Exotic Hidden-Heavy Hadrons

and Where to Find Them

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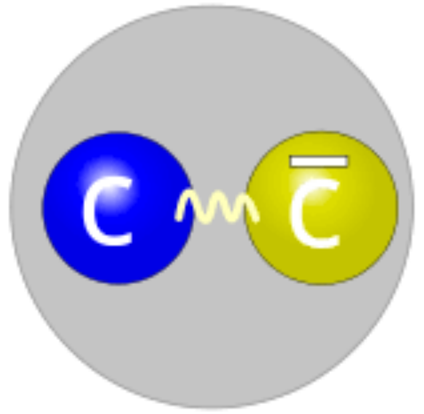
Braaten & Bruschini, arXiv:2409.00802

Peter Lepage

Effective field theories for Heavy Quarkonium



momentum scales: $\Lambda_{\rm QCD} < m_Q v^2 < m_Q v < m_Q$

Integrate out scale m_Q $\Longrightarrow$ NonRelativistic QCD (NRQCD)

Caswell & Lepage 1985

Integrate out scale $m_Q v$ $\Longrightarrow$ heavy quark and antiquark

interact through potentials

(Lepage, unpublished)

Effective field theory for Heavy Quarkonium

Integrate out scale $m_Q v$ $\Longrightarrow$ potential NRQCD (pNRQCD)

Brambilla, Pineda, Soto & Vairo 2000

Effective field theory for Double-Heavy Hadrons and Hidden-Heavy Hadrons

Born-Oppenheimer Effective Field Theory (BOEFT)

Berwein, Brambilla, Tarrus Castella & Vairo 2015
Oncala & Soto 2017
Brambilla, Krein, Tarrus Castella & Vairo 2017
Soto & Tarrus Castella 2020
Berwein, Brambilla, Mohapatra & Vairo 2024

Exotic Hidden-Heavy Hadrons

discovery of $X_c = \chi_{c1}(3872)$ Belle collaboration September 2003

hidden-charm tetraquark meson $(c\bar{c}q\bar{q})$

counting as of June 2023 Lebed 2023

44 $c\bar{c}$ tetraquark mesons
5 $c\bar{c}$ pentaquark baryons
5 $b\bar{b}$ tetraquark mesons
4 $cc\bar{c}\bar{c}$ tetraquark mesons

Challenge: understand Exotic Hidden-Heavy Hadrons based on QCD

No solution to the problem for over 20 years!

Exotic Hidden-Heavy Hadrons

Molecular models: constituents are color-singlet heavy hadrons
natural explanation for why many Exotic Hidden-Heavy Hadrons
have masses near thresholds for pairs of heavy hadrons

Models with colored constituents: quarks, gluons, diquarks, …
explosion in predicted but unobserved states

Constituent models can postdict most observed states
no compelling pattern
tenuous connections to fundamental theory QCD

Born-Oppenheimer Approximation for QCD

framework for Exotic Hidden-Heavy Hadrons firmly based on fundamental theory

Born-Oppenheimer approximation for QCD

Juge, Kuti & Morningstar 1999

heavy quarks: charm, bottom

Hidden-Heavy Hadron includes heavy quark and antiquark
plus light quarks, antiquarks, and gluons

Step 1

use Lattice QCD to calculate Born-Oppenheimer potentials:
discrete energy levels of QCD
in the presence of static *3* and *3** color sources
separated by distance *r*

Step 2

solve Schrödinger equation for heavy quark and antiquark
in Born-Oppenheimer potentials

behavior at small *r* and large *r* determined by simpler problem:
QCD with a single static color source

QCD with single static color source

symmetries: rotational, parity, charge conjugation

$\Longrightarrow$ discrete energy levels labeled by J^{PC}

_3_ or **_3*_** source:

discrete energy levels are called “static hadrons”

- spectrum can be calculated using lattice QCD
- spectrum can also be obtained by extrapolating charm hadrons and bottom hadrons to infinite quark mass

_8_ source:

discrete energy levels are called “adjoint hadrons”

SU(3)-flavor singlets are also called “gluelumps”

- spectrum can be calculated using lattice QCD
  gluelump spectrum in *SU(3) gauge theory*: Foster & Michael 1999
  Herr, Schlosser & Wagner 2023
  gluelump spectrum in QCD with 2 light flavors: Marsh & Lewis 2014
- same as spectrum for gluino-hadrons that contain long-lived heavy gluino

QCD with two static color sources

3 and ***3**** sources separated by distance *r*

discrete energy levels are "Born-Oppenheimer potentials"

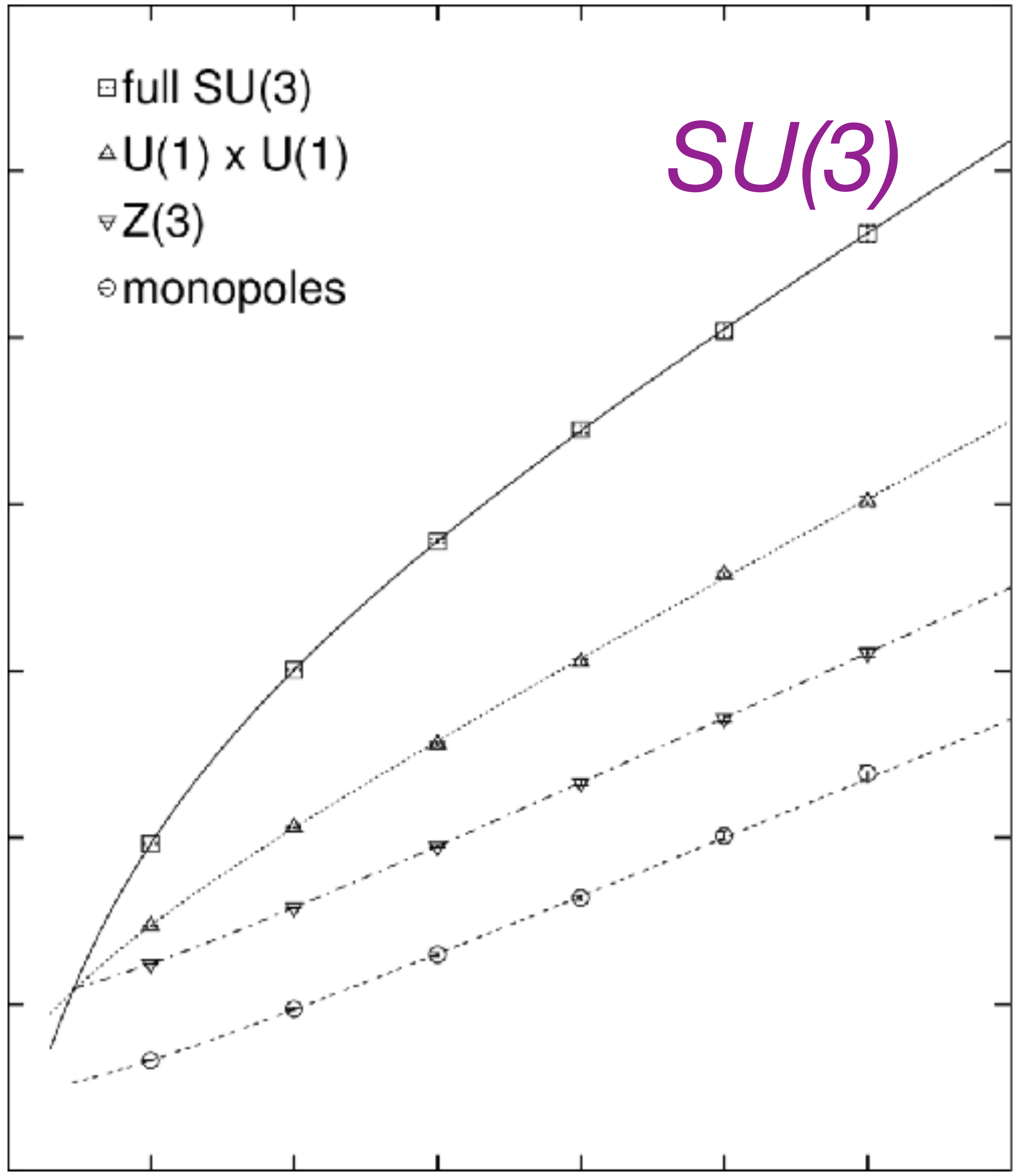
- B-O potentials can be calculated using lattice QCD

symmetries	quantum numbers
cylindrical	$\lambda = 0, \pm 1, \pm 2, \ldots$
$P \times C$	$CP = +1, -1$
reflection R	$R = +1, -1$

traditional Born-Oppenheimer quantum numbers from atomic physics: Λ_η^ϵ

- $\Lambda = \Sigma, \Pi, \Delta, \ldots$ for $|\lambda| = 0,1,2,\ldots$
- $\eta = g, u$ for $CP = +1, -1$
- $\epsilon = +, -$ for $R = +1, -1$ (only needed for $\Lambda = \Sigma$)

History of Born-Oppenheimer potentials



Stack 1984

first calculation of ground-state (quarkonium) B-O potential using pure *SU*(3) Lattice gauge theory

Σ_g^+ potential increases linearly at large *r*

$\Longrightarrow$ gluon flux tube connecting ***3*** and ***3**** sources

Σ_g^+ potential attractive color-Coulomb potential at small *r*

$\Longrightarrow$ perturbative QCD

What happens in QCD at large *r* ?

string breaking: two static mesons bound to ***3*** and ***3**** sources !

$\Longrightarrow$ ground-state Σ_g^+ potential must approach constant

= 2 × (energy of ground-state static meson)

History of Born-Oppenheimer potentials

Bali et al. 1995

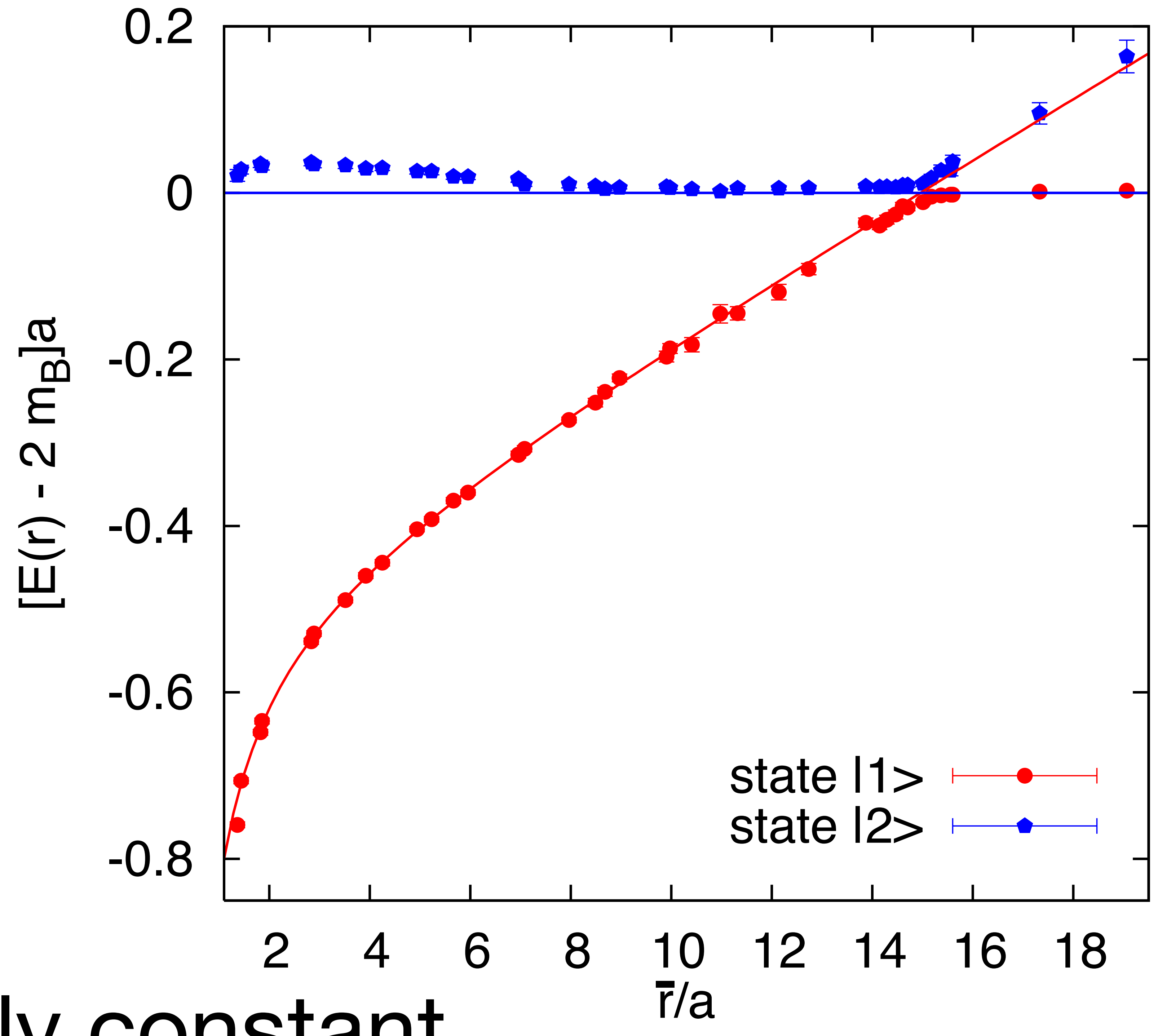
calculation of first two Σ_g^+ potentials

using Lattice QCD with 2 flavors of light quarks

narrow avoided crossing near 1.2 fm between

Σ_g^+ quarkonium potential that increases linearly

static-meson-pair potential that is approximately constant

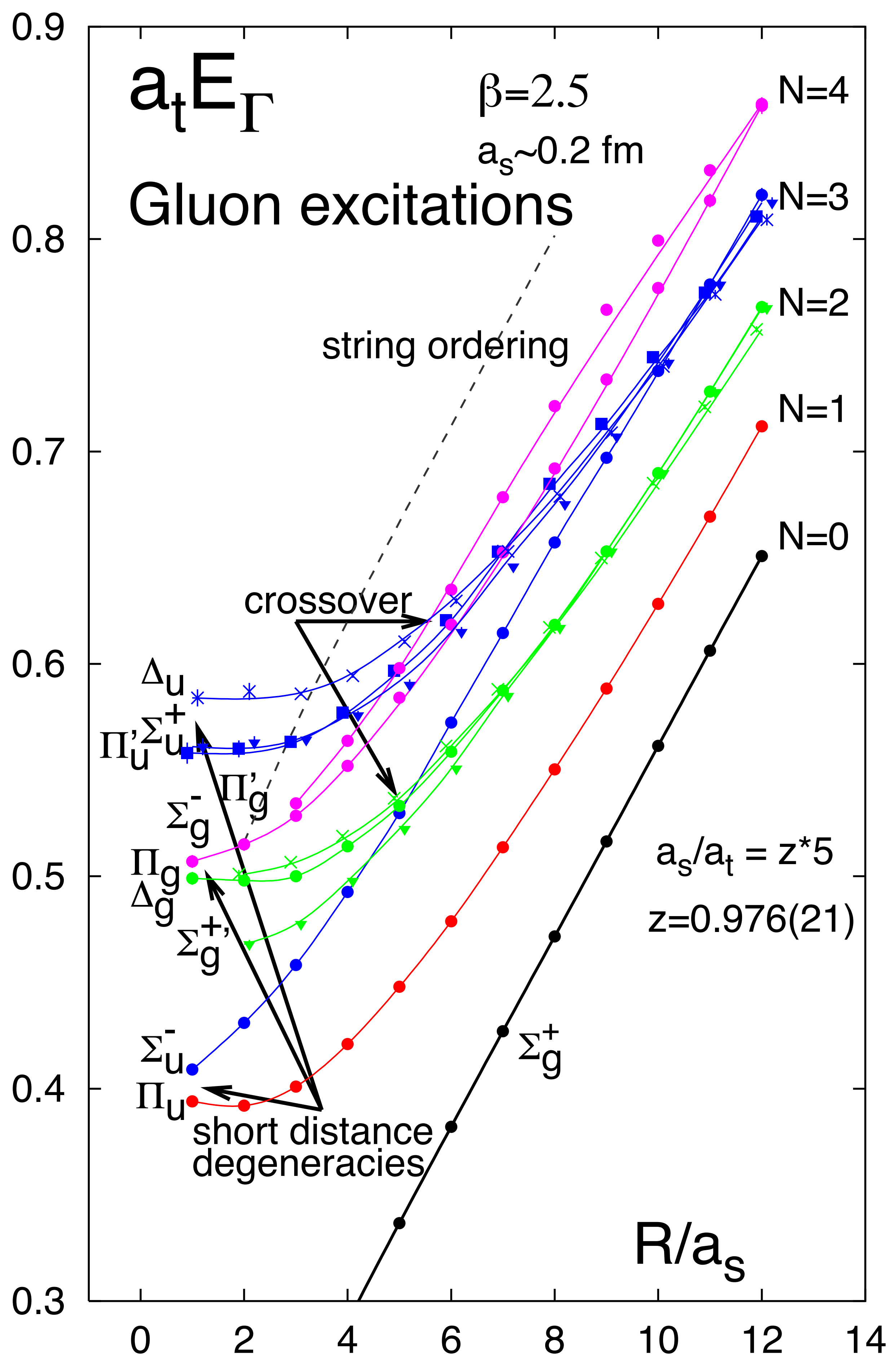


In hindsight

static-hadron-pair potential at large *r* for every pair of static hadrons !

what happens to them at small *r* ?

History of Born-Oppenheimer potentials



Juge, Kuti, and Morningstar 1999, 2002

excited B-O potentials in pure *SU*(*3*) Lattice gauge theory

$\Pi_u, \Sigma_u^-, \Sigma_g^{+\prime}, \ldots$ potentials: increase linearly at large *r*

$\Longrightarrow$ excited gluon flux tube connecting *3* and *3** sources

pNRQCD: **Brambilla, Pineda, Soto & Vairo 1999**

excited B-O potentials at small *r* :

repulsive color-Coulomb potentials offset by gluelump energy

In hindsight

adjoint-hadron potential at small *r* for every adjoint hadron !

(repulsive color-Coulomb potential offset by adjoint-hadron energy)

what happens to them at large *r* ?

History of Born-Oppenheimer potentials

Discovery of hidden-heavy tetraquark mesons with electric charge

$Z_b^+ = T_{b\bar{b}1}(10610)^+, Z_b^{+'} = T_{b\bar{b}1}(10650)^+$ Belle collaboration 2011

$Z_c^+ = T_{c\bar{c}1}(3900)^+$ BESIII, Belle collaborations 2013

Braaten, Langmack & Smith 2014

hidden-heavy tetraquark mesons with electric charge

are bound states in isospin-1 B-O potentials

Assumption:

isospin-1 B-O potentials have same qualitative behavior

as B-O potentials for pure *SU(3)* gauge theory

- repulsive color-Coulomb potentials at small *r* RIGHT!
- increasing linearly at large *r* WRONG!

but offset by isospin-1 adjoint-meson energy instead of gluelump energy

Born-Oppenheimer Potentials for QCD

Born-Oppenheimer potentials

behavior at small r and large r determined by QCD with single color source

- adjoint-hadron potentials at small *r*, but what happens at large *r* ?
- static-hadron-pair potentials at large *r*, but what happens at small *r* ?

Spectrum of QCD in the presence of color sources is smooth function of *r* !

$\Longrightarrow$ adjoint-hadron potentials at small *r*
must connect smoothly to static-hadron-pair potentials at large *r*

Berwein, Brambilla, Mohapatra & Vairo 2024
Braaten & Bruschini 2024

Behavior of B-O potentials for QCD whose light-quark flavor is not *SU(3)* singlet

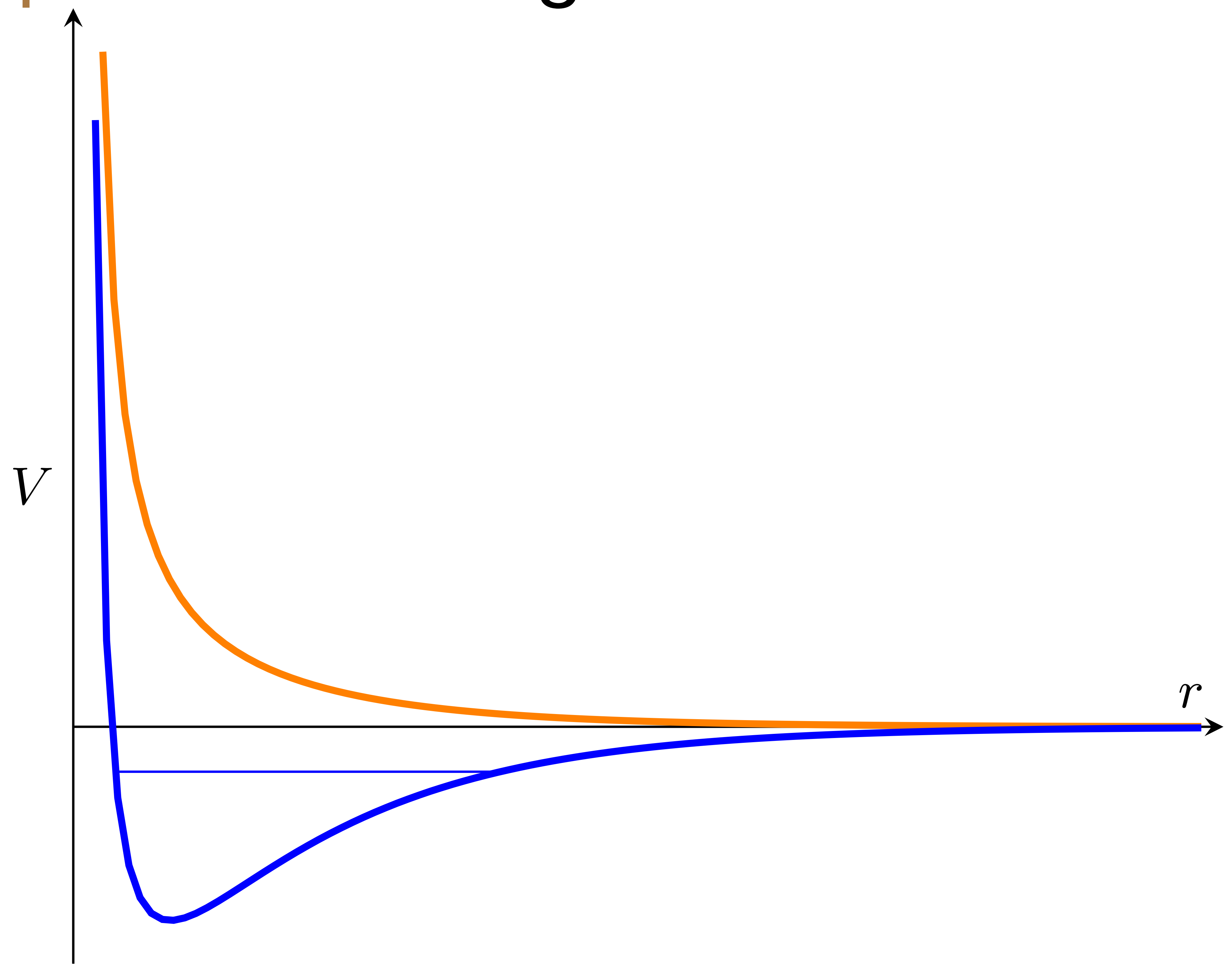
- repulsive color-Coulomb potential at small *r*
- connects smoothly to constant potential at large *r* (not confining)

completely different from pure SU(3) gauge theory

Born-Oppenheimer Potentials for QCD

Spectrum of QCD in the presence of color sources is smooth function of *r* !

$\Longrightarrow$ adjoint-hadron potential at small *r* must connect smoothly
to static-hadron-pair potential at large *r*



adjoint-hadron potential
can approach static-hadron-pair potential

either from above
in which case it cannot support bound states

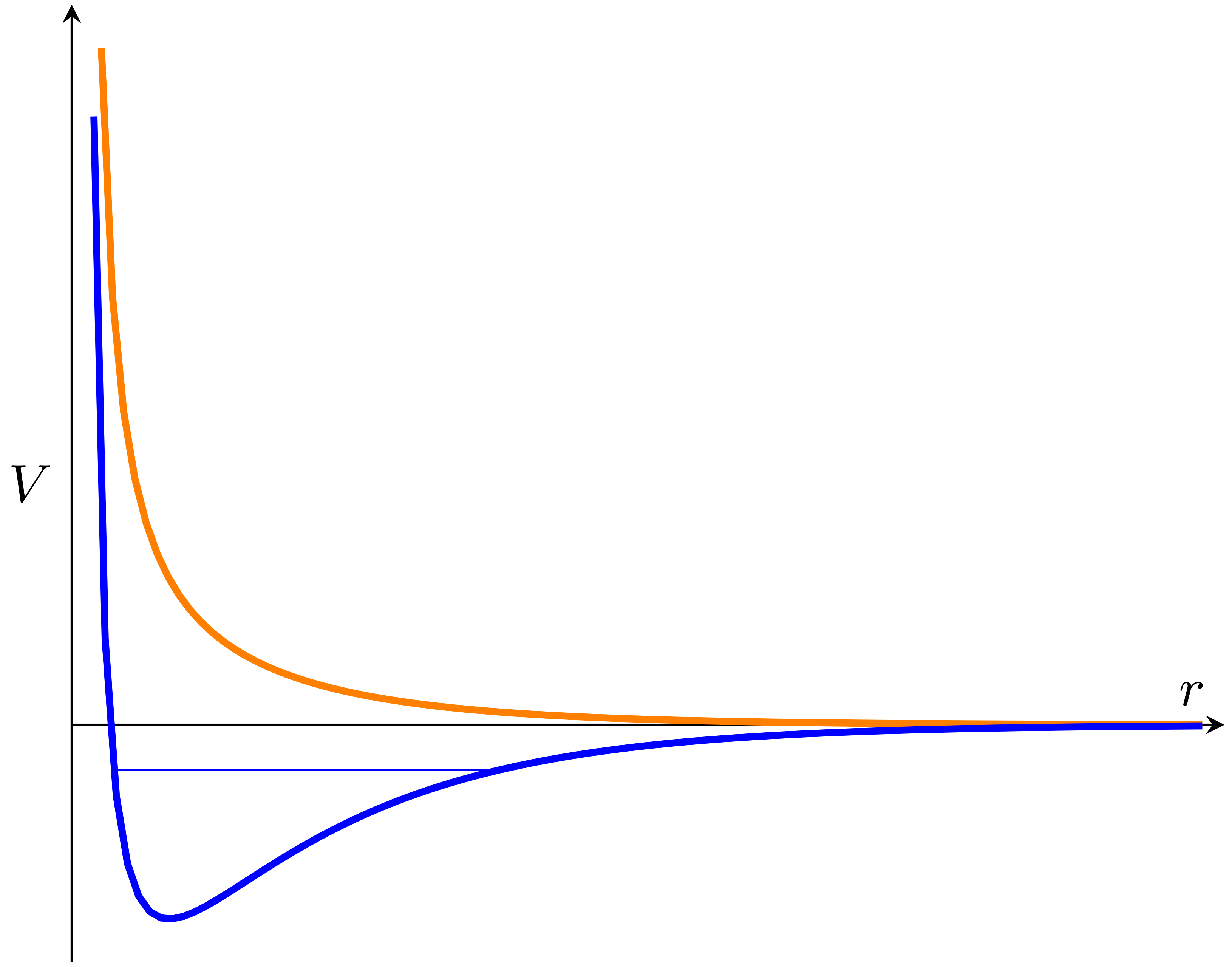
or from below
in which case it may support bound states
but only if the potential is deep enough
its depth is determined by adjoint-hadron energy

Solution to Exotic Hidden-Heavy Hadron problem

Braaten & Bruschini 2024

Exotic Hidden-Heavy Hadrons:

bound states or resonances in Born-Oppenheimer potentials
that are adjoint-hadron potentials at small r
and approach static-hadron-pair potentials from below

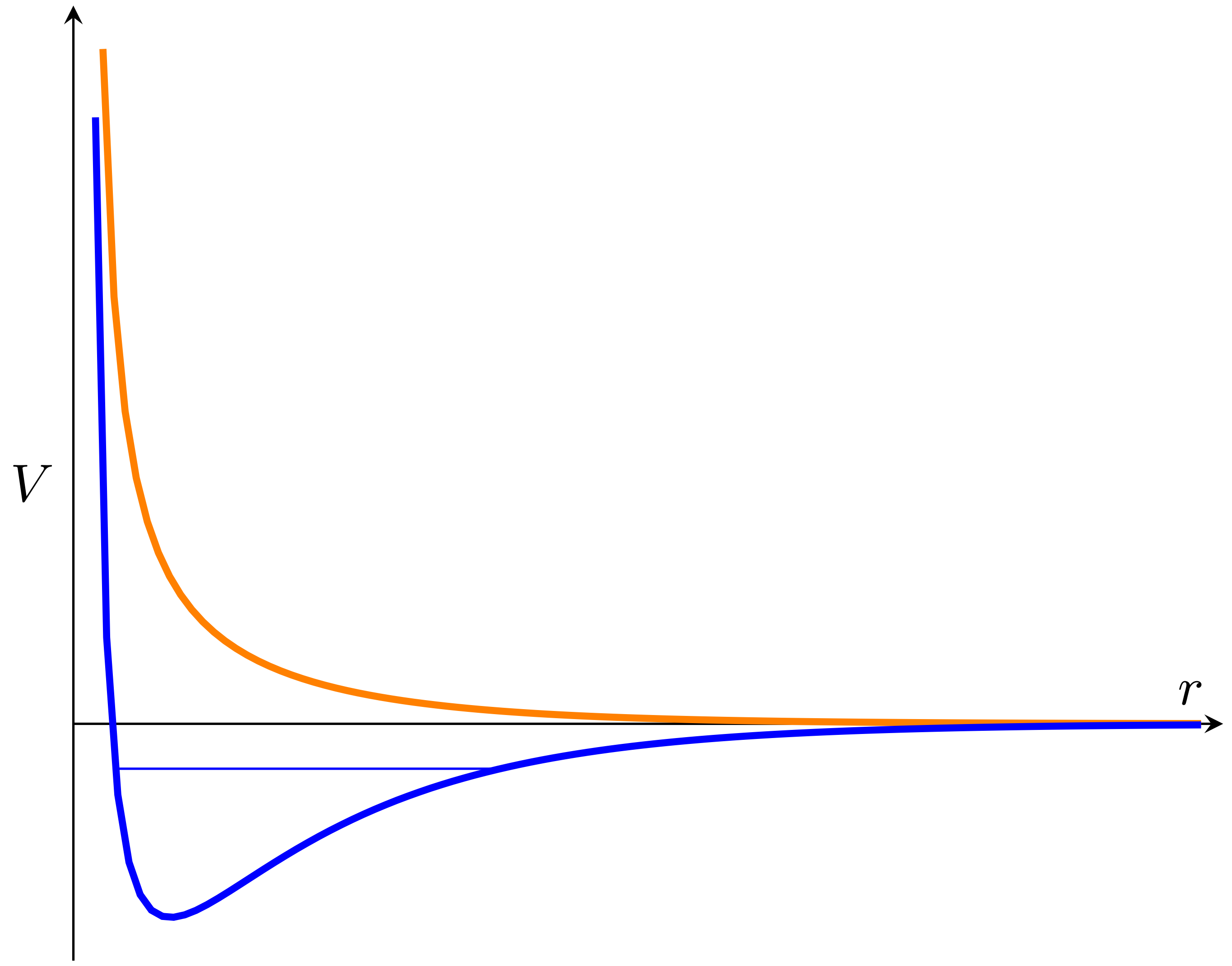


- explains why most Exotic Hidden-Heavy Hadrons
have mass near a heavy-hadron-pair threshold

Solution to Exotic Hidden-Heavy Hadron problem

typical splittings between adjoint hadrons with **8** source
are larger than those between static hadrons with **3** source

only those adjoint-hadron potentials
associated with the lowest-energy adjoint hadrons
will approach the static-hadron-pair potential from below



Exotic Hidden-Heavy Hadrons
are associated only with the lowest-energy adjoint hadrons !

- avoids explosion in number of Exotic Hidden-Heavy Hadrons

Solution to Exotic Hidden-Heavy Hadron problem

Amazing properties of some Exotic Hidden-Heavy Mesons
are determined by energies of adjoint mesons

$\chi_{c1}(3872)$ has mass within 100 keV of $D^{*0}\bar{D}^0$ threshold
explained by fine tuning of energy of isospin-0 $J^{PC} = 1^{--}$ adjoint meson

$T_{b\bar{b}1}(10610)^+$ has mass within 3 MeV of $B^*\bar{B}$ threshold
$T_{b\bar{b}1}(10650)^+$ has mass within 3 MeV of $B^*\bar{B}^*$ threshold
but it does not decay into $B^*\bar{B}$ or $B\bar{B}^*$
explained by fine tunings of energies of isospin-1 $J^{PC} = 1^{--}$ adjoint meson
and isospin-1 $J^{PC} = 0^{-+}$ adjoint meson

fine tunings of adjoint-meson energies can be verified by lattice QCD !

Solution to Exotic Hidden-Heavy Hadron problem

Short-term goals

$\chi_{c1}(3872)$ has $J^{PC} = 1^{++}$

heavy-quark spin-symmetry partners have $J^{PC} = 1^{+-}, 0^{++}, 2^{++}$

Use diabatic Born-Oppenheimer approximation

to predict their masses to within a few MeV

$T_{b\bar{b}1}(10610)^0$, $T_{b\bar{b}1}(10650)^0$ have $J^{PC} = 1^{+-}$

heavy-quark spin-symmetry partners have $J^{PC} = 0^{++}, 0^{++}, 1^{++}, 2^{++}$

Use diabatic Born-Oppenheimer approximation

to predict their masses to within a few MeV

Longer-term goal:

Use diabatic Born-Oppenheimer approximation

to predict the masses of all Exotic Hidden-Heavy Hadrons

Exotic Hidden-Heavy Hadrons:

bound state or resonances in Born-Oppenheimer potentials
that are repulsive color-Coulomb potentials at small *r*
and approach a static-meson-pair potential from below

Where to Find Them:

near the heavy-hadron-pair thresholds

associated with the lowest-energy adjoint hadrons

Braaten & Bruschini, arXiv:2409.00802

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