NRQCD and Quarkonium Production

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Key Ideas from Peter

Heavy Quarkonium: A Multi-Scale Problem

- Heavy quarkonium: a bound state of a heavy quark Q and heavy antiquark $\bar{Q}$ (charmonium, bottomonium).
- There are three important scales in in a heavy quarkonium:
  - m, the heavy-quark mass;
  - mv, the typical heavy-quark momentum;
  - mv^2, the typical heavy-quark kinetic energy and binding energy.
- v is the typical heavy-quark velocity in the quarkonium CM frame.
  - $v^2 \approx 0.3$ for charmonium.
  - $v^2 \approx 0.1$ for bottomonium.

- In theoretical analyses, it is useful to separate the perturbative scale m from the lower, nonperturbative scales.
  - $\alpha_s(m_c) \approx 0.25$ and $\alpha_s(m_b) \approx 0.18$
  - Approximate symmetries (*e.g.* heavy-quark spin symmetry) are valid at the lower scales.
  - Analytic calculations simplify when they involve only one scale at a time.
  - Lattice calculations can encompass only a limited range of scales, and so become more tractable after scale separation.

NRQCD

- The effective field theory NRQCD provides a convenient way to separate the scale m from the scale mv and lower scales.
- Generalization of NRQED Caswell, Lepage [PLB 167 (1986) 437].
- A valid description of physics with momenta $p \lesssim mv$.
- Construct by integrating out the modes in the QCD path integral that have $p \gtrsim m$.
- Leading terms in $p/m \sim v$ are just the Schrödinger action.

$$\mathcal{L}_0 = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2m} \right) \chi.$$

$$D_t = \partial_t + igA_0. \qquad \mathbf{D} = \boldsymbol{\partial} - ig\mathbf{A}.$$

  - ψ is the two-component (Pauli) spinor that annihilates a Q.
  - χ is the two-component spinor that creates a $\bar{Q}$.

- To reproduce QCD completely, we would need an infinite number of interactions. For example, at next-to-leading order in v^2 we have

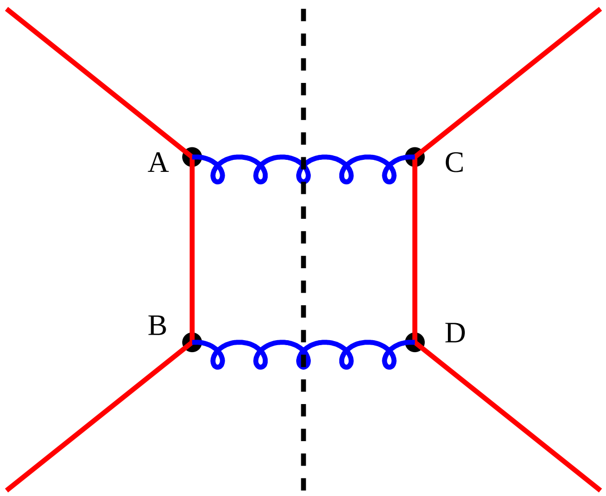
$$\begin{aligned}
\delta\mathcal{L}_{\rm bilinear} &= \frac{c_1}{8m^3}\left[\psi^\dagger(\mathbf{D}^2)^2\psi \;-\; \chi^\dagger(\mathbf{D}^2)^2\chi\right] \\
&+ \frac{c_2}{8m^2}\left[\psi^\dagger(\mathbf{D}\cdot g\mathbf{E}-g\mathbf{E}\cdot\mathbf{D})\psi+\chi^\dagger(\mathbf{D}\cdot g\mathbf{E}-g\mathbf{E}\cdot\mathbf{D})\chi\right] \\
&+ \frac{c_3}{8m^2}\left[\psi^\dagger(i\mathbf{D}\times g\mathbf{E}-g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi+\chi^\dagger(i\mathbf{D}\times g\mathbf{E}-g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right] \\
&+ \frac{c_4}{2m}\left[\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi \;-\; \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right].
\end{aligned}$$

- In practice, work to a given precision in v.
- The c_i are called short-distance coefficients.
  - They can be computed in perturbation theory by matching amplitudes in full QCD and NRQCD.
  - The c_i contain the effects from momenta $p \gtrsim m$.
- Momenta in NRQCD are cut off at a scale $\Lambda \sim mv$.
  - Λ plays the rôle of a factorization scale between the hard and soft physics.

Quarkonium Inclusive Decays in NRQCD

(GTB, Braaten, Lepage [hep-ph/9407339])

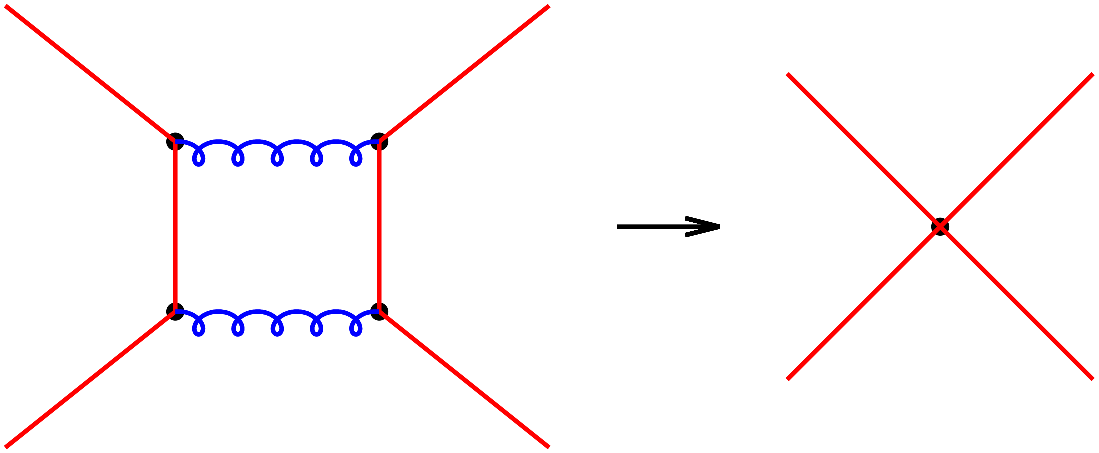
Space-time Picture of Heavy-Quarkonium Annihilation



- A(C) and B(D) are within $\sim 1/m$ of each other.
- Emission of a gluon with energy $\sim m$ puts heavy-quark propagator off shell by $\sim m$.

- A and C are within $\sim 1/m$ of each other.
  - The squared amplitude is insensitive to momenta $\lesssim m$ that run through the upper gluon.
  - Therefore, the Fourier transform has support only for A and C within $\sim 1/m$ of each other.

- Soft final-state interactions could spoil this argument.
  - But soft divergences cancel by the KLN thm. for inclusive processes.
- In NRQCD the annihilation is represented by local 4-fermion interactions.



- The finite size of the annihilation vertex is taken into account by including operators of higher order in v.

- The inclusive annihilation rate is given by the NRQCD factorization formula:

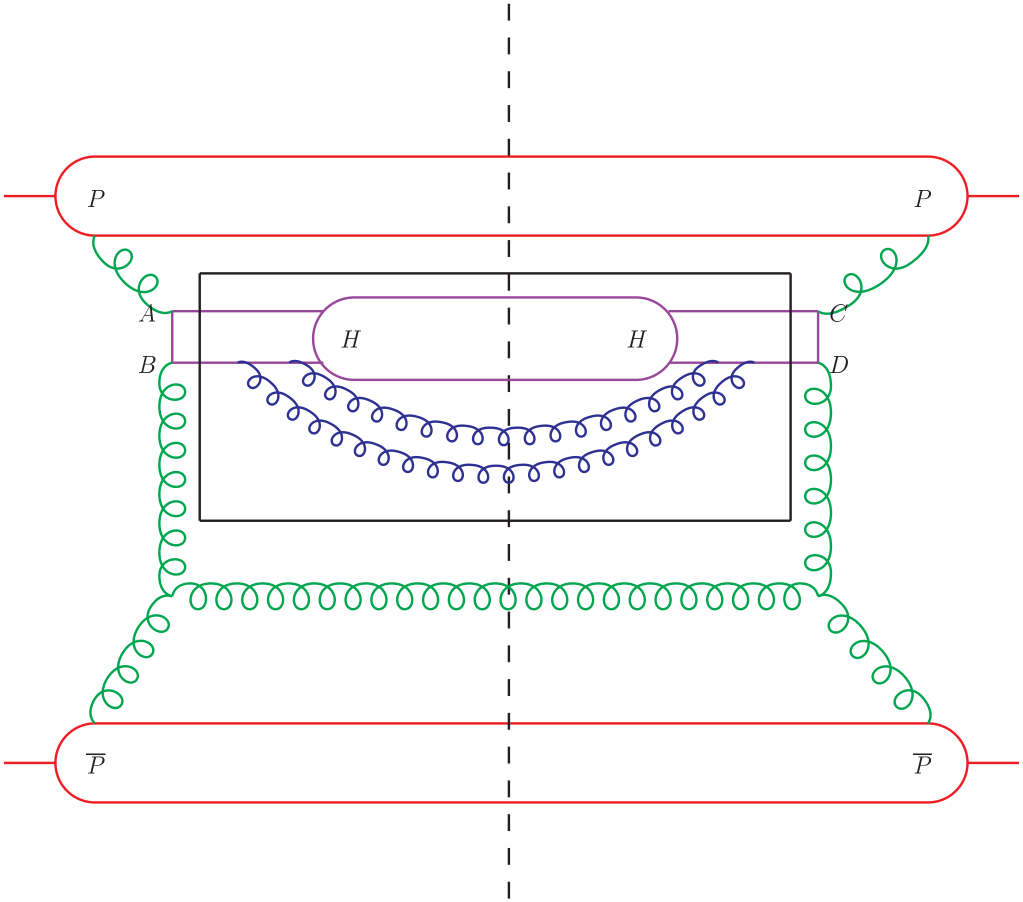
$$\Gamma(H \to \text{light hadrons}) = \sum_n \frac{2\,\mathrm{Im}\,f_n}{m^{d_n-4}} \langle H|\mathcal{O}_n|H\rangle.$$

- The f_n are short-distance coefficients (SDCs).
  - Determine the f_n by matching annihilation amplitudes between full QCD and NRQCD in perturbation theory.
  - The f_n have an expansion in powers of α_s.
- All of the nonperturbative physics is in the long-distance matrix elements (LDMEs) of the 4-fermion operators in the quarkonium state.
  - Calculate on the lattice or determine from experiments.
  - Universal—process independent.
- The matrix elements have a known scaling with v.
  - Truncate the sum over n at the desired accuracy.

- The $Q\bar{Q}$ pair can annihilate in a color-octet or a color-singlet state.
  - The color-singlet contribution alone is IR divergent for P-wave decays.
  - The color-octet contribution cures the IR divergence.
- The color-singlet contributions at leading order in v are called the color-singlet model (CSM).
- NRQCD factorization for inclusive quarkonium decays is believed to be valid to all orders in α_s and v.

Factorization of the Inclusive Cross Section

- An incoming p and $\bar{p}$ emit high-energy gluons that create a high-p_T $Q\bar{Q}$ pair.
- The high-p_T $Q\bar{Q}$ pair evolves into a quarkonium H by emitting soft gluons.



- $A(C)$ and $B(D)$ are within $\sim 1/m_Q$ of each other.
- Kinematics implies the heavy-quark propagators are off shell by $\sim m$.
- The points A and C are within $\sim 1/p_T$ of each other.
- The rate is insensitive to a change in the momentum through the upper energetic gluons if it is $\ll p_T$.

- The part of the diagram inside the box corresponds to an LDME.
- The remainder of the diagram is the SDC.

- The probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium can be calculated as a vacuum-matrix element in NRQCD:

$$\langle 0|\mathcal{O}_n^H|0\rangle = \langle 0|[\chi^\dagger \kappa_n \psi](0)\left(\sum_X |H+X\rangle\langle H+X|\right)[\psi^\dagger \kappa'_n \chi](0)|0\rangle.$$

The κ_n's are combinations of Pauli matrices and color matrices.

- This is the matrix element of a four-fermion operator, but with a projection onto an intermediate state of the quarkonium H plus anything.
- The production matrix elements are the crossed versions of quarkonium decay matrix elements.
  - Only the color-singlet production and decay matrix elements are simply related.

The NRQCD Factorization Conjecture

(GTB, Braaten, Lepage [hep-ph/9407339])

- The inclusive cross section for producing a quarkonium at large p_T can be written as a sum of short-distance coefficients (SDCs) times NRQCD long-distance matrix elements (LDMEs).

$$\sigma(H) = \sum_n F_n \langle 0|\mathcal{O}_n^H|0\rangle.$$

- The SDCs (F_n)
  - Determined by matching full QCD to NRQCD.
  - Essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair, convolved with the parton distributions.
  - Have an expansion in powers of α_s.
- The LDMEs ($\langle 0|\mathcal{O}_n^H|0\rangle$) are supposed to be universal (process independent).
  - That is what gives NRQCD factorization its predictive power.

- The LDMEs have a known scaling with v.
- The current phenomenology of J/ψ, $\psi(2S)$, and Υ production uses LDMEs through relative order v^4:

$$\begin{aligned}
&\langle\mathcal{O}^H({}^3S_1^{[1]})\rangle \quad (O(v^0)),\\
&\langle\mathcal{O}^H({}^1S_0^{[8]})\rangle \quad (O(v^3)),\\
&\langle\mathcal{O}^H({}^3S_1^{[8]})\rangle \quad (O(v^4)),\\
&\langle\mathcal{O}^H({}^3P_J^{[8]})\rangle \quad (O(v^4)).
\end{aligned}$$

  Notation is ${}^{2S+1}L_J^{[c]}$, where c is the color multiplet.

- Calculations show that the ${}^3S_1^{[1]}$ contributions are negligible for J/ψ hadroproduction.
- Three color-octet LDMEs need to be determined phenomenologically for each S-wave quarkonium state.
- The $\langle\mathcal{O}^H({}^3P_J^{[8]})\rangle$ ($J=0,1,2$) are related by the heavy-quark spin symmetry.

Modification of the LDMEs

(Nayak, Qiu, Sterman [hep-ph/0501235])

- The color-octet LDMEs must be modified by the inclusion of Wilson (eikonal) lines to make them gauge invariant.
- The Wilson lines are path integrals of the gauge field.
- Run from the $Q\bar{Q}$ creation points to infinity.
- Essential at two-loop order to allow certain soft contributions to be absorbed into the NRQCD LDMEs.

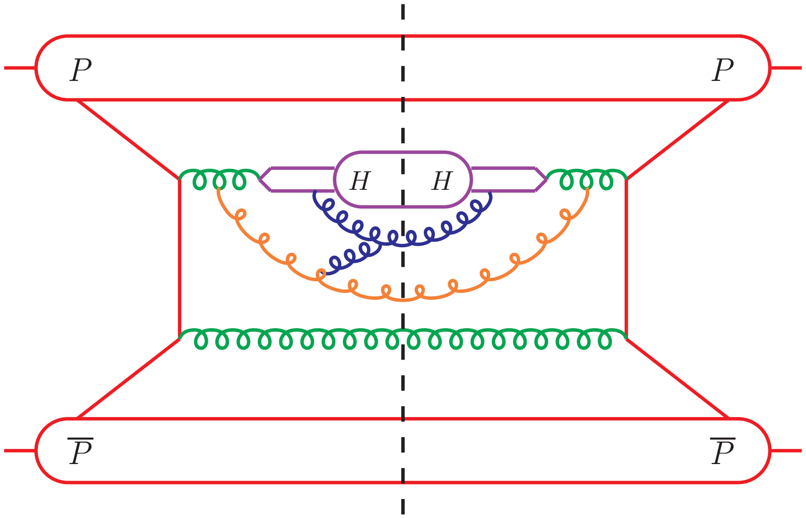
Ingredients of a Factorization Proof

In the collision CM frame:

- Collinear gluons (Nayak, Qiu, Sterman [hep-ph/0509021])
  - factor into parton distributions in the initial state,
  - cancel in the final state,
  - except for collinear gluons that are associated with the quarkonium jet, which factor into quarkonium fragmentation functions.
  - Collinear divergences on heavy-quark lines regulated by m.
- Soft gluons that connect the quarkonium jet to the other hadrons in the production process factor and cancel. (Nayak, Qiu, Sterman [hep-ph/0509021])
  - Holds only through relative order m^2/p_T^2.
    p_T is the quarkonium transverse momentum in the CM frame.
  - Suggests that factorization holds only for $p_T \gg m$.

A Key Difficulty in Proving Factorization

- How do we treat gluons with momenta of order m_Q in the quarkonium rest frame? (Nayak, Qiu, Sterman [hep-ph/0509021])



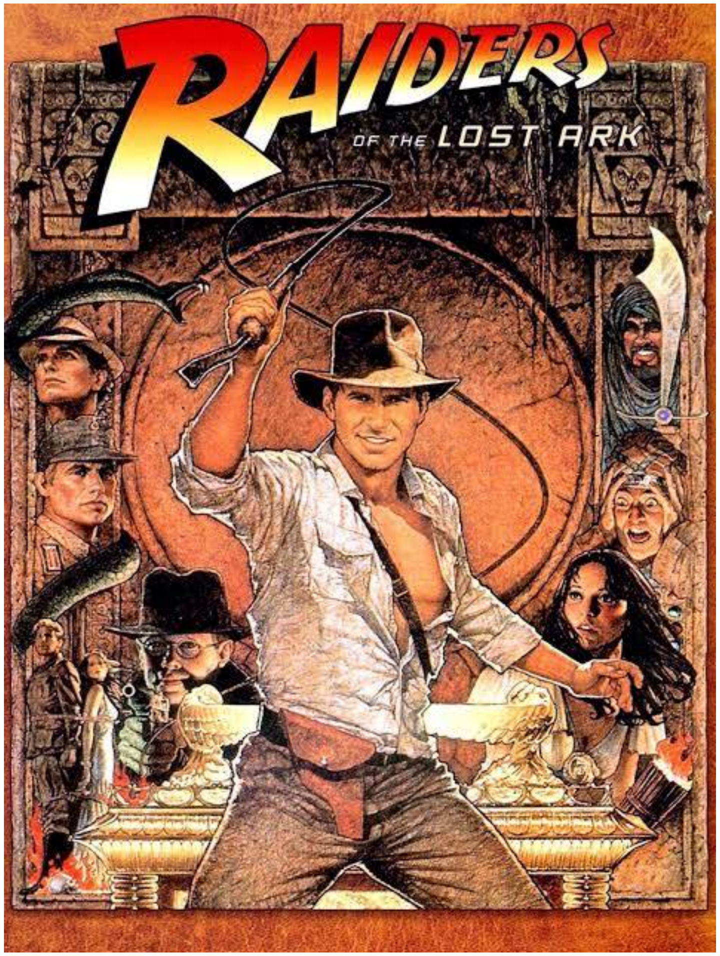
- If the orange gluon has momentum of order m_Q, it can't be absorbed into the NRQCD LDME.
- But the orange gluon can have non-vanishing soft exchanges with the quarkonium constituents.
- The orange gluon produces a "mini-jet" in the fragmentation function.

- The orange gluon can be approximated by a Wilson line.
  - It could then correspond to the Wilson line in the LDME.
  - But only if the LDME does not depend on the direction of the Wilson line, which is given by the direction of the gluon.
  - If the LDME depends on the direction of the Wilson line, then the universality of the LDME is spoiled.

- Through two loops, a “miracle” happens: The dependence on the direction of the Wilson line cancels.
  - Nayak, Qiu, Sterman (hep-ph/0501235, hep-ph/0509021, hep-ph/0608088): First established the two-loop result in a light-front calculation.
  - GTB, Chung, Ee, Kim (1910.05497): Confirmed using covariant methods.
  - Zhang, Meng, Ma, Chao (2011.04905): Confirmed in gluon fragmentation into a ${}^3P_J^{[1,8]}$ $Q\bar{Q}$ pair.
- No obvious generalization of this result to higher orders in α_s.
- Violations of factorization could first appear in N^{3}LO calculations of the LDMEs.
  - But would be parametrically suppressed only by $\alpha_s(m)$ relative to LO.
- An all-orders proof (or disproof) of the universality of the LDMEs is the essential theoretical problem in quarkonium production.

Peter’s Secret Identity

- Peter has always sought out new adventures (*e.g.* Grand Canyon hikes).



- After the initial NRQCD work, Raiders of the Lost Arc appeared.
- Peter started talking about bringing a bullwhip to lectures.
- But Peter never revealed his secret identity. . .

- As Manny the Shark and Alligator wrestler!

Manny Puig Swims With Mako Sharks | Savage Wild | Real Wild

Real Wild · 34K views · 2 years ago

Phenomenology of Quarkonium Production

- Will focus on inclusive J/ψ production
- Interesting theoretical and experimental work that I will not discuss:
  - $\psi(2S)$, χ_c production and polarization
  - Exclusive charmonium production
  - Diffractive charmonium production
  - Double charmonium production
  - Bottomonium production and polarization

SDCs for S-Wave Quarkonia at NLO in α_s

- Hamburg Group: Butenschön, Kniehl
  - J/ψ hadroproduction cross sections and polarizations
  - η_c and $\psi(2S)$ hadroproduction cross sections
  - Cross sections for J/ψ production in ep (photoproduction) and $\gamma\gamma$ collisions
  - Cross sections for J/ψ production in association with a W or Z boson
- PKU Group: Chao, Ma, Shao, Wang, Zhang *et al.*
  - J/ψ and $\psi(2S)$ hadroproduction cross sections and polarizations
  - η_c and $\Upsilon(nS)$ hadroproduction cross sections
- IHEP Group: Gong, Wan, Wang
  - J/ψ, $\psi(2S)$, and $\Upsilon(nS)$ hadroproduction cross sections and polarizations
- These three groups agree on the hadroproduction-cross section and polarization SDCs.

- ANL Group (GTB, Chung, Kim, Lee) and ANL-PKU Group (GTB, Chao, Chung, Kim, Lee, Ma)
  - Resummed leading logs of p_T^2/m_c^2 to all orders for charmonium cross sections and polarizations, using PKU SDCs.

Determinations of the Color-Octet LDMEs from Fits to Data

- Representative, but not all inclusive.
- The color-singlet LDMEs
  - Can be determined from decays to leptons/photons.
  - Give negligible contributions, except in the case of the η_c cross section.
- The Hamburg Group (1105.0920)
  - Made use of their extensive NLO calculations of SDCs.
  - Determined all 3 color-octet LDMEs by making a global fit to cross sections with $p_T > 3$ GeV from the Tevatron, LHC, RHIC, HERA, LEP II, KEKB.
  - Uncorrected for feeddown.

- The ANL Group (1403.3612)
  - Used their SDCs that include resummation of leading logs of p_T^2/m_c^2
  - Fit the CDF and CMS prompt J/ψ cross sections for $p_T > 10$ GeV.
- The ANL-PKU Group (1509.07904)
  - Used their SDCs that include resummation of leading logs of p_T^2/m_c^2.
  - Fit the CDF and CMS J/ψ, $\psi(2S)$, and χ_{cJ} cross sections for $p_T > 10$ GeV.
  - Included NLO feeddown contributions from $\psi(2S)$ and χ_{cJ} in their fit.
- The PKU Group (1009.3655, 1012.1030)
  - Fit the CDF J/ψ cross sections for $p_T > 7$ GeV.
  - Determined only 2 linear combinations of LDMEs unambiguously.
  - Resolved the ambiguity by using the LHCb η_c cross section plus the heavy-quark spin symmetry.
  - Experimental data were used to subtract feeddown contributions.

- The TUM Group (Brambilla, Chung, Vairo, Wang [2210.17345])
  - Used relations for the LDMEs from Potential NRQCD (pNRQCD) to reduce the number of free parameters (corrections of order v^2, $1/N_c^2$):

$$\begin{aligned}
\langle\mathcal{O}^V({}^3S_1^{[1]})\rangle &= \frac{3N_c}{2\pi}|R_V^{(0)}(0)|^2,\\
\langle\mathcal{O}^V({}^3P_J^{[8]})\rangle &= \frac{2J+1}{18N_c}\mathcal{E}_{00}\frac{3|R_V^{(0)}(0)|^2}{4\pi},\\
\langle\mathcal{O}^V({}^1S_0^{[8]})\rangle &= \frac{1}{6N_cm^2}\frac{3|R_V(0)|^2}{4\pi}c_F^2(m;\Lambda)\mathcal{B}_{00}(\Lambda),\\
\langle\mathcal{O}^V({}^3S_1^{[8]})\rangle(\Lambda) &= \frac{1}{2N_cm^2}\frac{3|R_V^{(0)}(0)|^2}{4\pi}\mathcal{E}_{10;10}(\Lambda),
\end{aligned}$$

    $R_V^{(0)}(0)$ is the wave function at the origin

    $\mathcal{E}$ and $\mathcal{B}$ are chromoelectric and chromomagnetic gluonic correlators.

    c_F is the SDC of the spin-flip term in the NRQCD action.

  - Wavefunctions at the origin from the decays to e^+e^-.
  - Gluon correlators from fits to the CMS J/ψ and $\psi(2S)$ and the ATLAS $\Upsilon(2S)$ and $\Upsilon(3S)$ cross sections, taking $p_T/(2m_Q) > 3$ and $p_T/(2m_Q) > 5$.
  - Corrected for feeddown using experimental data.

- The groups extract very different color-octet LDMEs.

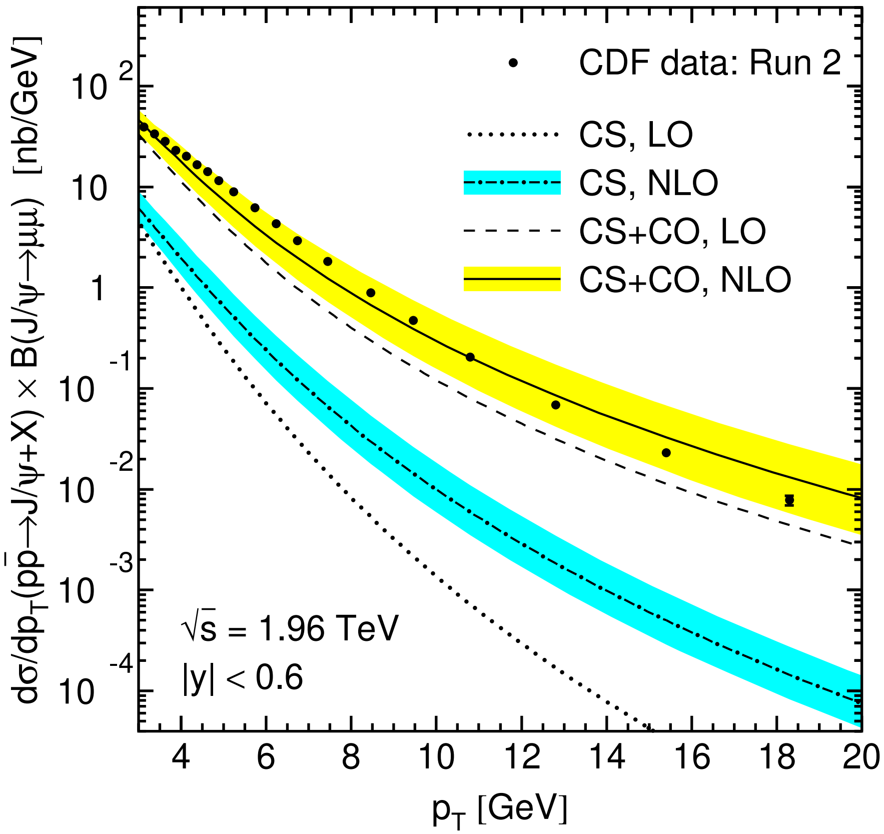
Group	$\langle \mathcal{O}^V(^3S_1^{[8]}) \rangle$	$\langle \mathcal{O}^V(^1S_0^{[8]}) \rangle$	$\langle \mathcal{O}^V(^3P_0^{[8]}) \rangle / m_c^2$
Hamburg	0.16 ± 0.35	3.04 ± 0.35	-0.40 ± 0.07
ANL	1.1 ± 1.0	9.9 ± 2.2	0.49 ± 0.44
ANL-PKU	-0.71 ± 0.36	11.0 ± 1.4	-0.31 ± 0.15
PKU	1.01 ± 0.29	0.73 ± 0.73	1.71 ± 0.52
TUM $(p_T/(2m_c) > 3)$	1.72 ± 0.18	-4.70 ± 1.55	3.14 ± 0.35
TUM $(p_T/(2m_c) > 5)$	1.57 ± 0.45	-2.73 ± 3.64	2.89 ± 0.87

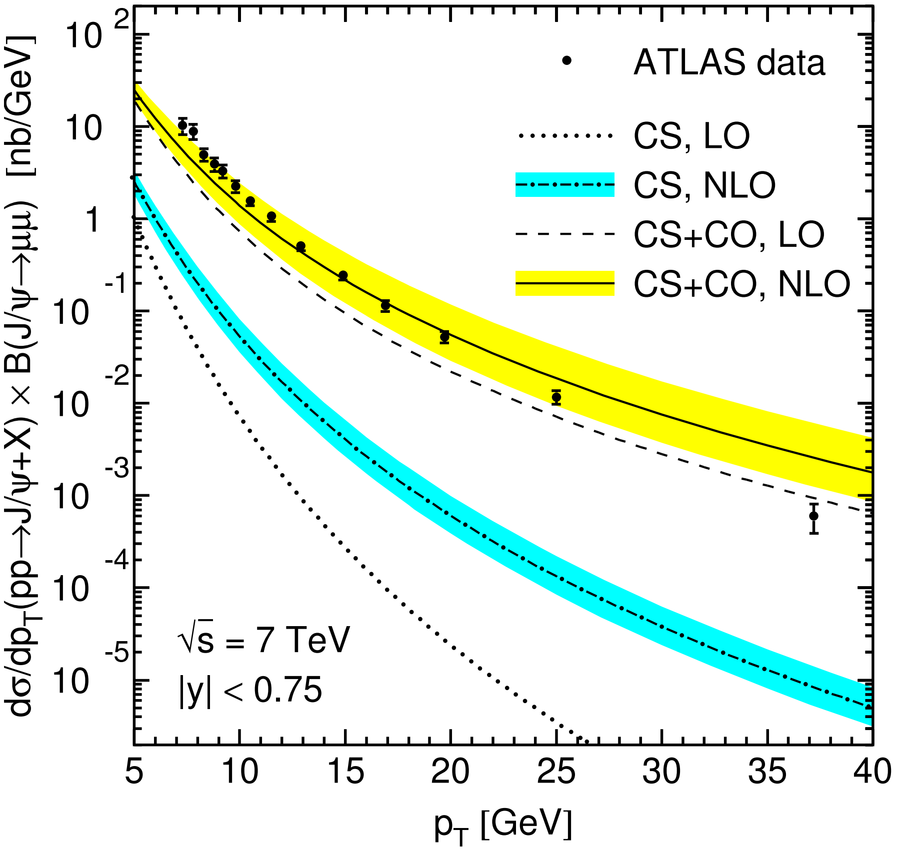
Units of 10^{-2} GeV^3.

- Transverse polarization tends to cancel if $\langle \mathcal{O}^V(^3S_1^{[8]}) \rangle$ and $\langle \mathcal{O}^V(^3P_0^{[8]}) \rangle / m_c^2$ have the same signs.
- True for all but the Hamburg Group.

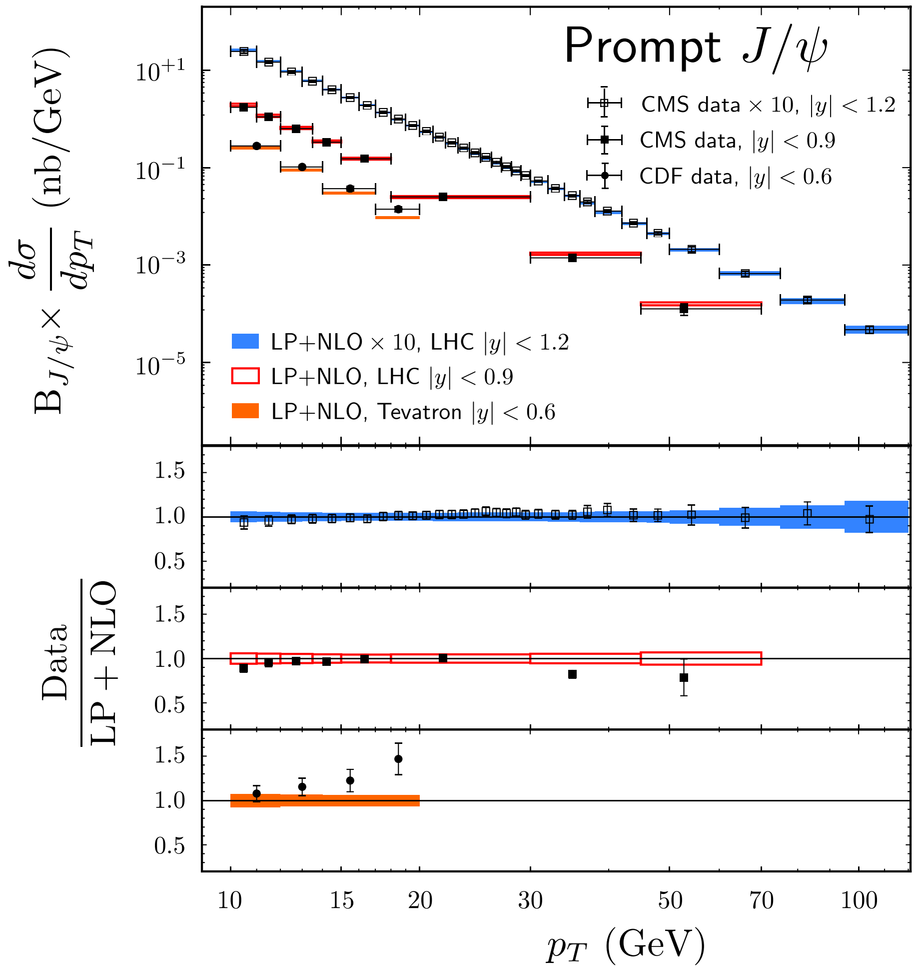
J/ψ Cross Section

- The Hamburg-Group predictions agree with the data, within uncertainties, but are tensions in the shapes that are most apparent at high p_T.



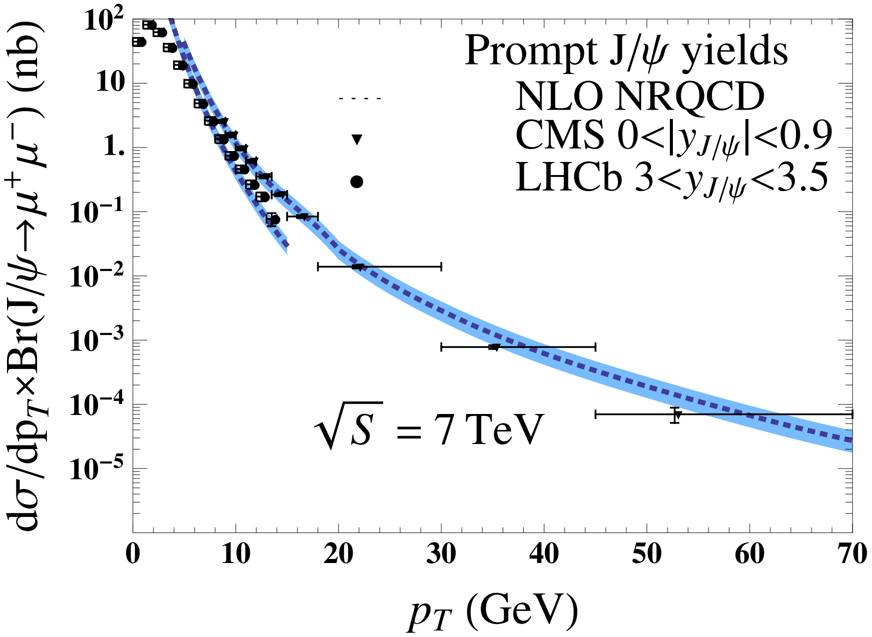


- The ANL-PKU-Group prediction agrees with the CMS data over a large p_T range:



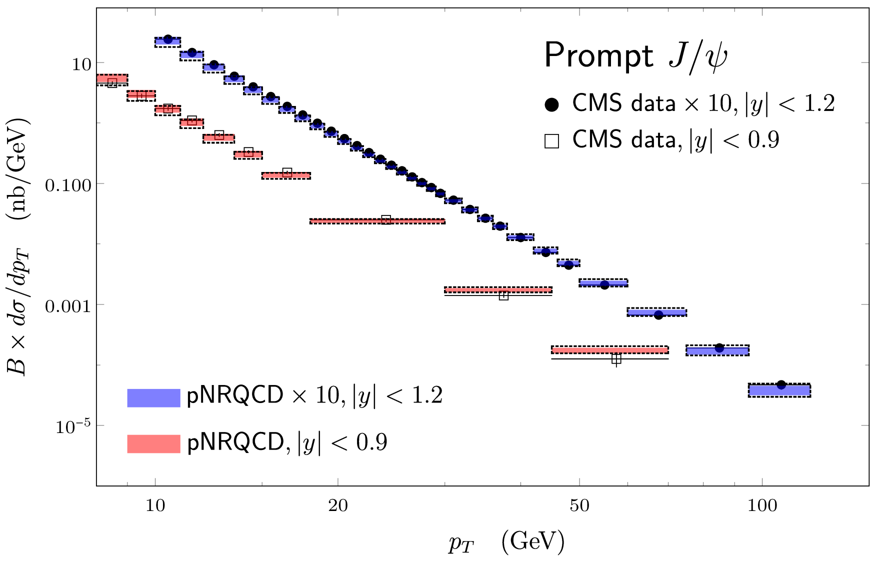
- There is some tension with the CDF data (hep-ex/0412071).
- The ANL-Group fit is similar.

- The PKU Group predictions agree with the CMS and LHCb data.



- Slight tension with the LHCb data.
- Prediction does not agree with data at low p_T, where NRQCD factorization is expected to fail.

- The TUM Group predictions agree with the CMS data.

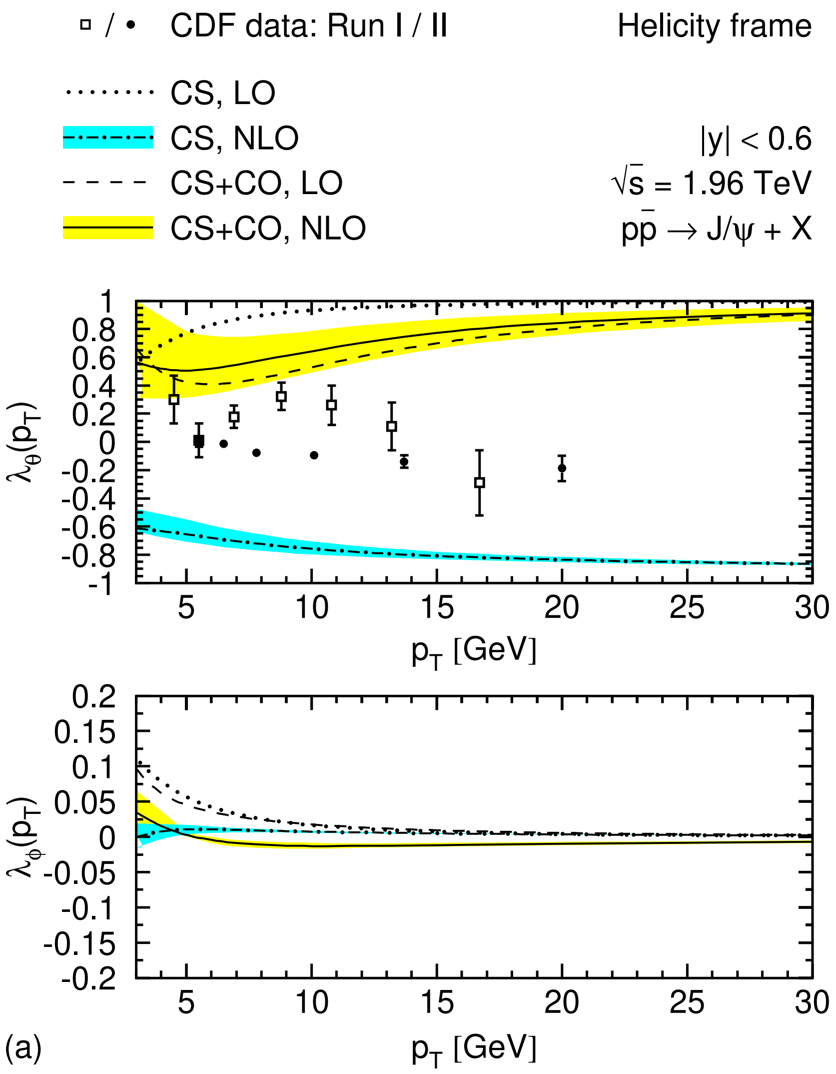


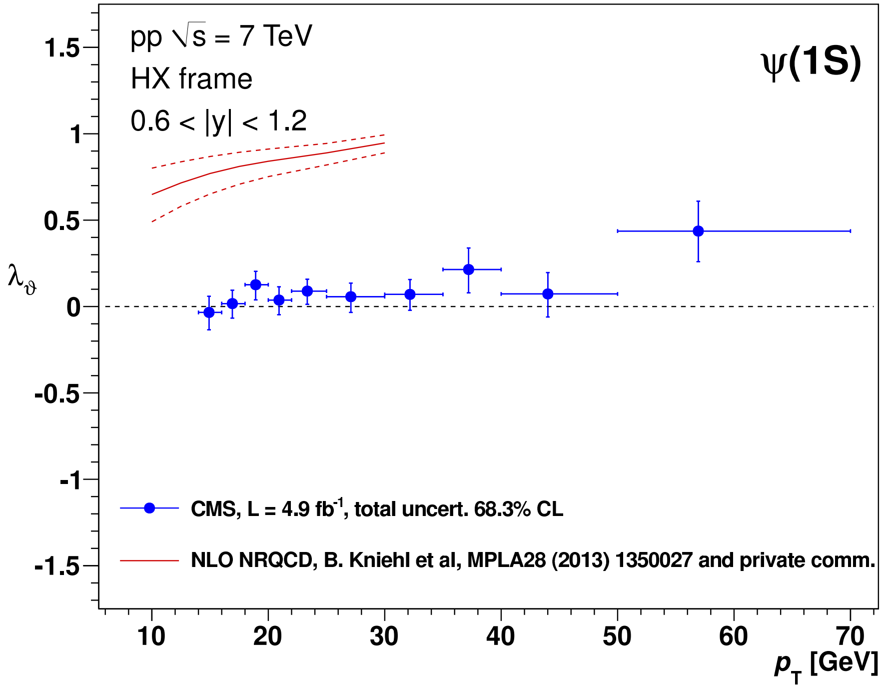
- Heights of Vertical bands determined by differences in fits with $p_T/(2m_c) > 3$ and $p_T/(2m_c) > 5$.

J/ψ Polarization

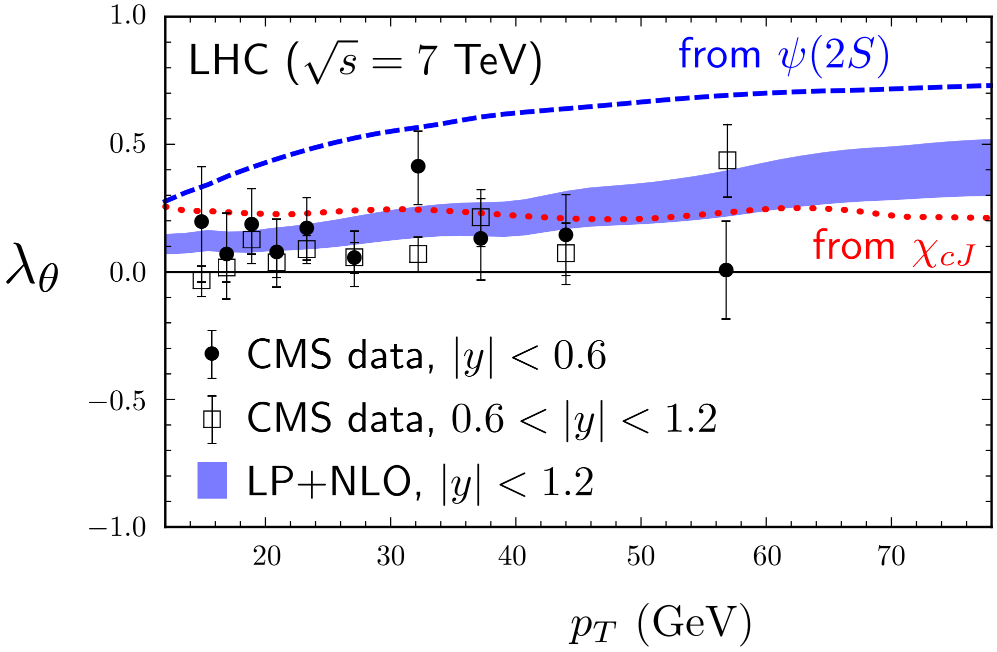
- At high p_T, the ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels are transversely polarized.
- The ${}^3P_J^{[8]}$ and ${}^3S_1^{[8]}$ channels tend to cancel if the LDMEs have the same signs.

- The Hamburg-Group predictions (1201.3862) (yellow and red bands) disagree with the CDF and CMS data.
- $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channels add.



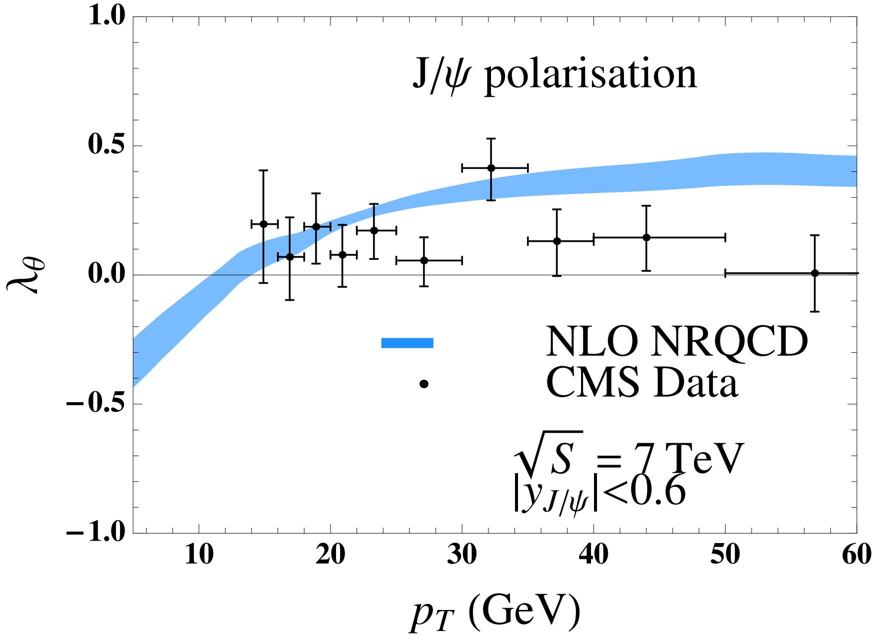


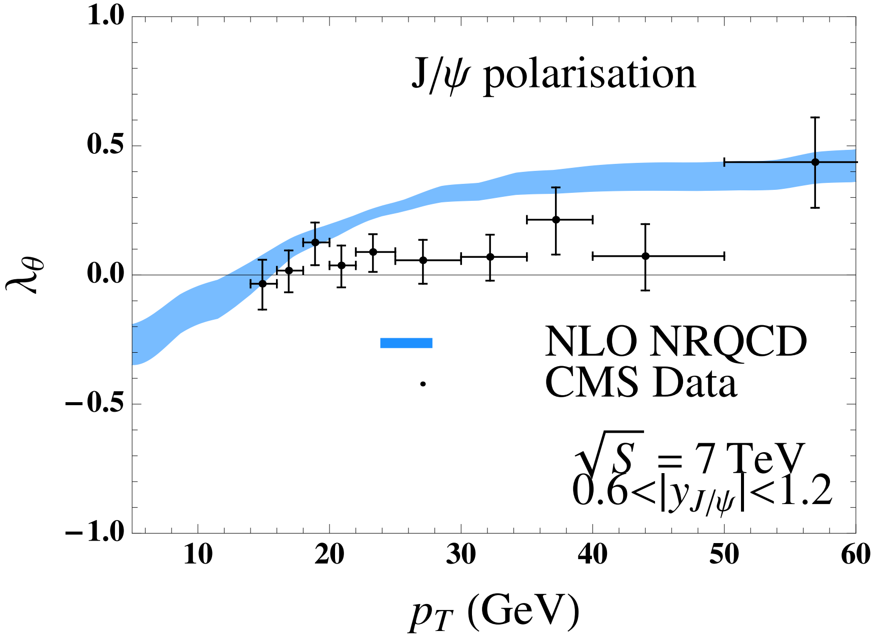
- ANL-PKU-Group prediction (1509.07904) (blue band) agrees with the CMS data.

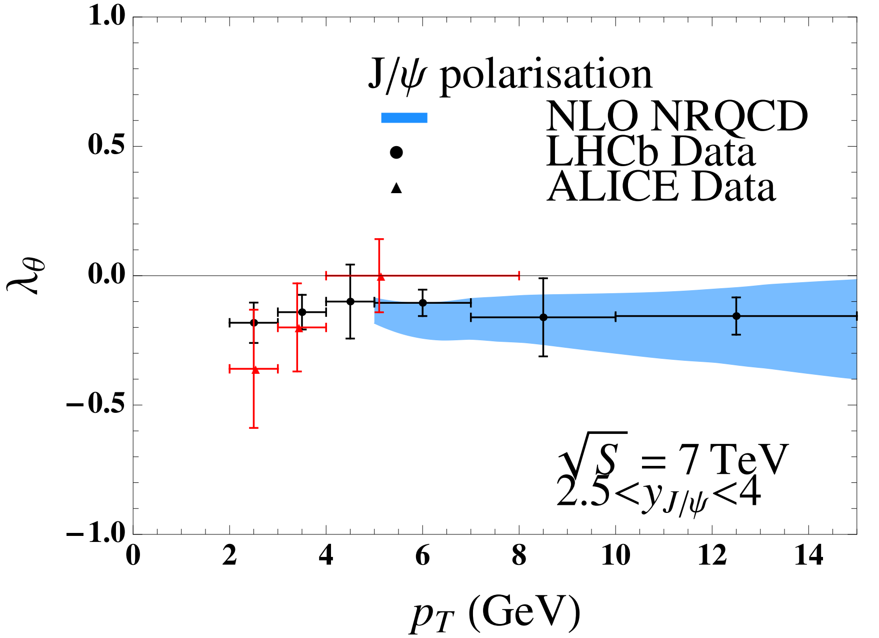


- ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels largely cancel.
- The unpolarized ${}^1S_0^{[8]}$ channel dominates.
- The ANL-Group fit is similar.

- The PKU-Group predictions agree, with some tension, with the CMS, LHCB, and ALICE data.

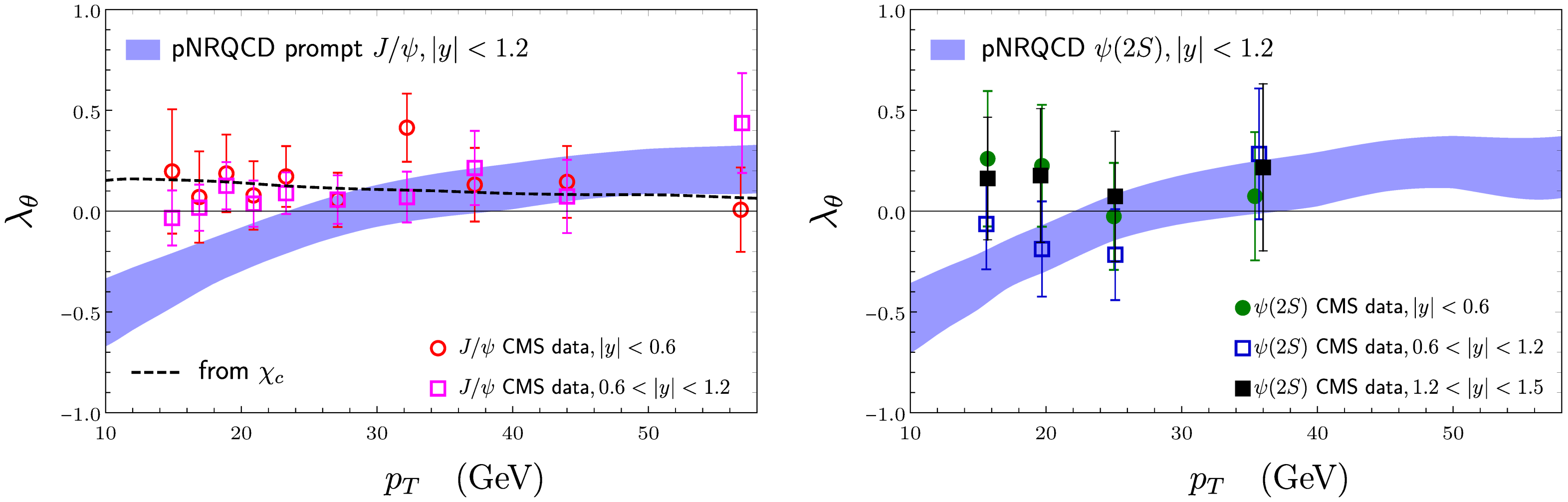






- ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels largely cancel.

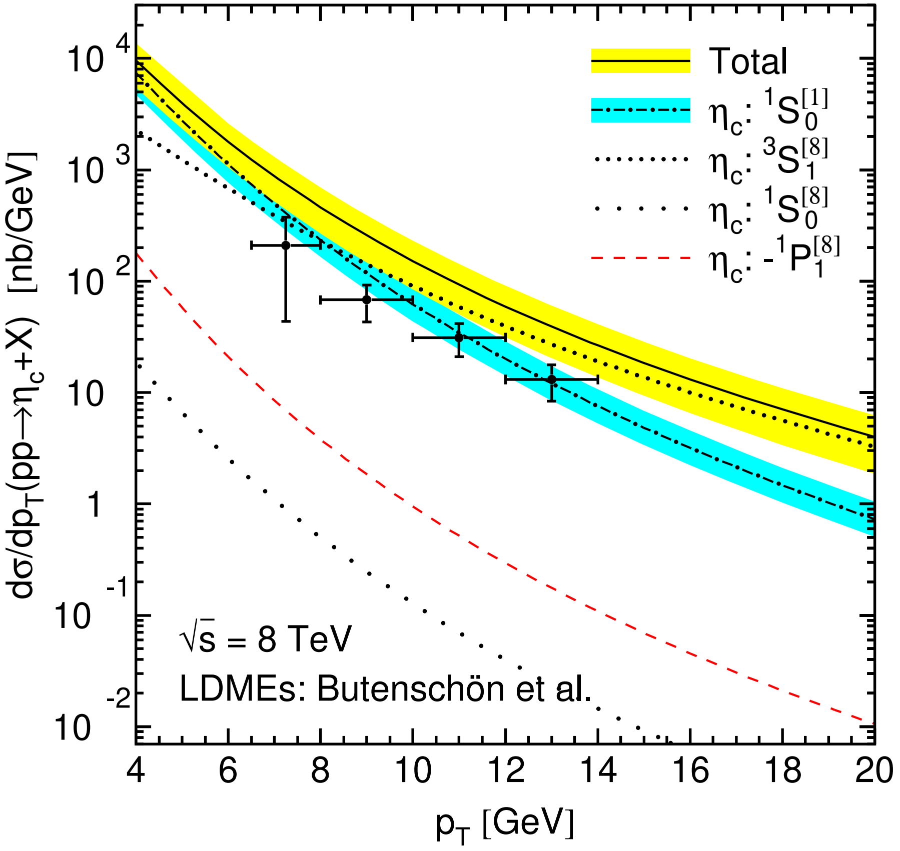
- The TUM predictions agree, with some tension, with the CMS data.



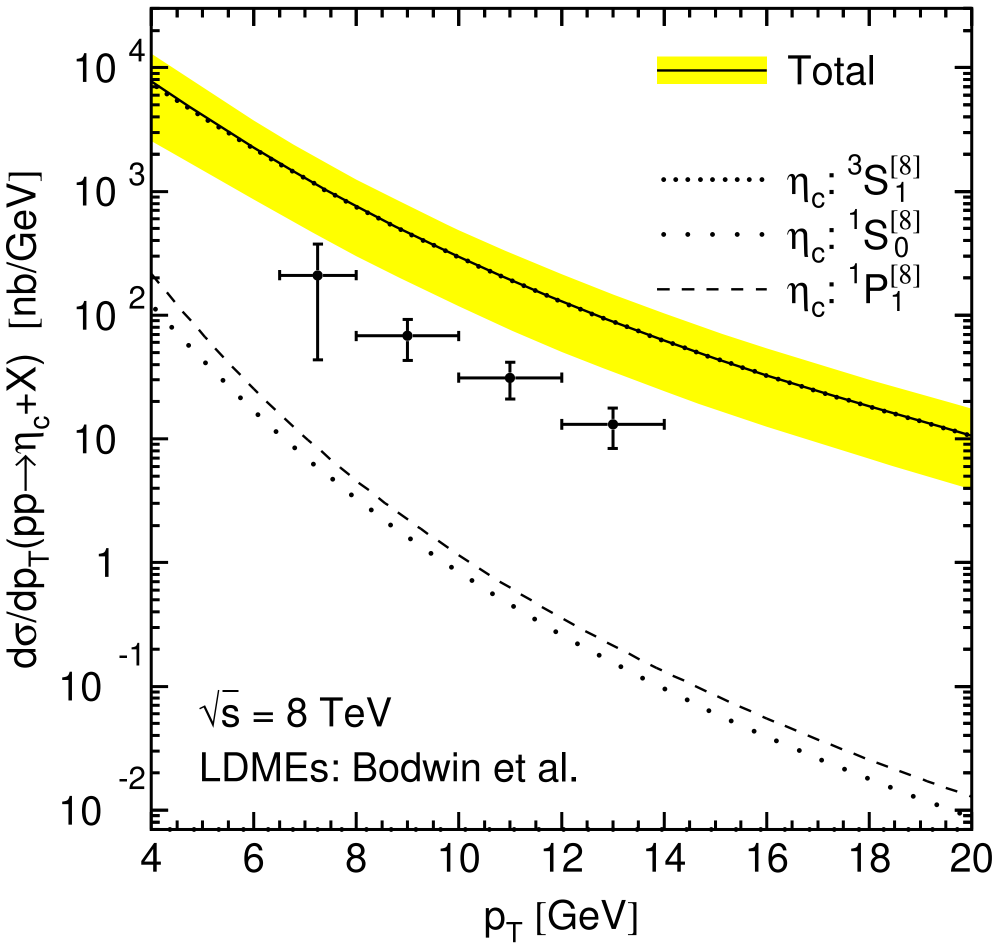
- ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels largely cancel.

η_c Production at LHCb (1409.3612)

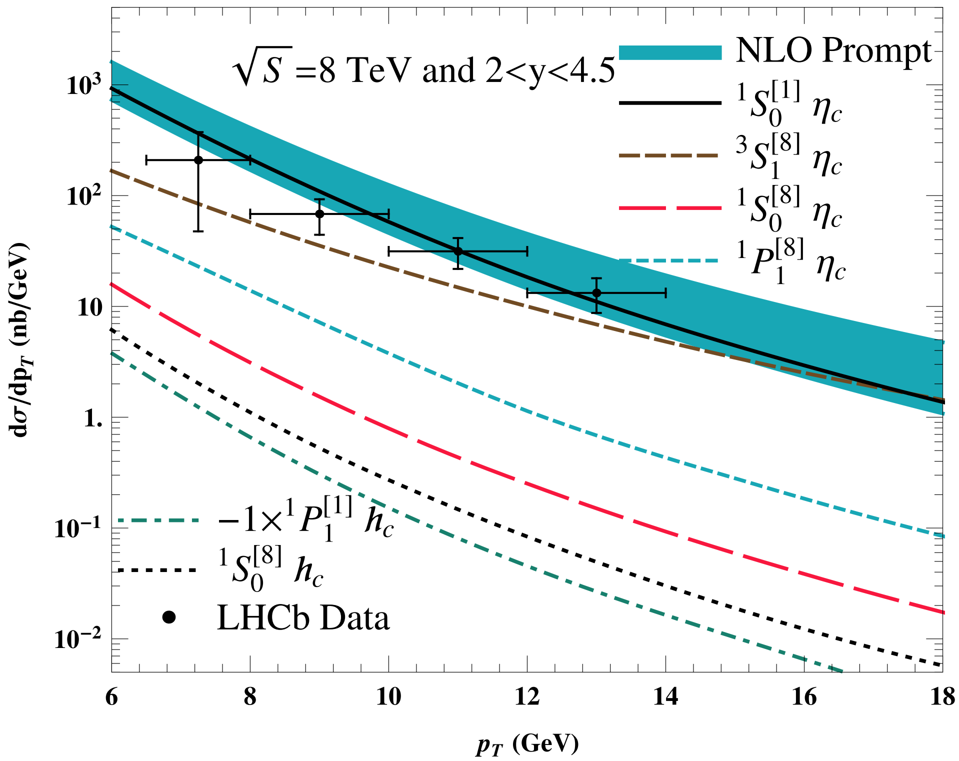
- The η_c LDMEs are fixed by using the heavy-quark spin symmetry of NRQCD to relate them to the J/ψ LDMEs.
- Good up to corrections of relative order v^2.
- The color-singlet contribution alone accounts for the measured cross section.
- The Hamburg-Group prediction overshoots the data by a factor of about 6.



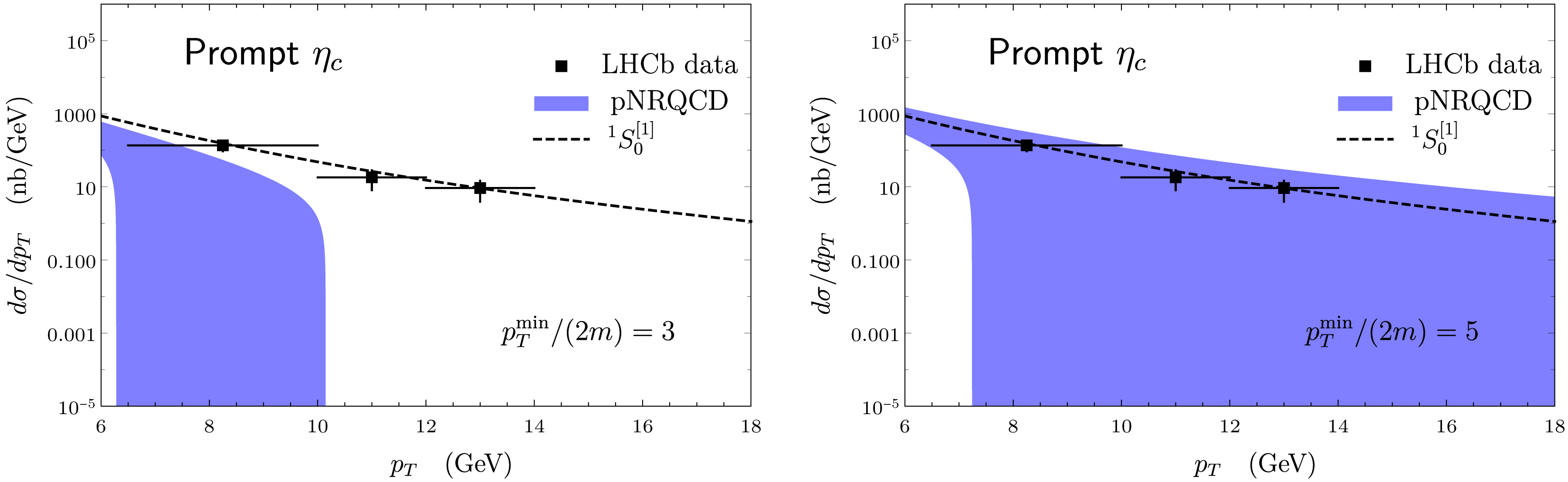
- The ANL-Group prediction is even worse.



- Not surprisingly, the PKU Group prediction, which used the η_c cross section as a constraint, agrees fairly well with the LHCb data.



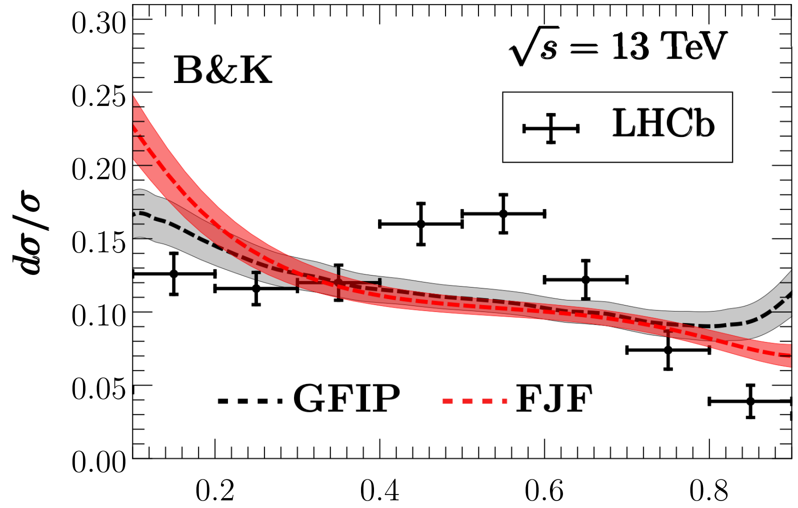
- The TUM-Group predictions are in agreement with the data for the LDMEs in which the cut $p_T/(2m_c) > 5$ was taken.

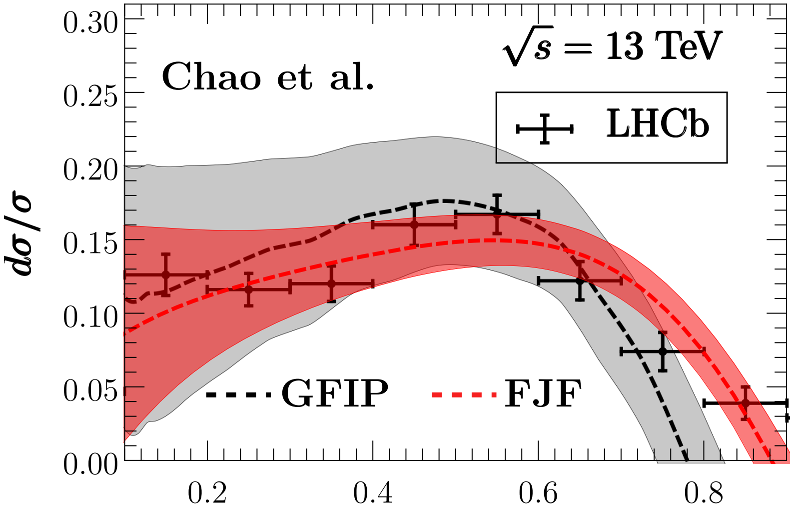


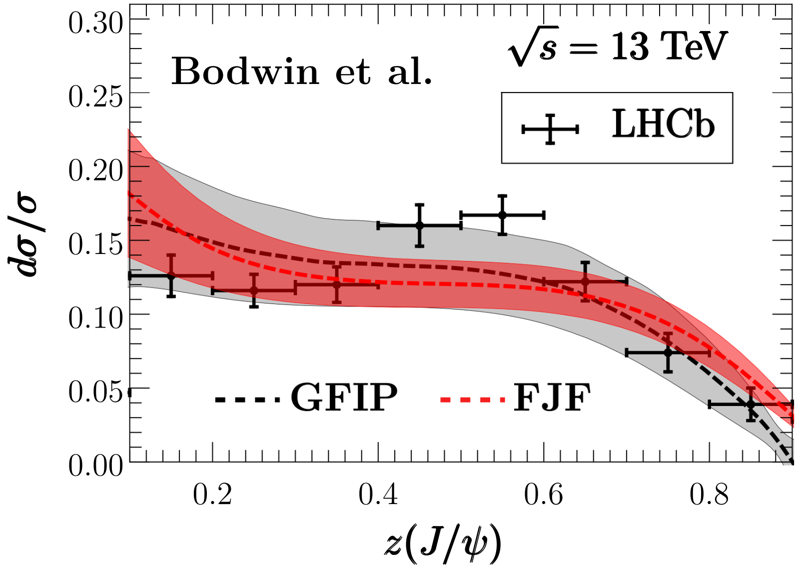
- A check of the LHCb result by a different group would be useful.

J/ψ Energy Fraction in a Jet

- Bain, Dai, Makris, Leibovich, Mehen (1702.05525) used NRQCD factorization to compute
  $z(J/\psi)$, the energy of the J/ψ divided by the energy of the accompanying jet.
- A measure of the gluon radiation that is expected to accompany the quarkonium in color-octet production.
  - Gets to the heart of the color-octet production mechanism.
- Computed using a corrected version of Pythia (GFIP) and the fragmenting jet function (FJF).
- Compared with the LHCb (1701.05116) data with predictions from different LDME sets.

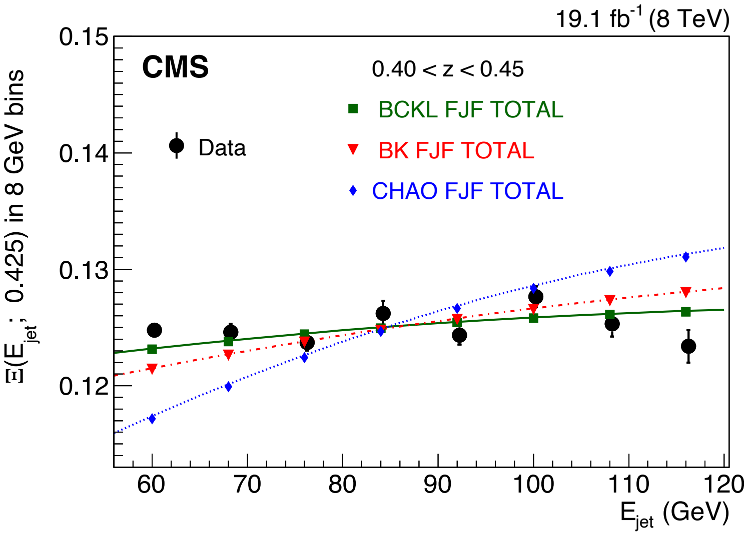


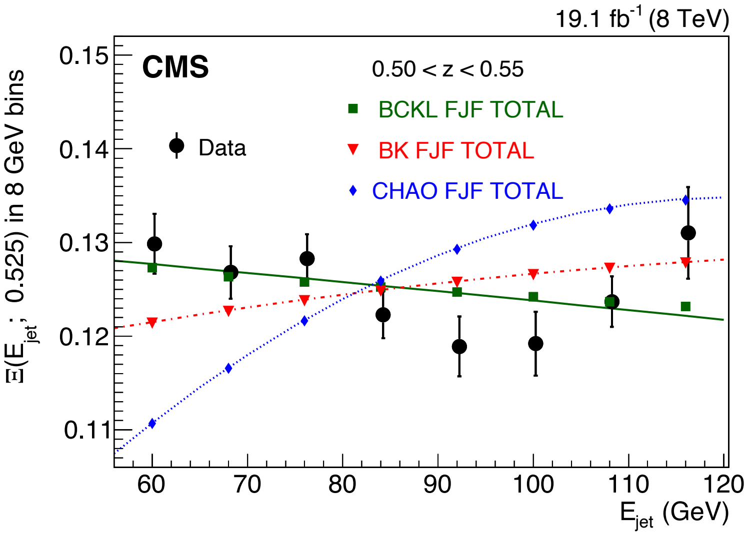


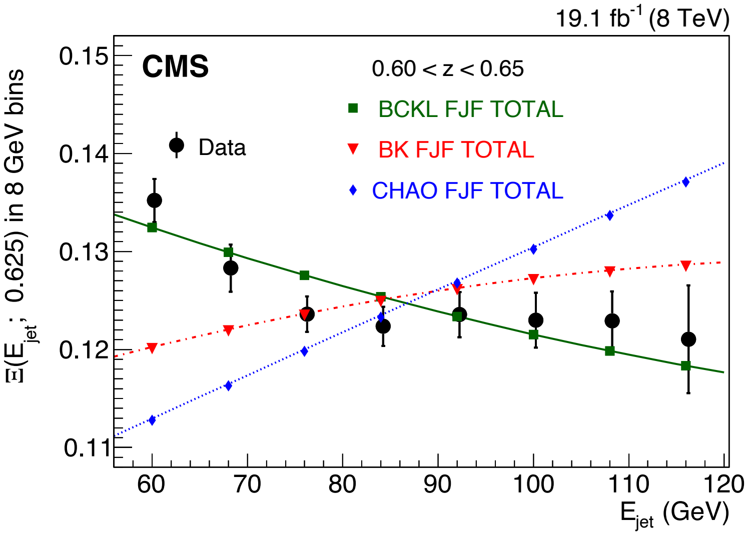


- BK is the Hamburg Group.
- Chao, Ma, Shao, Wang, Zhang, (Chao *et al.*) LDMEs are from a polarization-constrained fit (1201.2675).
- Bodwin *et al.* is the ANL-Group.
- The Chao *et al.* and ANL-Group LDMEs give reasonable fits.

- CMS (1910.01686) measured events in three z bins as a function of E_{jet}.



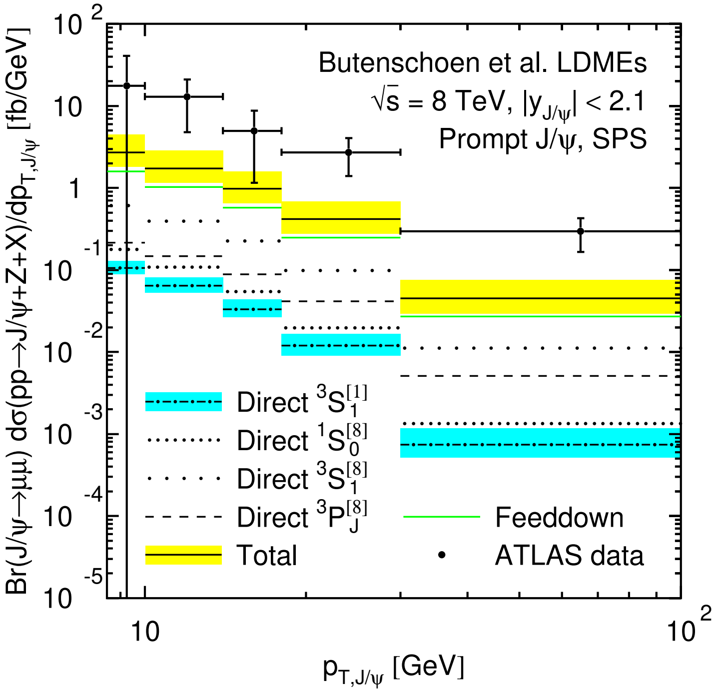


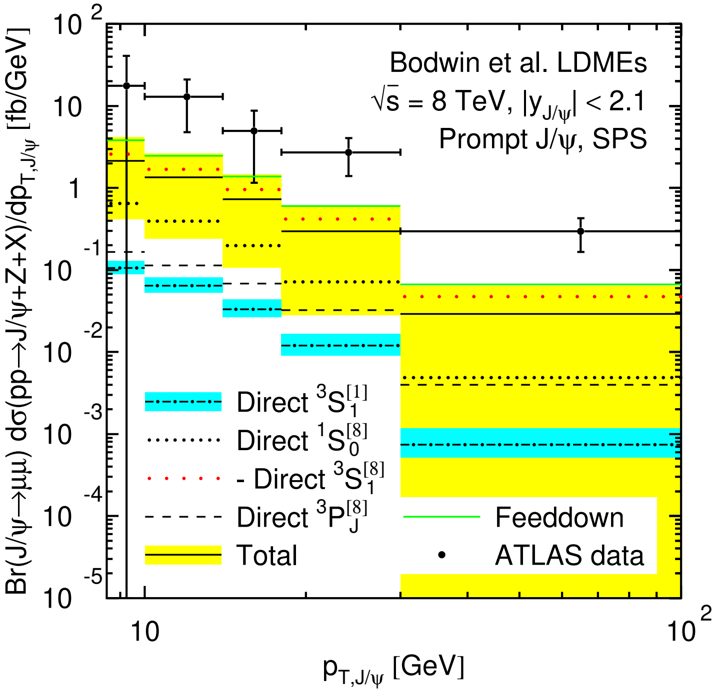


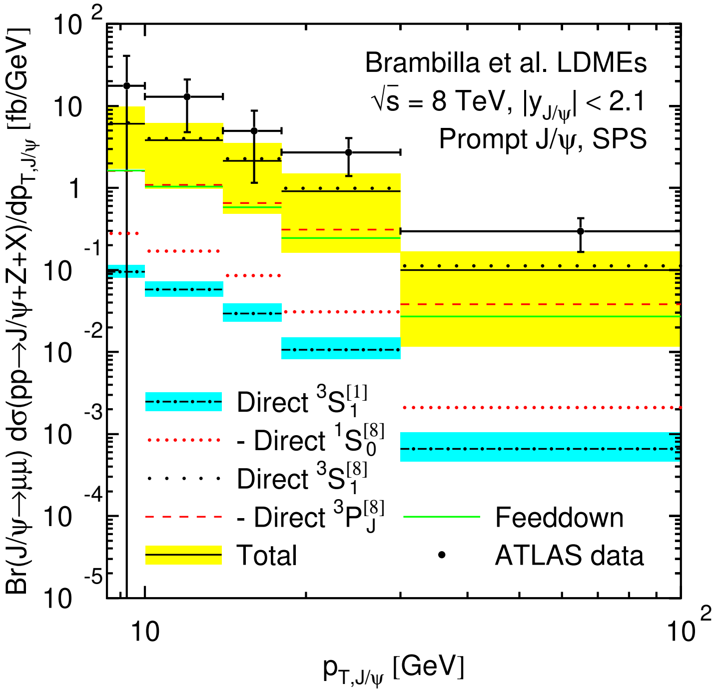
- $\Xi(E_{\mathrm{jet}}; z)$ is number of events in a z bin divided by number of events with $0.3 \le z \le 0.8$.
- z bins at $z = 0.425$, 0.525, 0.635.
- Only the BCKL (ANL-Group) LDMEs give a good fit.

J/ψ Production in Association with a Z or a W

- NLO calculation from Butenschön and Kniehl (2207.09366).
- The calculation is expected to be more reliable in the Z case because some of the $c\bar{c}$ channels open in the W case only at NNLO.
- Comparison with ATLAS data.
- All of the predictions undershoot the data.
- Double-parton scattering might account for some of the discrepancy.



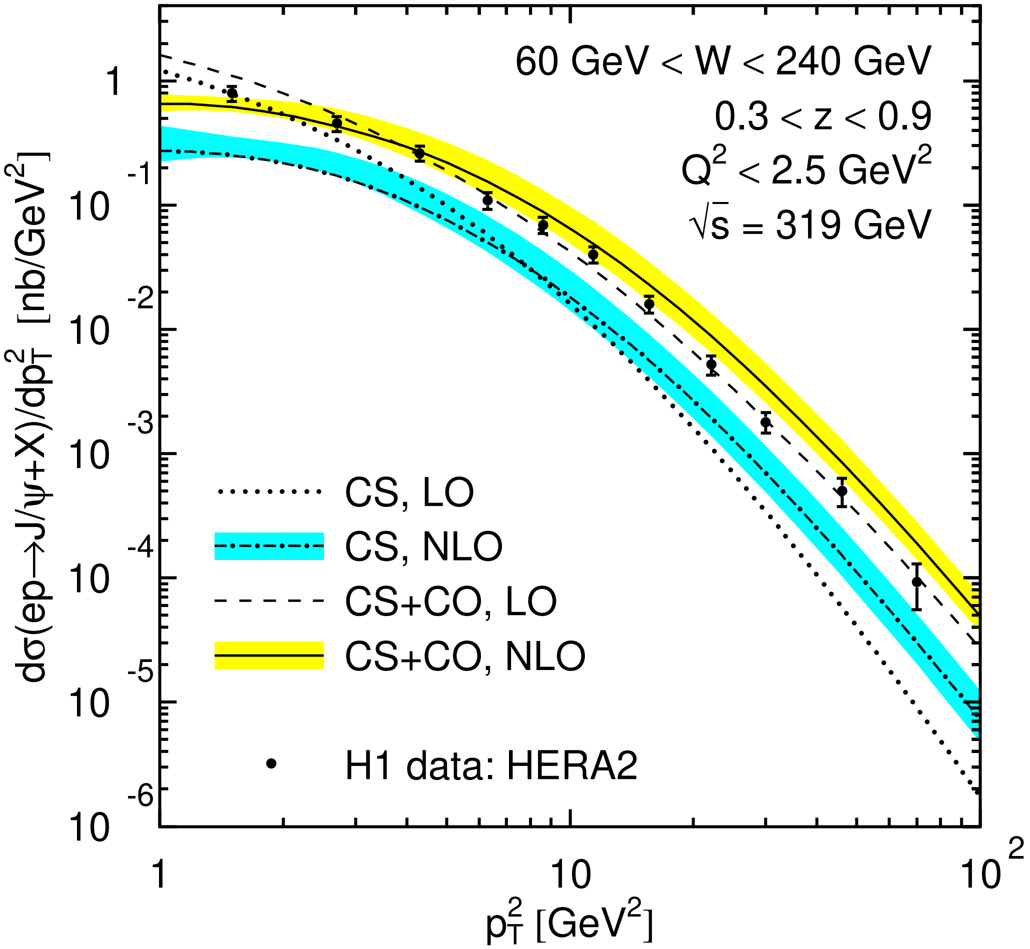




- $J/\psi + Z$ production
- Butenschoen *et al.* is the Hamburg Group.
- Bodwin *et al.* is the ANL-PKU Group.
- Brambilla *et al.* is the TUM Group ($p_T/(2m_c) > 3$).
- The TUM-Group prediction undershoots the data by only a about a factor of 3.

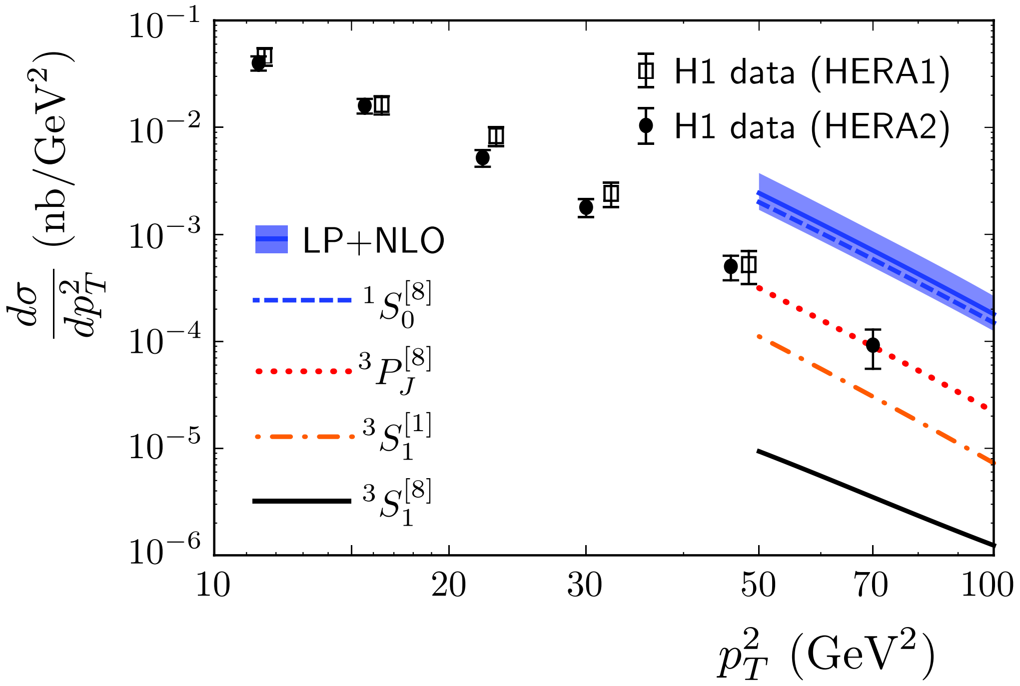
J/ψ Photoproduction at HERA

- The Hamburg Group used the HERA H1 photoproduction data (hep-ex/0205064, hep-ex/0510016) to fit their LDMEs.



- Some tension in the fit.
- The H1 data lie at $p_T \lesssim 8$ GeV.
- Is NRQCD factorization valid at such low values of p_T?

- The ANL-Group (1504.06019) resummation of logs of p_T^2/m_c^2 (blue curve) does not resolve the discrepancy.



- The p_T of the highest measured point is only about 8 GeV.
- But theory and data are not trending toward each other as p_T increases.

- The Chao *et al.* polarization-constrained LDMEs are incompatible with the H1 photoproduction data.

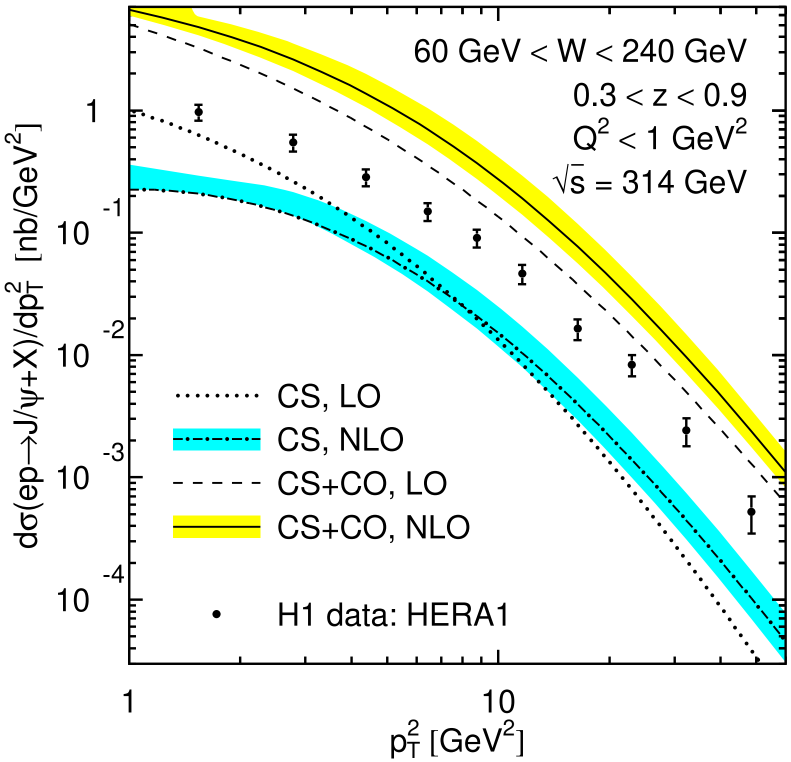


Figure courtesy of Mathias Butenschön.

Score Card

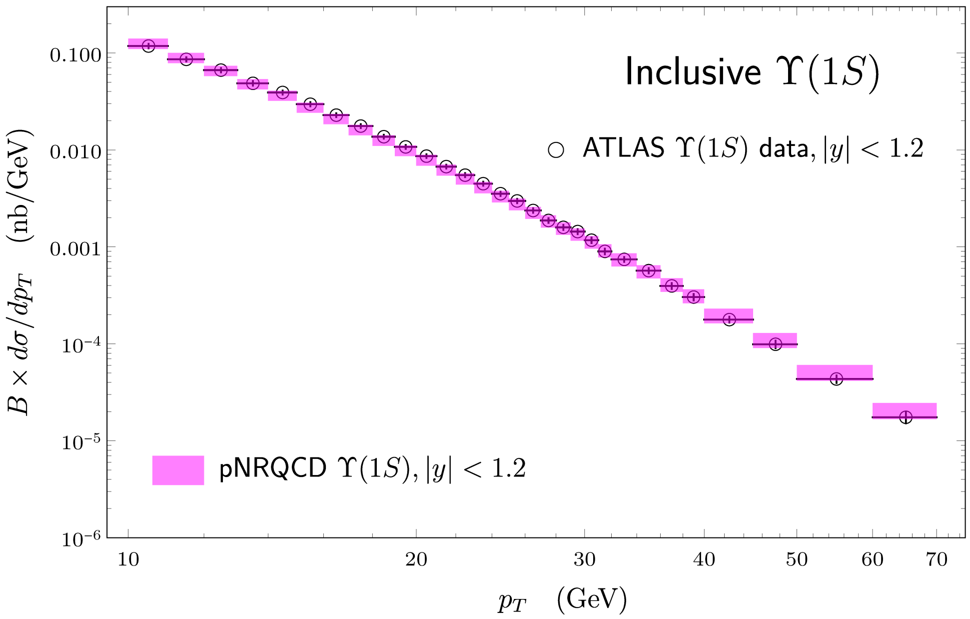
Group	J/ψ rate	J/ψ pol	η_c rate	jet shape	W/Z	ep
Hamburg	?	X	X	X	X	?
ANL	✓	✓	X	✓	-	X
ANL-PKU	✓	✓	-	-	X	-
PKU	✓	?	✓	-	-	-
TUM ($p_T/(2m_c) > 3$)	✓	✓	X	-	?	-
TUM ($p_T/(2m_c) > 5$)	✓	✓	✓	-	-	-

- No single LDME set describes all of the data.
- The TUM-Group LDMEs from the higher p_T cut look very promising.
- It would be good to have calculations of J/ψ jet shape and photoproduction (ep) of J/ψ for the TUM-Group LDMEs.

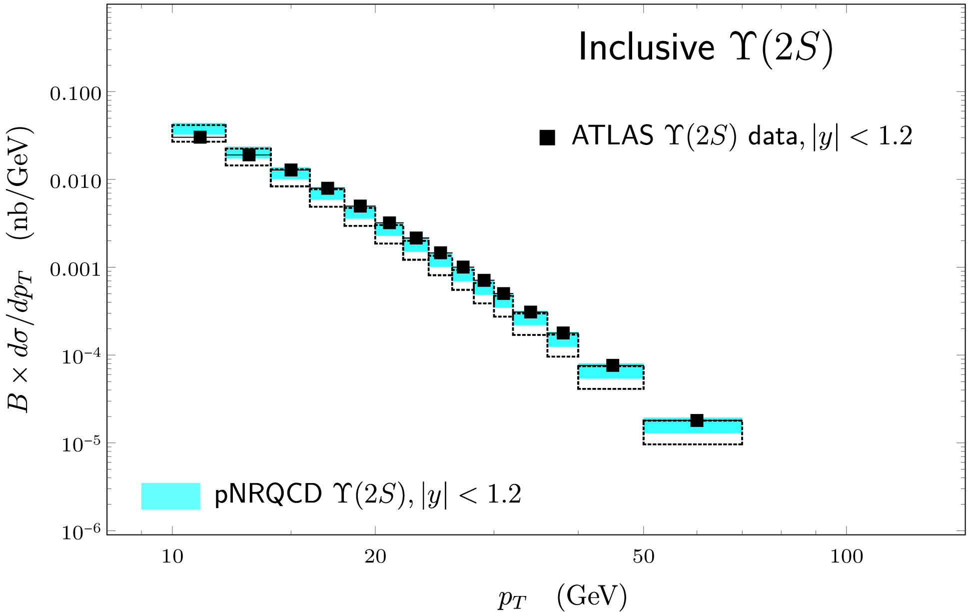
Universality of TUM-Group LDMEs

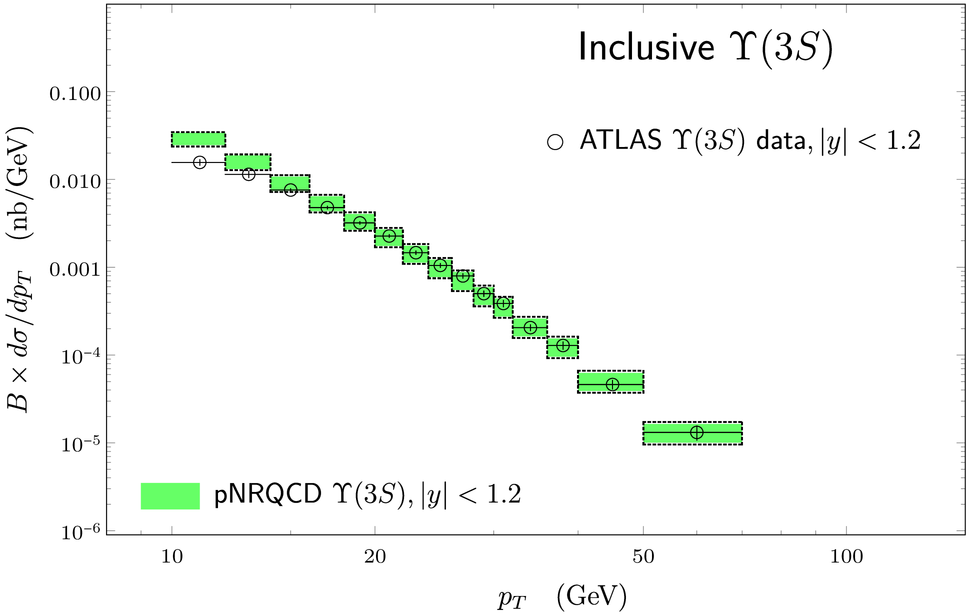
- The TUM Group uses just three the gluon correlators to
  - Fit the J/ψ, $\psi(2S)$, and $\Upsilon(nS)$ cross sections,
  - Predict their polarizations.
- Involves evolving the correlators to the scales m_c and m_b.

Cross Sections

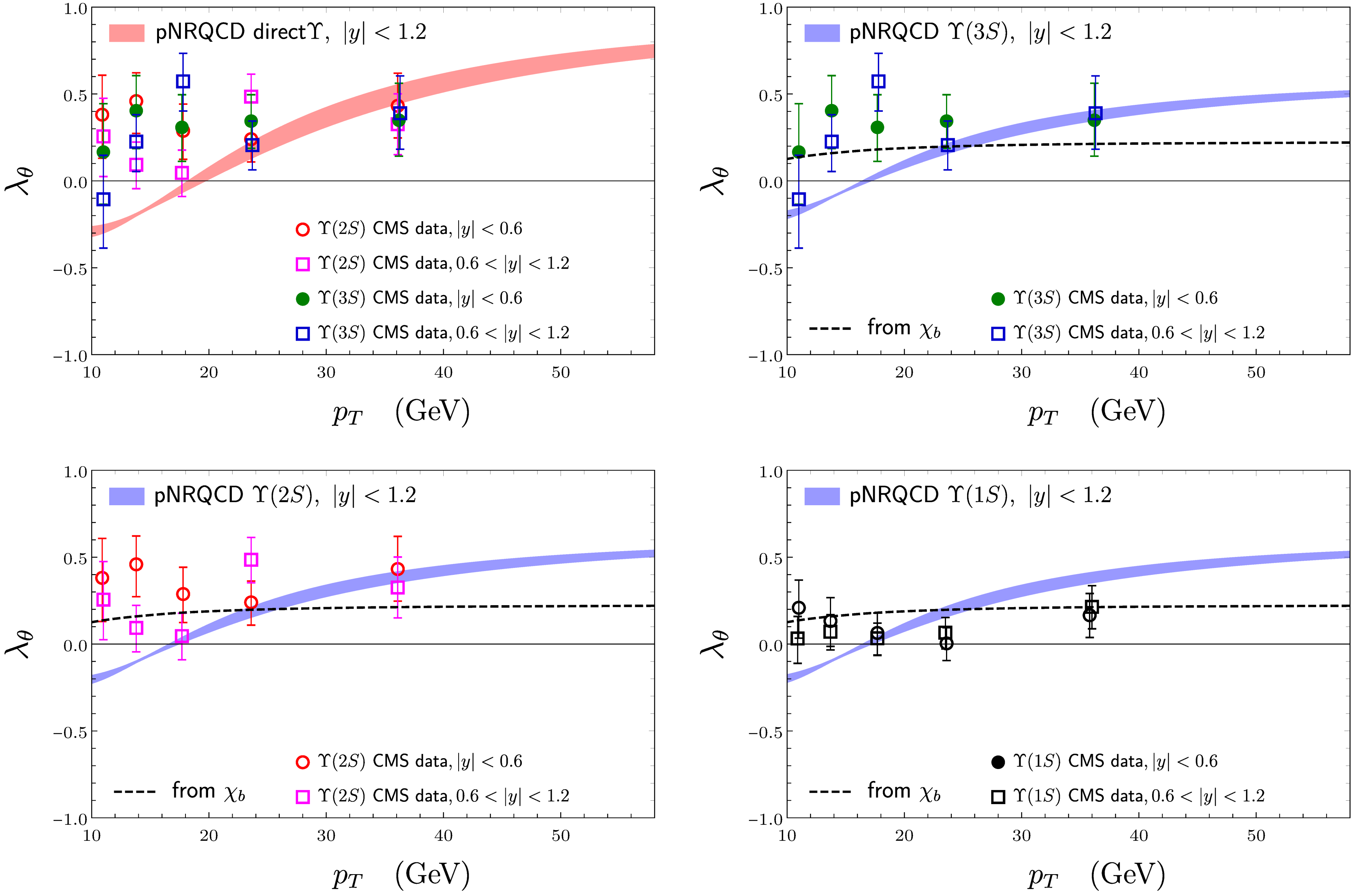


- $\Upsilon(1S)$ cross section is a prediction, not a fit.
- Remarkable agreement with the data.


Inclusive $\Upsilon(2S)$
■ ATLAS $\Upsilon(2S)$ data, $|y| < 1.2$
pNRQCD $\Upsilon(2S)$, $|y| < 1.2$
$B \times d\sigma/dp_T$ (nb/GeV)
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
10
20
50
100
p_T (GeV)


Inclusive $\Upsilon(3S)$
○ ATLAS $\Upsilon(3S)$ data, $|y| < 1.2$
pNRQCD $\Upsilon(3S)$, $|y| < 1.2$
$B \times d\sigma/dp_T$ (nb/GeV)
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
10
20
50
100
p_T (GeV)

Polarizations



Summary

- Peter's ideas on heavy quarkonia have spawned a rich field of theoretical and experimental work. (≈ 3000 citations on iNSPIRE)
- High-quality, high-p_T measurements from the LHC have enabled meaningful comparisons with theory:
  - J/ψ cross sections and polarization,
  - $\psi(2S)$ cross sections and polarization,
  - χ_{cJ} cross sections and polarization,
  - J/ψ jet energy fraction,
  - J/ψ production in association with a W or Z,
  - $\Upsilon(nS)$ cross sections and polarizations.
- The theory of quarkonium production has seen a number of experimental confirmations, but no single LDME set describes all of the data.

Future Directions

- A proof (or disproof) of the universality of the LDMEs is urgently needed.
- Lattice measurements of the gluon correlators are in progress (TUM Group).
  - First determination of the LDMEs directly from QCD.
- Tests of NRQCD in $b\bar{b}$ systems should be carried out.
  - The v expansion should converge faster.
  - Precision measurements of χ_b cross sections and polarizations are needed.
  - Extensions $\Upsilon(nS)$ cross sections and polarizations to higher p_T and higher precision are needed.
- Resummation of logs of p_T^2/m^2 should be incorporated into all calculations.
- A cure for the problem of negative cross sections at large p_T ($\gtrsim 140$ GeV) is needed.
  - Chen *et al.* (2304.04552): Soft-gluon factorization (shape functions).
  - Chung, Kim, Lee (2408.04255): Resummation of threshold logs.
- NNLO calculation of the SDCs may be feasible with modern amplitude technology.

- A check of the LHCB η_c cross section is needed.
- Jet energy fractions should be calculated for all of the LDME sets.
- Energy correlators (Chen *et al.* [2405.10056]) are another promising avenue of investigation.
- Higher-order calculations of gluon fragmentation to quarkonium could improve precision.
  - ${}^1S_0^{[1,8]}$: Artoisenet and Braaten (1810.02448)
  - ${}^1S_0^{[1,8]}$: Feng, Jia (1810.04138)
  - ${}^3P_J^{[1,8]}$: Zhang, Meng, Ma, Chao (2011.04905)

Backup Slides

Constraints from $e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}}$ at the B factories.

- Zhang, Ma, Wang, Chao (0812.5106, 0911.2166) computed the cross section for $e^+e^- \to c\bar{c}g$ through the ${}^1S_0^{[8]}$ and ${}^3P_0^{[8]}$ channels at NLO.
- Comparison with the Belle (0901.2775) measurement of $\sigma(e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}})$ leads to a bound on the color-octet LDMEs:

$$M_{4.0} = \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle + 4.0\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) < (2.0 \pm 0.6) \times 10^{-2}\ \text{GeV}^3$$

- Bound comes from neglecting the color-singlet contribution, which saturates the measured cross section by itself.
- Bound is in conflict with the LDMEs extracted from the hadron-collider data, except for the Hamburg LDMEs:

$$M_{3.9}^{\mathrm{PKU}} = (7.4 \pm 1.9) \times 10^{-2}\ \text{GeV}^3 \qquad M_{3.9}^{\mathrm{Hamburg}} = (2.17 \pm 0.56 \times 10^{-2}\ \text{GeV}^3$$
$$M_{3.9}^{\mathrm{IHEP}} = (6.00 \pm 0.98) \times 10^{-2}\ \text{GeV}^3 \qquad M_{3.9}^{\mathrm{ANL-PKU}} = (9.78 \pm 1.52) \times 10^{-2}\ \text{GeV}^3$$

- But, the Belle (0901.2775) measurement

$$\sigma(e^+e^- \to J/\psi + X) = (1.17 \pm 0.02 \pm 0.07)\ \text{pb}$$

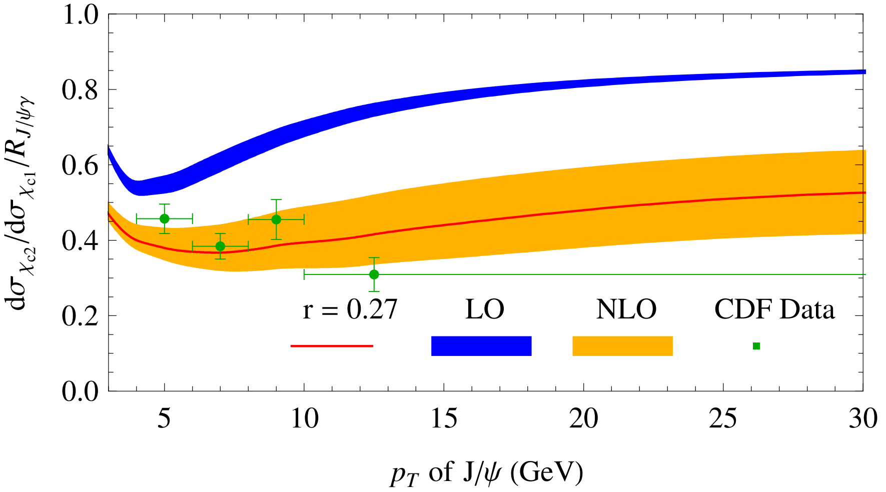
is more than a factor 2 smaller than the BaBar (hep-ex/0106044) measurement

$$\sigma(e^+e^- \to J/\psi + X) = (2.52 \pm 0.21 \pm 0.21)\ \text{pb.}$$

- Most of the data are at $p_T \lesssim 3$ GeV.
- Is p_T too small for NRQCD factorization to be valid?

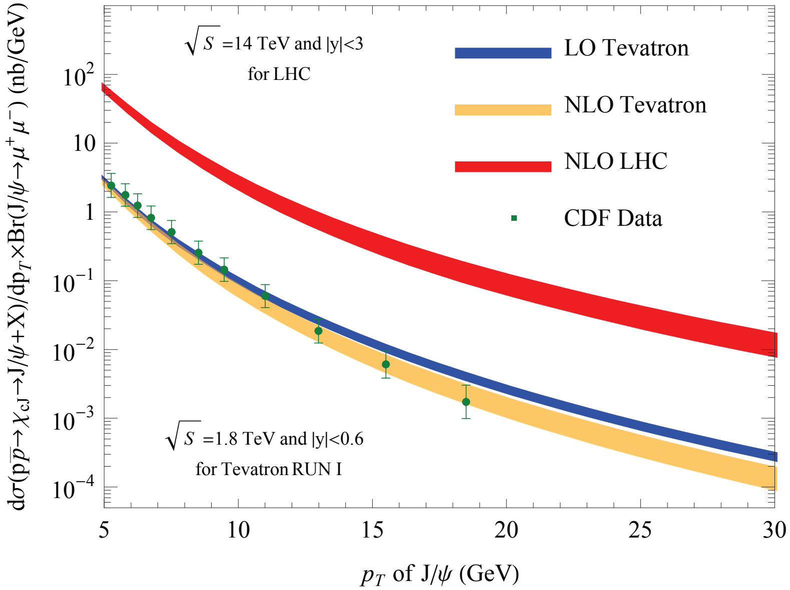
χ_{c1} and χ_{c2} Production

- Ma, Wang, Chao (1002.3987) fit the ratio $d\sigma_{\chi_{c1}}/d\sigma_{\chi_{c2}}$:



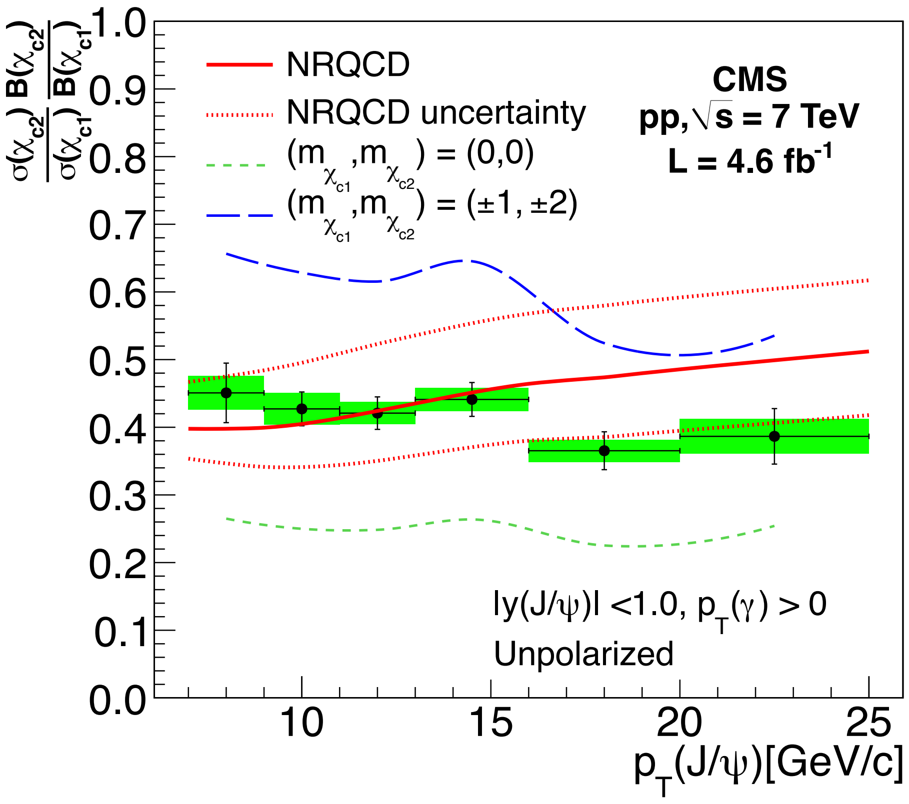
- Used CDF (hep-ex/0703028) data.
- Fit, plus a potential-model value of $\langle {}^3P_J^{[1]} \rangle$ (Eichten, Quigg (PRD 52, 1726)), allowed them to fix $\langle {}^3S_1^{[8]} \rangle$.

- That allowed them to predict the cross section:



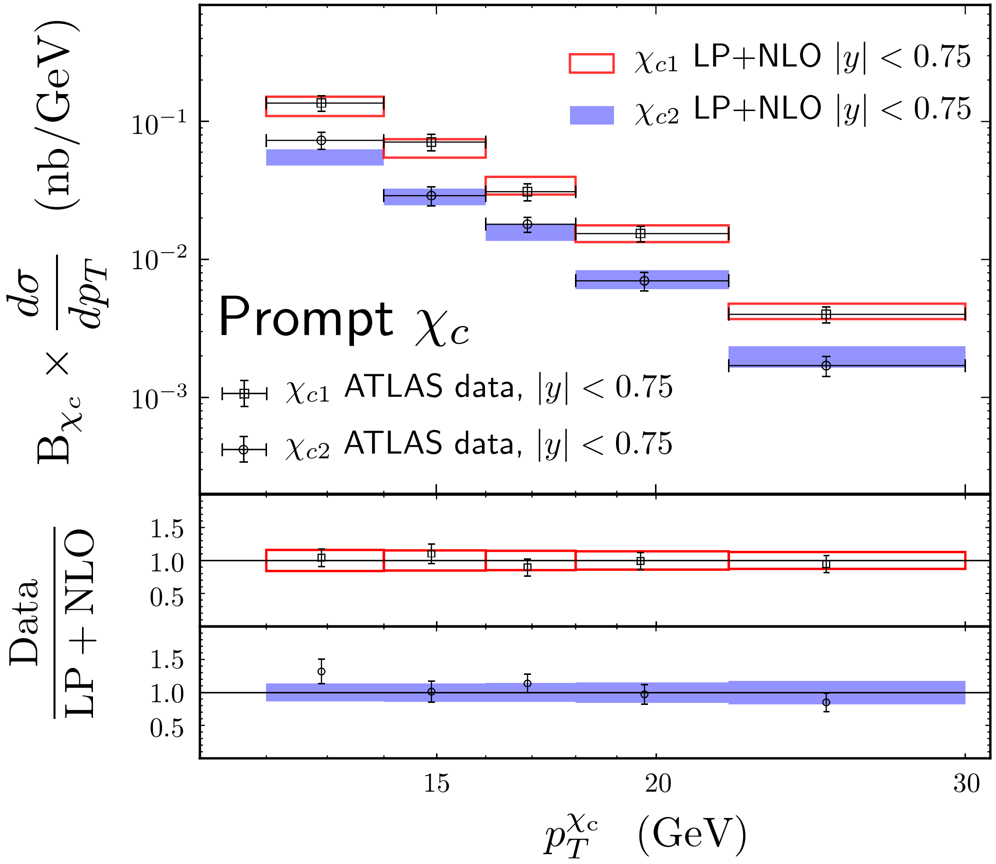
- In good agreement with the CDF data.

- The prediction of Ma, Wang, Chao also agrees with the CMS data (1210.0875):



- The dashed green and blue lines are the shifts in the ratio for extreme values of the χ_{cJ} polarizations.

- The ANL-PKU group fit the ATLAS (1404.7035) data:

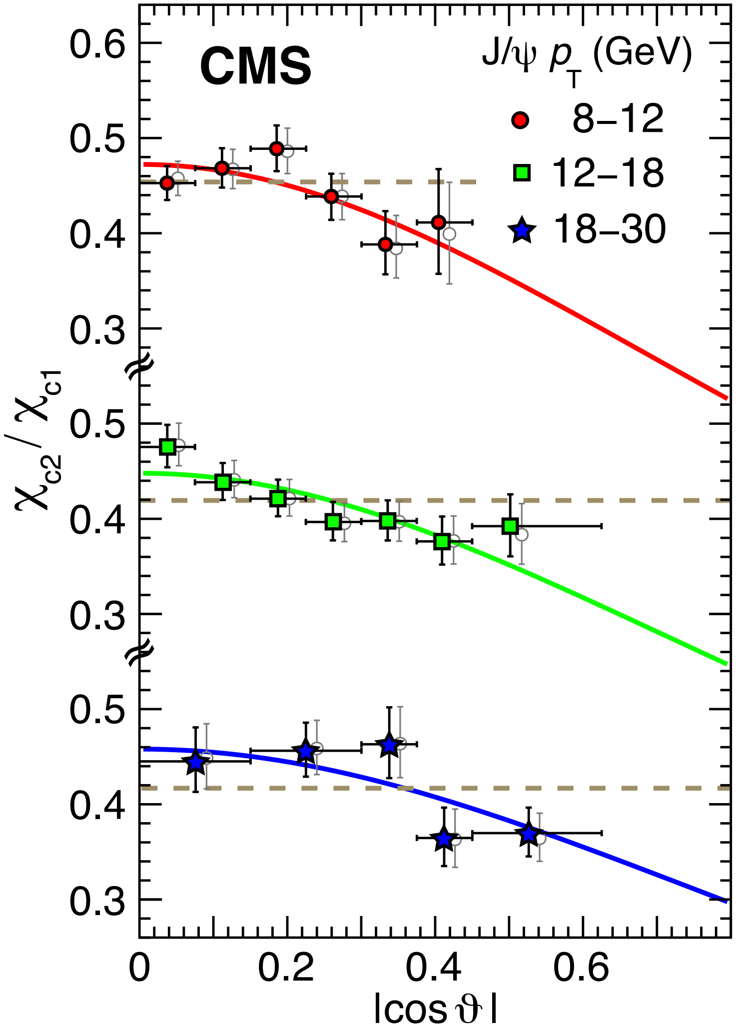


- The fitted value of the color-singlet LDME: $\langle \mathcal{O}^{\chi c1}(^3P_J^{[1]})\rangle = (7.9\pm2.4)\times10^{-2}$ GeV^5.
- Potential-model value [Eichten, Quigg (PRD 52, 1726)]: $\langle \mathcal{O}^{\chi c1}(^3P_J^{[1]})\rangle = 10.7 \times 10^{-2}$ GeV^5.
- Value from two-photon decays of the χ_{c1} and χ_{c2} [Chung, Lee, Yu (0808.1625)]: $\langle \mathcal{O}^{\chi c1}(^3P_J^{[1]})\rangle = 6.0^{+4.3}_{-2.9} \times 10^{-2}$ GeV^5.

- Good agreement among LDMEs from different physical processes strongly suggests that NRQCD factorization is more than just curve fitting.

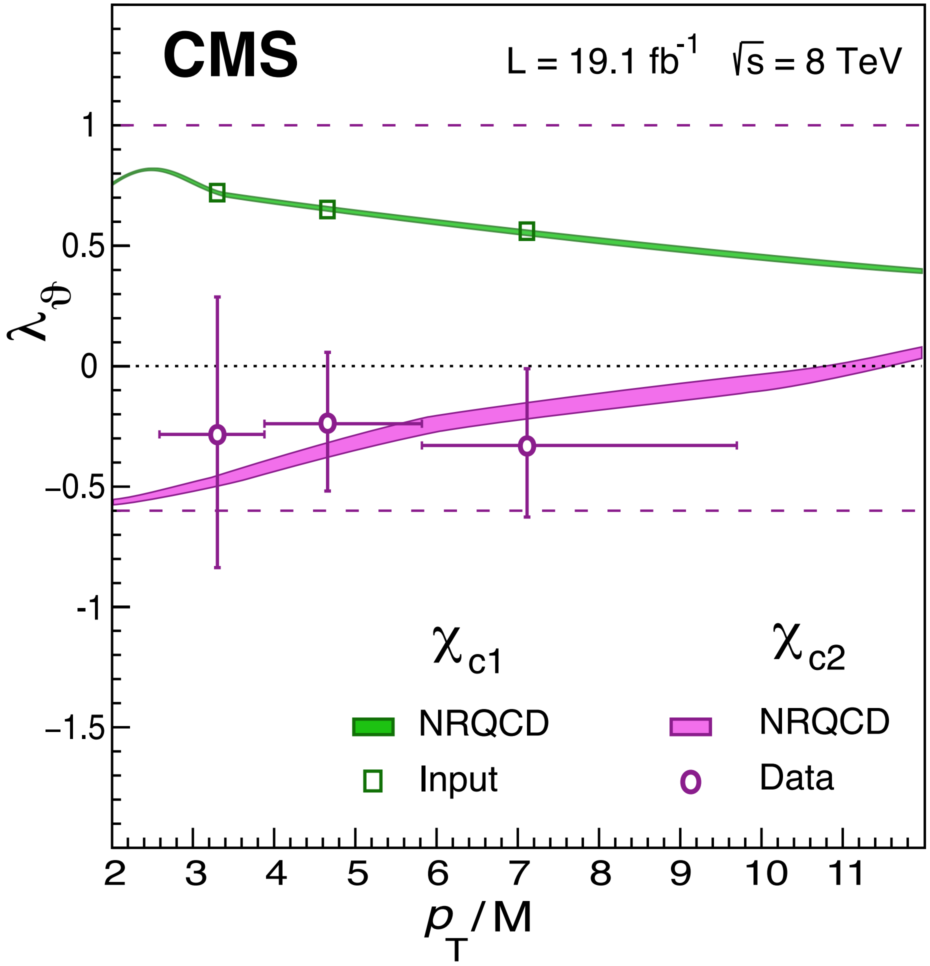
χ_{c1} and χ_{c2} Polarizations

- The muon angular distribution in $\chi_{cJ} \to J/\psi \to \mu^+\mu^-$ is a proxy for the χ_{cJ} polarization.



- In order to reduce systematics, CMS measures the ratio of angular distributions.
- The NRQCD curves [Faccioli, Lourenço, Araújo, Seixas, Krätschmer, Knünz (1802.01106)] use the SDCs from the PKU group.

- Faccioli et al. used the CMS measurement of $d\sigma_{\chi_{c2}}/d\sigma_{\chi c1}$ to fix the ratio of NRQCD LDMEs and used the PKU-group SDCs to predict the polarizations.



- They determined the χ_{c2} polarization (purple "data" points) by using
  - the prediction for the χ_{c1} polarization,
  - the measured ratio of angular distributions.
- "Out of the box" prediction of NRQCD works well.

Color-Singlet Model (CSM)

- The CSM is theoretically inconsistent.
  - Uncanceled infrared divergences at leading order in v for production of P-wave states and at higher orders in v for other states.
- The CSM predictions in NLO fall well below the observed cross sections.

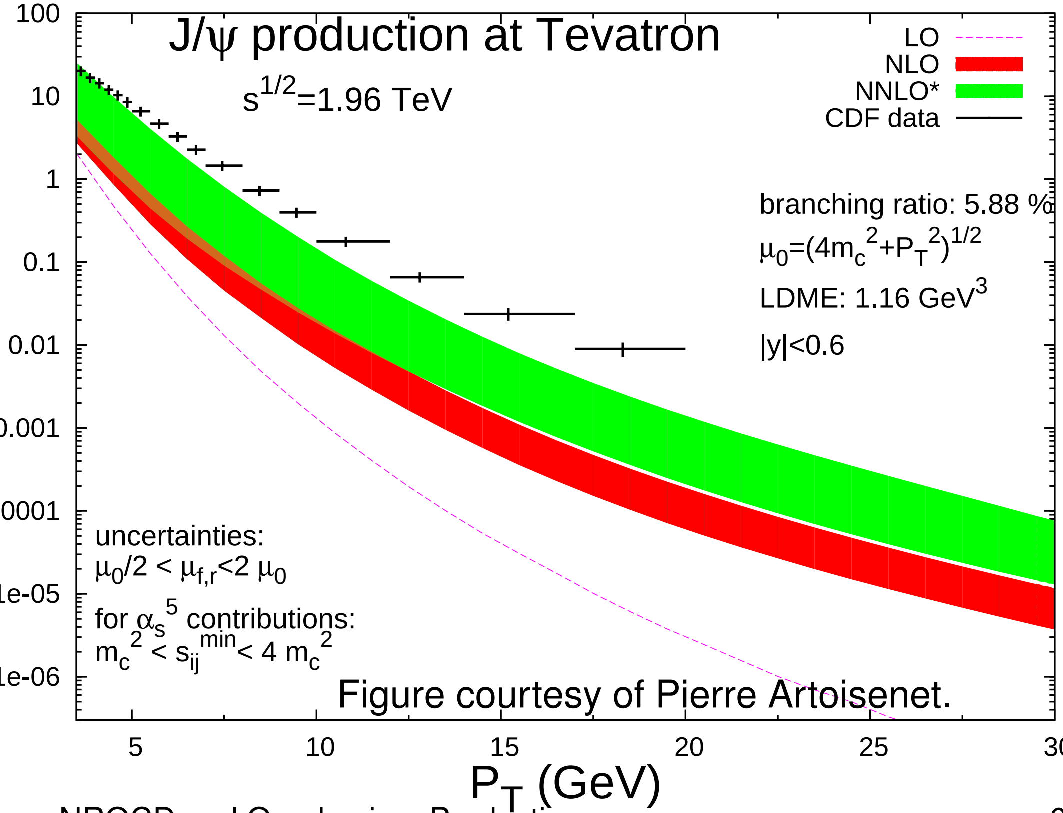
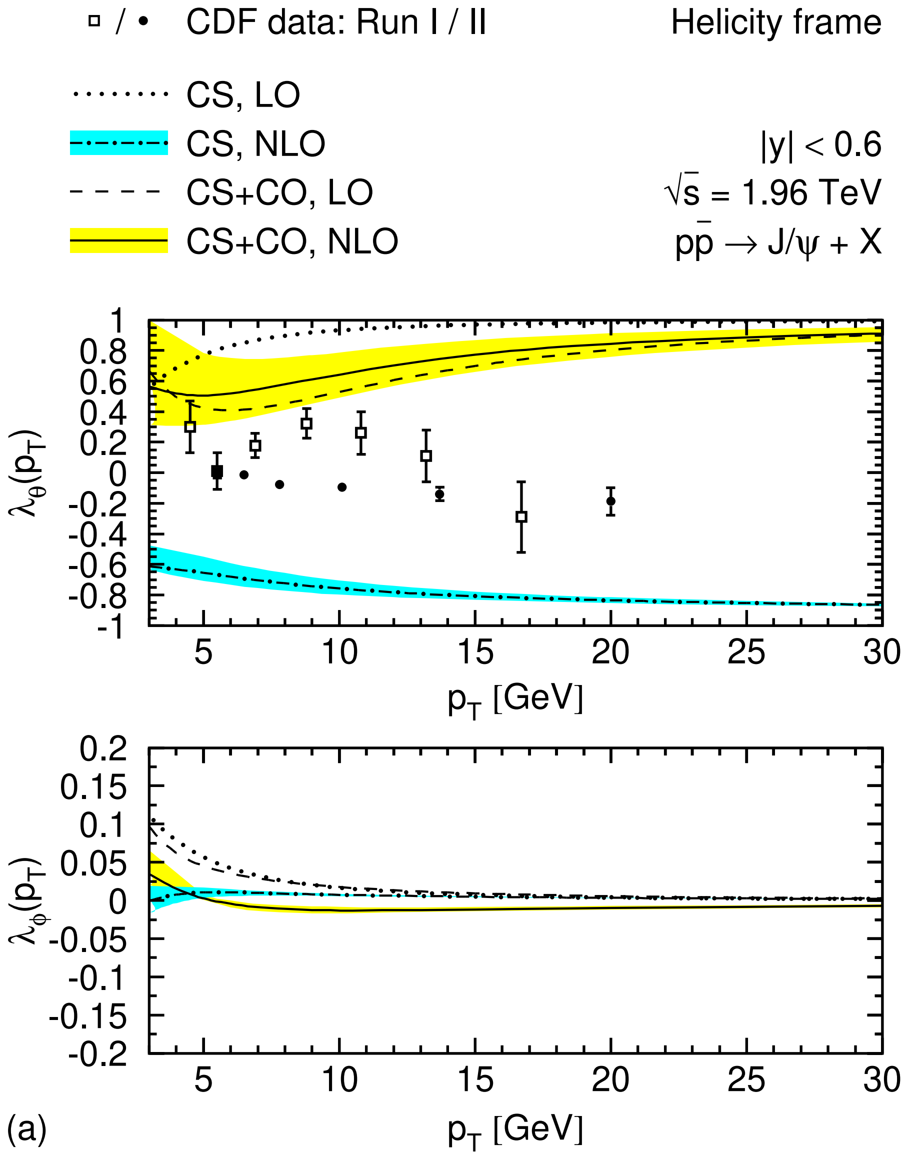


Figure courtesy of Pierre Artoisenet.

- The NNLO* calculation is an estimate based on real-emission contributions only.
- When the virtual contributions are added, the true NNLO contribution will likely be smaller.

- The CSM predictions in NLO do not describe the polarization data.



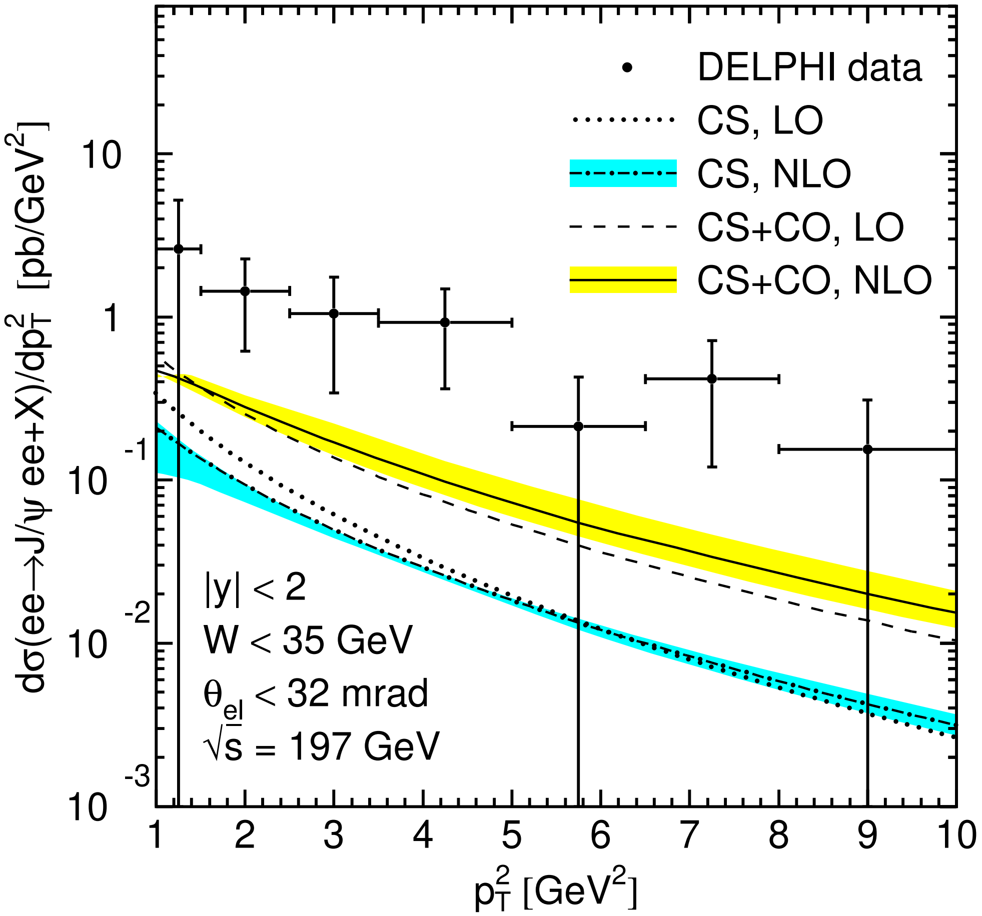
Color-Evaporation Model (CEM)

- The Color Evaporation Model (CEM) says that rate to produce a quarkonium is proportional to the rate to produce a $Q\bar{Q}$ pair, regardless of the quantum numbers of the $Q\bar{Q}$ pair or the quarkonium.
  - Not plausible in quantum field theory: Different $Q\bar{Q}$ states will have different overlaps with a given quarkonium state.
- The CEM requires an *ad hoc* modification, k_T smearing, in order to describe the data reasonably well.
- Nevertheless, because of its simplicity, the CEM is a useful way to describe production when a fundamental theory is not necessary, *e.g.* in studies of production in media.

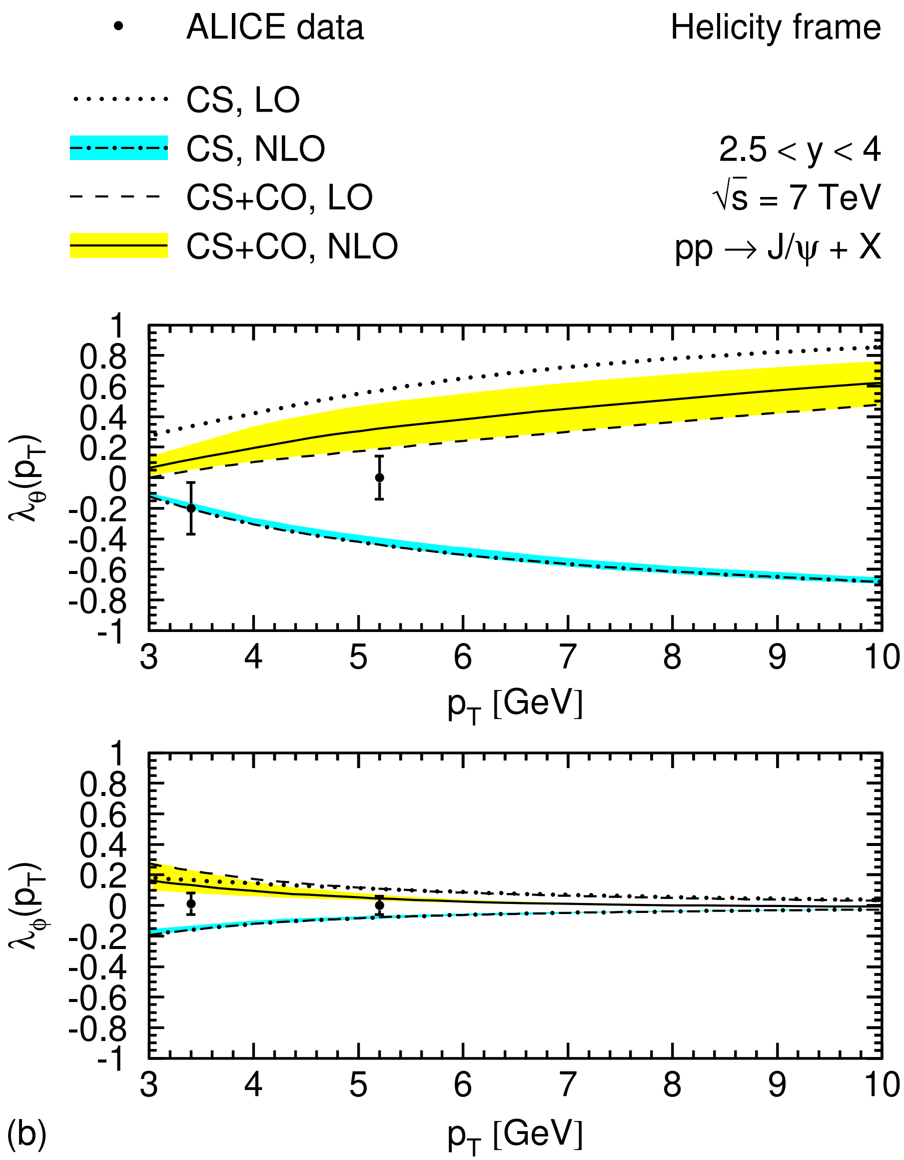
k_T-Factorization Approach

- The k_T-Factorization Approach could, in principle, yield valid results. But…
- it relies on k_T-dependent parton distributions, which are poorly determined;
- calculations are usually carried out within the CSM;
- calculations are usually carried out only in LO.

J/ψ Production in $\gamma\gamma$ Scattering at LEP II

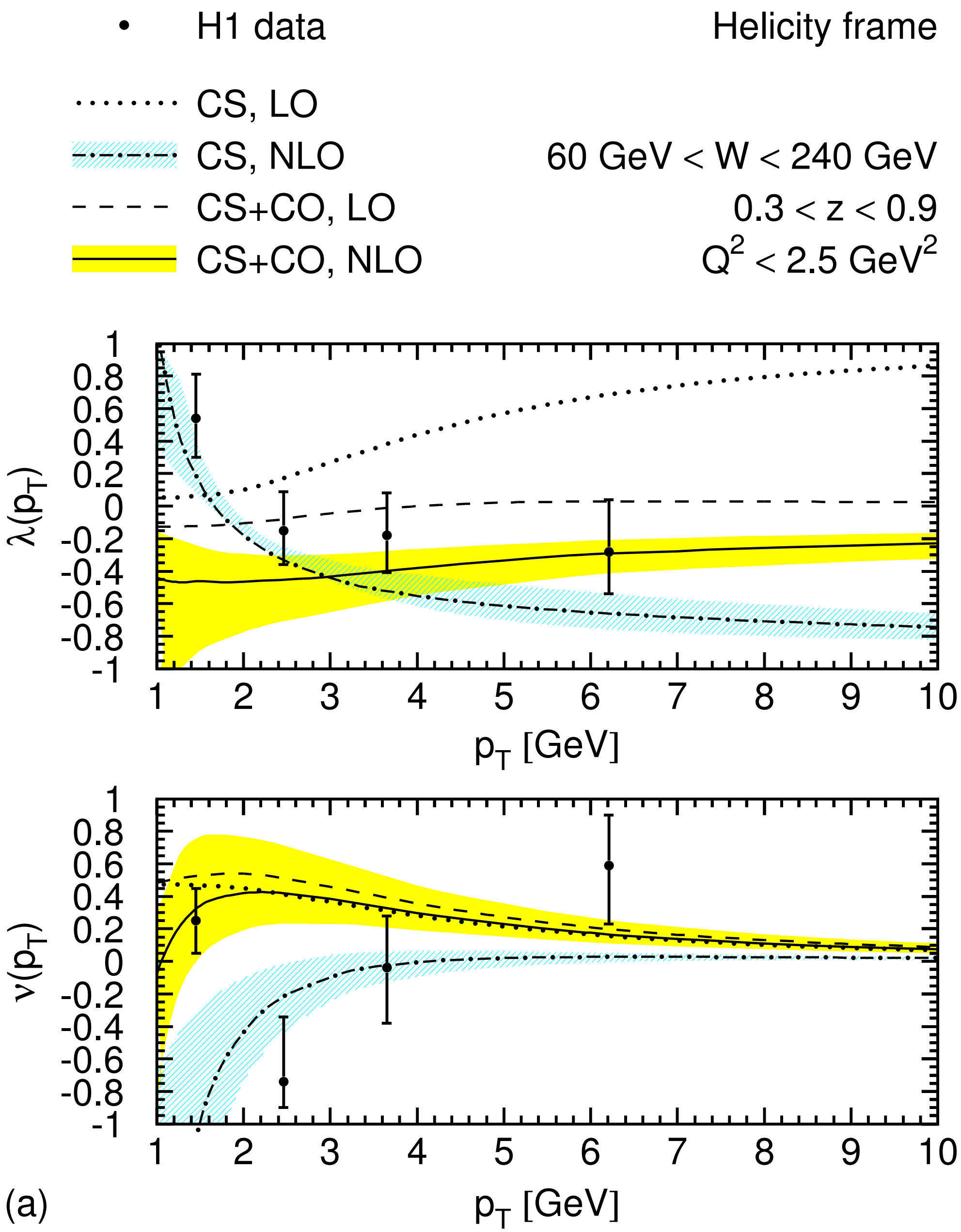


- The DELPHI (2003) data are slightly incompatible with the prediction of the Butenschön and Kniehl (2011) global fit.
- The error bars are large, especially at high p_T.
- Factorization may not hold at low values of p_T.

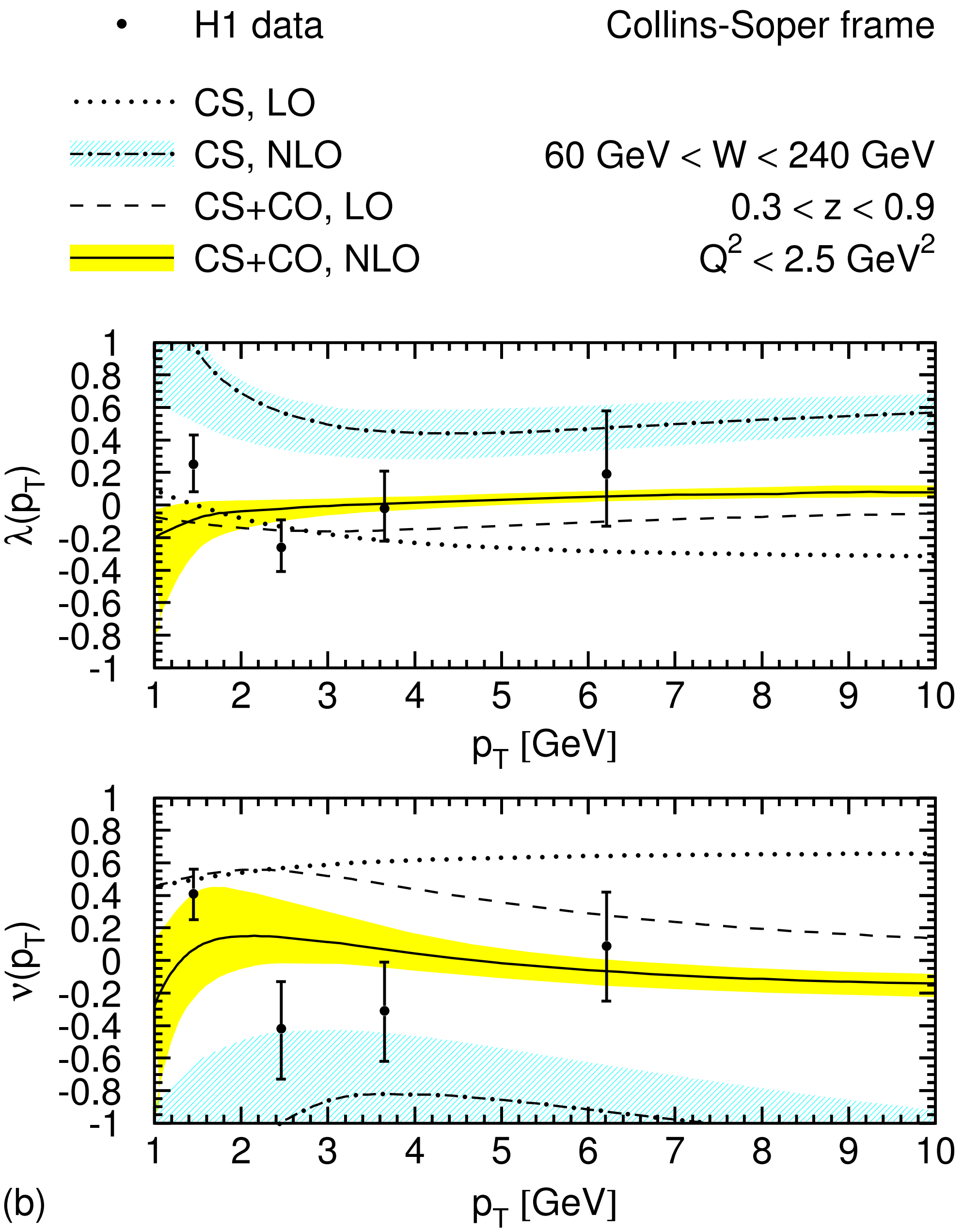


- The prediction from the Butenschön and Kniehl (2011) global fit is in agreement with the ALICE (2012) data.
- But the theory is for direct production, while the ALICE data includes production in B-meson decays and feed-down from χ_{cJ} states and the $\psi(2S)$.

- The Butenschön and Kniehl (2011) global fit can also be used to predict the polarization in inelastic J/ψ photoproduction at HERA.

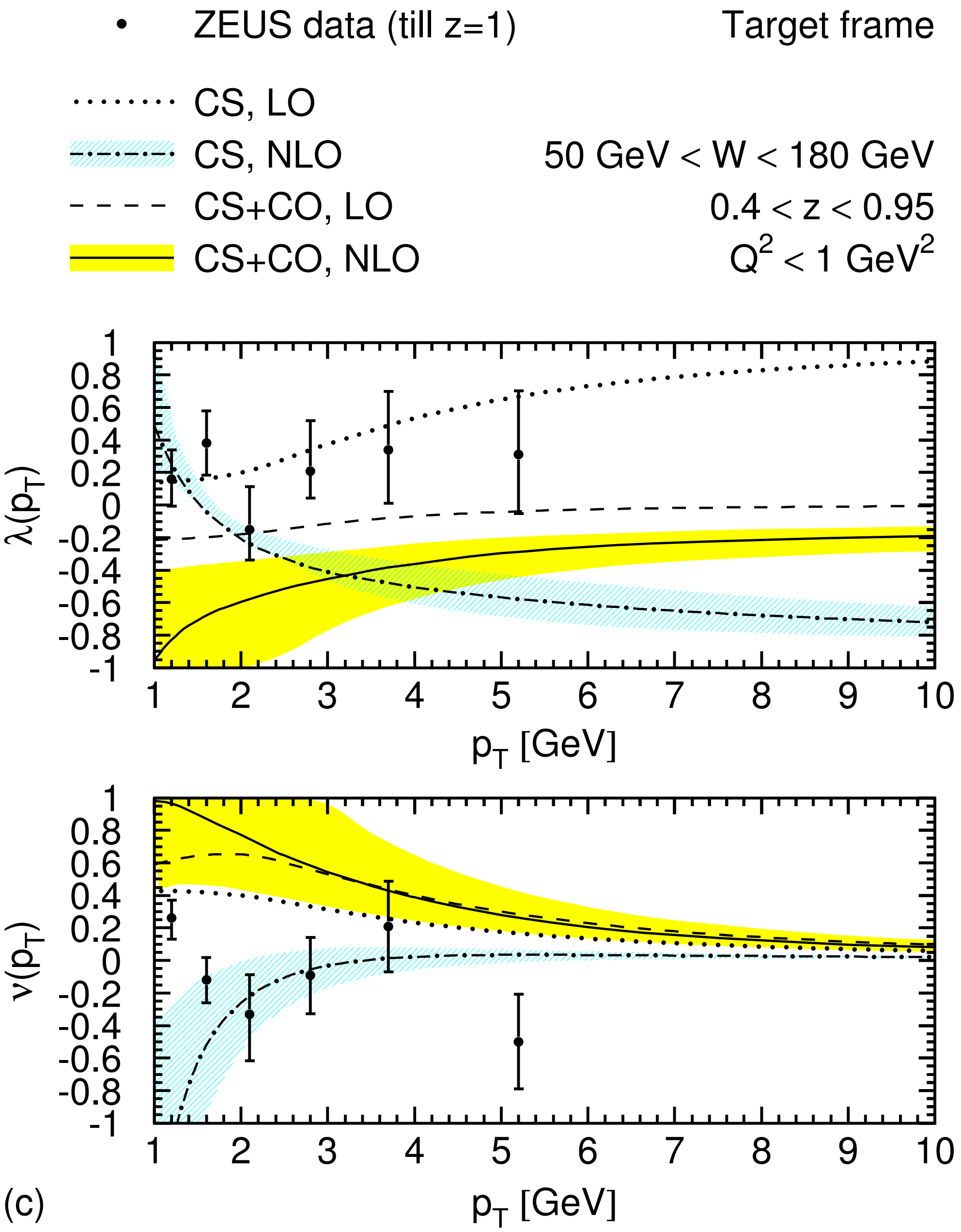


- The data are roughly compatible with the theory at large p_T, but the error bars are large.



H1 data
Collins-Soper frame
CS, LO
CS, NLO
60 GeV < W < 240 GeV
CS+CO, LO
0.3 < z < 0.9
CS+CO, NLO
Q^2 < 2.5 GeV^2
λ(p_T)
ν(p_T)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
1 2 3 4 5 6 7 8 9 10
p_T [GeV]

(b)



ZEUS data (till z=1)
Target frame
CS, LO
CS, NLO
50 GeV < W < 180 GeV
CS+CO, LO
0.4 < z < 0.95
CS+CO, NLO
Q^2 < 1 GeV^2
λ(p_T)
ν(p_T)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
1 2 3 4 5 6 7 8 9 10
p_T [GeV]

(c)