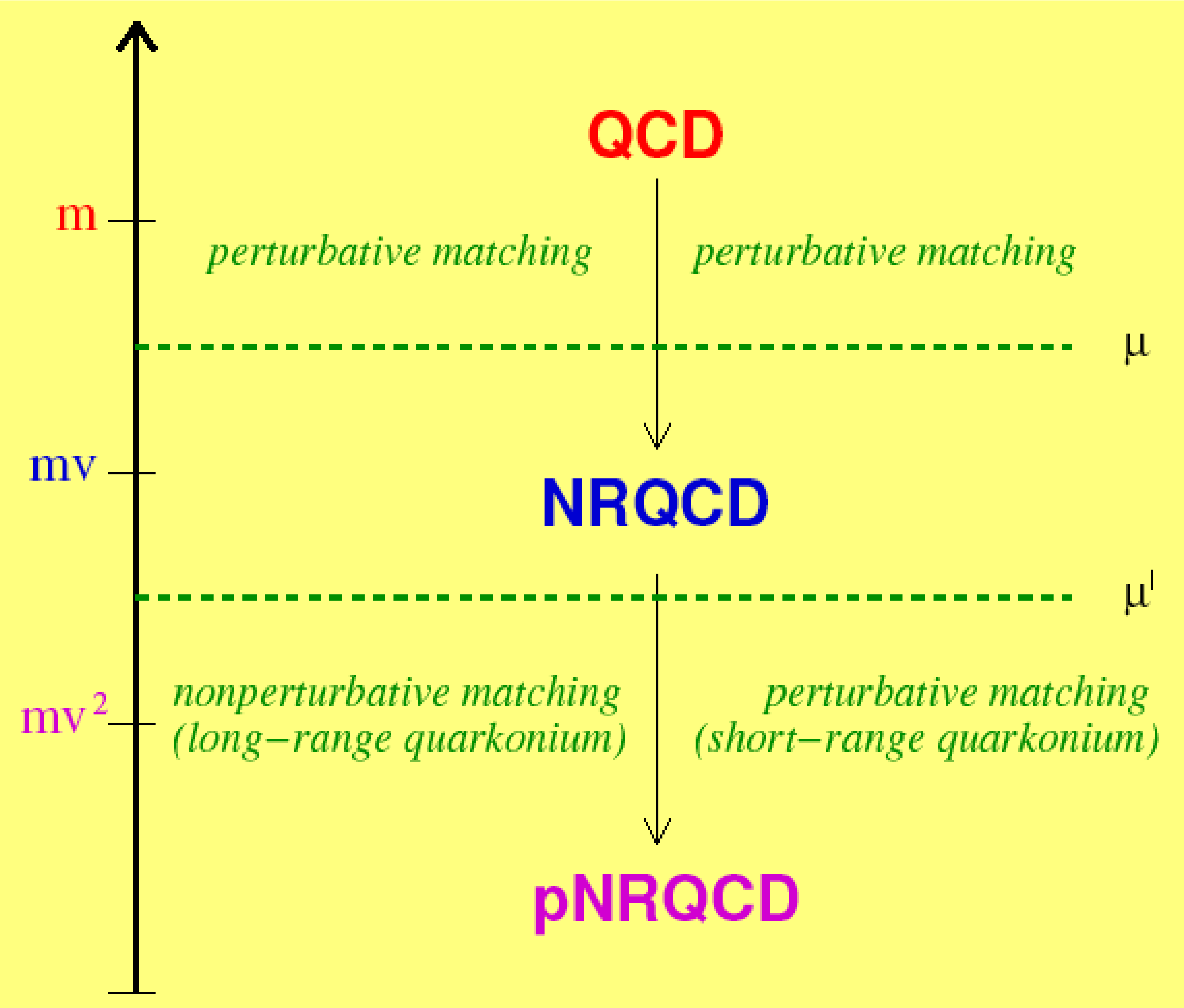
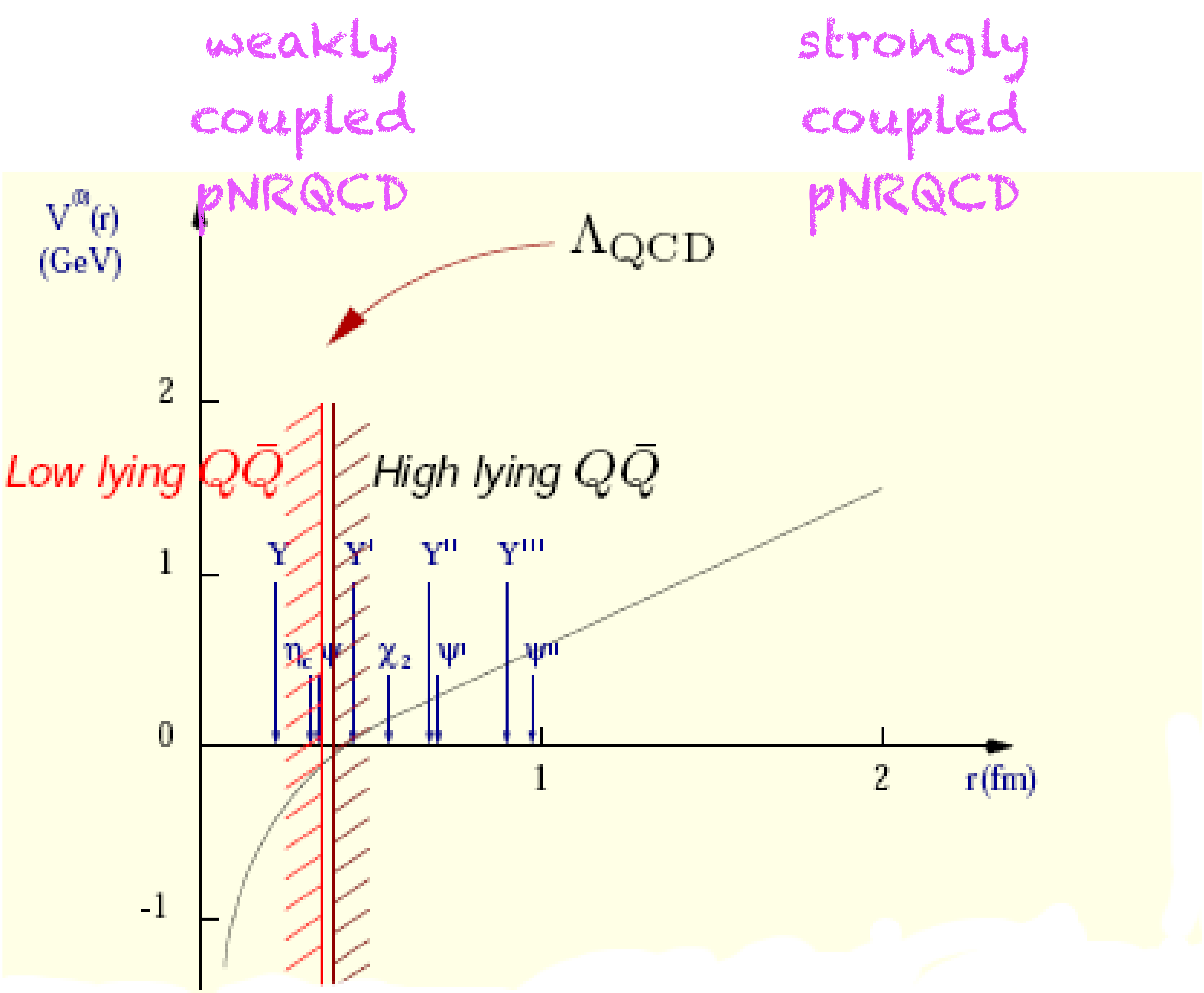
Quarkonium with NREFT: pNRQCD



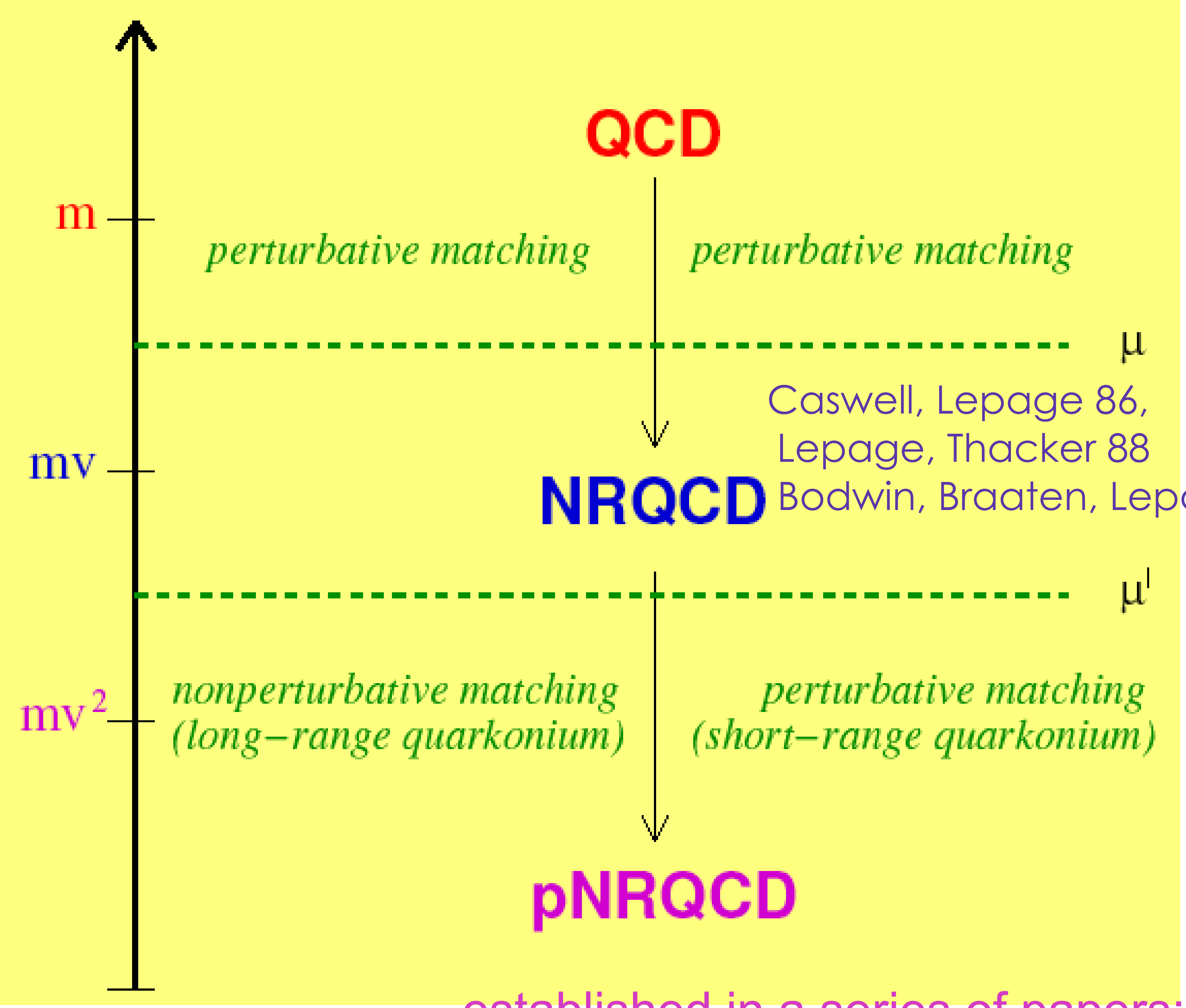


A potential picture arises at the level of pNRQCD:

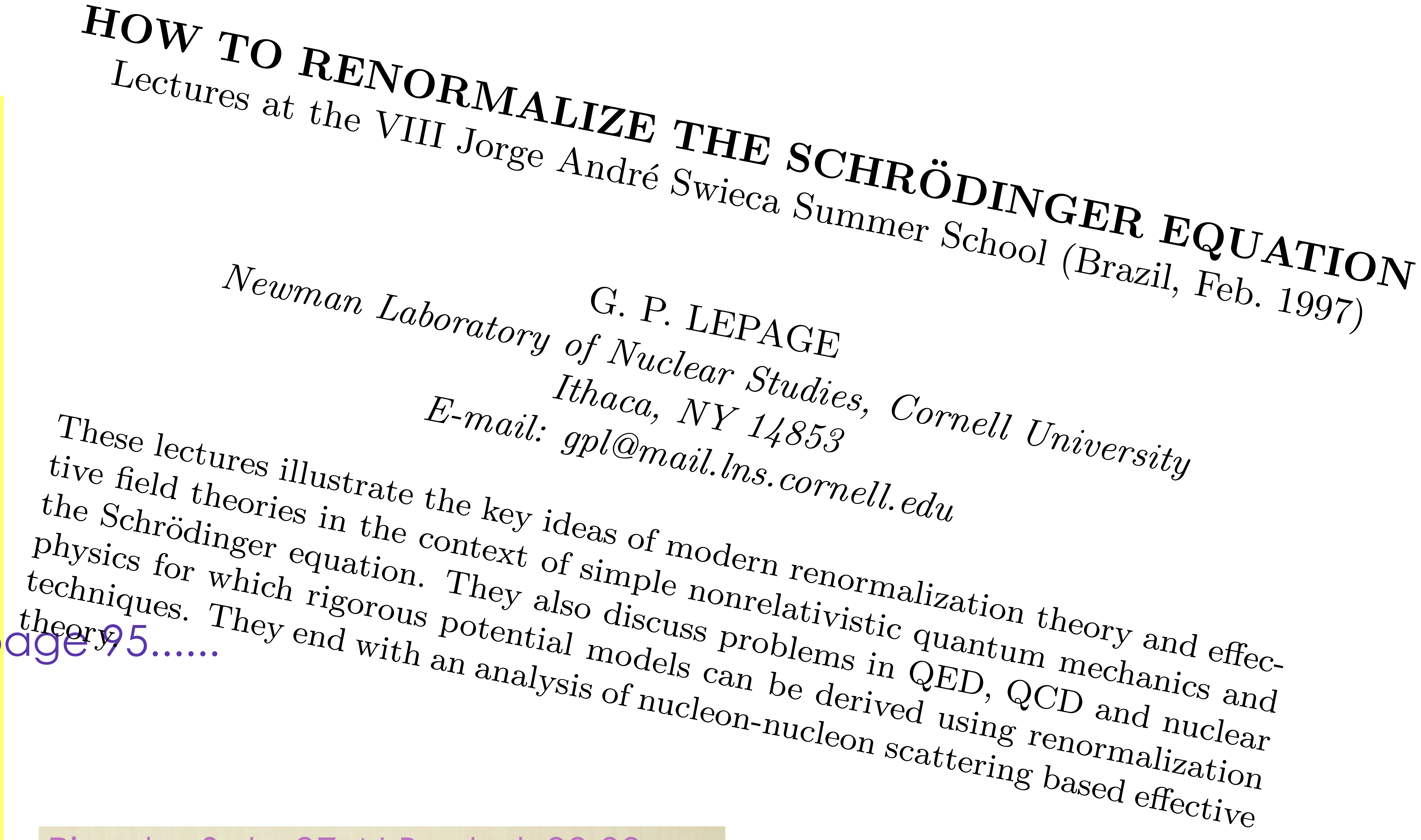
- the potential is perturbative if $mv \gg \Lambda_{\rm QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{\rm QCD}$

In QCD another scale is relevant $\Lambda_{\rm QCD}$

Quarkonium with NREFT



Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95......



HOW TO RENORMALIZE THE SCHRÖDINGER EQUATION

Lectures at the VIII Jorge André Swieca Summer School (Brazil, Feb. 1997)

G. P. LEPAGE

Newman Laboratory of Nuclear Studies, Cornell University
Ithaca, NY 14853
E-mail: gpl@mail.lns.cornell.edu

These lectures illustrate the key ideas of modern renormalization theory and effective field theories in the context of simple nonrelativistic quantum mechanics and the Schrödinger equation. They also discuss problems in QED, QCD and nuclear physics for which rigorous potential models can be derived using renormalization techniques. They end with an analysis of nucleon-nucleon scattering based effective theory.

Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

NRQED pert
Labelle Lepage 1993
but cutoff

Manohar 1997
Matching in DR

established in a series of papers:
Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, et al. 00–024
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

Bound systems with a typical radius smaller than $\Lambda_{\mathrm{QCD}}^{-1}$

- QFT = QCD
- It is obtained by integrating out hard and soft gluons with p or E scaling like m, mv.
- The d.o.f. are $Q\bar{Q}$ pairs (sometimes cast in color singlet S and color octet O) and ultrasoft modes (e.g. light quarks, low-energy gluons):
  $\phi = S$

- The Lagrangian is organized as an expansion in $1/m$ and r.
- The form of $\Delta\mathcal{L}$ and of the ultrasoft modes depends on the low energy dynamics.
- The power counting is
  $\rightarrow p \sim 1/r \sim mv$ (soft scale),
  $\rightarrow E \sim \mathbf{p}^2/2m \sim V^{(0)} \sim \mathbf{P}_{\mathrm{cm}} \sim 1/\mathbf{R}_{\mathrm{cm}} \sim mv^2$ (ultrasoft scale),
  $\rightarrow$ operators in $\Delta\mathcal{L}$ scale like $(mv^2)^{\mathrm{dimension}}$.

Weakly coupled pNRQCD

- Pineda Soto NP PS 64 (1998) 428
  Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:

$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla}A(R,t) + \dots$

$\mathbf{R}$ = center of mass

$\mathbf{r}$ = $Q\bar{Q}$ distance

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r\, {\rm Tr}\,\{S^\dagger(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots)S + O^\dagger(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots)O +$$

LO in r

$$+V_A(S^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger\mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\} + \dots$$

NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

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NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

The matching coefficients are the Coulomb potential

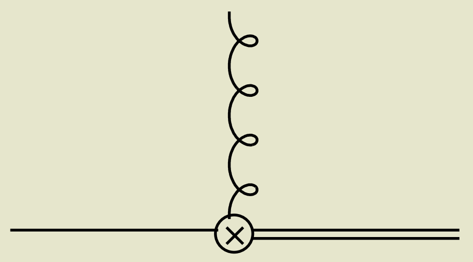
$$V_S(r) = -C_F\frac{\alpha_{\rm s}}{r} + \ldots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_{\rm s}}{r} + \ldots,$$

$$V_A = 1 + \mathcal{O}(\alpha_{\rm s}^2),\ V_B = 1 + \mathcal{O}(\alpha_{\rm s}^2).$$

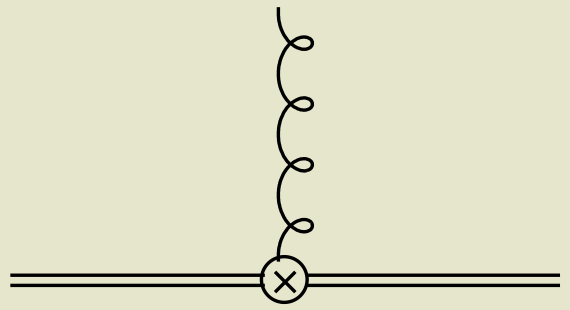
Feynman rules

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V)}$$

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$$



$= {\rm O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm S}$



$= {\rm O}^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, {\rm O}\}$

Matching the potential

Brambilla Pineda Soto Vairo PRD 60 (1999) 091502

$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \dots$$

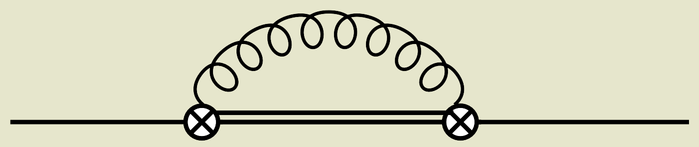
- The potential is a matching coefficient of the EFT that may be computed from first principle by matching Green's functions in QCD with Green's function in pNRQCD, it is scheme and scale dependent, and undergoes renormalization.
  It may be organized as an expansion in $1/m$:
  - The interaction terms contained in $\Delta\mathcal{L}$ provide corrections to the quantum mechanical picture.

- The static potential:
  in weakly coupled pNRQCD

$$V^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \ln \langle \Box \rangle - \Delta\mathcal{L}\,\text{effects}; \qquad \Box = \exp\left\{ ig \oint_{r\times T} dz^\mu A_\mu \right\}$$

Wilson loops (as matching Green's functions) guarantee gauge invariance.

$$V^{(0)}(r,\mu') = \lim_{T\to\infty} \frac{i}{T} \ln \langle \Box \rangle - \quad [\text{diagram}] \quad + \cdots$$



$$= E_0(r) + \frac{i}{N} \int_0^\infty dt\, e^{-it(V_o - V)} \langle \mathrm{Tr}\, \mathbf{r}\cdot g\mathbf{E}(t)\, \mathbf{r}\cdot g\mathbf{E}(0)\rangle(\mu') + \cdots$$

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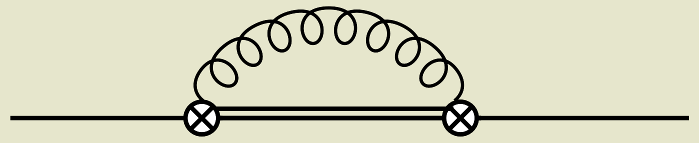

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The QCD static potential at N^4LO

$$V^{(0)}(r,\mu') = -C_F \frac{\alpha_s(1/r)}{r}\left\{ 1 + \frac{\alpha_s(1/r)}{4\pi} a_1 + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 a_2 \right.$$
$$+ \left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 \left[\frac{16\,\pi^2}{3} C_A^3 \ln r\mu' + a_3 \right]$$
$$\left. + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^4 \left[a_4^{L2} \ln^2 r\mu' + \left(a_4^L + \frac{16}{9}\pi^2 C_A^3 \beta_0(-5 + 6\ln 2) \right) \ln r\mu' + \cdots \right] \right\}$$

a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $\ln r\mu$ N.B. Pineda, Soto, Vairo 99

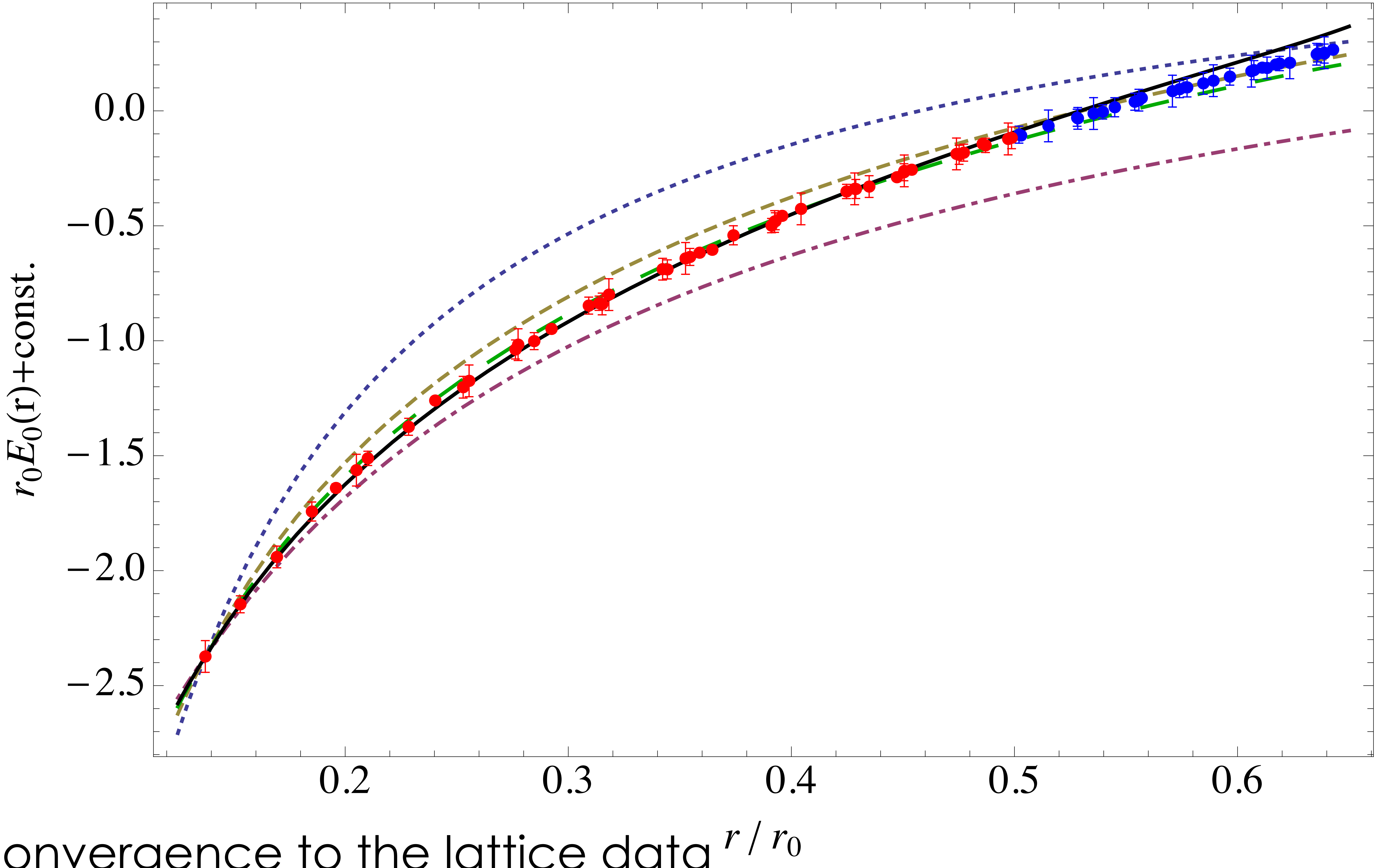
a_4^{L2}, a_4^L N.B., Garcia, Soto, Vairo 06

α_s

from quarkonium

QQbar singlet static energy at NNNLL in pNRQCD in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014, with Weber 2019



Good convergence to the lattice data

Can exclude linear nonperturbative corrections at short distance

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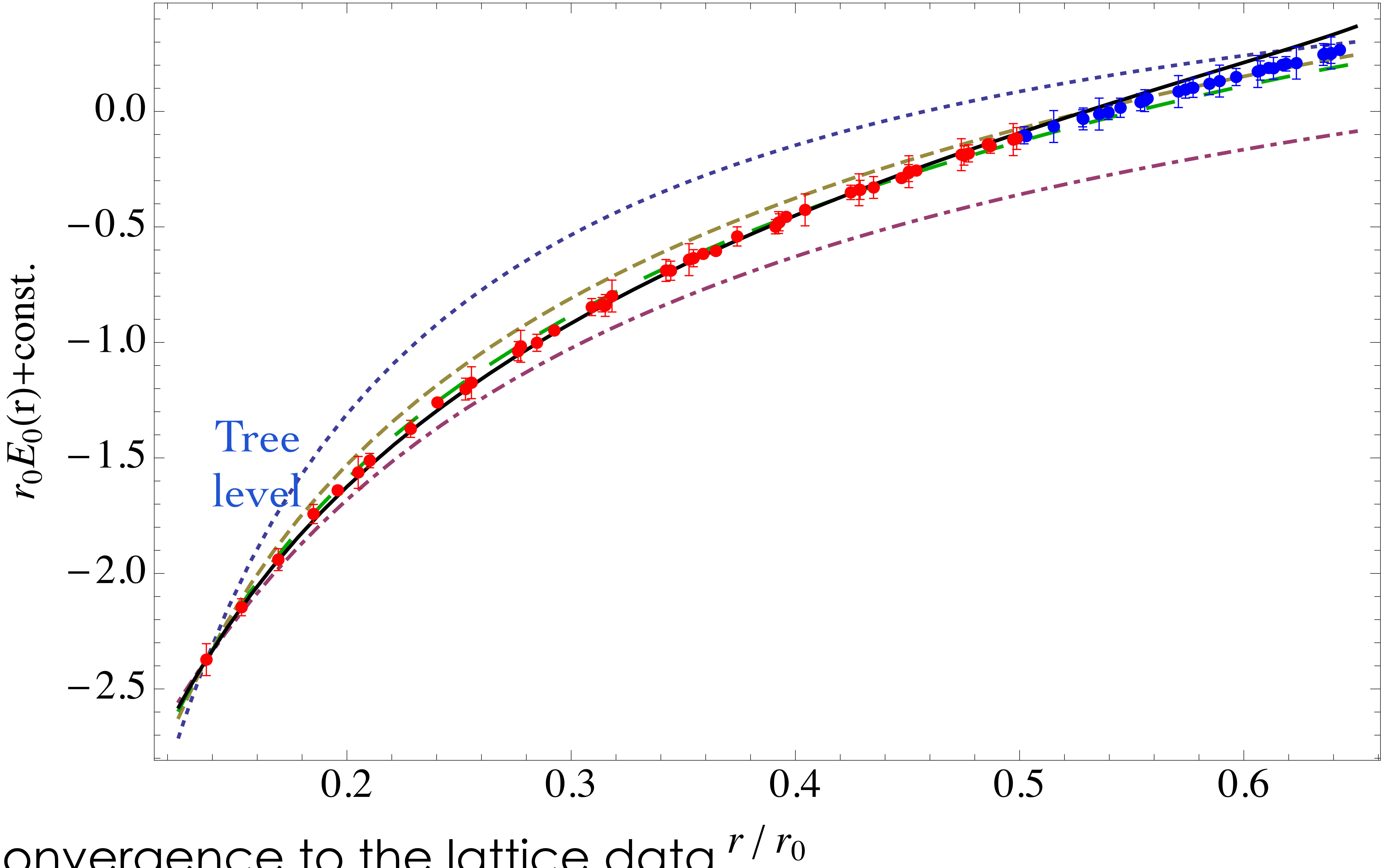

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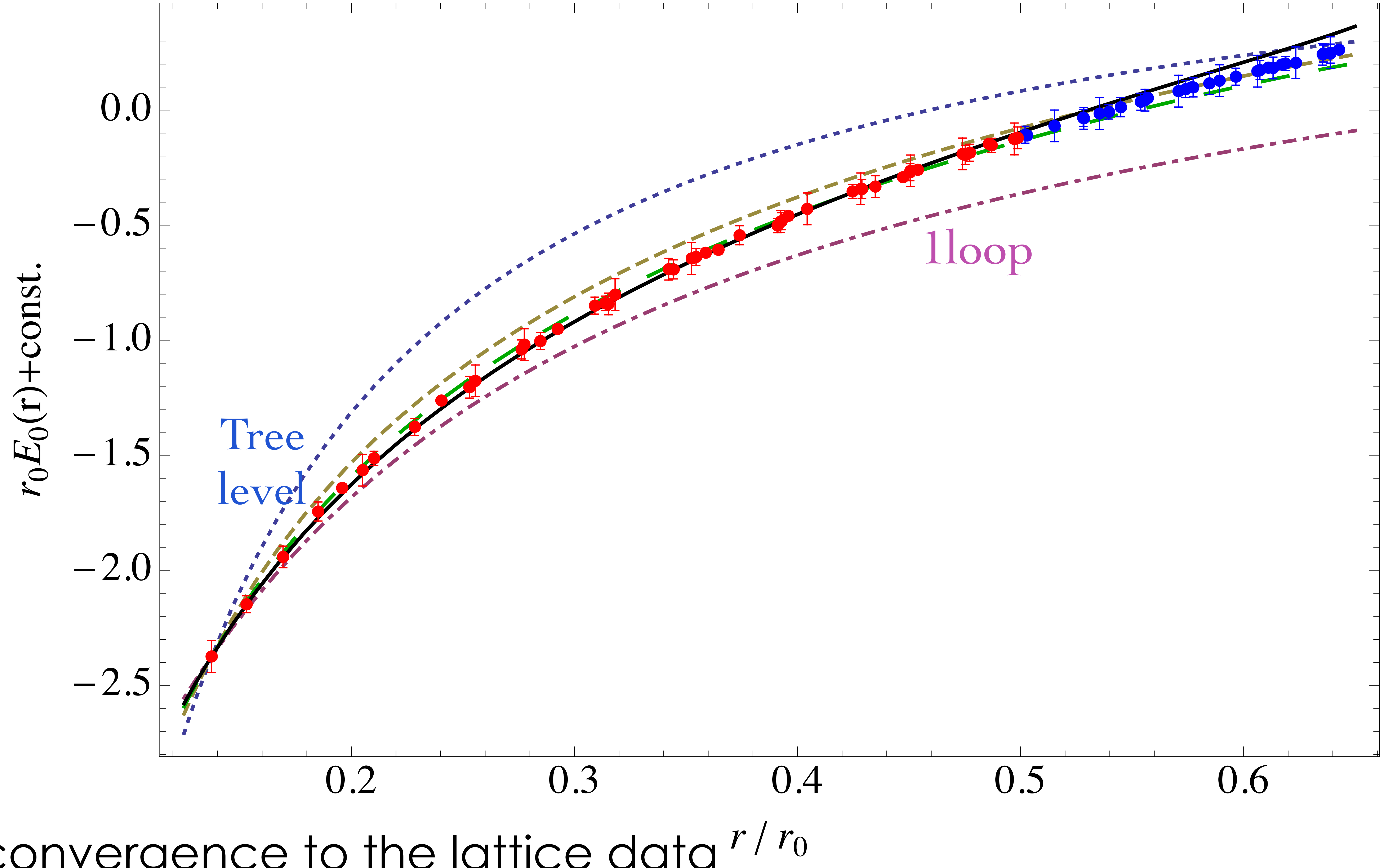

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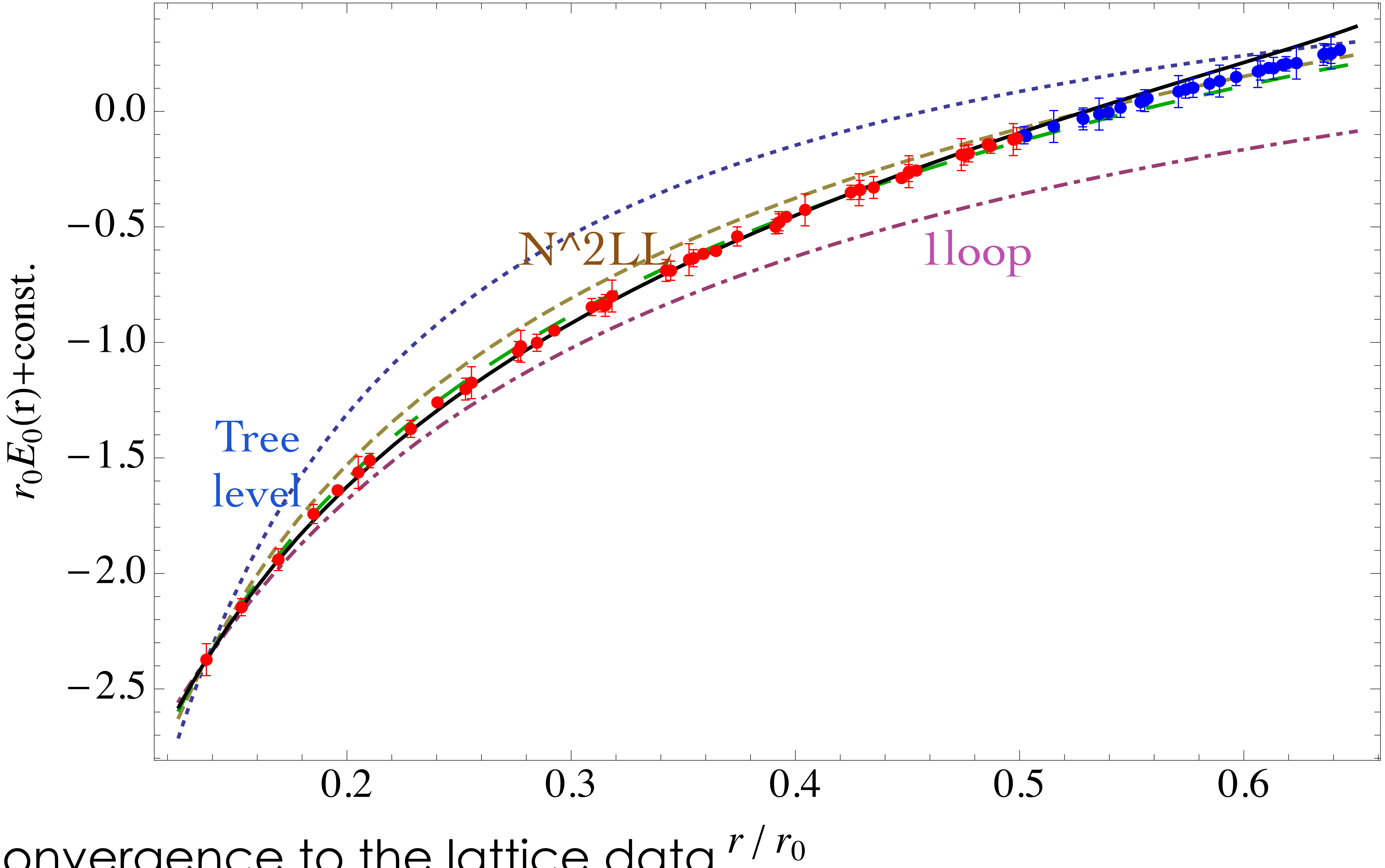

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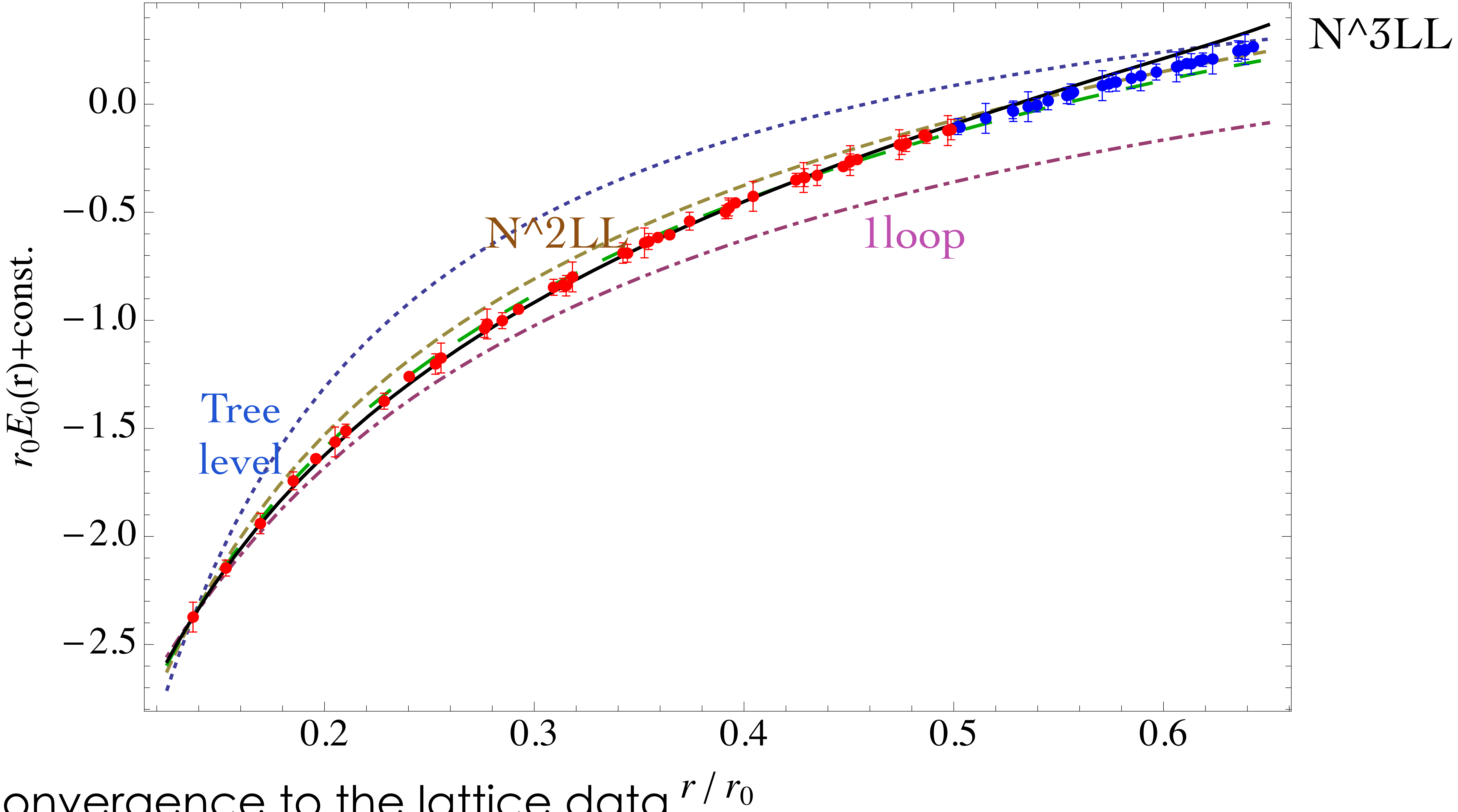

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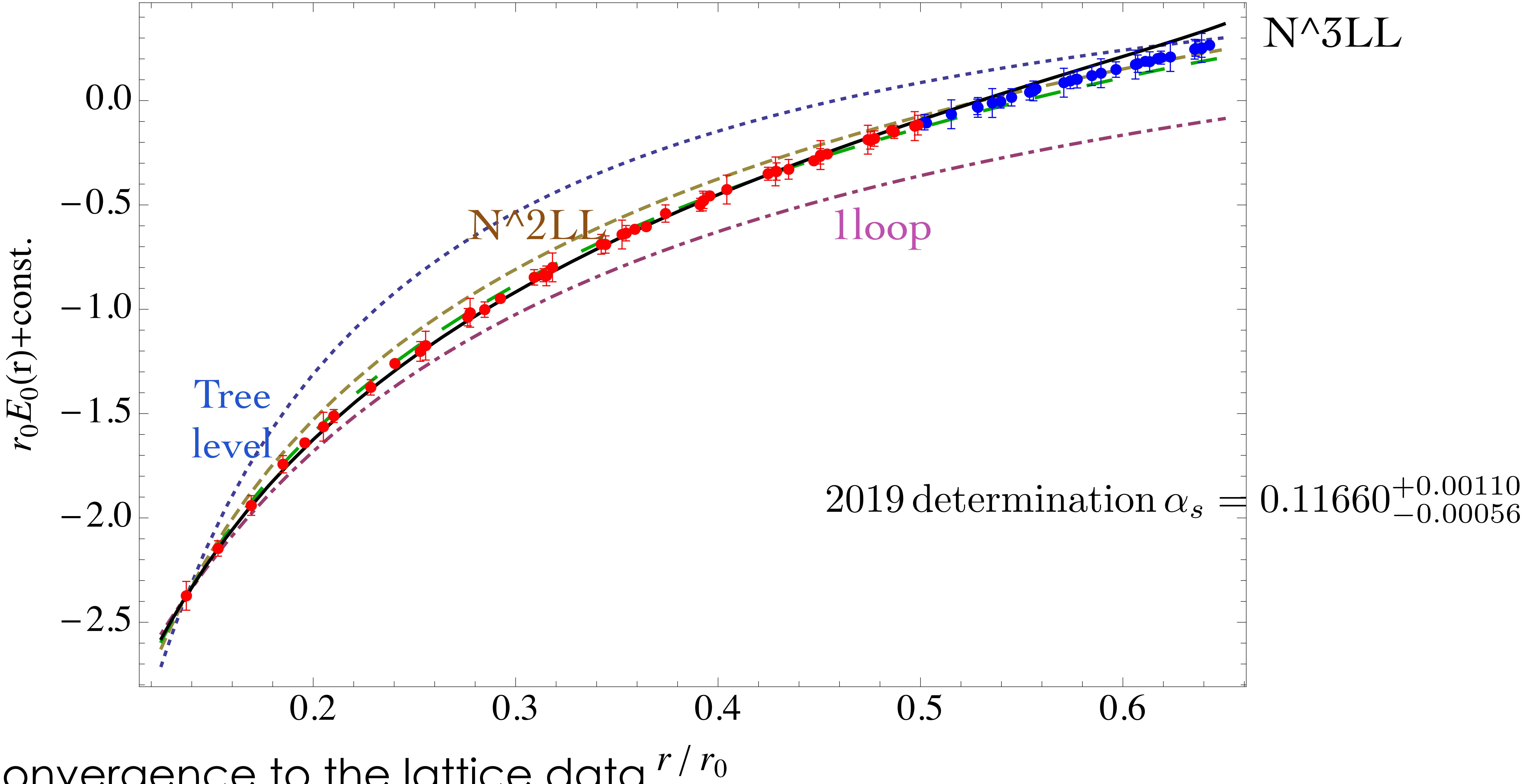


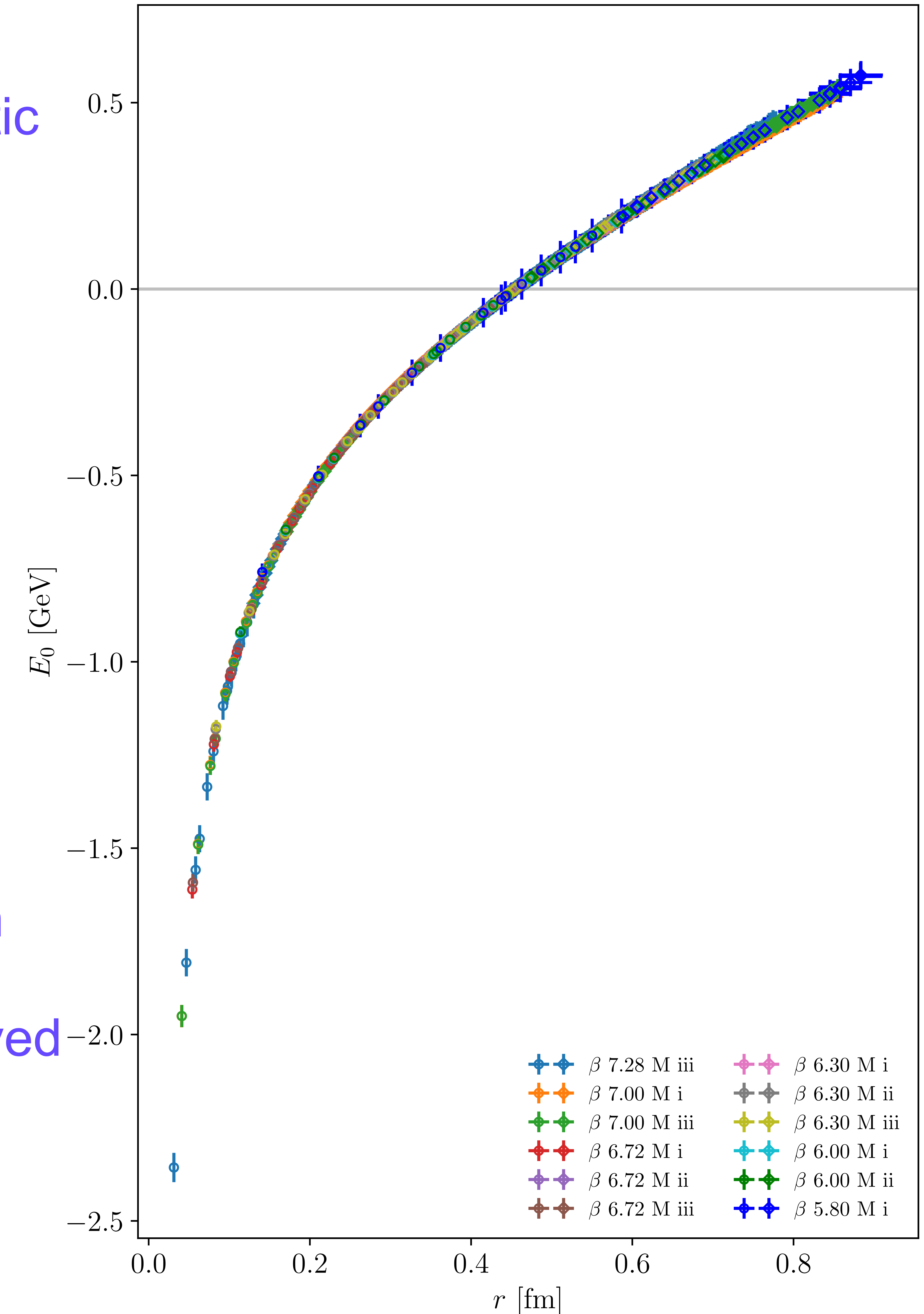
Good convergence to the lattice data

Can exclude linear nonperturbative corrections at short distance

First lattice calculation of the QCD static energy with n_f=2+1+1, I.e. with charm effects

Can be used to extract alpha_s with n_f=2+1+1

Finite charm mass effect Should be implemented in perturbation theory
Charm decoupling is observed



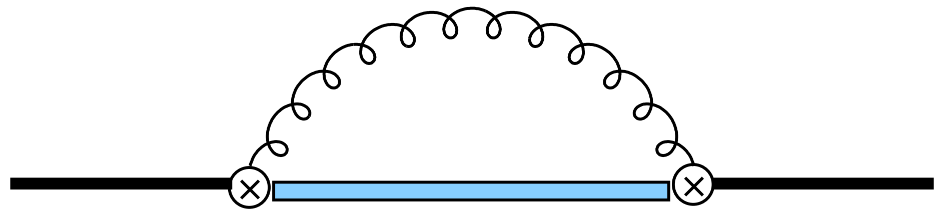
TUMQCD
N.B., Delgado, Kronfeld, Leino, Petreczky, Steinbeisser, Vairo, Weber *Phys.Rev.D* 107 (2023) 7, 074503 •

TUMQCD
Our lattice QCD collaboration with mission to complement EFT methods and lattice QCD to obtain strong interaction observables

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

$\sim e^{i\Lambda_{\rm QCD} t}$

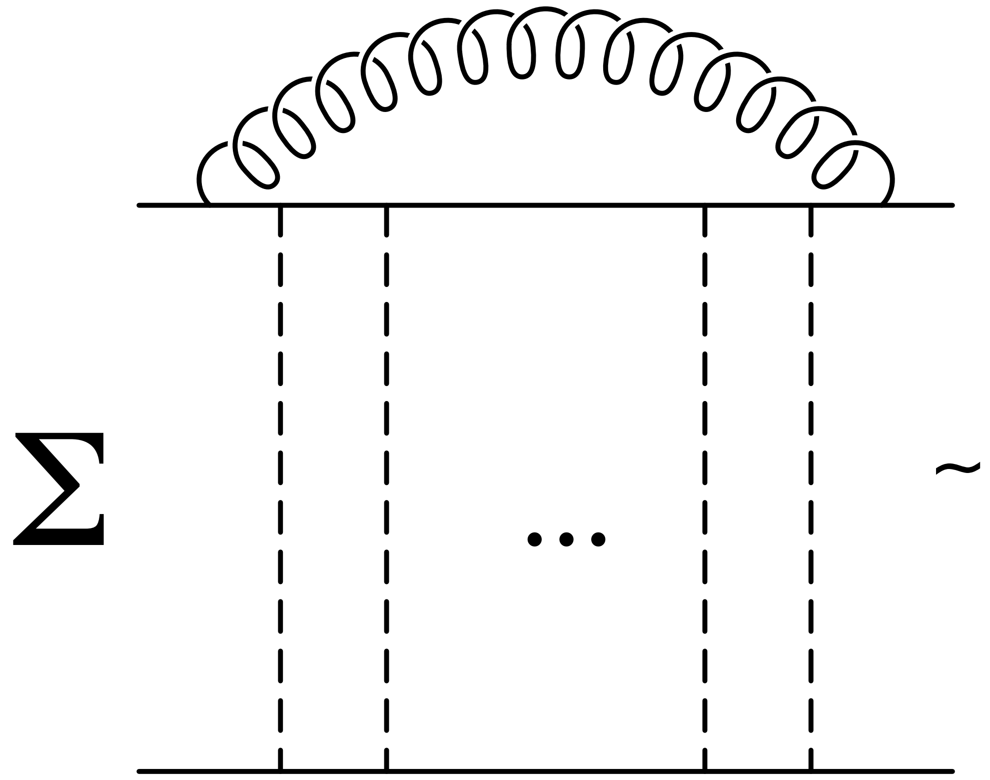
$$E_n = \langle n|H_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\,\langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\,\langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

—>used to extract precise (NNNLO) determination of m_c and m_b

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

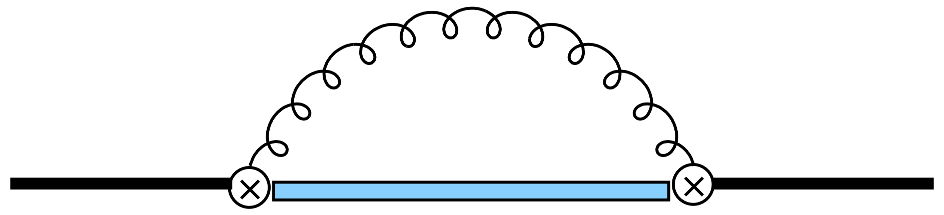


$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n | \frac{p^2}{m} + V_s | n \rangle + \langle n |$$  $$| n \rangle$$

$$\sim e^{i\Lambda_{\rm QCD} t}$$

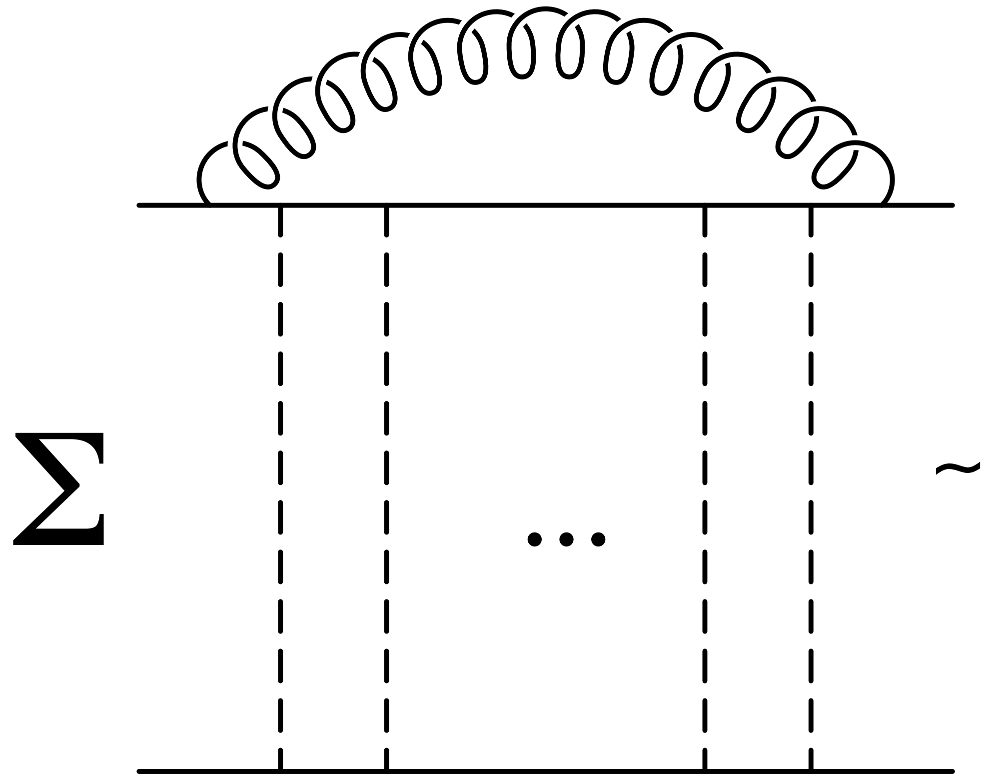
$$E_n = \langle n | H_s(\mu) | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \, \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \, \langle \mathbf{E}(t) \, \mathbf{E}(0) \rangle (\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0) \rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

—>used to extract precise (NNNLO) determination of m_c and m_b

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED



$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

Applications of weakly coupled pNRQCD include:
ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

Peter's Legacy

Precision Υ Spectroscopy and Fundamental Parameters From NRQCD

The NRQCD Collaboration *

We present results from a high precision NRQCD simulation of the quenched Υ system at $\beta = 6$. We demonstrate a variety of important lattice techniques, including the perturbative improvement of actions, tadpole improvement, and multicorrelated fits for extracting the spectrum of excited states. We present new determinations of $\alpha_s(Mz)$ and M_b, two fundamental parameters of the Standard Model.

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Andreas Kronfeld, distinguished Fermilab scientist,

TUM-IAS Fellow, TUM Ambassador

Member of the TUMQCD lattice collaboration

Bound systems with a typical radius $\sim \Lambda_{\mathrm{QCD}}^{-1}$

Strongly coupled pNRQCD and Born Oppenheimer EFT

A nonperturbative problem: construct a pNREFT description on the basis of scale separations and symmetries

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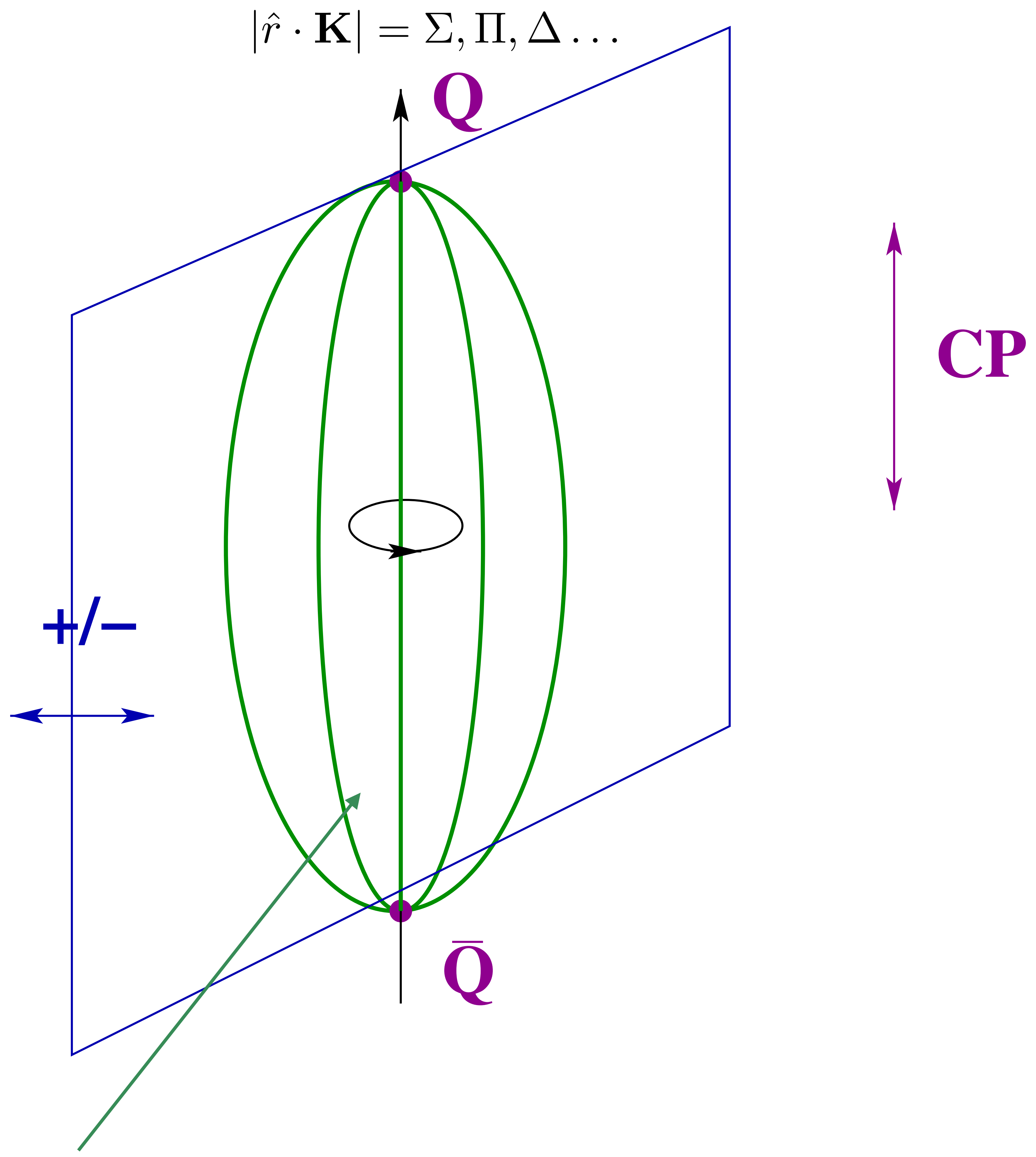
Two heavy quarks with large mass $m \gg \Lambda_{\rm QCD}$ and residual scale separation $\Lambda_{\rm QCD} \gg E$



Nonperturbative light degrees of freedom
glue and light quarks

produce a hierarchy of NRQCD static energies identified by the quantum number of $D_{\infty h}$

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \; (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g), -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

$$\Lambda_\eta^\sigma$$

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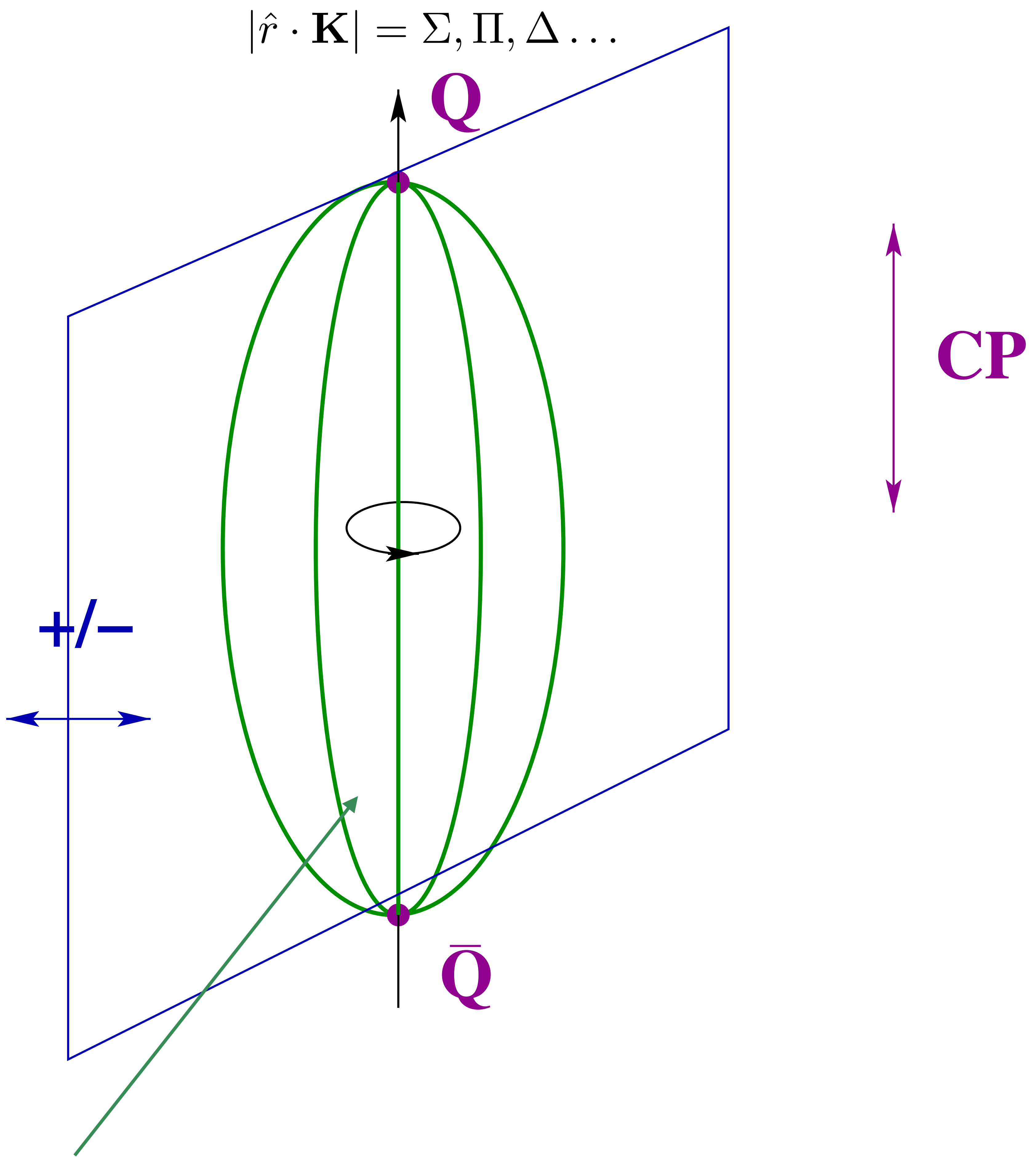

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$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\,\frac{1}{2}\left(\boldsymbol{\Pi}^a\boldsymbol{\Pi}^a + \mathbf{B}^a\mathbf{B}^a\right) - \sum^{n_f} \bar{q}\, i\mathbf{D}\cdot\boldsymbol{\gamma}\, q$$

$$\mathcal{H}^{(0)}|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1,\mathbf{x}_2)|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n;\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

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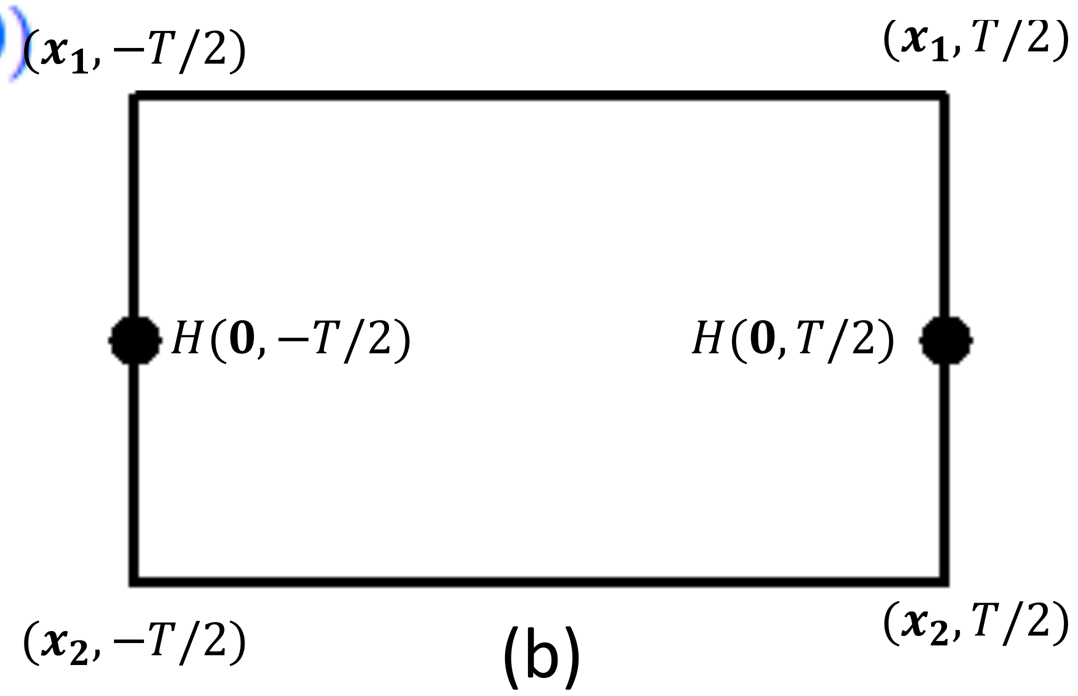

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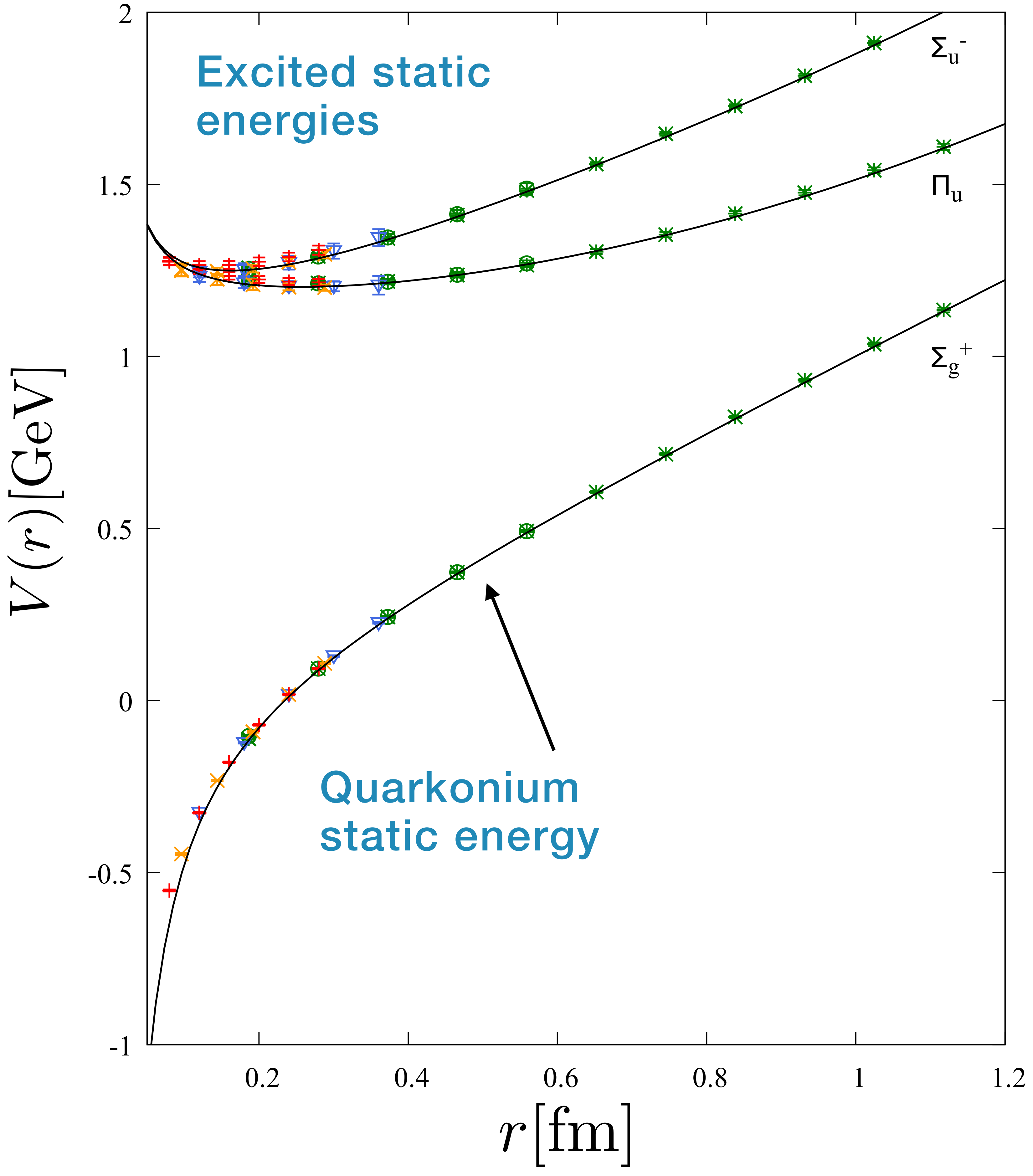

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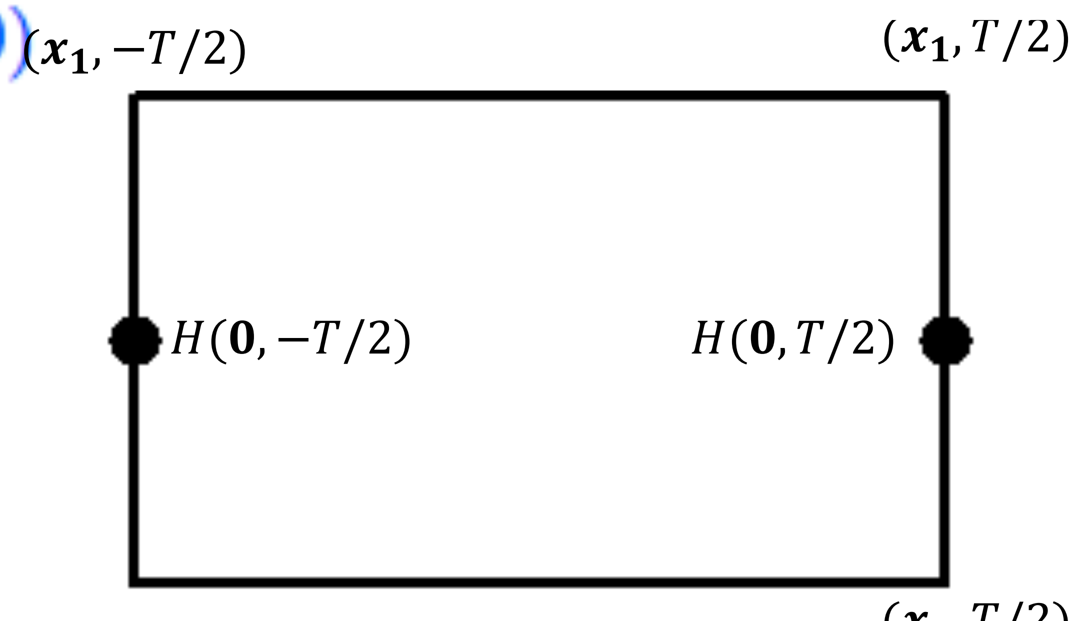
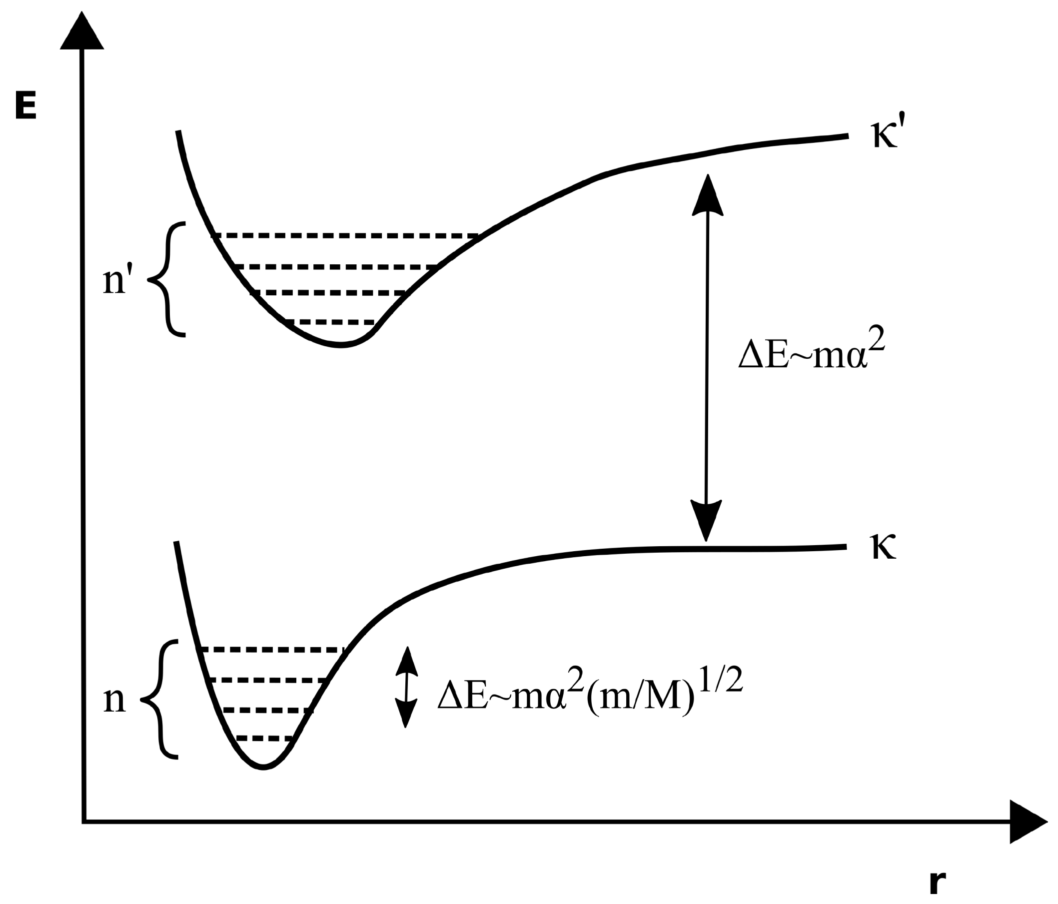


(b)

Phi =Wilson lines and H= gluonic and light quarks

Kuti, Morningstar et al 1997, Pineda, Vairo; Wagner et al



QED —

$\Lambda_{QCD} > mv^2$

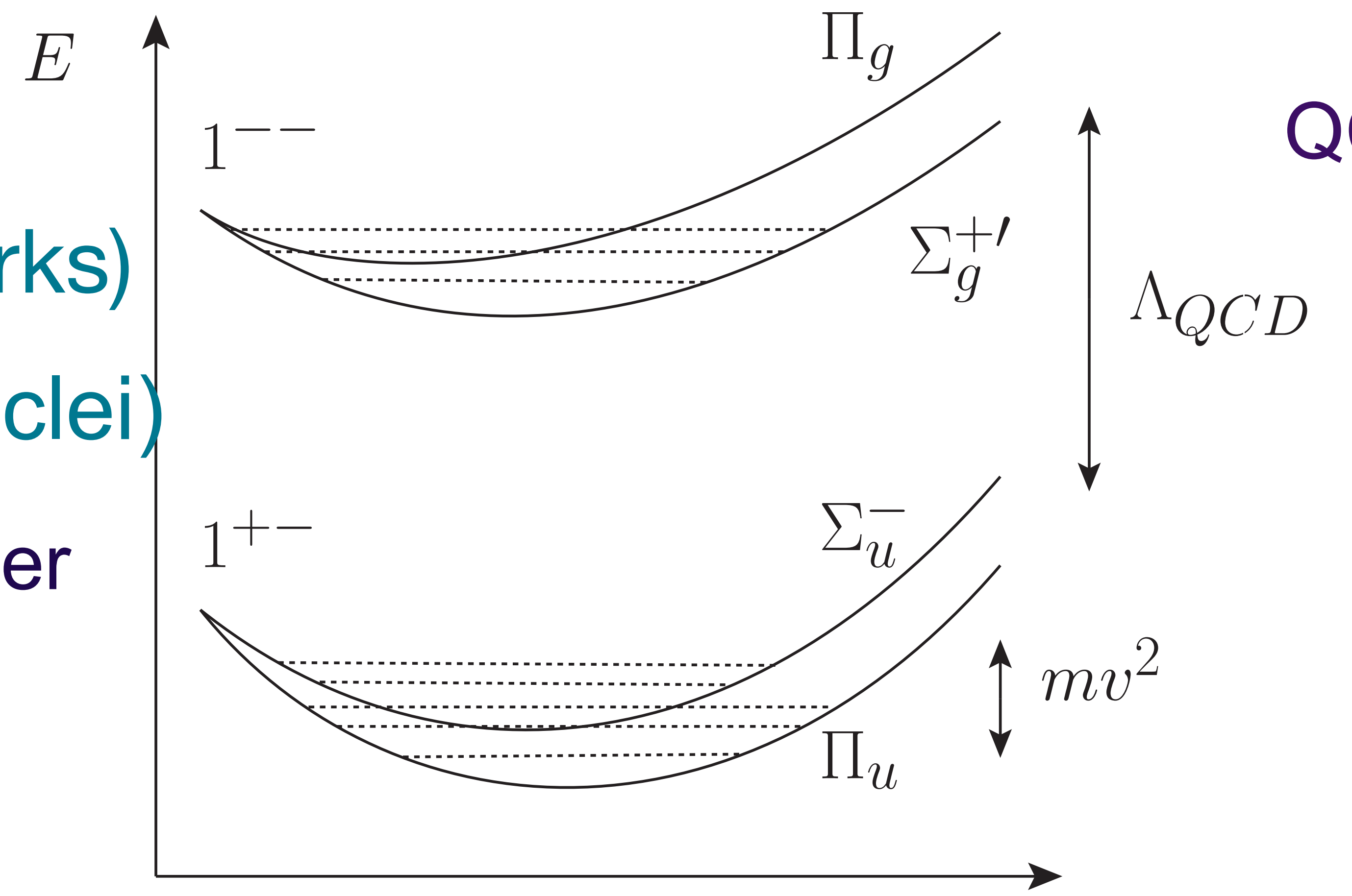
fast (gluons, light quarks) and slow (heavy quarks)

like in molecular physics (fast-electrons, slow nuclei)

Born Oppenheimer Description

Braaten PRL 111 (2013) 162003

Braaten Langmack Smith PRD 90 (2014) 014044

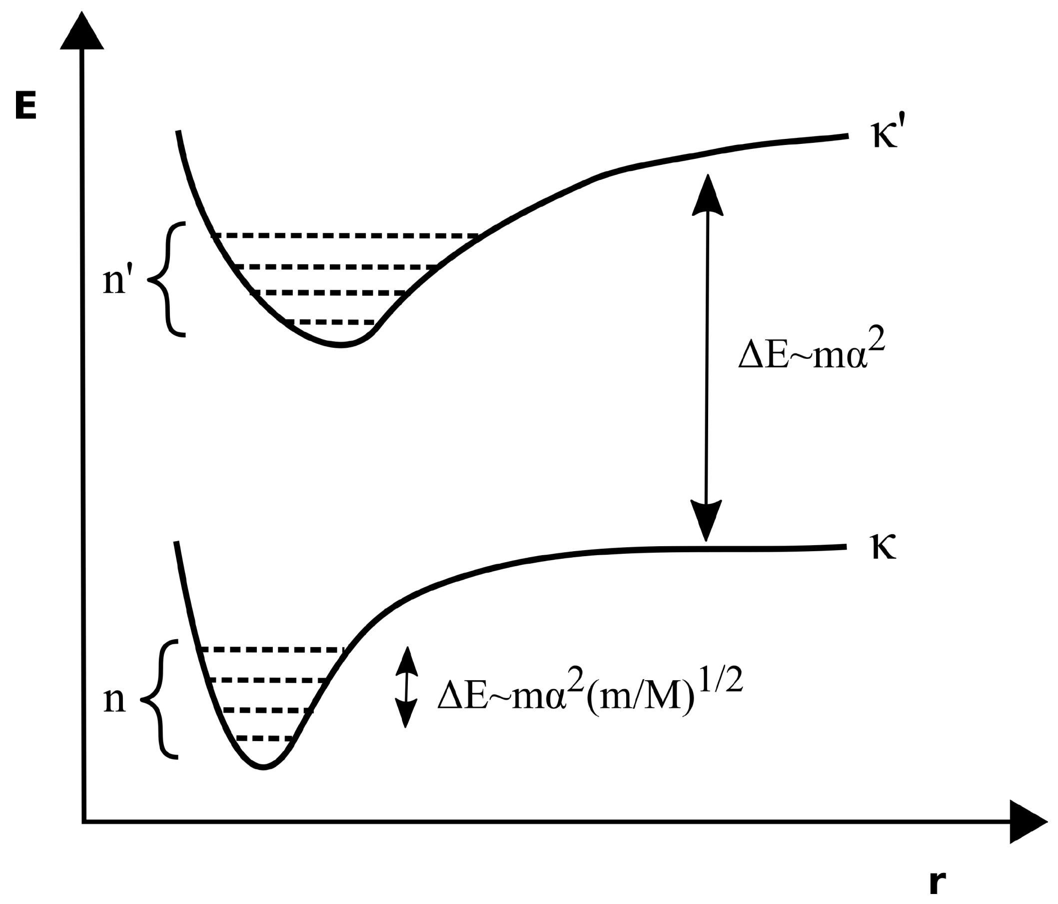


QCD

Higher excitations develop a gap of order Lambda_QCD

Introducing a finite mass m:

- The spectrum of the mv^2 fluctuations around the lowest static energy is the quarkonium spectrum
- The spectrum of the mv^2 fluctuations around the higher excitations is the exotic spectrum (hybrids and tetraquarks)



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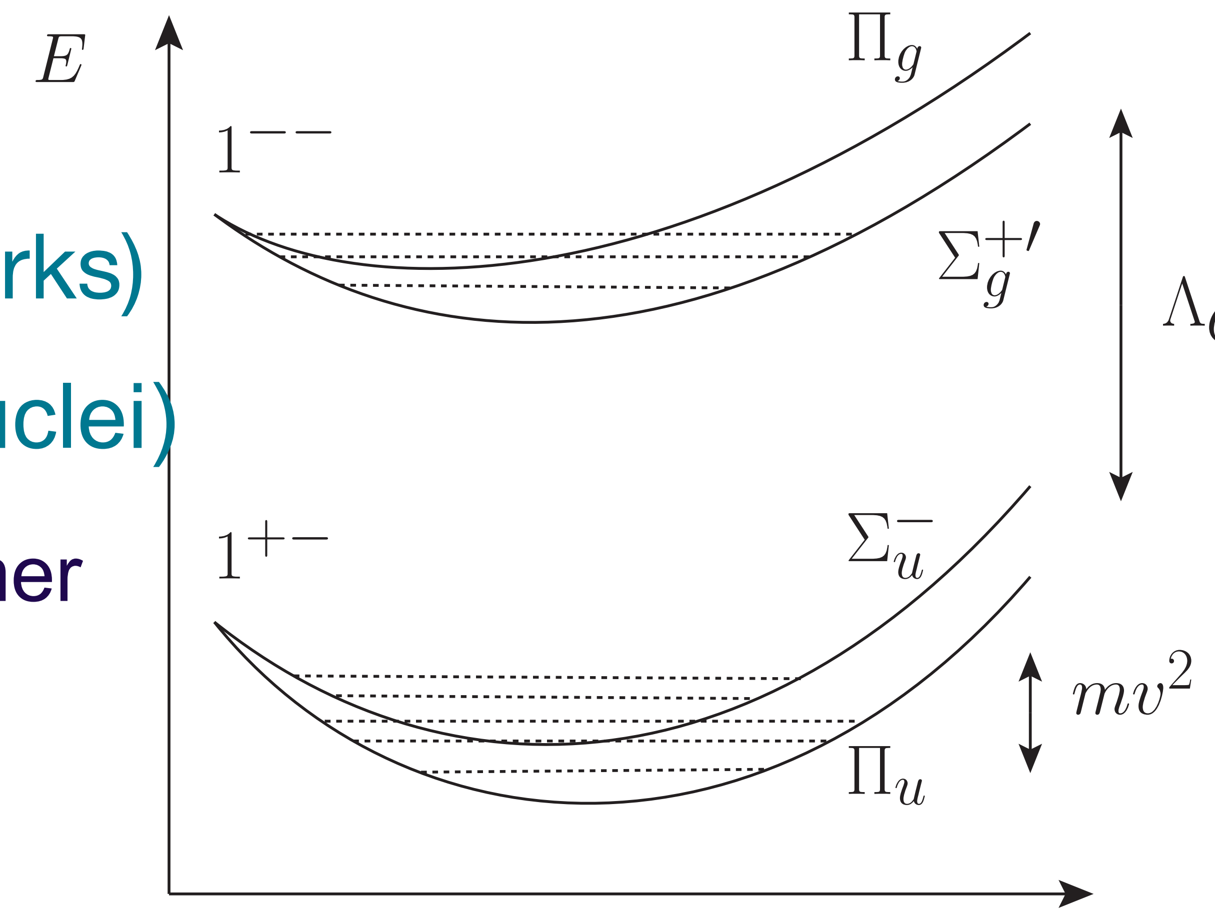

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Nonperturbative matching to the pNREFT

systematically

$$\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$$

expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$|\underline{0}; \mathbf{x}_1\mathbf{x}_2\rangle -> |(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

$E_0(r) -> V_0(r)$ pNRQCD N.B, Pineda, Soto, Vairo 1999—

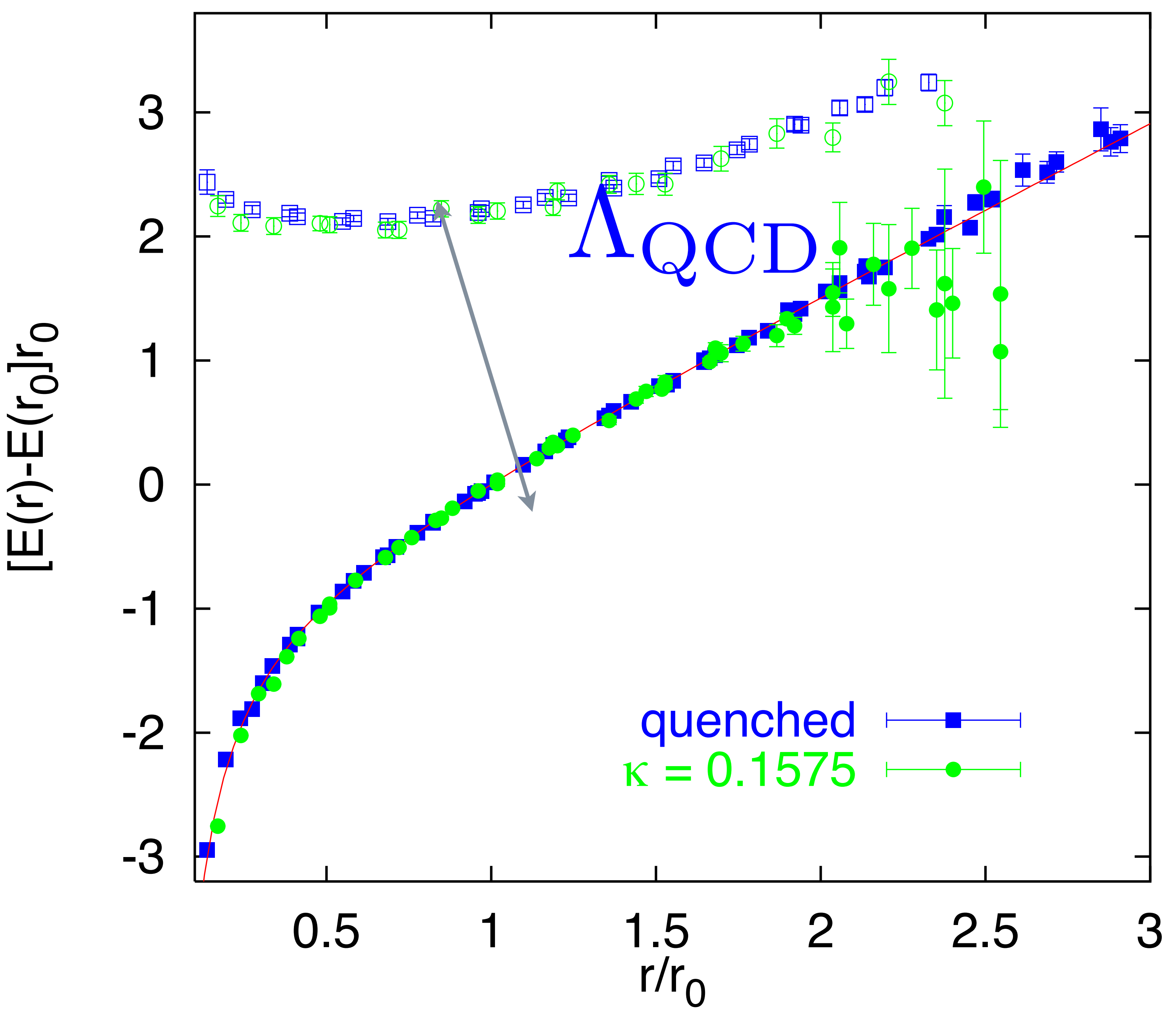
$|\underline{n} > 0; \mathbf{x}_1\mathbf{x}_2\rangle -> |(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

$|Q\bar{Q}q\bar{q}\rangle$ Tetraquarks

$E_n^{(0)}(r) -> V_n^{(0)}(r)$ BOEFT Berwein, N.B, Tarrus, Vairo 2015

Strongly coupled pNRQCD for quarkonium

$mv \sim \Lambda_{QCD}$



Bali et al. 98

pNRQCD and the potentials come from integrating out all scales up to mv^2

- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

Brambilla Pineda Soto Vairo 00

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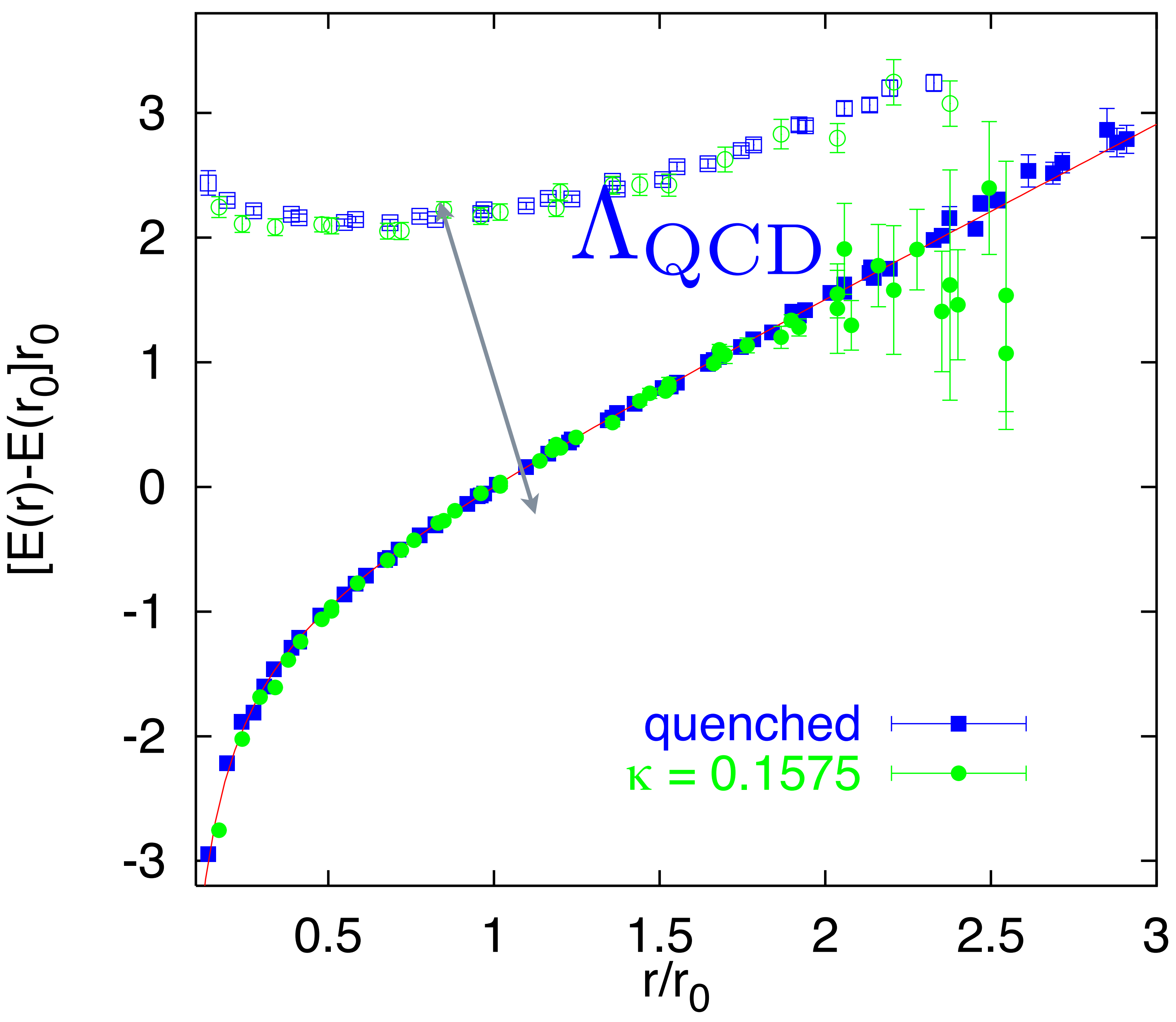

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$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

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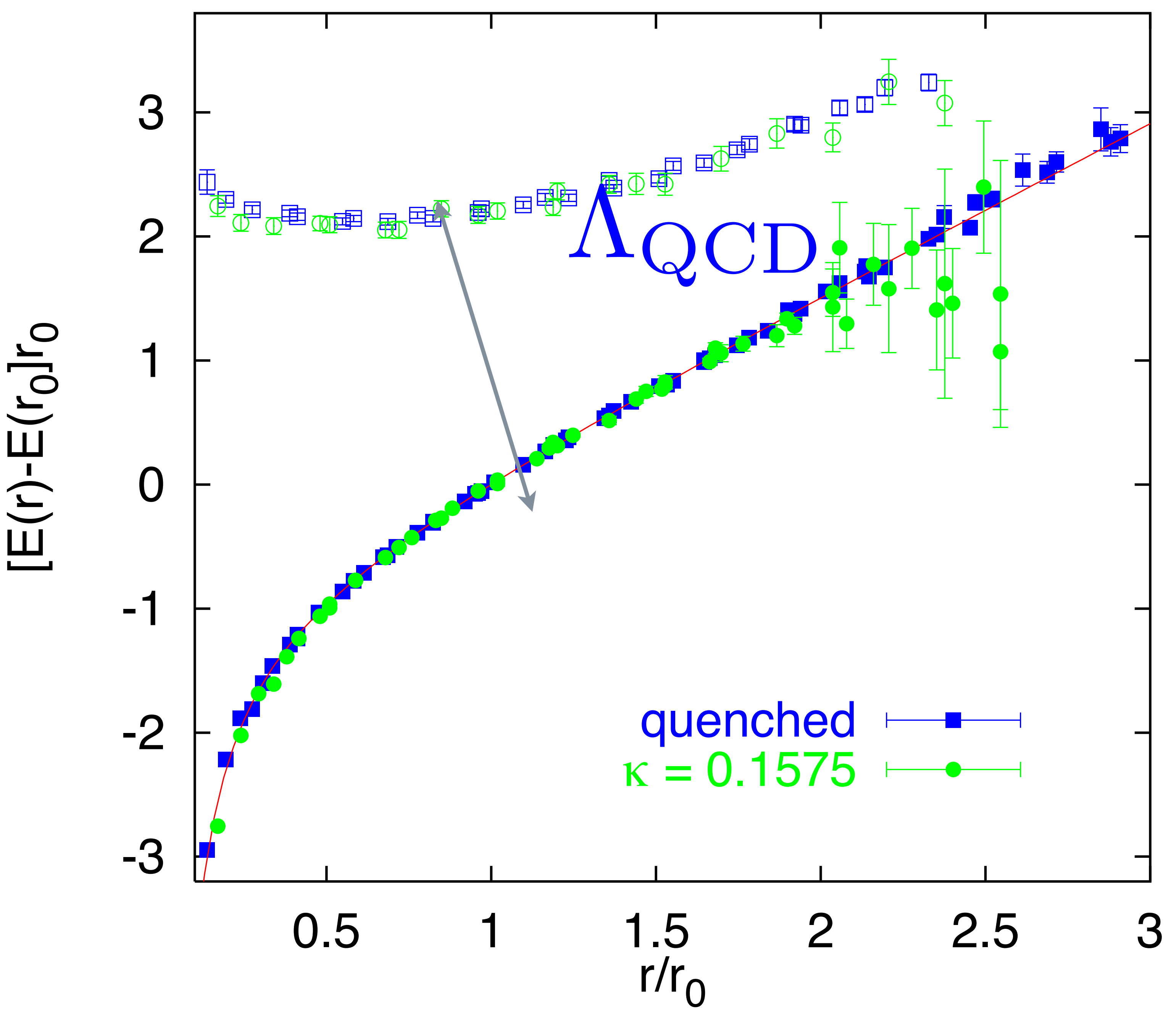

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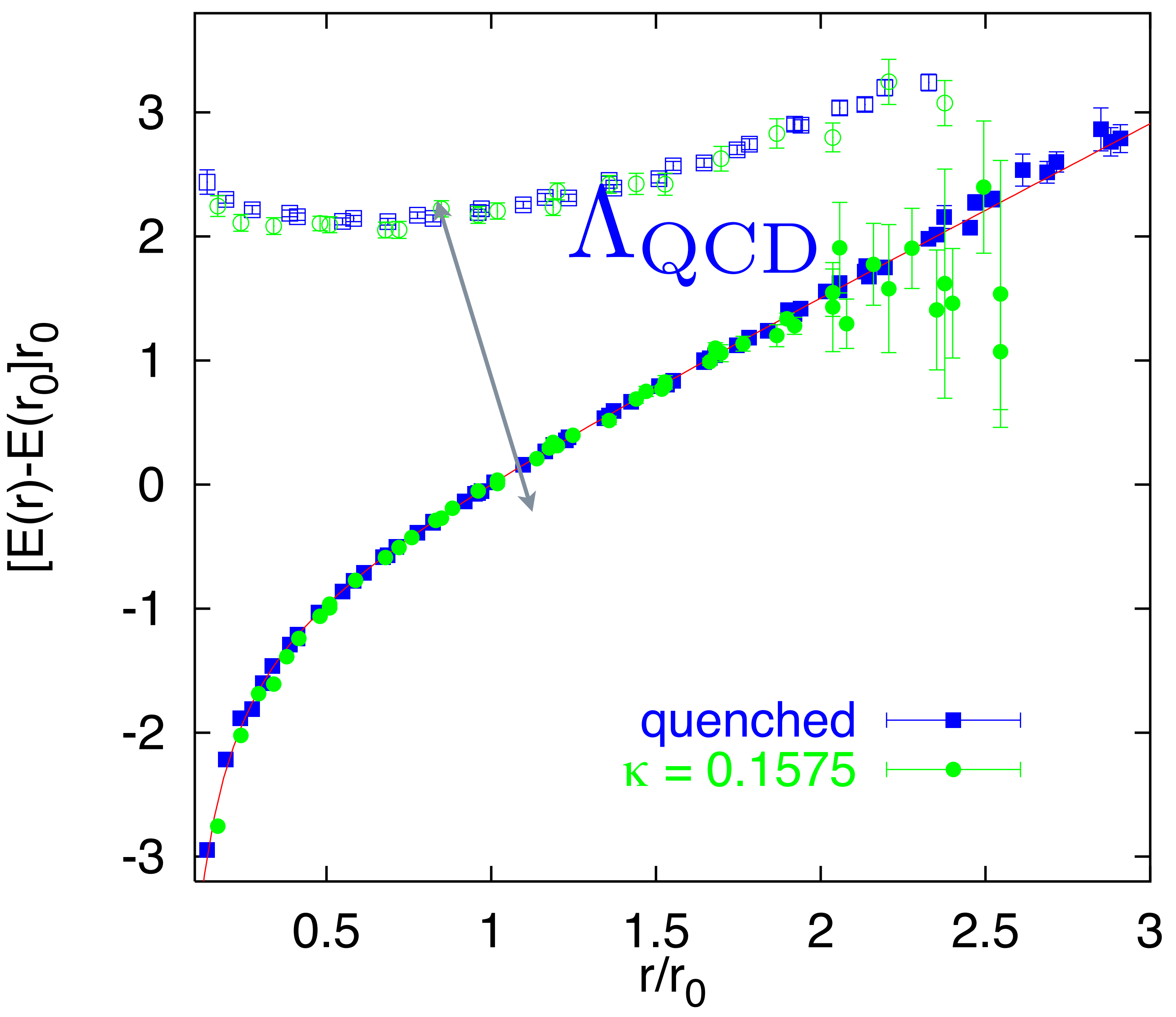

Bali et al. 98

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Applications regard: Spectrum, decays, production at LHC, studies of confinement

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static (V_0), spin dependent (V_{SD}), velocity dependent (V_{VD})

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

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$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t\, \langle \square^{E(t)\,E(t)} \rangle$$

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

$$V^{(2)}_{\rm SD} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle \square^{E(t)}_{i\; j\,B} \rangle\; \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2)\; |V^{(2)}_{LS}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t\, \langle \square_{i\; j} \rangle - \frac{2c_F - 1}{2} \nabla^k V^{(0)} \right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2)\; |V^{(1)}_{LS}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \square_{i\; j} \rangle - \frac{\delta_{ij}}{3} \langle \square \rangle \right) \left(\mathbf{S}_1 \cdot \mathbf{S_2} - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) \right)\; |V_T$$

$$+\left(\frac{2}{3} c_F^2 i \int_0^\infty dt \langle \square \rangle - 4\left(d_{sv} + \frac{4}{3} d_{vv} \right) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S_2}\; |V_S$$

Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

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Pineda Vairo PRD 63 (2001) 054007

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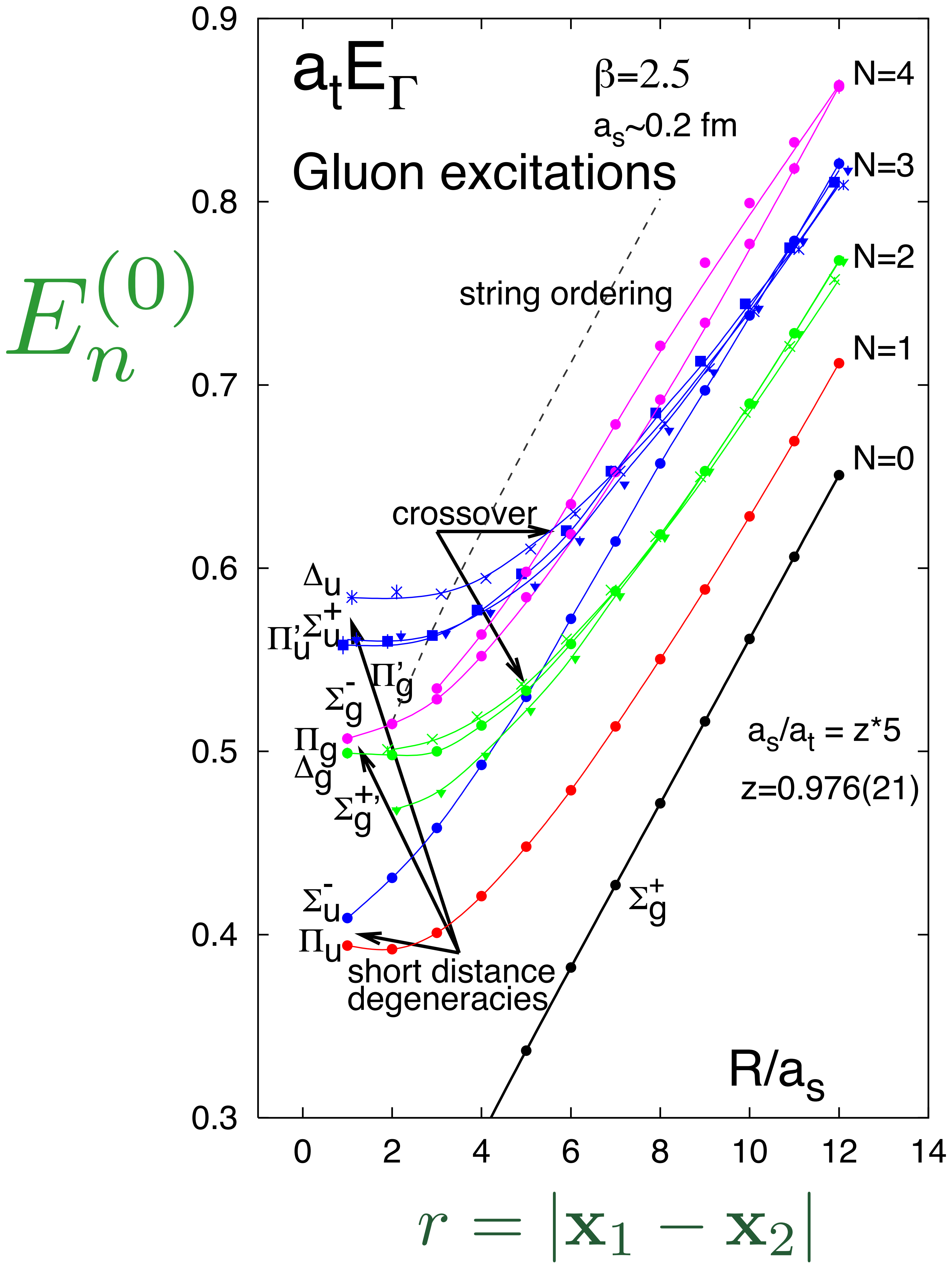
- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cancel any QM divergences, good UV behaviour
- the flavour dependent part is extracted in the NRQCD matching coefficients
- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

Exotics: Hybrids

Lattice Spectrum of NRQCD hybrid static energies E^0_n



Juge Kuti Mornigstar 98-06

Schlosser, Wagner 2111.00741, Bali Pineda 2004

○ Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 034502

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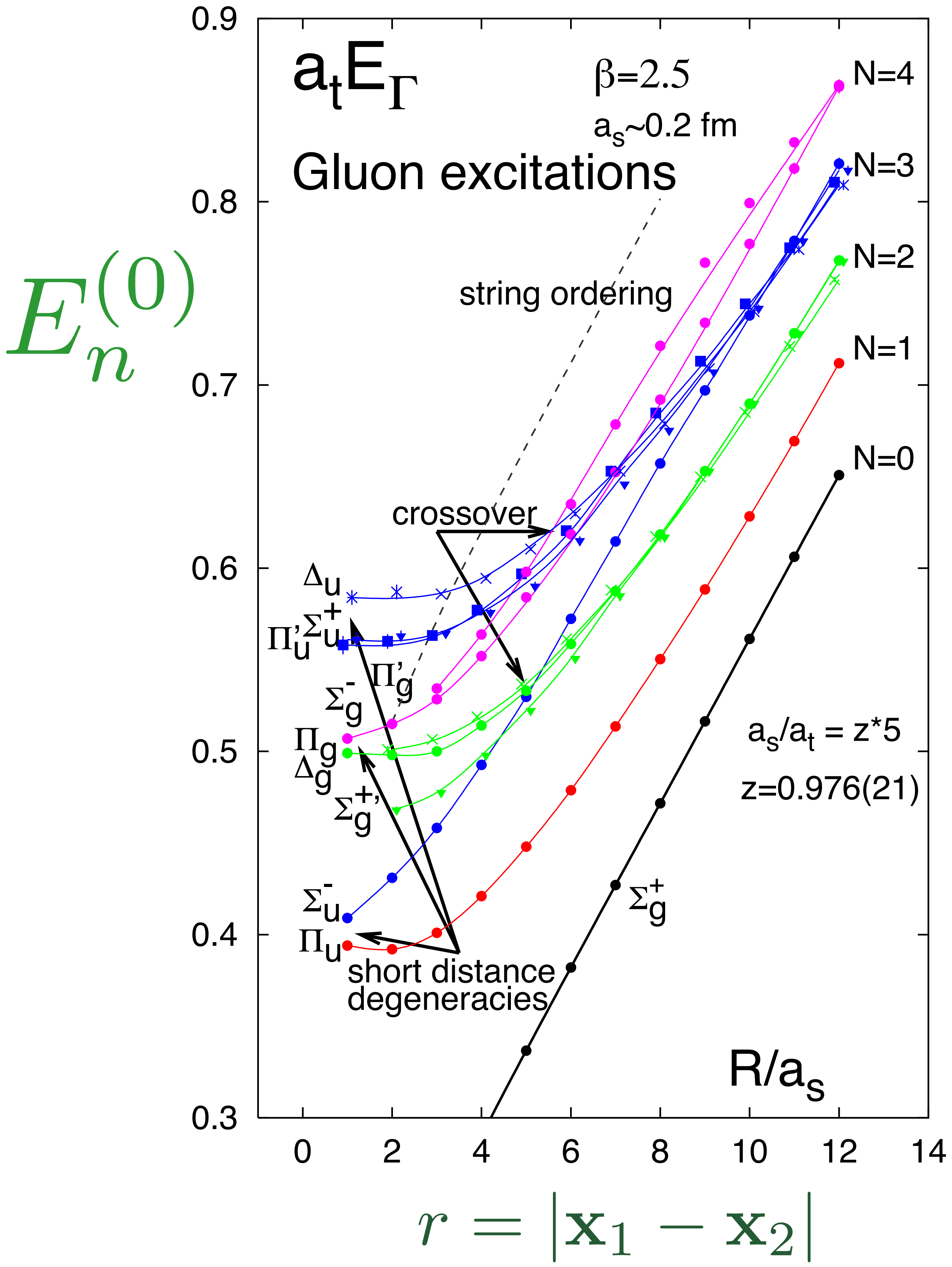

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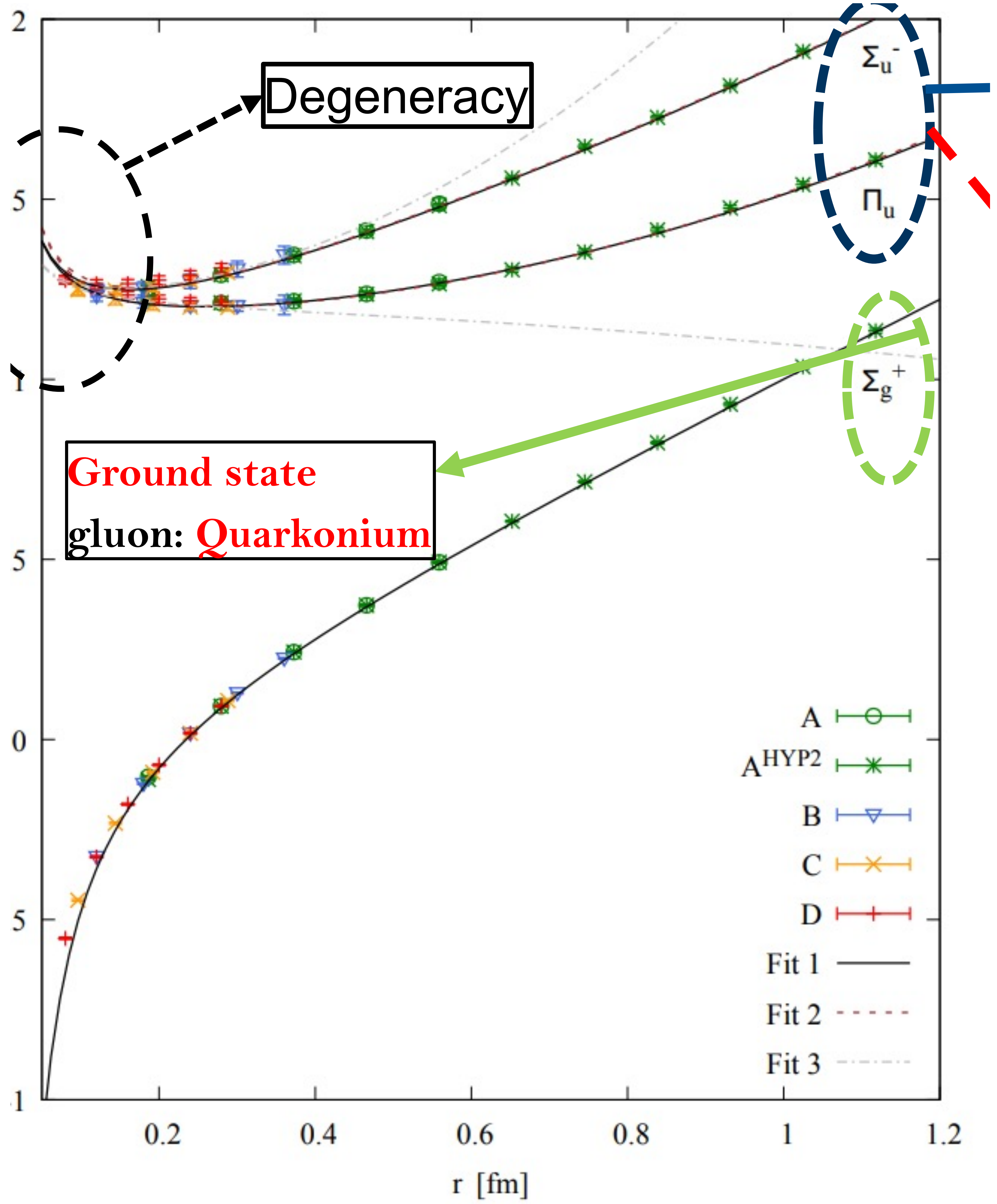
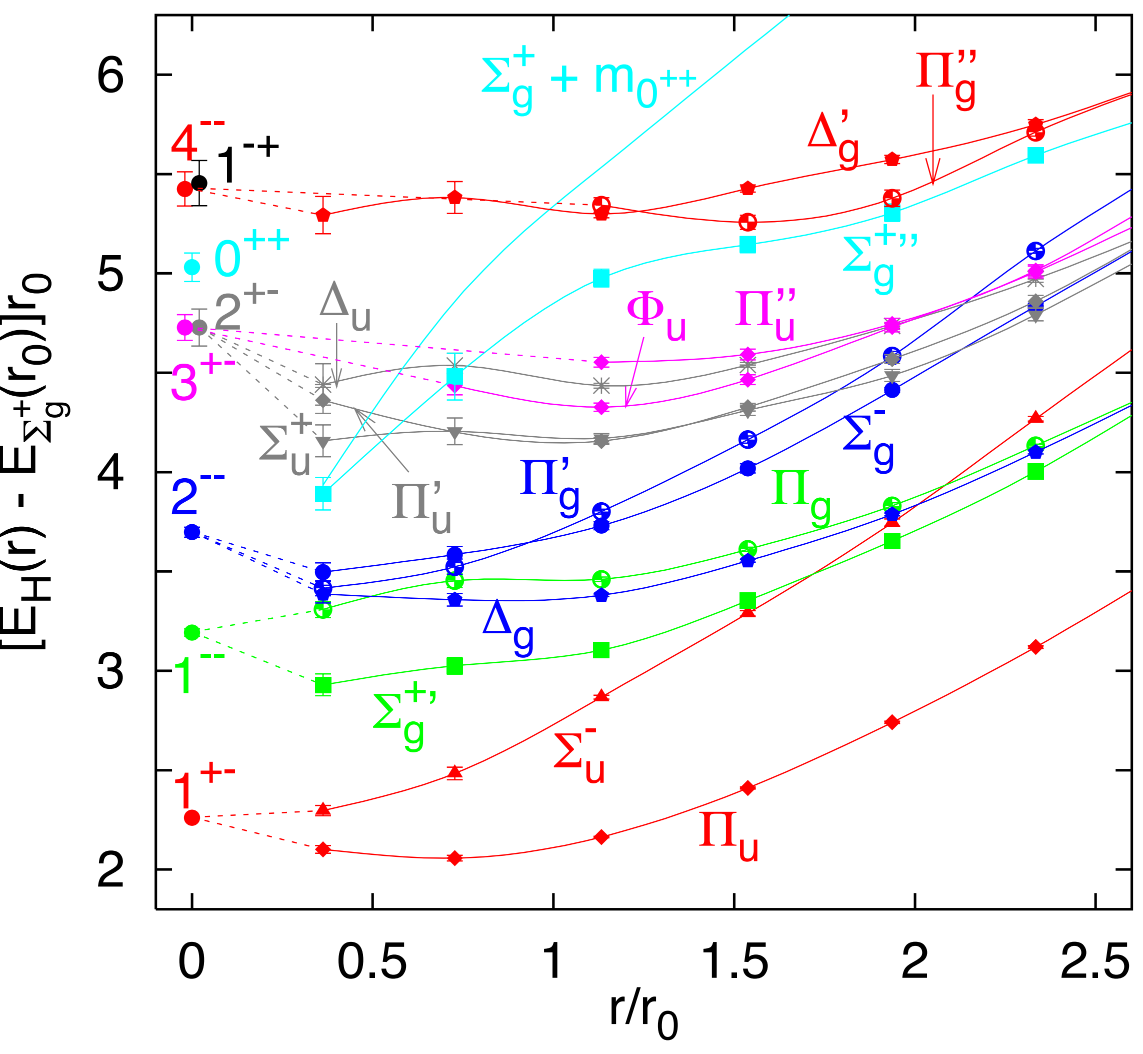


Schlosser, Wagner 2111.00741, Bali Pineda 2004

Schlosser and Wagner Phys. Rev. D. 105, (2022)

- Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 034502

Hybrids static energies at short distances



The BOEFT characterises the hybrids static energy for short distance

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.

the hybrid static energy can be written as a (multipole) expansion in r:

octet potential — non perturbative coefficient

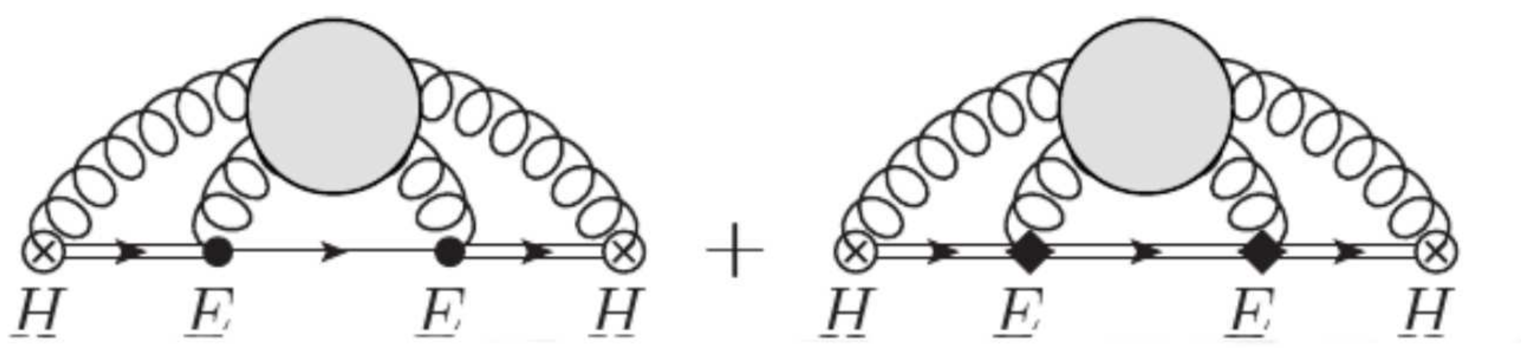
$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \dots$$

Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi^{\text{adj}}_{ab}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,



In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π_g', Δ_g; Σ_u^+, Π_u', Δ_u are degenerate.

Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

The first hybrid static energy excitation

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$P^{i\dagger}_{\kappa\lambda} O^a(\mathbf{r},\mathbf{R},t) H^{ia}_\kappa(\mathbf{R},t) = Z_\kappa \Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{ \Psi^\dagger_{1+-\lambda} \left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m} \hat{r}^i_{\lambda'} \right) \Psi_{1+-\lambda'} \right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp \left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \frac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \frac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$

fitted from the lattice hybrids static energies

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$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
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fitted from the lattice hybrids static energies

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0 \Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}^2_r}{m} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda\left(\frac{\boldsymbol{\nabla}^2_r}{m}\right)\hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}^2_r}{m} + C^{\text{nad}}_{1+-\lambda\lambda'}$$

with $C^{\text{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda\left[\frac{\boldsymbol{\nabla}^2_r}{m}, \hat{r}^i_{\lambda'}\right]$ called the nonadiabatic coupling.

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

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$$\mathcal{L}_{\text{BOEFT for } 1^{+-}} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{ \Psi^{\dagger}_{1^{+-}\lambda} \left(i\partial_0 - V_{1^{+-}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_{\lambda} \frac{\boldsymbol{\nabla}^2_r}{m} \hat{r}^i_{\lambda'} \right) \Psi_{1^{+-}\lambda'} \right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1^{+-}\lambda\lambda'} = V^{(0)}_{1^{+-}\lambda\lambda'} + \frac{V^{(1)}_{1^{+-}\lambda\lambda'}}{m} + \frac{V^{(2)}_{1^{+-}\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1^{+-}\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1^{+-}\lambda}$, with $V^{(0)}_{1^{+-}0} = E_{\Sigma_u^-}$, $V^{(0)}_{1^{+-}\pm 1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

$$\left[-\frac{1}{mr^2}\,\partial_r r^2 \partial_r + \frac{1}{mr^2} \begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E^{(0)}_{\Sigma} & 0 \\ 0 & E^{(0)}_{\Pi} \end{pmatrix} \right] \begin{pmatrix} \psi^{(N)}_{\Sigma} \\ \psi^{(N)}_{-\Pi} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_{\Sigma} \\ \psi^{(N)}_{-\Pi} \end{pmatrix}$$

$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{mr^2} + E^{(0)}_{\Pi} \right] \psi^{(N)}_{+\Pi} = \mathcal{E}_N\, \psi^{(N)}_{+\Pi}$$

Mixing remove the degeneration among opposite parity states: ->Lambda doubling existing also in molecular physics

- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$

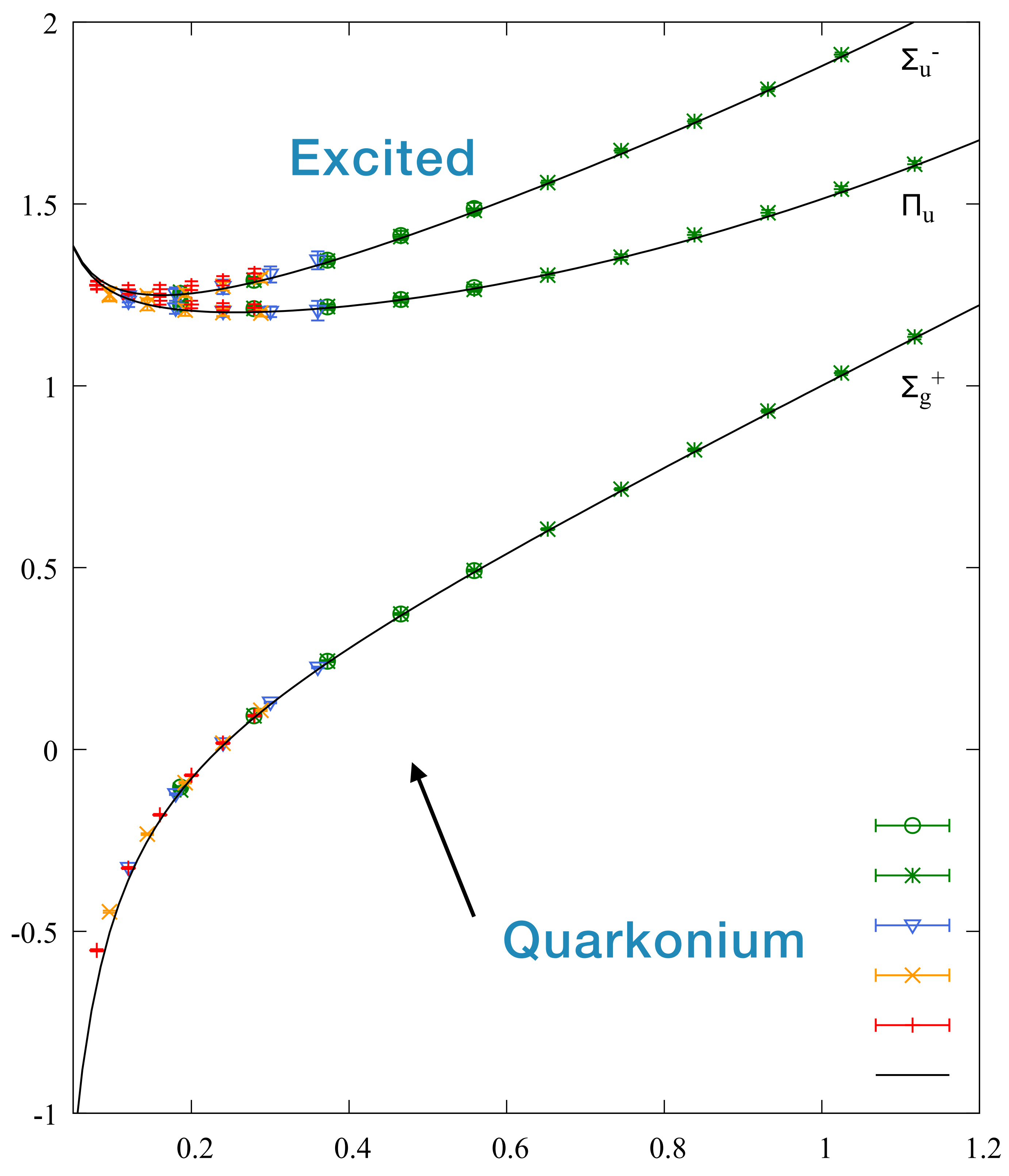
One Born–Oppenheimer Effective Theory to rule them all: hybrids, tetraquarks, pentaquarks, doubly heavy baryons and quarkonium

Matthias Berwein,[1] Nora Brambilla,[1,2,3] Abhishek Mohapatra,[1,*] and Antonio Vairo[1]

The discovery of XYZ exotic states in the hadronic sector with two heavy quarks, represents a significant challenge in particle theory. Understanding and predicting their nature remains an open problem. In this work, we demonstrate how the Born–Oppenheimer (BO) effective field theory (BOEFT), derived from Quantum Chromodynamics (QCD) on the basis of scale separation and symmetries, can address XYZ exotics of any composition. We derive the Schrödinger coupled equations that describe hybrids, tetraquarks, pentaquarks, doubly heavy baryons, and quarkonia at leading order, incorporating nonadiabatic terms, and present the predicted multiplets. We define the static potentials in terms of the QCD static energies for all relevant cases. We provide the precise form of the nonperturbative low-energy gauge-invariant correlators required for the BOEFT: static energies, generalized Wilson loops, gluelumps, and adjoint mesons. These are to be calculated on the lattice and we calculate here their short-distance behavior. Furthermore, we outline how spin-dependent corrections and mixing terms can be incorporated using matching computations. Lastly, we discuss how static energies with the same BO quantum numbers mix at large distances leading to the phenomenon of avoided level crossing. This effect is crucial to understand the emergence of exotics with molecular characteristics, such as the $\chi_{c1}(3872)$. With BOEFT both the tetraquark and the molecular picture appear as part of the same description.

XYZ are a formidable opportunity to learn more about the fundamental strong force!

confinement force



$r[\mathrm{fm}]$

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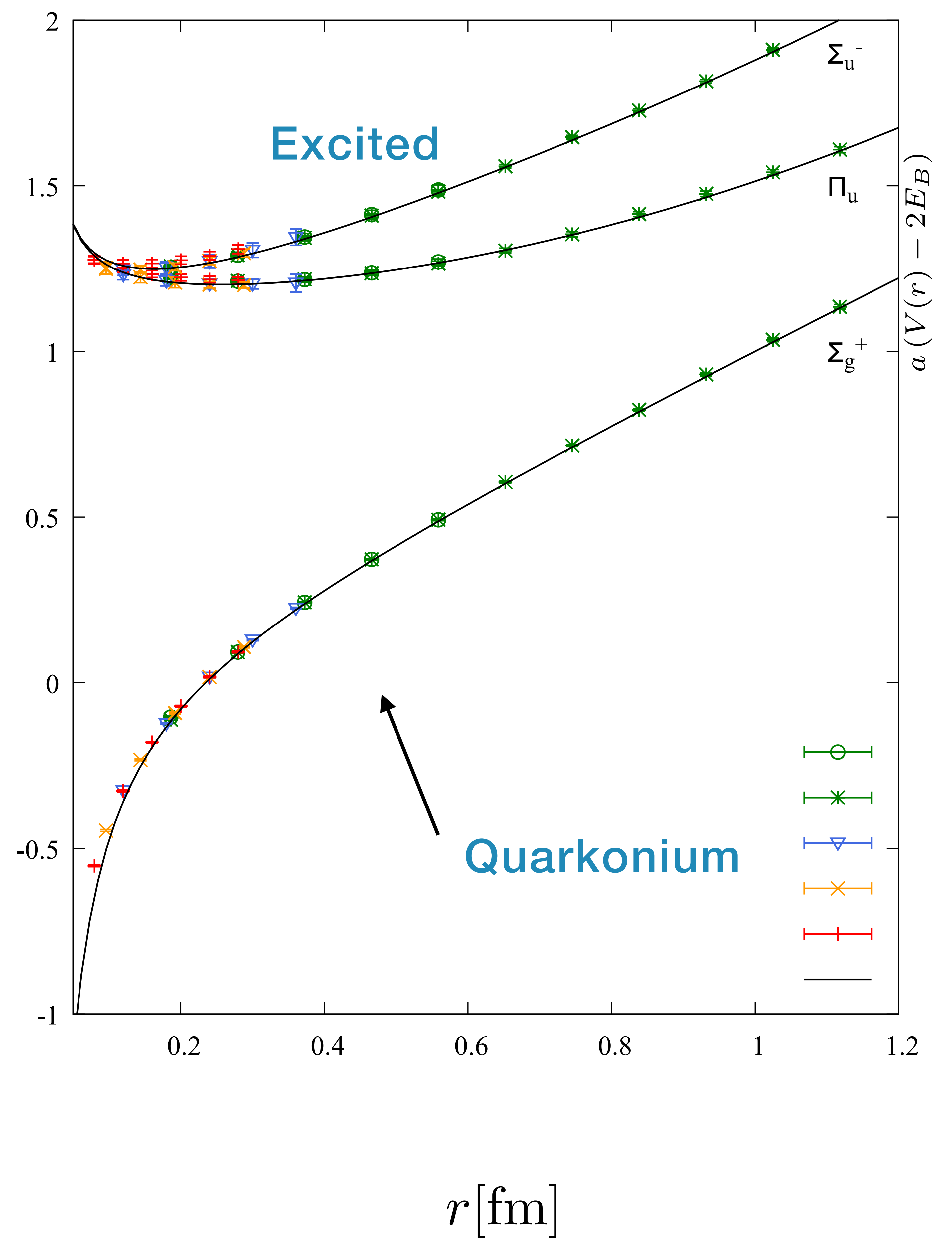
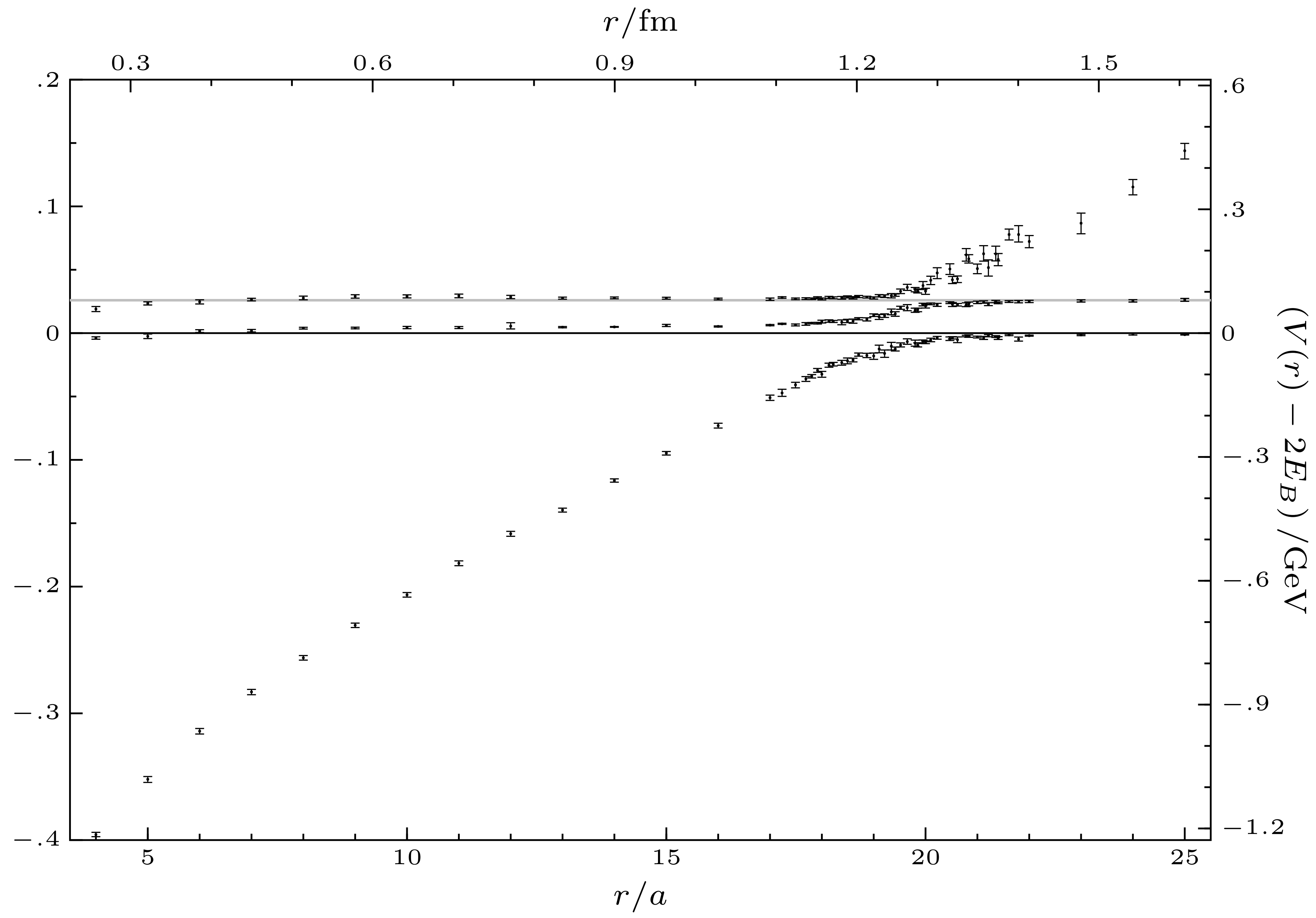
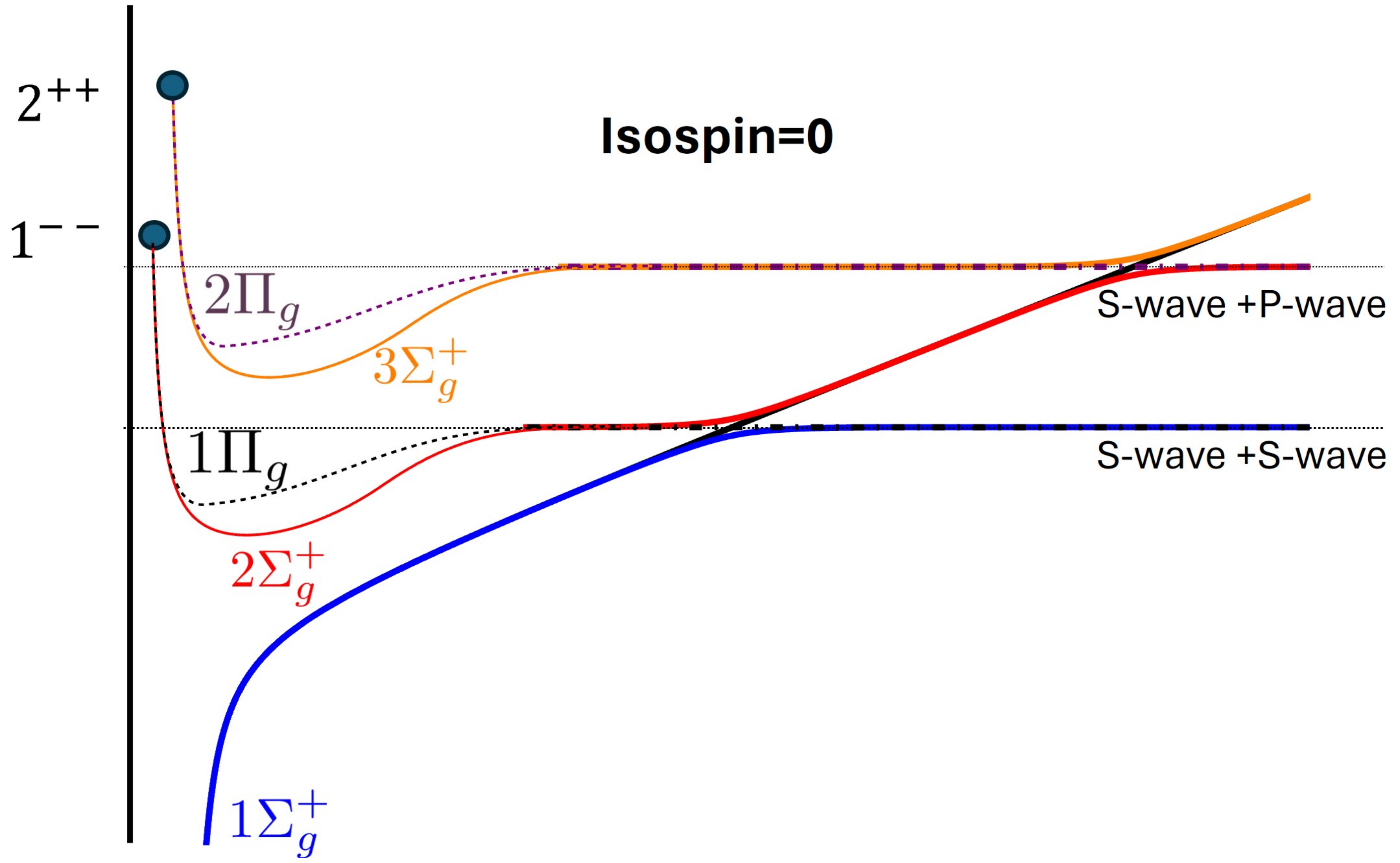




tetraquarks and
heavy-light overlap at
large distance

avoided level crossing
between quarkonium
and tetraquarks

Quarkonium, Tetraquarks
and heavy-light pairs



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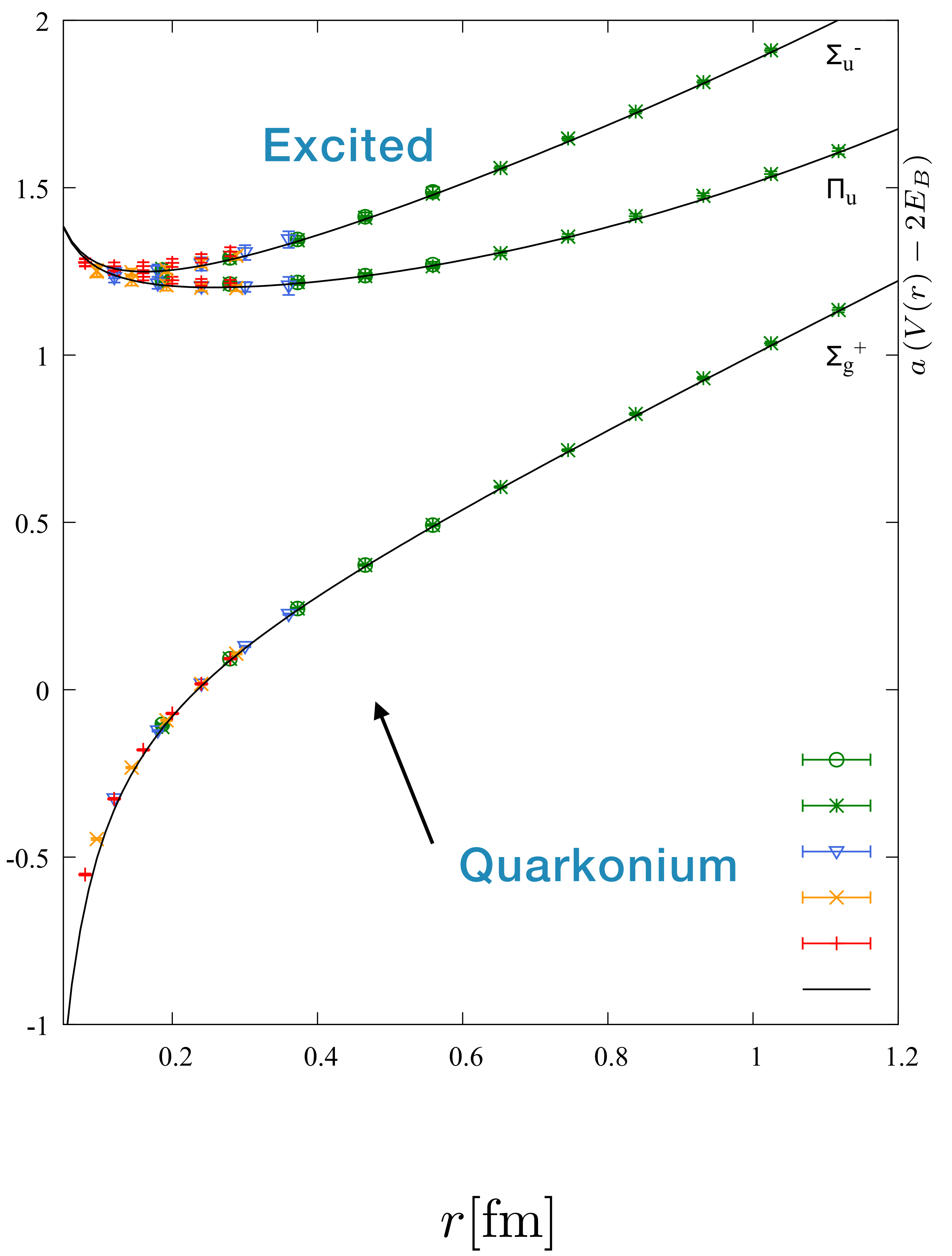

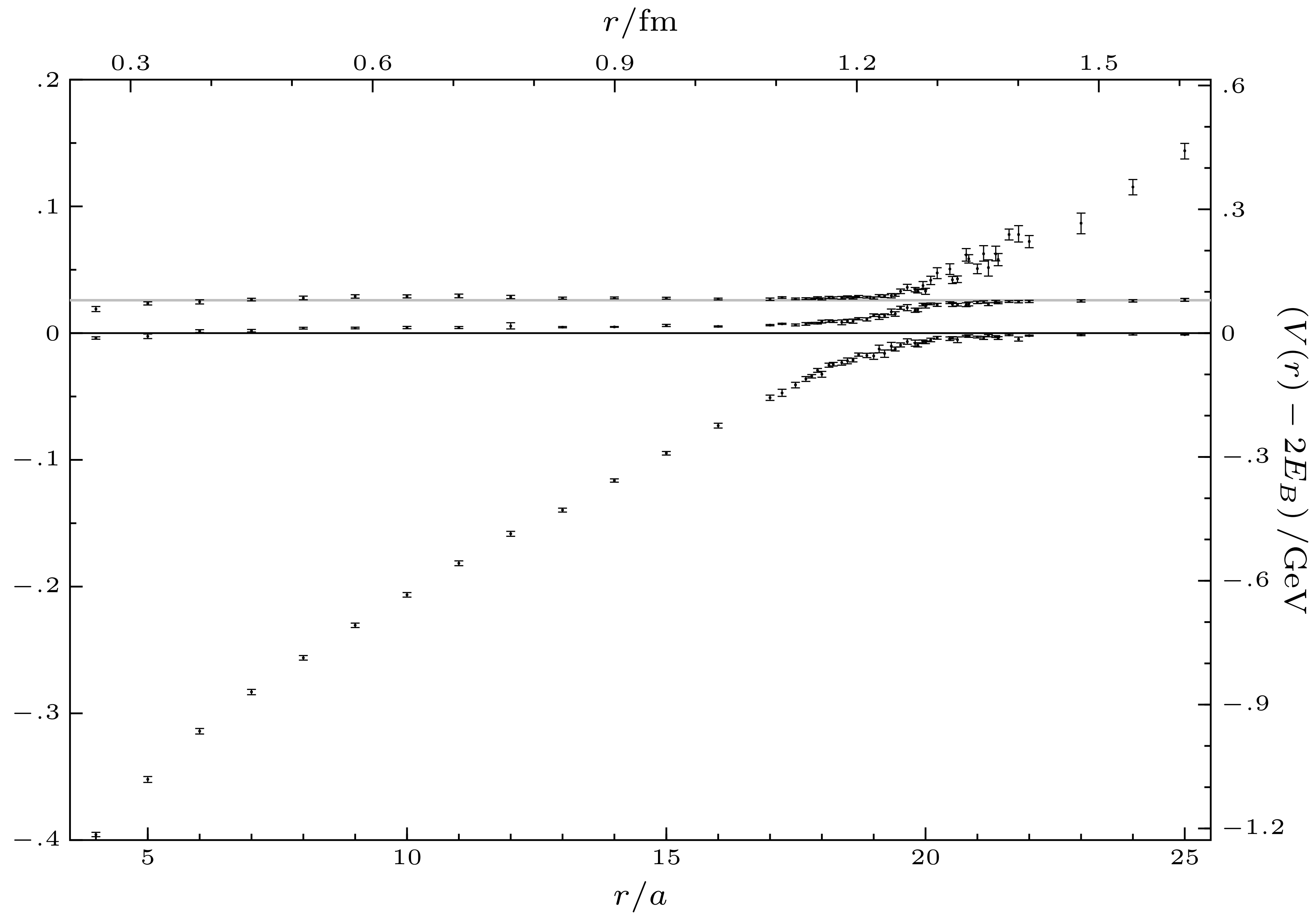

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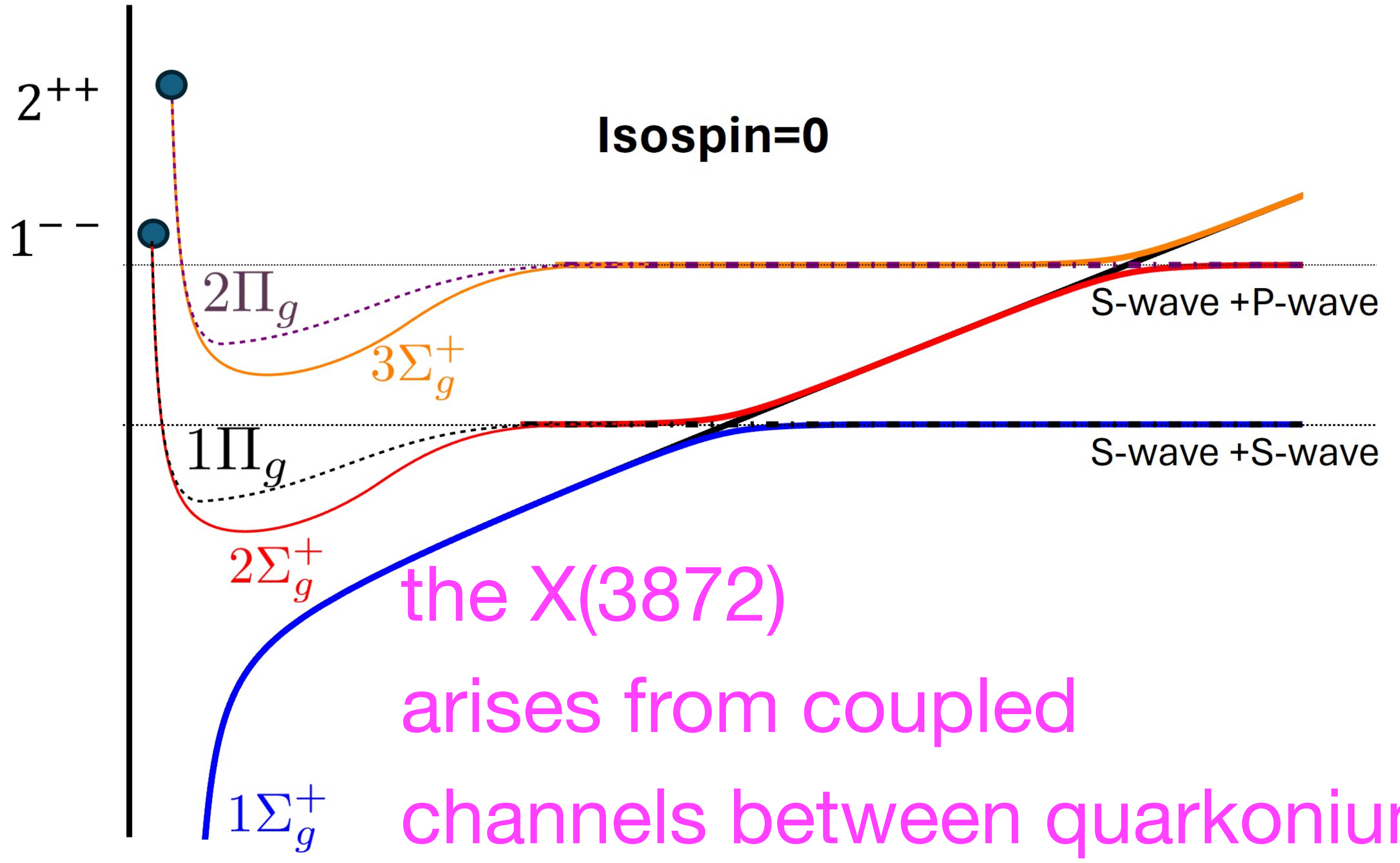
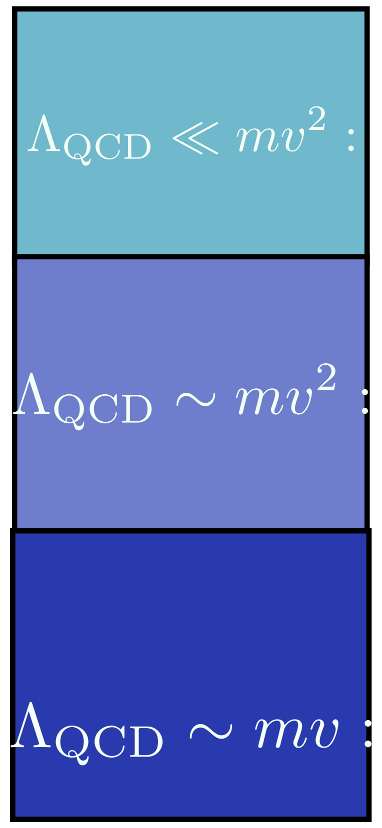


the X(3872) arises from coupled channels between quarkonium and the two first tetraquarks

Low energy (nonperturbative) factorized effects depend on the size of the physical system

The EFT factorizes the low energy nonperturbative part.
Depending on the physical system:



local condensates

$\langle F^2(0) \rangle$ $\Upsilon(1S)$, ...

non local condensates

$\Lambda_{QCD} \sim t$ $\langle F(t)F(0) \rangle$ annihilations, short range $c\bar{c}$, $b\bar{b}$, gluelumps, ...

$\Lambda_{QCD} \sim loop$ $\langle e^{ig \oint dx A(x)} \rangle$ long range $c\bar{c}$, $b\bar{b}$, hybrids, glueballs, ...

Wilson loops

The more extended the physical object, the more we probe the non-perturbative vacuum.

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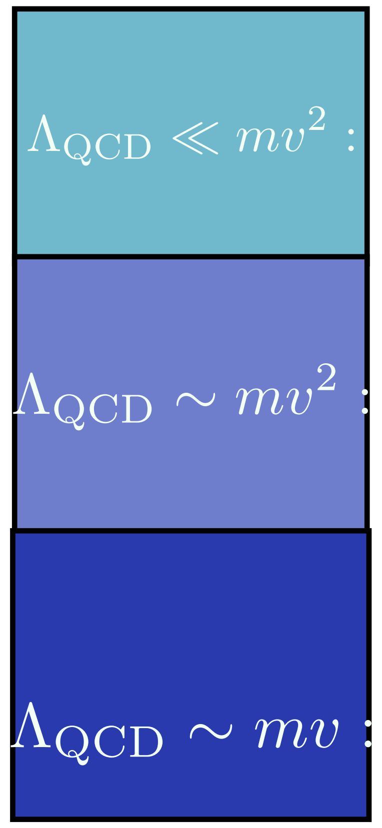

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GAIN:

Inside the EFT:Model independent predictions, power counting

Lattice Calculation of only few nonperturbative objects, universal and depending only on the glue—> at variance with the state dependent calculation of each single observable with the full dynamics!

Inside the EFT: flexible phenomenological applications, understanding of the underlying degrees of freedom and dynamics

CHALLENGE:

Need techniques to reduce noise and improve convergence to continuum for calculation of chromelectric and chomomagnetic fields—> Gradient flow

Avoid change of scheme between continuum and lattice (cutoff) regularization> Gradient flow (composite operators renormalisation in cutoff scheme is painful)

problem of slow convergence to continuum—> cured in gradient flow!

As TUMQCD Lattice collaboration we are addressing theseproblems

Outlook

Nonrelativistic multiscale systems are formidable tools to probe strong interactions and play a key role in many processes at the frontier in particle physics

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pNREFT gives us a novel interpretation of the non relativistic problem and the Schroedinger equation (QM) from field theory and **allow to describe NR multi scales systems in QFT**

Allow us to make calculations with unprecented precision and to systematically factorize short from long range contributions —> **in particular we can explore new characteristics of the strong force**

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Combining pNREFTs and other EFTs (Chiral, HTL..) , lattice calculations and new concepts (open quantum systems) we can address relevant contemporary problems:

- The XYZ world
- Quarkonium production: we can factorize the LDMEs in low energy correlators to be calculated on the lattice!
- Quarkonium potential and spectrum at finite temperature: pNRQCD +HTL , new paradigm on suppression
- Nonequilibrium evolution of quarkonium in QGP: pNRQCD + HTL+open quantum systems+ lattice: Linblad eqs
- Dark matter pairs in early universe: pNREFT + HTL+open quantum systems: cross section and evolution
- Applications to Jets, neutrinos, cosmology, quantum information

Thanks Peter!

The future of heavy quarks is bright because

You have been tremendously inspirational for us and your legacy is terrific!

Backup

LDMEs in pNRQCD

The pNRQCD factorization formulas for P-wave quarkonium em production are

$$
\begin{aligned}
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]};\text{em})|\Omega\rangle &= (2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\left[1+\frac{2}{3}\frac{i\mathcal{E}_2}{m}+O\left(v^2\right)\right]\\
\langle\Omega|\mathcal{T}^{\chi_{QJ}}({}^3P_J^{[8]};\text{em})|\Omega\rangle &= (2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\frac{4}{3}\frac{\mathcal{E}_1}{m}\\
\langle\Omega|\mathcal{P}^{\chi_{QJ}}({}^3P_J^{[1]};\text{em})|\Omega\rangle &= (2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\left[m\varepsilon-\frac{2}{3}\mathcal{E}_1+O\left(v^3\right)\right]
\end{aligned}
$$

$R'(0)$ is the derivative of the radial wavefunction at the origin, and ε the binding energy.

$$
\mathcal{E}_n=\frac{1}{2N_c}\int_0^\infty dt\, t^n\langle\Omega|gE^{i,a}(t)\Phi^{ab}(0,t)gE^{i,b}(0)|\Omega\rangle
$$

$\Phi^{ab}(0,t)$ is a Wilson line in the adjoint representation connecting $(t,\mathbf{0})$ with $(0,\mathbf{0})$.

Chromoelectric correlators for electromagnetic production

The wavefunctions at the origin may be computed solving the equation of motion of pNRQCD with potentials determined from lattice QCD or via phenomenological models.

The correlators can be fitted on data for $\chi_{c0}(1P) \to \gamma\gamma$, $\chi_{c2}(1P) \to \gamma\gamma$ and $\sigma(e^+e^- \to \chi_{c1}(1P) + \gamma)$ ($= 17.3^{+4.2}_{-3.9} \pm 1.7$ fb at $\sqrt{s}$ =10.6 GeV from Belle).

- Belle coll PRD 98 (2018) 092015

The correlators are universal: they do not depend neither on the flavor of the heavy quark nor on the quarkonium state:

$$\mathcal{E}_1 = -\,0.20^{+0.14}_{-0.14} \pm 0.90 \text{ GeV}^2$$
$$i\mathcal{E}_2 = 0.77^{+0.98}_{-0.86} \pm 0.85 \text{ GeV}$$

The universal nature of the correlators allows to use them to compute cross sections (and decay widths) for quarkonia with different principal quantum number and bottomonia.

LDMEs in pNRQCD

The pNRQCD factorization formulas for P-wave quarkonium hadroproduction are

$$\begin{aligned}
\langle\Omega|\mathcal{O}^{h_Q}({}^1P_1^{[1]})|\Omega\rangle =& 3\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\\
\langle\Omega|\mathcal{O}^{h_Q}({}^1S_0^{[8]})|\Omega\rangle =& 3\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\frac{1}{9N_c m^2}\mathcal{E}\\
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})|\Omega\rangle =& (2J+1)\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\\
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle =& (2J+1)\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\frac{1}{9N_c m^2}\mathcal{E}
\end{aligned}$$

$R^{(0)\prime}(0)$ is the derivative of the radial wavefunction at the origin at leading order in v.
LDMEs are polarization summed in the case of χ_{QJ} states.

The above expressions imply (at leading order in v) the universality of the ratios

$$\frac{m^2\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle}{\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})|\Omega\rangle}=\frac{m^2\langle\Omega|\mathcal{O}^{h_Q}({}^1S_0^{[8]})|\Omega\rangle}{\langle\Omega|\mathcal{O}^{h_Q}({}^1P_1^{[1]})|\Omega\rangle}=\frac{\mathcal{E}}{9N_c}$$

$$\mathcal{E}=\frac{3}{N_c}\int_0^\infty dt\,t\int_0^\infty dt'\,t'\,\langle\Omega|\Phi_\ell^{\dagger ab}\Phi^{\dagger ad}(0;t)gE^{d,i}(t)gE^{e,i}(t')\Phi^{ec}(0;t')\Phi_\ell^{bc}|\Omega\rangle$$

$$\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\frac{3|R_V^{(0)}(0)|^2}{4\pi}\mathcal{E}_{10;10}(\mu)$$

$$\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R_V^{(0)}(0)|^2}{4\pi}c_F^2(\mu)\,\mathcal{B}_{00}(\mu)$$

$$\langle \mathcal{O}^V({}^3P_0^{[8]})\rangle = \frac{1}{18N_c}\frac{3|R_V^{(0)}(0)|^2}{4\pi}\mathcal{E}_{00}$$

$$\mathcal{E}_{10;10} = \Big| d^{dac}\int_0^\infty dt_1\, t_1 \int_{t_1}^\infty dt_2\, gE^{b,i}(t_2) \times \Phi_0^{bc}(t_1;t_2)gE^{a,i}(t_1)\Phi_0^{df}(0;t_1)\Phi_\ell^{ef}|\Omega\rangle\Big|^2$$

$$\mathcal{B}_{00} = \Big|\int_0^\infty dt\, gB^{a,i}(t)\Phi_0^{ac}(0;t)\Phi_\ell^{bc}|\Omega\rangle\Big|^2$$

$$\mathcal{E}_{00} = \Big|\int_0^\infty dt\, gE^{a,i}(t)\Phi_0^{ac}(0;t)\Phi_\ell^{bc}|\Omega\rangle\Big|^2$$