Semileptonic B decays on the lattice

Judd Harrison,

Cornell, October 2024

Background

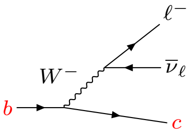
In the Standard Model (SM), bottom quarks decay to lighter c, s, u and d quarks via two types of weak interaction:

Flavour-Changing Charged Currents:

- $b \to cW^-$, $b \to uW^-$ - tree-level, decay rates related to Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, V_{cb}, V_{ub}
- virtual W^- can decay to leptons $\ell\bar{\nu}$, with $\ell = \mu, e$ or heavy τ
- test of equal weak couplings for $\ell = \mu, e, \tau$
- $|V_{cb}|$ and $|V_{ub}|$, tests of CKM unitarity

e.g. $b \to c\, \ell^- \bar{\nu}_\ell$, $\quad \ell = e,\ \mu$ or τ

$$\sim \frac{4}{\sqrt{2}} G_F V_{cb} \bar{c}_L \gamma_\mu b_L \bar{\ell}_L \gamma^\mu \nu_L ,$$

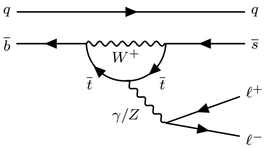


Meson decays: $B \to \pi\ell\bar{\nu}$, $B \to \rho\ell\bar{\nu}$, $B_s \to K^{(*)}\ell\bar{\nu}$, $B_{(s)} \to D_{(s)}^{(*)}\ell\bar{\nu}$, $B_c \to J/\psi\ell\bar{\nu}$

Flavour-Changing Neutral Currents:

- 1-loop SM amplitudes
- small amplitudes mean new physics effects may stand out more clearly
- sensitivity to new physics effects coupling to top quark in the loop

e.g. "Penguin" diagrams, $\sim V_{tb}V_{ts}^*$



Meson decays: $B \to K^{(*)}\bar{\ell}\ell$, $B_s \to \phi\bar{\ell}\ell$, $B_c \to D_s^{(*)}\bar{\ell}\ell$

b quarks on the lattice

In lattice QCD, the full theory of quantum chromodynamics is discretised and solved numerically

$$\int dx^4 \mathcal{L}_{\rm QCD}(\bar\psi(x), \psi(x), A_\mu(x))$$
$$\downarrow$$
$$\sum_n a^4 \mathcal{L}^{\rm Lattice}_{\rm QCD}(\bar\psi(x_n), \psi(x_n), U_\mu(x_n)) \rightarrow$$

$$\xrightarrow[a\to 0]{\lim}$$ continuum theory predictions

To avoid large discretisation effects, we require $am_q << 1$, but we also require $m_\pi L >> 1$ to avoid finite-volume effects.

- b quarks require very small lattice spacings, and a very large number of lattice points, $\sim (L/a)^4 \Rightarrow$ computationally expensive

One option: use effective action e.g. nonrelativistic QCD (NRQCD), Thacker+Lepage, Phys Rev D43 (1991) 196

- initial-value problem, numerical solution is very fast and efficient
- but need to match action and currents perturbatively $\rightarrow$ systematic $\sim \mathcal{O}(\frac{\alpha_s \Lambda_{\rm QCD}}{m_b}) \approx 5\%$ uncertainties

b quarks on the lattice

As the available computing power has increased, so have the masses of particles which can be accurately included in lattice QCD calculations.

A second option: use fully relativistic heavy quarks with masses $m_h < m_b$ and extrapolate up to the physical b quark (Shigemitsu, Davies, Follana, Gámiz, Gregory, Lepage, Na, Wingate, 2009)

- Highly Improved Staggered Quark (HISQ) action (Follana, Mason, Davies, Hornbostel, Lepage, Shigemitsu, Trottier, Wong, 2006) ideal for this
  - $\rightarrow$ staggered quarks are very numerically efficient, enabling high statistics
  - $\rightarrow$ high level of improvement leads to greatly reduced discretisation effects, particularly those due to the heavy quark mass
- some new challenges:
  - $\rightarrow$ disentangling $(am_h)^2$ effects from physical dependence on $\Lambda_{\rm QCD}/m_h$ can be tricky
  - $\rightarrow$ use of multiple values of am_h on each ensemble makes correlator fits more challenging
- but many benefits:
  - $\rightarrow$ use of HISQ for all valence quarks enables precise nonperturbative renormalisation of lattice currents, eliminating dominant systematic uncertainty
  - $\rightarrow$ statistics limited, and therefore systematically improvable
  - $\rightarrow$ modern MILC ensembles have light, strange and charm sea quarks
  - $\rightarrow$ access to higher momenta $\rightarrow$ good coverage of full kinematic range of decays

This approach to semileptonic B decays has been enormously fruitful in recent years

Semileptonic B decays

Matrix elements, parameterised using form factors, can be extraced from multi-exponential fits to 2-point and 3-point correlation functions. E.g. for $B \to D^*$:

$$\langle \hat{\mathcal{O}}_{D^*}(t)\hat{\mathcal{O}}^\dagger_{D^*}(0)\rangle \to \sum_{n}^{N_{\rm exp}} |A_n^{D^*}|^2 e^{-E_n^{D^*}t} + ...,$$

$$\langle \hat{\mathcal{O}}_{B}(t)\hat{\mathcal{O}}^\dagger_{B}(0)\rangle \to \sum_{n}^{N_{\rm exp}} |A_n^{B}|^2 e^{-E_n^{B}t} + ...,$$

$$\langle \hat{\mathcal{O}}_{D^*}(T)\hat{\mathcal{J}}(t)\hat{\mathcal{O}}^\dagger_{B}(0)\rangle \to \sum_{n,m}^{N_{\rm exp}} A_n^{D^*} A_m^{B\dagger} V_{nm} e^{-tE_m^B-(T-t)E_n^{D^*}} + ...$$

Ground state matrix element, $V_{00} \sim \langle D^*|\hat{\mathcal{J}}|B\rangle$, can be related to form factors and fed into physical-continuum extrapolation and subsequent phenomenology analyses.

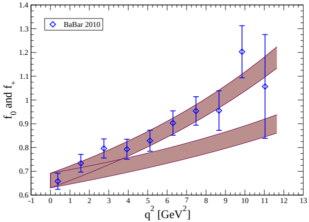
Peter Lepage's freely available **gvar**, **lsqfit** and **corrfitter** Python packages have been absolutely vital to a large number of lattice QCD calculations, providing (among many things):

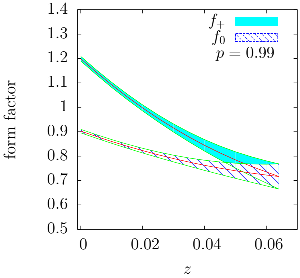
- calculation of means and covariances, error propagation, SVD analysis methods, resampling procedures
- constrained nonlinear least-squares fitting procedures with implementations of a large number of different minimisation algorithms
- Lattice QCD-specific fit functions, GEVP methods

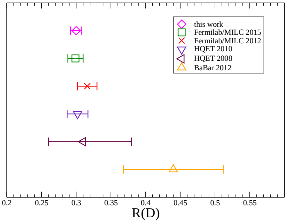
Results for flavour-changing charged current decays

$B \to D\ell\nu$

Left: $B \to D$ f_0 and f_+ form factors using NRQCD (Na, Bouchard, Lepage, Monahan, Shigemitsu, 2015) + BaBar 2010 data. Right: $B \to D$ form factors from Fermilab-MILC collaboration using improved Wilson fermions.



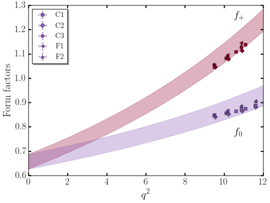


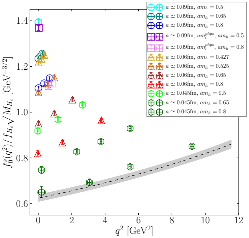


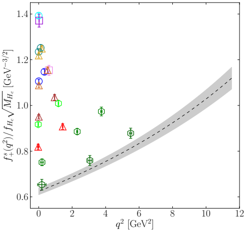
(Na, Bouchard, Lepage, Monahan, Shigemitsu, 2015)

$B_s \to D_s \ell \bar{\nu}$

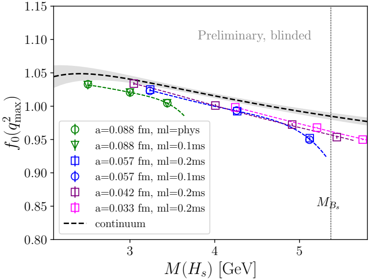
Left: $B_s \to D_s$ (Monahan, Na, Bouchard, Lepage, Shigemitsu, 2017). Middle and right: Lattice data for $B_s \to D_s$ form factors f_0 and f_+. (McLean, Davies, Koponen, Lytle, 2019)







Ongoing work by Fermilab-MILC collaboration using HISQ b quarks (2403.03959)



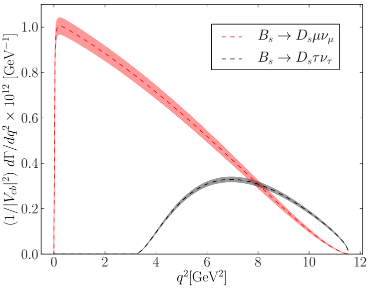
$B_s \to D_s \ell \bar{\nu}$

HPQCD continuum results for $B_s \to D_s \ell \bar{\nu}$ branching fraction computed from for $\ell = \mu, \tau$, giving

$$R(D_s) = \frac{\mathcal{B}(B_s \to D_s \tau \bar{\nu})}{\mathcal{B}(B_s \to D_s \ell \bar{\nu})} = 0.2987(46).$$

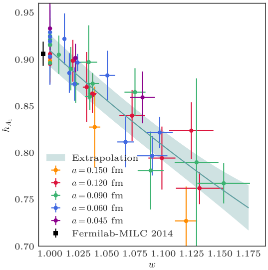
Recently used by LHCb to provide first determination of V_{cb} using $B_s \to D_s$ (2001.03225):

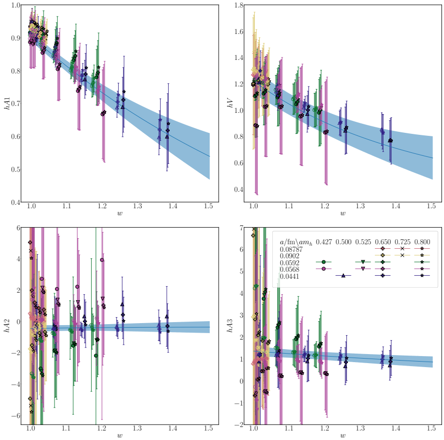
$$V_{cb}^{B_s \to D_s} = (42.3 \pm 0.8 \pm 0.9 \pm 1.2) \times 10^{-3}$$

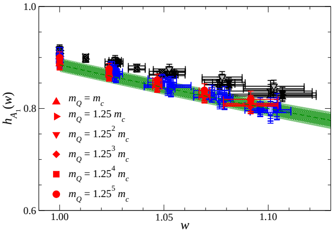


$B \to D^* \ell \nu$

Top left: $B \to D^*$ form factor h_{A_1} from Fermilab-MILC using Wilson clover action for b (Bazavov et al. 2022). Right: $B \to D^*$ vector and axial vector form factors computed using HISQ b (Harrison, Davies, 2024). Bottom left: $B \to D^*$ form factor h_{A_1} from JLQCD using Möbius domain wall fermions (Aoki et al. 2024).

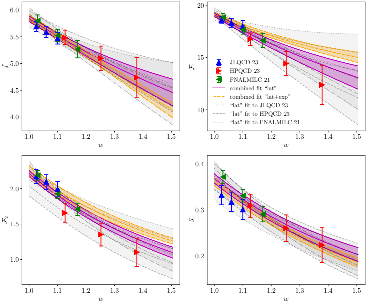


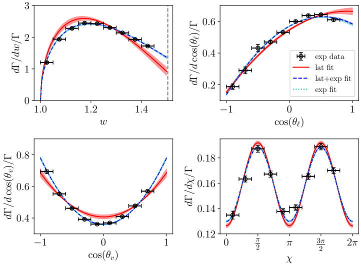




$B \to D^* \ell \bar{\nu}$

Continuum $B \to D^*$ form factors from combined BGL fit to lattice data, together with normalised differential rates using HPQCD and Fermilab-MILC results (Bordone, Jüttner, 2024)

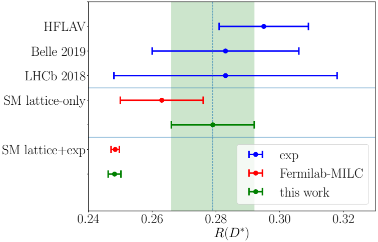




Statistically acceptable combined fit, gives

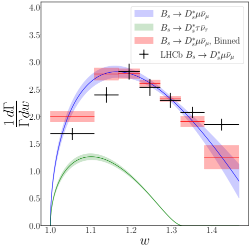
$$V_{cb} = 0.04025(71).$$

Lattice-only $R(D^*) = \frac{\mathcal{B}(B \to D^* \tau \bar{\nu})}{\mathcal{B}(B \to D^* \ell \bar{\nu})}$ consistent with experiment - but find tension in the normalised differential rate



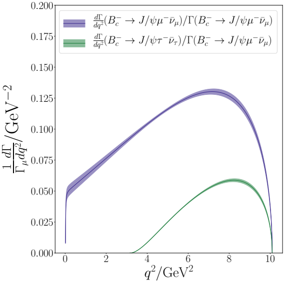
$B_c \to J/\psi \ell \bar{\nu}$ and $B_s \to D_s^*$

$B_s \to D_s^*$ results from HPQCD using HISQ b can also be compared to LHCb, which shows similar shape discrepancy in normalised differential rate.



HPQCD HISQ b results for $B_c \to J/\psi \ell \bar{\nu}$ normalised differential decay rate (Harrison, Davies, Lytle, 2020) for $\ell = \mu, \tau$, gives

$$R(J/\psi) = \frac{\mathcal{B}(B_c \to J/\psi \tau \bar{\nu})}{\mathcal{B}(B_c \to J/\psi \ell \bar{\nu})} = 0.2582(38).$$



LHCb found (1711.05623)

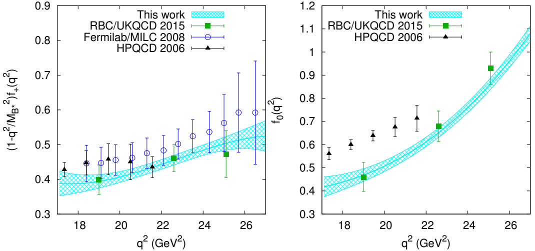
$$R(J/\psi) = 0.71 \pm 0.17_{\rm stat} 0.18_{\rm sys}$$

now using these results to improve ongoing updated analysis.

$B \to \pi \ell \nu$

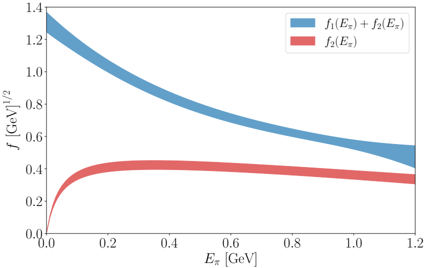
For $B \to \pi \ell \bar{\nu}$, different calculations showed some disagreement.

- Form factors from Fermilab-MILC collaboration (Bailey et al. 2015), HPQCD using NRQCD (Colquhoun, Dowdall, Koponen, Davies, Lepage, 2015), and RBC/UKQCD (Arthur et al. 2014).



Fully relativistic results for $B \to \pi$ also available from JLQCD using Möbius domain wall fermions (2203.04938).

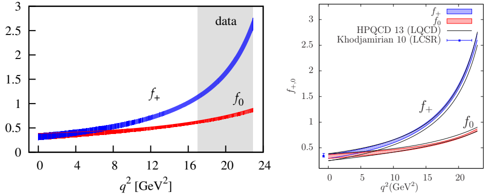
Ongoing calculations by HPQCD and Fermilab-MILC



Results for flavour-changing neutral current B-meson decays

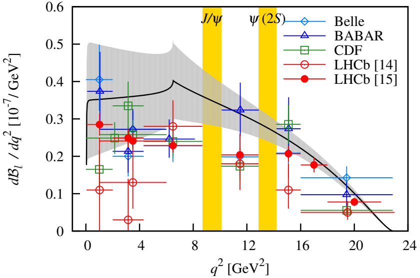
Rare $B \to K\bar{\ell}\ell$ with nonrelativistic b quarks

NRQCD predictions for $B \to K$ form factors were found to be consistent with later unquenched results from the Fermilab-MILC collaboration using the Fermilab clover action.

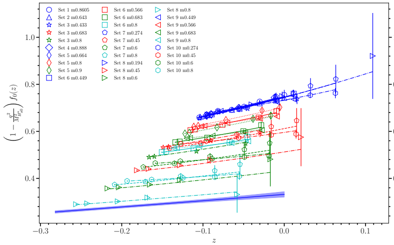


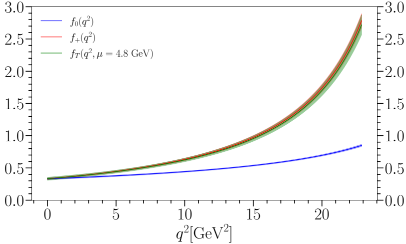
Left: $B \to K$ form factors from NRQCD (Bouchard, Lepage, Monahan, Na, Shigemitsu, 2014).
Right: form factors using Fermilab clover action (Bailey et al. 2016).

$B \to K\bar{\ell}\ell$ branching fraction with $\ell = e, \mu$, together with experimental data from LHCb, BABAR, CDF and Belle. (Bouchard, Lepage, Monahan, Na, Shigemitsu, 2014)



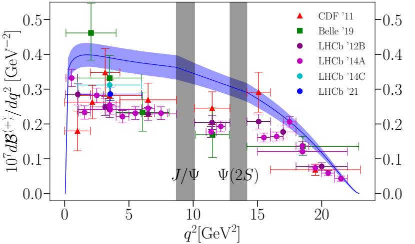
$B \to K\bar{\ell}\ell$ with HISQ b quarks





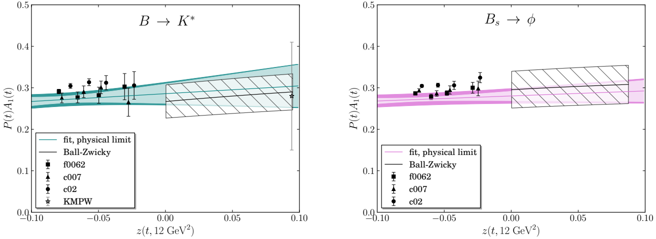
Left: lattice data for $B \to K$ form factor f_0 with the H_s^* pole removed, as a function of $z(q^2)$, at multiple a, am_h and m_h, together with extrapolated continuum form factor. Right: continuum results for f_0, f_+ and f_T form factors across the full kinematical range. (Parrott, Bouchard, Davies, 2023)

New predictions for $B^+ \to K^+\ell^+\ell^-$, with $\ell = e,\ \mu$, show clear tension at the level of 4.2σ with experimental results.

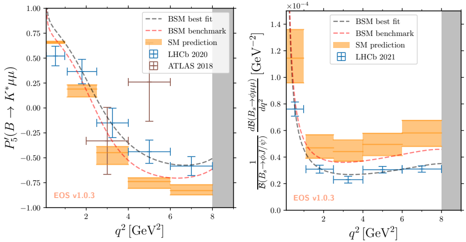


Rare $B \to K^* \bar{\ell}\ell$ and $B_s \to \phi \bar{\ell}\ell$ with nonrelativistic b quarks

NRQCD predictions for $B \to K^* \bar{\ell}\ell$ and $B_s \to \phi \bar{\ell}\ell$ form factors (Horgan, Liu, Meinel, Wingate, 2014) have remained key inputs to global $b \to s\bar{\ell}\ell$ new physics analyses for nearly

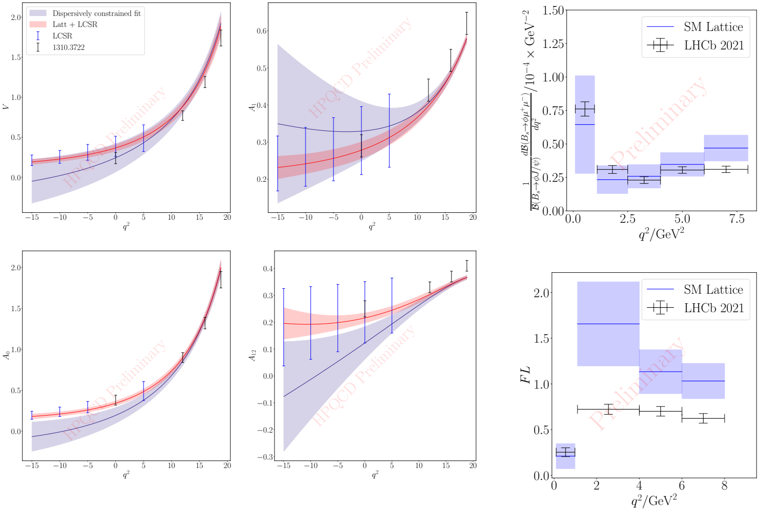


Predictions for $B \to K^* \bar{\ell}\ell$ and $B_s \to \phi \bar{\ell}\ell$ including NRQCD LQCD, dispersive bounds and LCSR, together with experimental data (Gubernari, Reboud, van Dyk, Virto, 2022).



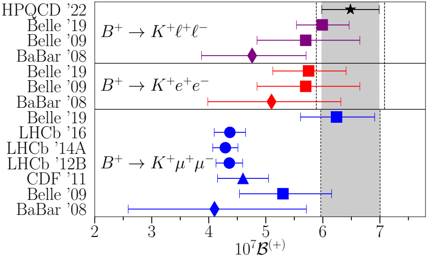
$B_s \to \phi\bar{\ell}\ell$ with HISQ b quarks (Preliminary)

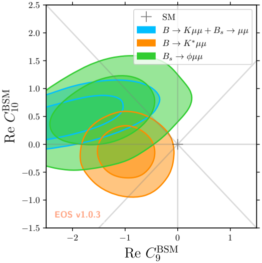
Fully relativistic $B_s \to \phi\bar{\ell}\ell$ well underway. preliminary results
show good agreement with older NRQCD results and light-cone sum-rules (2011.09813).

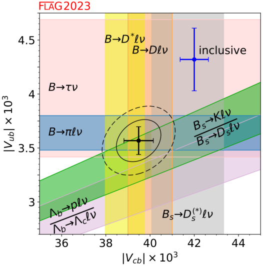


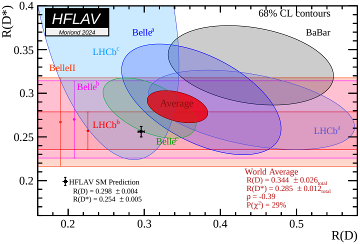
Outlook

- Persistent tensions with SM predictions for semileptonic B decays remain, e.g.





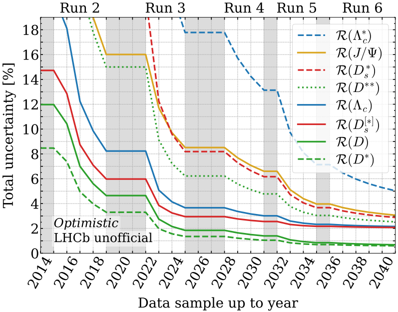


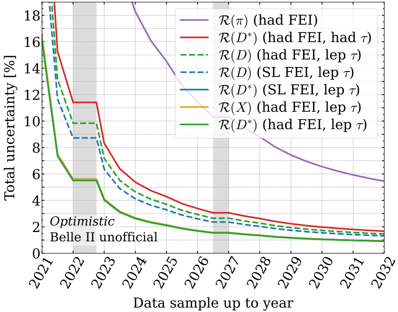


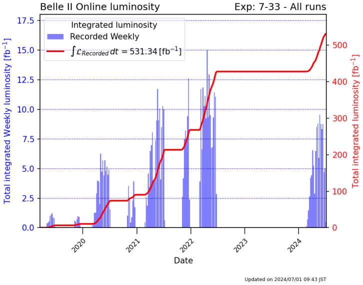
2207.13371, 2206.03797, FLAG2023, HFLAV 2024 Moriond update

Outlook

Excellent experimental prospects for the next decade, e.g. for ratios $R(X)$







Challenges for the lattice

Baryons

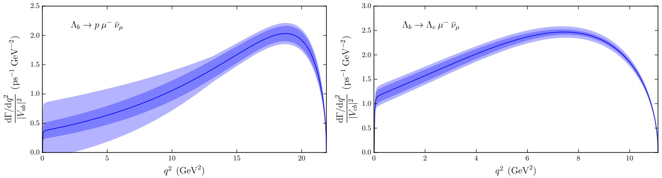
Baryonic decays can provide important tests of the SM complimentary to meson decays, e.g.

$$R(\Lambda_c) = \frac{\Gamma(\Lambda_b \to \Lambda_c \tau \bar{\nu})}{\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu})},$$

and $|V_{ub}|/|V_{cb}|$ from $\Lambda_b \to p \ell \bar{\nu}$ and $\Lambda_b \to \Lambda_c \ell \bar{\nu}$.

On the lattice, baryon decays are more challenging due - three quark states lead to many more Wick contractions, increased computational cost and worse statistics.

Despite this, excellent progress has been made using heavy-quark actions for the b and c (Detmold, Lehner, Meinel, 2015)



with significant updates in progress (2309.01821).

Calculations using HISQ present additional challenges, including constructing suitable in-flight staggered baryon operators.

Challenges for the lattice

Decays to multiple hadrons

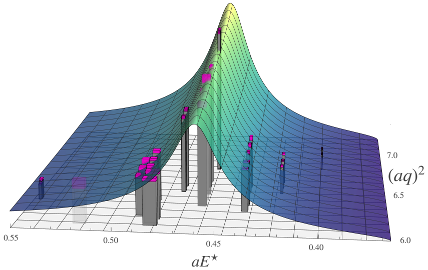
Decays such as $B \to K^* \bar{\ell}\ell$ involving final states which are not stable in QCD (e.g. $K^* \to K\pi$) should strictly be treated as $B \to K\pi\bar{\ell}\ell$ with $K\pi$ in P-wave close to the K^* resonance.

On the lattice, this requires the use of the Lellouch-Lüscher finite-volume formalism, which accounts for the interactions between the final state hadrons.

This requires careful extraction of the finite volume energy spectrum from lattice operators in irreducible representations subduced from continuum $J = 1$ and higher states. These are then used to determine the scattering phases for the 2-hadron final state.

Preliminary results for $B \to \pi\pi\ell\bar{\nu}$ from Leskovec et al. (2403.19543)

$$\langle \pi\pi, \epsilon(P,s)|V^\mu|B(p)\rangle = \frac{2iV(E^\star, q^2)}{m_B + 2m_\pi} \varepsilon^{\mu\nu\alpha\beta}\epsilon(P,s)^*_\nu P_\alpha p_\beta$$



Long-distance effects in rare decays

Tensions in global $b \to s\bar{\ell}\ell$ analyses now approaching 5σ (Gubernari, Reboud, van Dyk, Virto, 2022), with uncertainties from lattice QCD form factors still dominant.

However, the matrix elements for rare $b \to s\bar{\ell}\ell$ decays include non-local interactions of the form

$$\int dx^4 e^{iq\cdot x} \langle M_s | T\left\{ \hat{\bar{q}}\gamma_\mu \hat{q}(x) \hat{\mathcal{O}}_{\bar{q}q\bar{s}b}(0) \right\} |B\rangle .$$

Computing these non-local matrix elements on the lattice requires the evaluation of 4-point functions, which have a Euclidean spectral decomposition of the form

$$\begin{aligned} \mathcal{I}^{\text{Euclidean}} = & -\int_0^\infty dE \frac{\rho_1(E)}{2E} \frac{1 - e^{-T_a(E-E_B)}}{E_B - E} \langle K(p)|\hat{\bar{q}}\gamma_\mu \hat{q}(0)|E(k)\rangle \langle E(k)|\hat{\mathcal{O}}_{\bar{q}q\bar{s}b}(0)|B(k)\rangle \\ & + \int_0^\infty dE \frac{\rho_2(E)}{2E} \frac{1 - e^{-T_b(E-E_K)}}{E_B - E} \langle K(p)|\hat{\mathcal{O}}_{\bar{q}q\bar{s}b}(0)|E(p)\rangle \langle E(p)|\hat{\bar{q}}\gamma_\mu \hat{q}(0)|B(k)\rangle . \end{aligned}$$

The limit $T_b \to \infty$ is safe, but several states, such as KJ/ψ, with $E < E_B$ give rise to divergences in limit $T_a \to \infty$.

These must be carefully removed, either by explicit calculation or fitting to the T_a dependence of 4-point functions as for $K \to \pi$ (1608.07585 Christ, Feng, Jüttner, Lawson, Portelli, Sachrajda)

Conclusions

- NRQCD and heavy-quark actions have enabled a wide range of predictions for B semileptonic decays, with many hints of new physics
- Fully relativistic methods such as heavy-HISQ have confirmed, and in many cases strengthened, tensions between SM and experiment
- LHCb and Belle II will greatly reduce uncertainties and measure many new observables over the next decade
- SM predictions for local hadronic form factors will be systematically improved in the near future
- However, still lots of work to do to understand rare decays, decays to resonances, baryonic decays

Stay tuned, and thanks for listening!