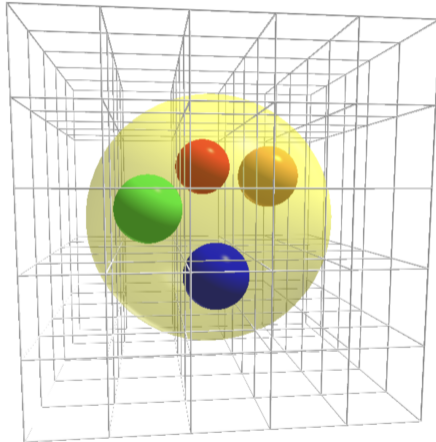


# Exotic heavy hadrons from lattice QCD

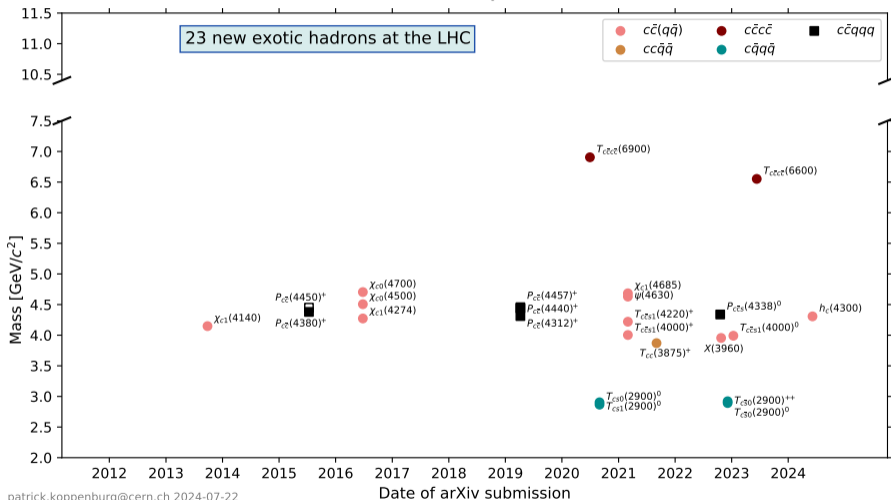
LepageFest, 2024



Randy Lewis, York University, Toronto

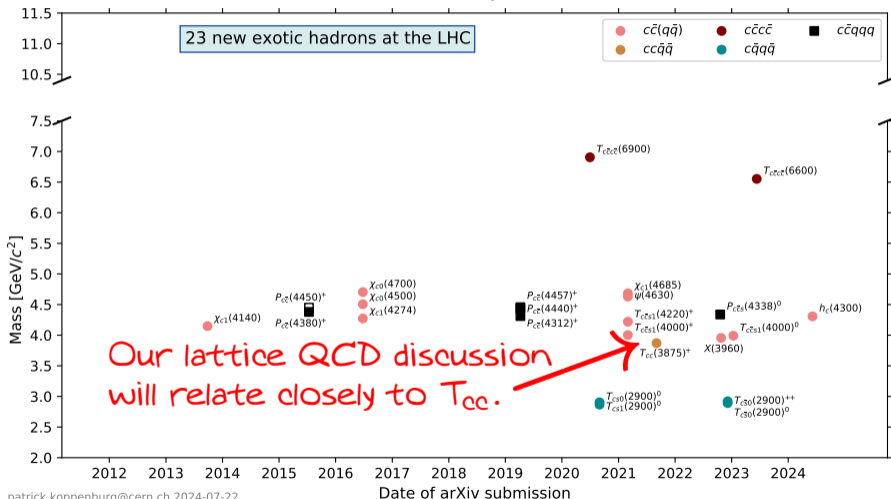
# New exotic hadrons are being discovered every year

For example,



# New exotic hadrons are being discovered every year

For example,

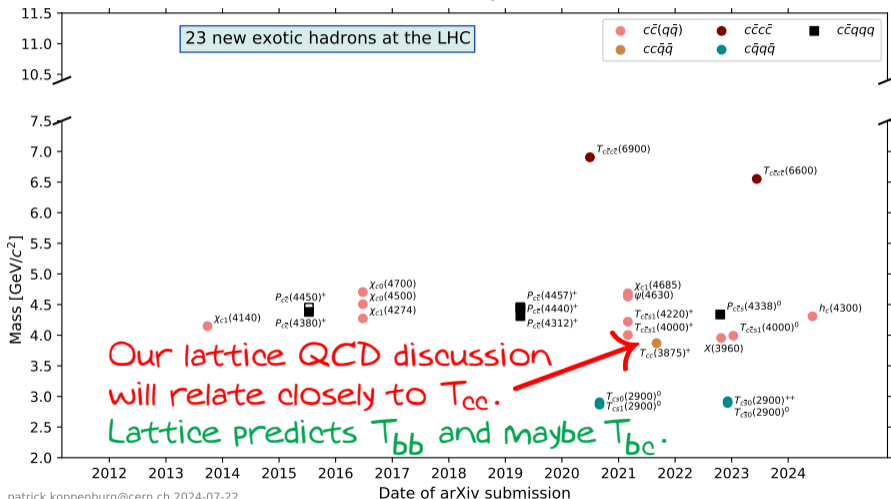


patrick.koppenburg@cern.ch 2024-07-22

P. Koppenburg (LHCb collaboration), [www.nikhef.nl/~pkoppenb/particles.html](http://www.nikhef.nl/~pkoppenb/particles.html)

# New exotic hadrons are being discovered every year

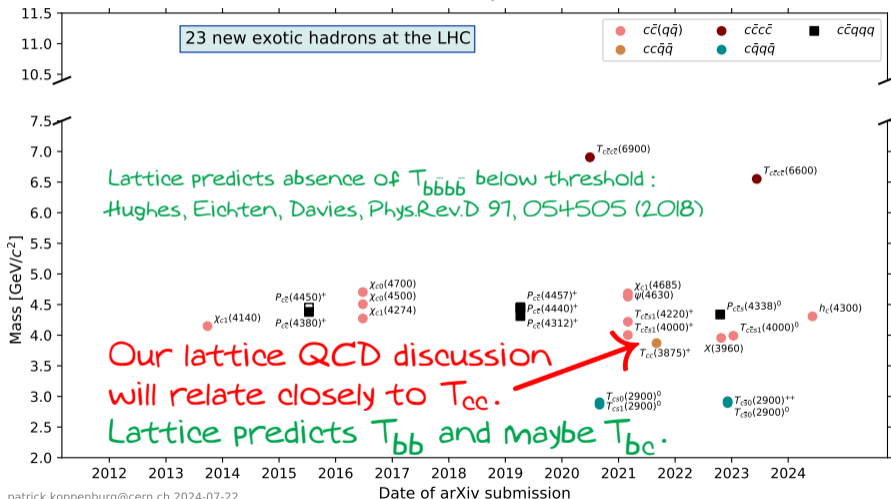
For example,



patrick.koppenburg@cern.ch 2024-07-22

# New exotic hadrons are being discovered every year

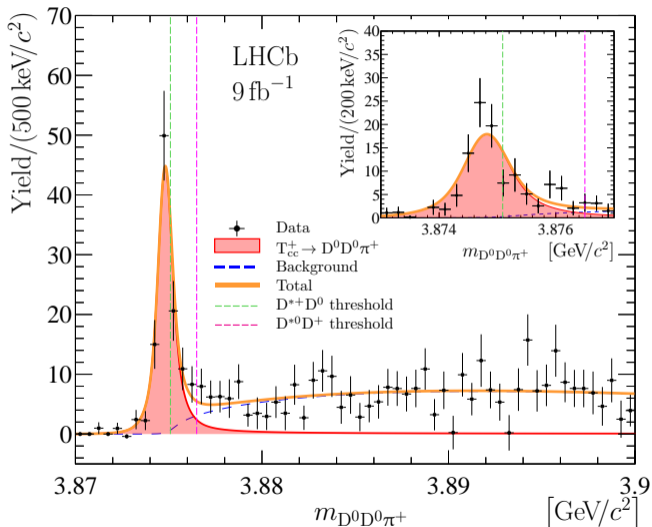
For example,



patrick.koppenburg@cern.ch 2024-07-22

P. Koppenburg (LHCb collaboration), [www.nikhef.nl/~pkoppenb/particles.html](http://www.nikhef.nl/~pkoppenb/particles.html)

## Experimental observation of $T_{cc}$



- $T_{cc}$  is slightly below threshold:

$$\delta m \sim \frac{1}{4} \text{ MeV}$$

- Predictions varied widely:

$$-300 \text{ MeV} \lesssim \delta m \lesssim 300 \text{ MeV}$$

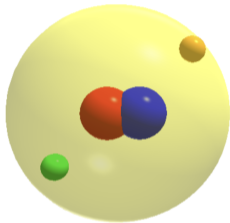
- **Directly from lattice QCD?**

$T_{cc}$  is an extreme challenge

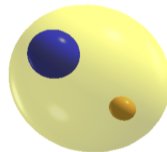
$T_{bb}$  and  $T_{bc}$  are more accessible

## Intuition for a very heavy diquark

tetraquark

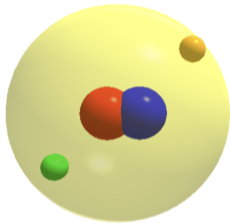


pair of mesons



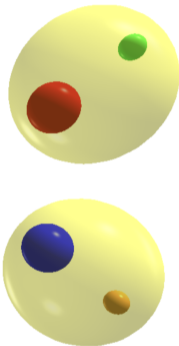
## Intuition for a very heavy diquark

tetraquark



The heavy diquark is compact with binding energy  $\sim \alpha_s^2 m_Q \sim \frac{m_Q}{\ln^2(m_Q)}$ .

pair of mesons

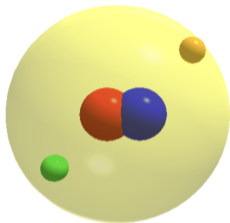


Therefore, for  $m_Q \rightarrow \infty$ , the tetraquark is a stable particle in QCD.



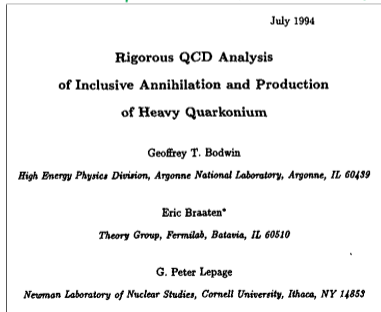
# Intuition for a very heavy diquark

tetraquark

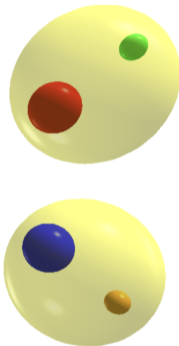


The heavy diquark is compact with binding energy  $\sim \alpha_s^2 m_Q \sim \frac{m_Q}{\ln^2(m_Q)}$ .

Recall the quarkonium discussion :



pair of mesons

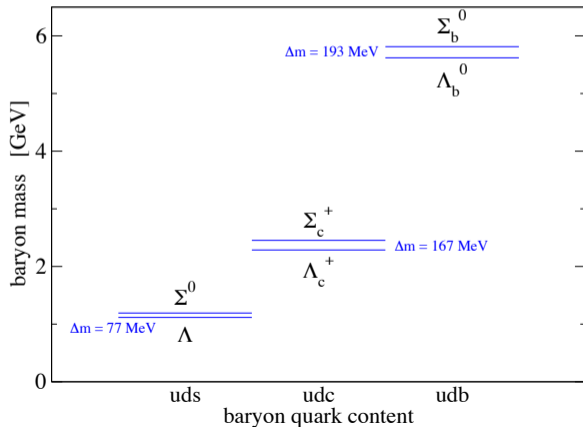


Therefore, for  $m_Q \rightarrow \infty$ , the tetraquark is a stable particle in QCD.

## Intuition for a good light diquark

Recall some standard heavy baryons.

Each  $\Lambda_Q$  is more deeply bound than its  $\Sigma_Q$  partner, especially as  $m_Q \rightarrow \infty$ .



baryon

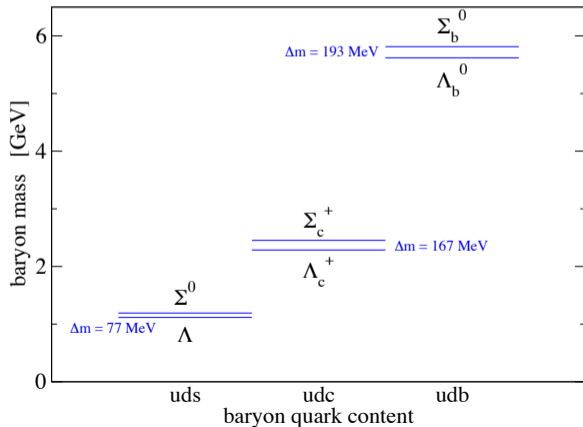


## Intuition for a good light diquark

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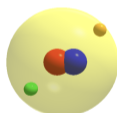
A tetraquark can have this same  $\Lambda$ -type light diquark.



baryon

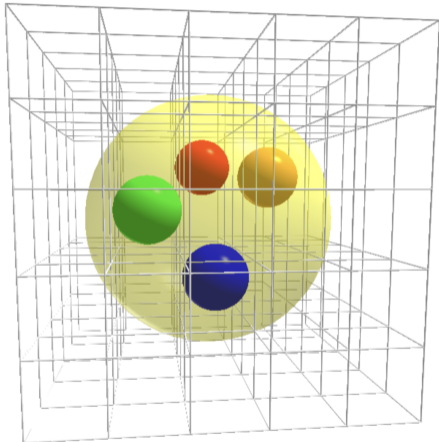


tetraquark



In both hadrons, the light diquark sees a heavy color triplet.

## Roles for lattice QCD



Computing directly from QCD is the goal.

Extrapolations are required in lattice spacing, volume and some quark masses.

Choices made by different authors give valuable insight into systematic effects.

A consensus from the lattice community will provide confidence for tetraquark physics.

# $T_{bb}$ binding energy for $ud\bar{b}\bar{b}$ from lattice QCD

$M(T_{bb}) - M(B) - M(B^*)$	
1209.6274	— Wagner, Bicudo
1210.1953	— Brown, Orginos
1510.03441	— Bicudo, Cichy, Peters, Wagner
1612.02758	— Bicudo, Scheunert, Wagner
1607.05214	— Francis, Hudspith, Lewis, Maltman
1810.12285	— Junnarkar, Mathur, Padmanath
1904.04197	— Leskovec, Meinel, Pflaumer, Wagner
2008.11146	— Mohanta, Basak
2303.17295	— Hudspith, Mohler
2306.03565	— Aoki, Aoki, Inoue
2404.03588	— Alexandrou, Finkenrath, Leontiou, Meinel, Wagner
2407.08816	— Colquhoun, Francis, Hudspith, Lewis, Maltman, Parrott

arXiv number

static  $b$  quarks

NRQCD  $b$  quarks

# $T_{bb}$ binding energy for $ud\bar{b}\bar{b}$ from lattice QCD

$$M(T_{bb}) - M(B) - M(B^*)$$

static $b$ quarks		NRQCD $b$ quarks	
1209.6274	Wagner, Bicudo	1607.05214	Francis, Hudspith, Lewis, Maltman
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arXiv number

## Defining lattice NRQCD

CLNS 92/1136  
OHSTPY-HEP-T-92-001  
February, 1992

**Improved Nonrelativistic QCD  
for Heavy Quark Physics**

G. Peter Lepage,<sup>1</sup> Lorenzo Magnea<sup>2</sup> and Charles Nakhleh  
Newman Laboratory of Nuclear Studies  
Cornell University, Ithaca, NY 14853

Ulrika Magnea  
State University of New York at Stony Brook  
Stony Brook, NY 11794

Kent Hornbostel  
The Ohio State University  
Columbus, OH 43210

Fermilab-Pub-91/355-T  
(Revised!)

On the Viability of Lattice Perturbation Theory

G. Peter Lepage  
Newman Laboratory of Nuclear Studies  
Cornell University, Ithaca, NY 14853

and

Paul B. Mackenzie  
Theoretical Physics Group  
Fermi National Accelerator Laboratory  
P.O. Box 600, Batavia, IL 60610

September 1992 (Revised November 1992)

# $T_{bb}$ binding energy for $ud\bar{b}\bar{b}$ from lattice QCD

$$M(T_{bb}^*) - M(B) - M(B^*)$$

	static $b$ quarks	NRQCD $b$ quarks
1209.6274	— Wagner, Bicudo	
1210.1953	— Brown, Orginos	
1510.03441	— Bicudo, Cichy, Peters, Wagner	
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arXiv number

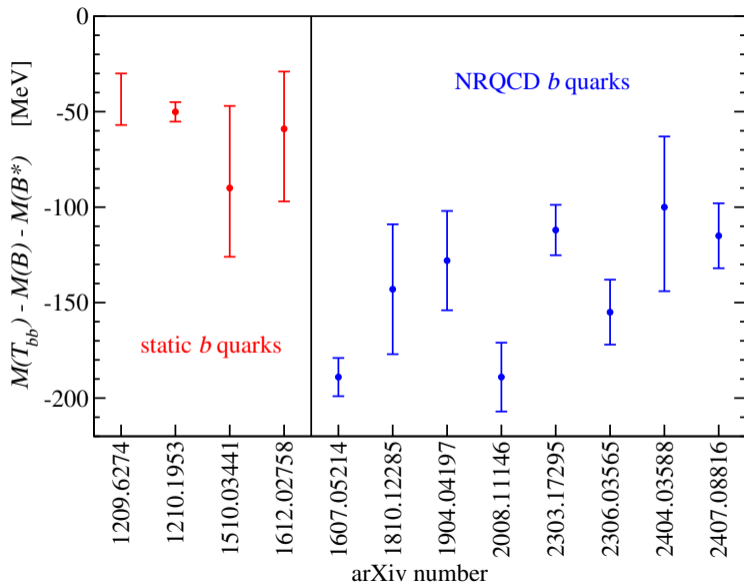
Thanks to Peter Lepage in 2024

## ACKNOWLEDGEMENTS

The authors wish to thank G. P. Lepage for considerable input and advice regarding correlator fitting with his software (described in text). WP, RL and KM are supported by grants from the Natural Sciences and Engineering Research Council of Canada. AF

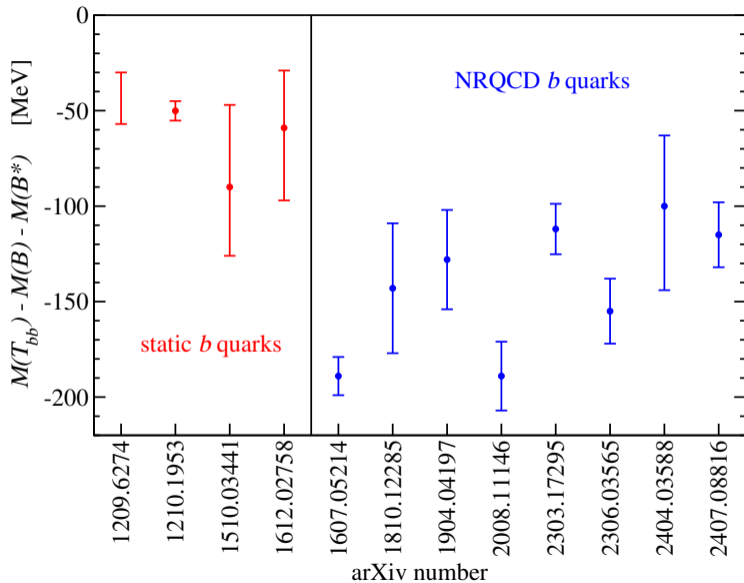
2407.08816

# $T_{bb}$ binding energy for $ud\bar{b}\bar{b}$ from lattice QCD





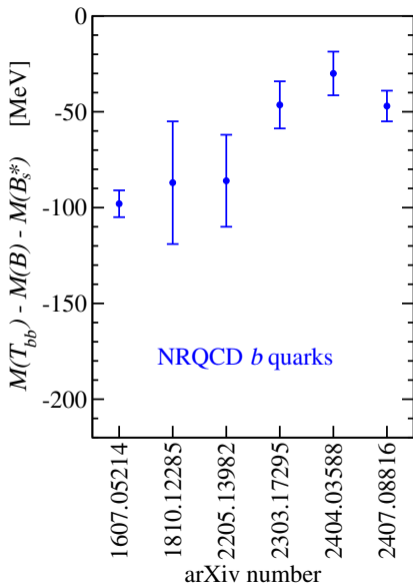
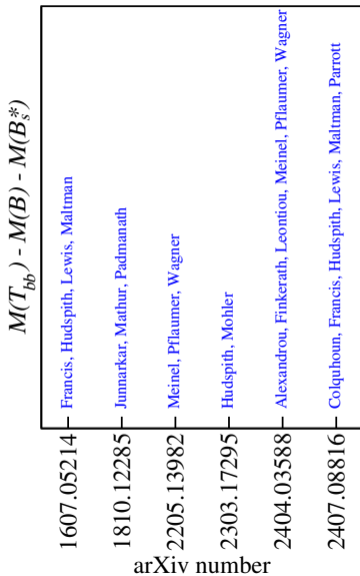
# $T_{bb}$ binding energy for $ud\bar{b}\bar{b}$ from lattice QCD



## Systematic effects:

- chosen operator set (local, nonlocal, smeared, meson pair, diquark pair, scattering states)
- tuning of NRQCD
- extrapolations (lattice spacing, volume, quark masses)
- chosen lattice action (Wilson, HISQ, overlap, domain wall)

# $T_{bb}$ binding energy for $us\bar{b}\bar{b}$ from lattice QCD



# $T_{bc}$ binding energies for $ud\bar{c}\bar{b}$ from lattice QCD

	$J^P = 1^+$	$J^P = 0^+$
1810.10550	Francis, Hudspith, Lewis, Maltman	
2006.14294	Hudspith, Colquhoun, Francis, Lewis, Maltman	
2205.13982	Meinel, Pflaumer, Wagner	
2307.14128	Padmanath, Radhakrishnan, Mathur	
2312.02925	Alexandrou, Finkenrath, Leontiou, Meinel, Pflaumer, Wagner	
2006.14294		Hudspith, Colquhoun, Francis, Lewis, Maltman
2205.13982		Meinel, Pflaumer, Wagner
2312.02925		Alexandrou, Finkenrath, Leontiou, Meinel, Pflaumer, Wagner
2404.08109		Radhakrishnan, Padmanath, Mathur

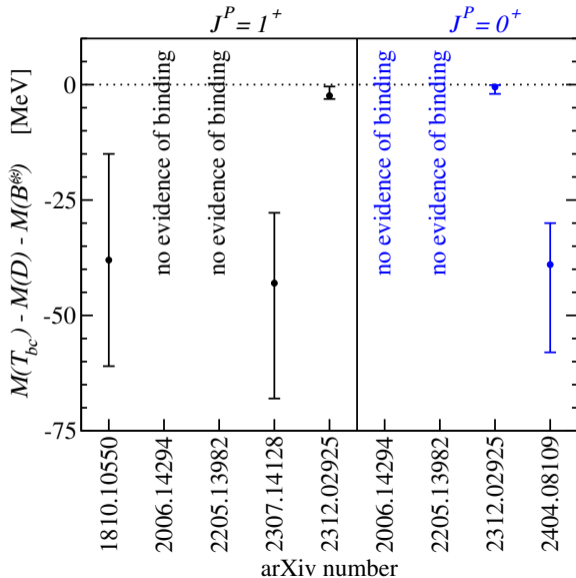
arXiv number

Additional systematics:

- larger threshold effects
- relativistic charm quark

[variants of El-Khadra, Kronfeld, Mackenzie  
Phys.Rev.D55, 3933 (1997)]

# $T_{bc}$ binding energies for $ud\bar{c}\bar{b}$ from lattice QCD



Additional systematics:

- larger threshold effects
- relativistic charm quark

[variants of El-Khadra, Kronfeld, Mackenzie  
Phys.Rev.D55, 3933 (1997)]

# Operators and energies for lattice $T_{bc}$ with $J^P = 0^+$

Alexandrou, Finkenrath, Leontiou, Meinel, Pflaumer, Wagner, PhysRevLett132,151902 = 2312.02925

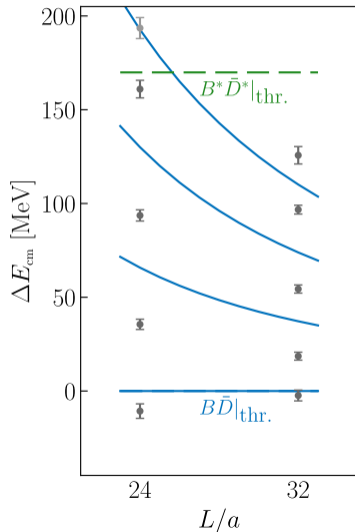
- $\sum_{\vec{x}} \left[ \bar{b}^a(\vec{x}) \gamma_5 C \bar{c}^{b,T}(\vec{x}) \right] \left[ u^{a,T}(\vec{x}) C \gamma_5 d^b(\vec{x}) \right] - (d \leftrightarrow u)$
- $\sum_{\vec{x}} B^+(\vec{x}) D^-(\vec{x}) - \sum_{\vec{x}} B^0(\vec{x}) \bar{D}^0(\vec{x})$
- $\sum_{\vec{x},j} B_j^{*+}(\vec{x}) D_j^{*-}(\vec{x}) - \sum_{\vec{x},j} B_j^{*0}(\vec{x}) \bar{D}_j^{*0}(\vec{x})$
- $B^+(\vec{q}) D^-(\vec{q}) - B^0(\vec{q}) \bar{D}^0(\vec{q})$  for  $\frac{|\vec{q}|L}{2\pi} \in \{0, 1, \sqrt{2}, \sqrt{3}\}$

where

$$B^+(x) = \bar{b}(x) \gamma_5 u(x)$$

$$B^+(\vec{q}) = \frac{1}{\sqrt{V}} \sum_{\vec{x}} B^+(x) e^{2\pi i \vec{q} \cdot \vec{x} / L}$$

etc.



Curves are non-interacting  $B \bar{D}$  energy levels.

## Toward the $T_{cc}$ binding energy from lattice QCD

An early attempt:

1810.12285	Junnarkar, Mathur, Padmanath	$-23 \pm 11$ MeV
------------	------------------------------	------------------

Recent studies: ( $DD^*$  scattering via volume dependence or effective potentials)

2202.10110	Padmanath, Prelovsek	$-9.9_{-7.1}^{+3.6}$ MeV
2206.06185	Chen, Shi, Chen, Gong, Liu, Sun, Zhang	$I = 0$ attractive but $I = 1$ repulsive
2302.04505	Lyu, Aoki, Doi, Hatsuda, Ikeda, Meng	$-59 \begin{pmatrix} +53 \\ -99 \end{pmatrix} \begin{pmatrix} +2 \\ -67 \end{pmatrix}$ keV
2402.14715	Collins, Nefediev, Padmanath, Prelovsek	"a very delicate fine tuning" is observed
2405.15741	Whyte, Wilson, Thomas	$-41 \pm 31$ MeV (virtual bound state)

Reminder:

Physical pion masses will be necessary because pion exchange contributions are significant.

Note: Available lattice studies of  $T_{cc}$  use  $m_\pi > m_{D^*} - m_D$ , meaning  $D^*$  mesons are stable.

## Dreams for the distant future

Real-time evolution in lattice QCD?

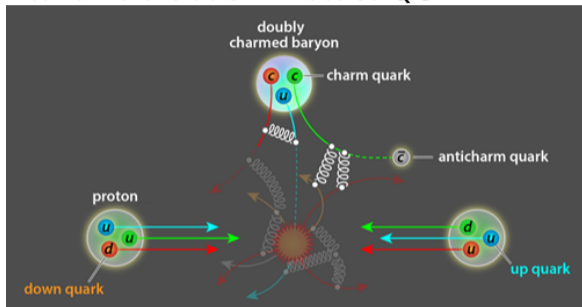


Image from <https://physics.aps.org/articles/v10/100>

Dense matter in lattice QCD?

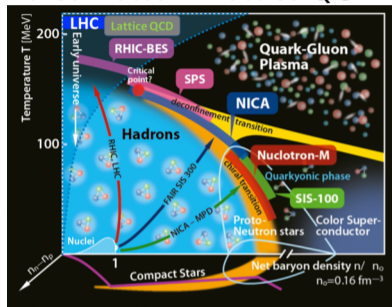



Image from arXiv:2201.00202

One small step for tetraquarks. . .

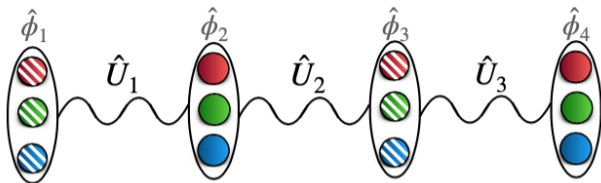
PHYSICAL REVIEW RESEARCH 5, 033184 (2023)

**Simulating one-dimensional quantum chromodynamics on a quantum computer:  
Real-time evolutions of tetra- and pentaquarks**

Yasar Y. Atas <sup>1,2,\*</sup>,† Jan F. Haase,<sup>1,2,3,\*</sup>‡ Jinglei Zhang,<sup>1,2,§</sup> Victor Wei <sup>1,4</sup> Sieglinde M.-L. Pfaendler <sup>5</sup>  
Randy Lewis,<sup>6</sup> and Christine A. Muschik<sup>1,2,7</sup>

## QCD in one spatial dimension

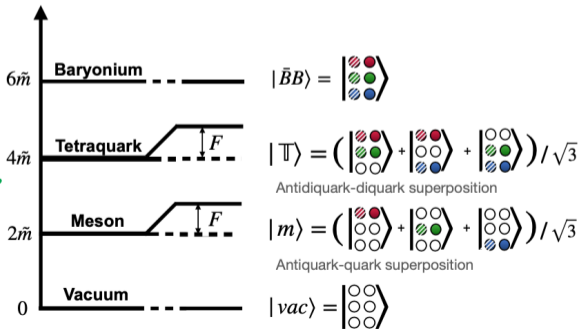
Staggered fermions:



Each site needs 3 qubits. Gauge fields provide interactions among them.

Two staggered sites in the strong coupling limit:

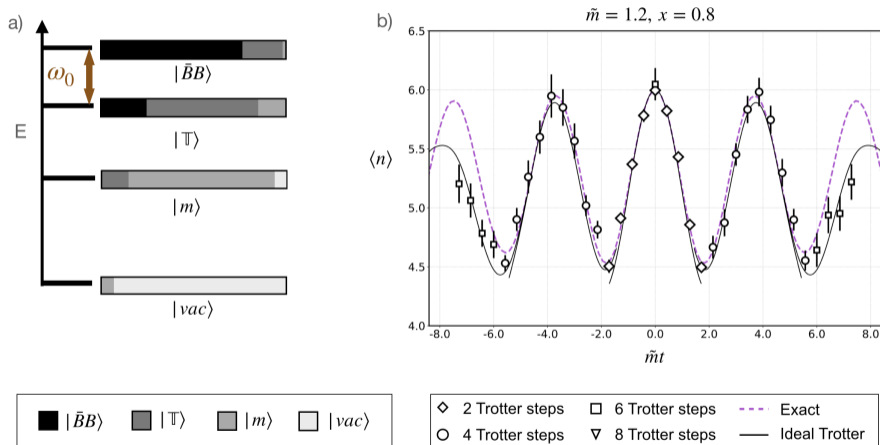
Mesons and tetraquarks have electric flux,  $F$





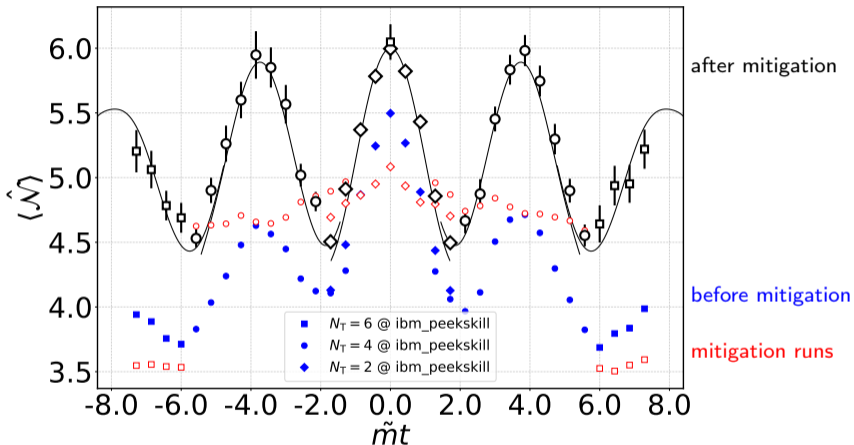
## Time evolution at intermediate coupling

Oscillations among the eigenstates were computed on `ibm_peekskill`.



This study required the mitigation of hardware errors.

## Practical error mitigation was crucial



The mitigation method was developed with three students at York University:

A Rahman, Lewis, Mendicelli, Powell, Phys.Rev.D 106, 074502 (2022).

## Conclusions

- Lattice NRQCD (by Lepage et al) is the foundation for  $T_{bb}$  and  $T_{bc}$  studies.
- There is clearly a bound  $T_{bb}$  with  $ud\bar{b}\bar{b}$  and with  $us\bar{b}\bar{b}$ .  
The systematic errors are being determined through multiple lattice studies.
- $T_{bc}$  does not yet have a consensus among lattice groups.  
Several studies exist and real progress has been made.
- $T_{cc}$  is a challenge for lattice QCD because binding energy  $\sim \frac{1}{4}$  MeV  $\ll \Lambda_{QCD}$ .  
Nevertheless, impressive efforts have been reported.
- Quantum computers have opened new opportunities for the coming decades.