Some highlights of heavy quark physics from lattice QCD

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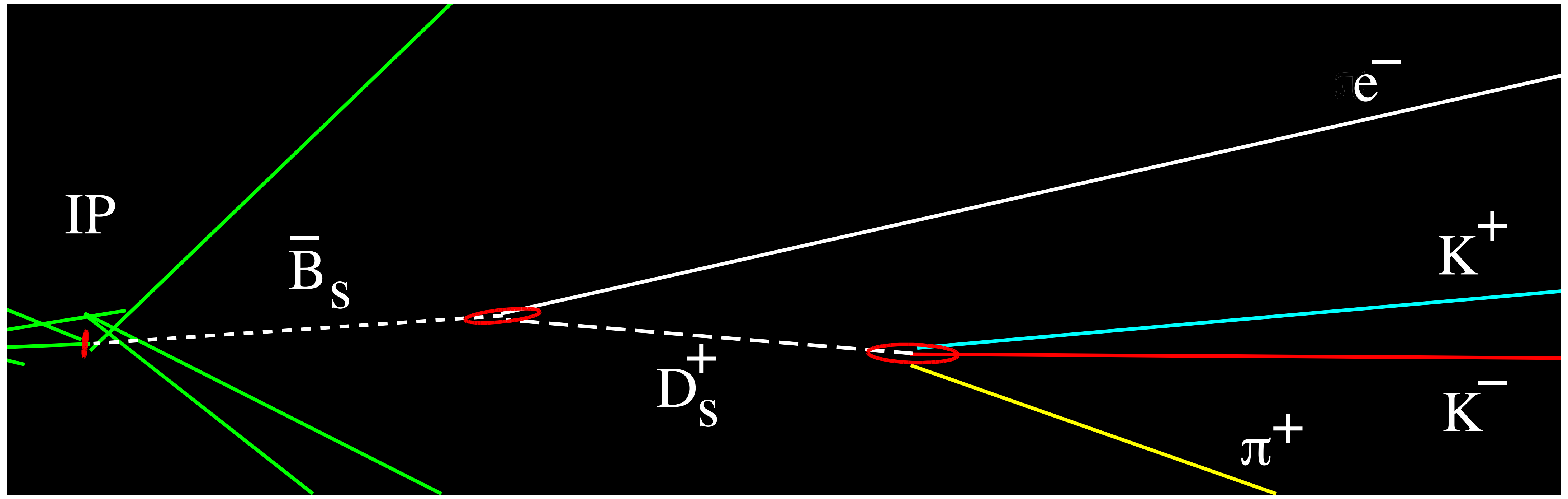
Cornell, October 2024

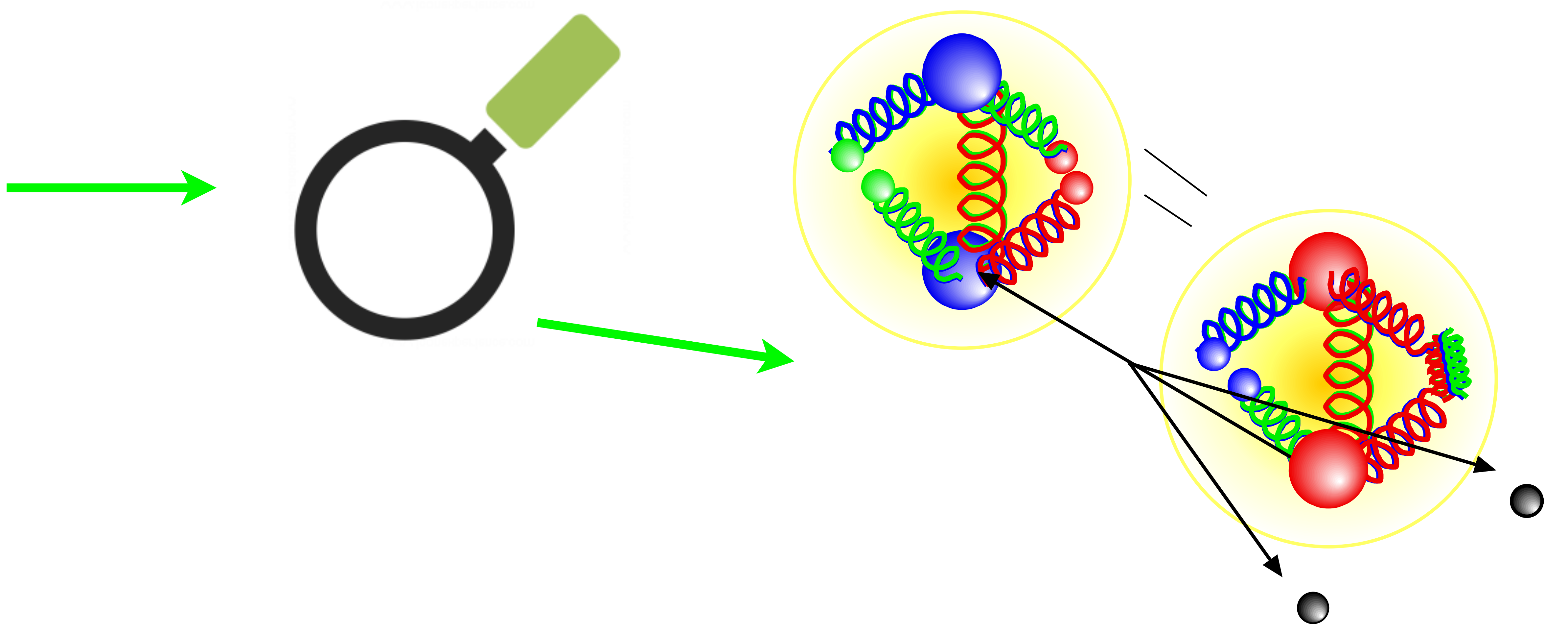
Importance of heavy quark physics and of lattice QCD

Heavy c, b quarks are ‘copies’ of u, d, s but expand hugely the range and variety of physics phenomena that allow tests of the strong interaction/ Standard Model and searches for new physics.

Lattice QCD is a ‘first principles’ approach to QCD that is now producing the ‘go-to’ accurate results for masses and matrix elements for c and b physics



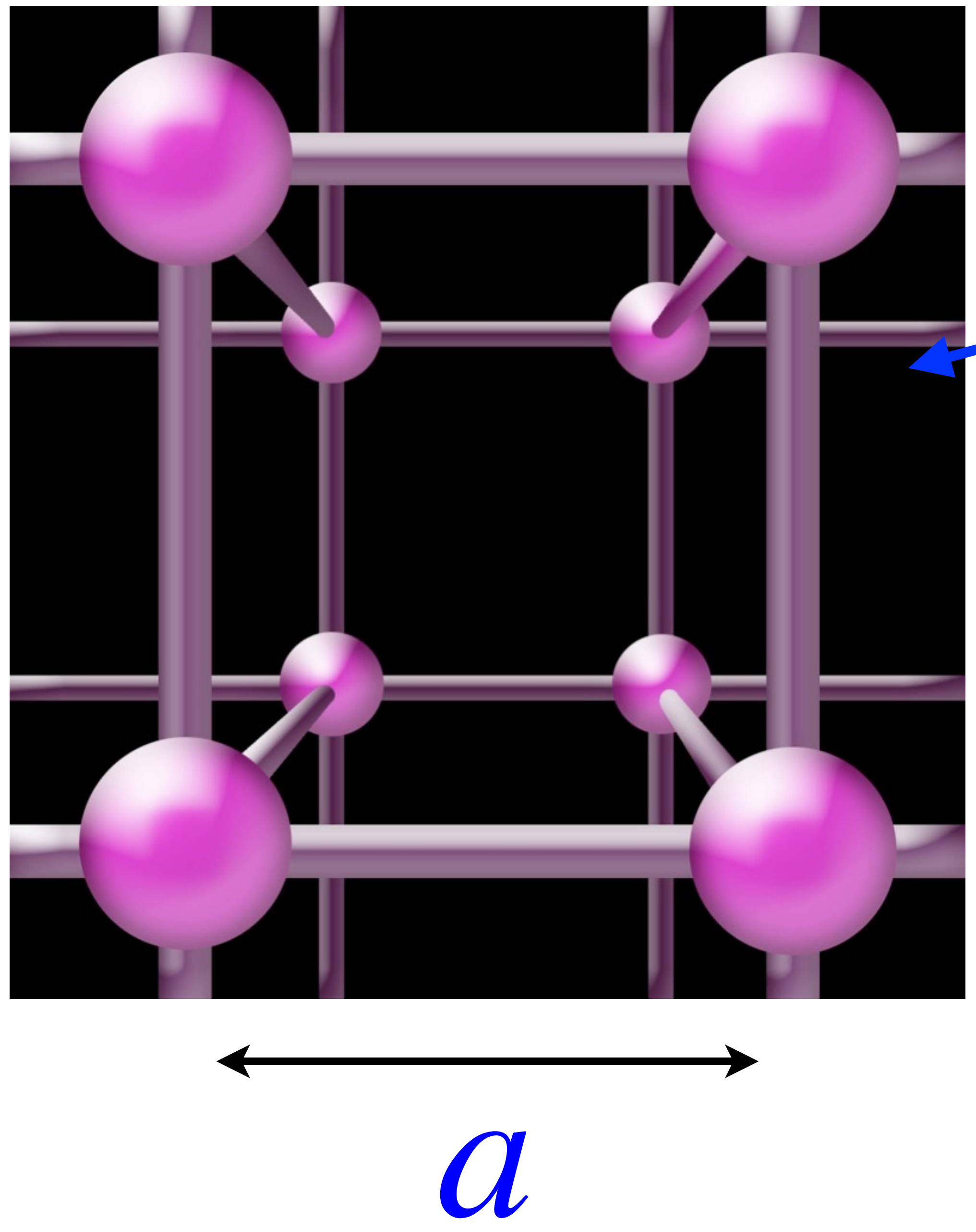
ALEPH@LEP



But it has taken a long time to get here …..

Lattice QCD provides an ‘ab initio’ approach to QCD.

15th October!



Determine Feynman Path Integral on a Euclidean space-time lattice.

$$\langle \mathcal{C} \rangle = \frac{1}{Z}\int \mathcal{D}U\, \mathcal{C}[U, M^{-1}]\, \underbrace{\det M\, e^{-(\int \frac{1}{2}\mathrm{Tr}F_{\mu\nu}^2 d^4x)}}$$

Integral over gluon field configurations U(x,y,z,t) = average over samples

a=0.1fm, N=50^4 lattice, gives multi-million dimensional integral

‘Measure’ hadron correlation functions on the configs. by combining quark propagators

Generate gluon field configurations with this probability distribution using Markov chain Monte Carlo

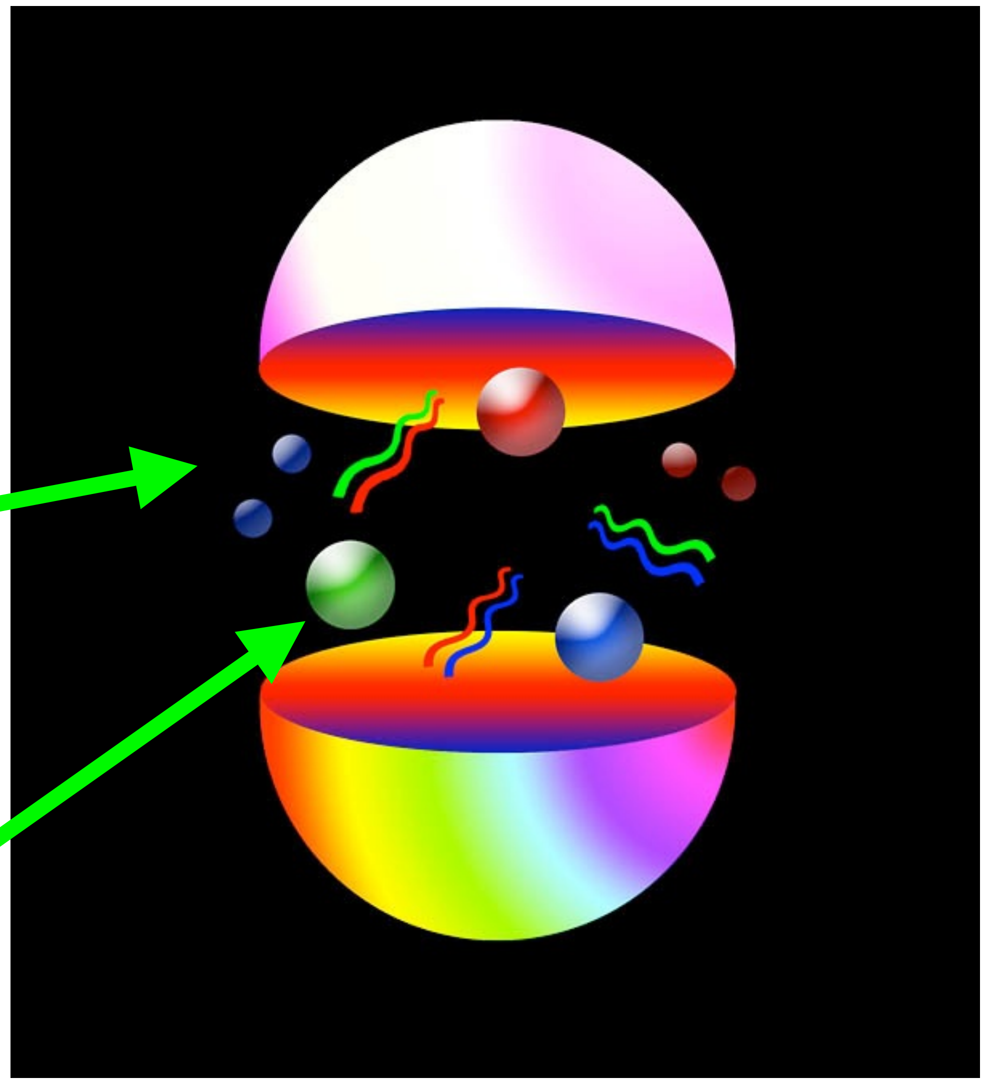
Quark fields are not explicit - appear through factors of the Dirac matrix, M

$$M = \gamma \cdot D + m_q$$

12Nx12N sparse matrix, must calculate det(M) and M^{-1}

det M gives effect of sea quarks in gluon field configurations

M^{-1} gives valence quark ‘propagators’ on the gluon field configurations



Lattice QCD = multi-step procedure

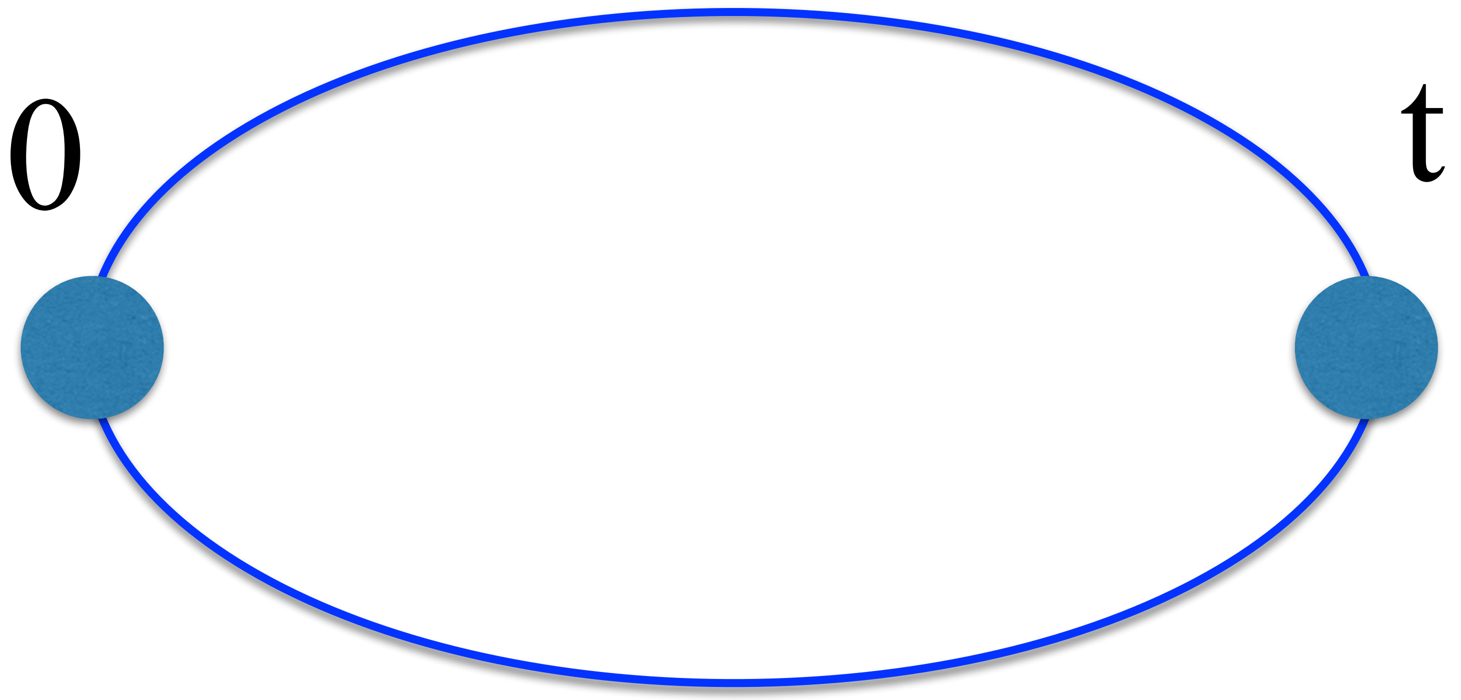
1) Generate sets of gluon fields (including effect of u, d, s, (c) sea quarks)

2) Solve Dirac eq. for valence quark propagators and combine to make “hadron correlation functions” - average these results over the set of gluon fields for $\langle \mathcal{C} \rangle$

3) Fit* :

$$\langle \mathcal{C} \rangle = A_0^2 e^{-E_0 t} + A_1^2 e^{-E_1 t} + \ldots$$

Amplitude, Energy/mass of ground-state
in units of the lattice spacing



4) Determine a and fix m_q for each quark using calibration hadron masses.

Repeat on sets with multiple a and extrapolate results in physical units to a=0.

Final accuracy depends on :

- statistical accuracy i.e. number of gluon field configurations
- control of lattice spacing dependence/ how well quark masses are tuned
- normalisation of operators (for decay amplitudes)

*github.com/gplepage/corrfitter

Late 1980s : introduction of lattice NRQCD

Thacker+Lepage, Phys Rev D43 (1991) 196

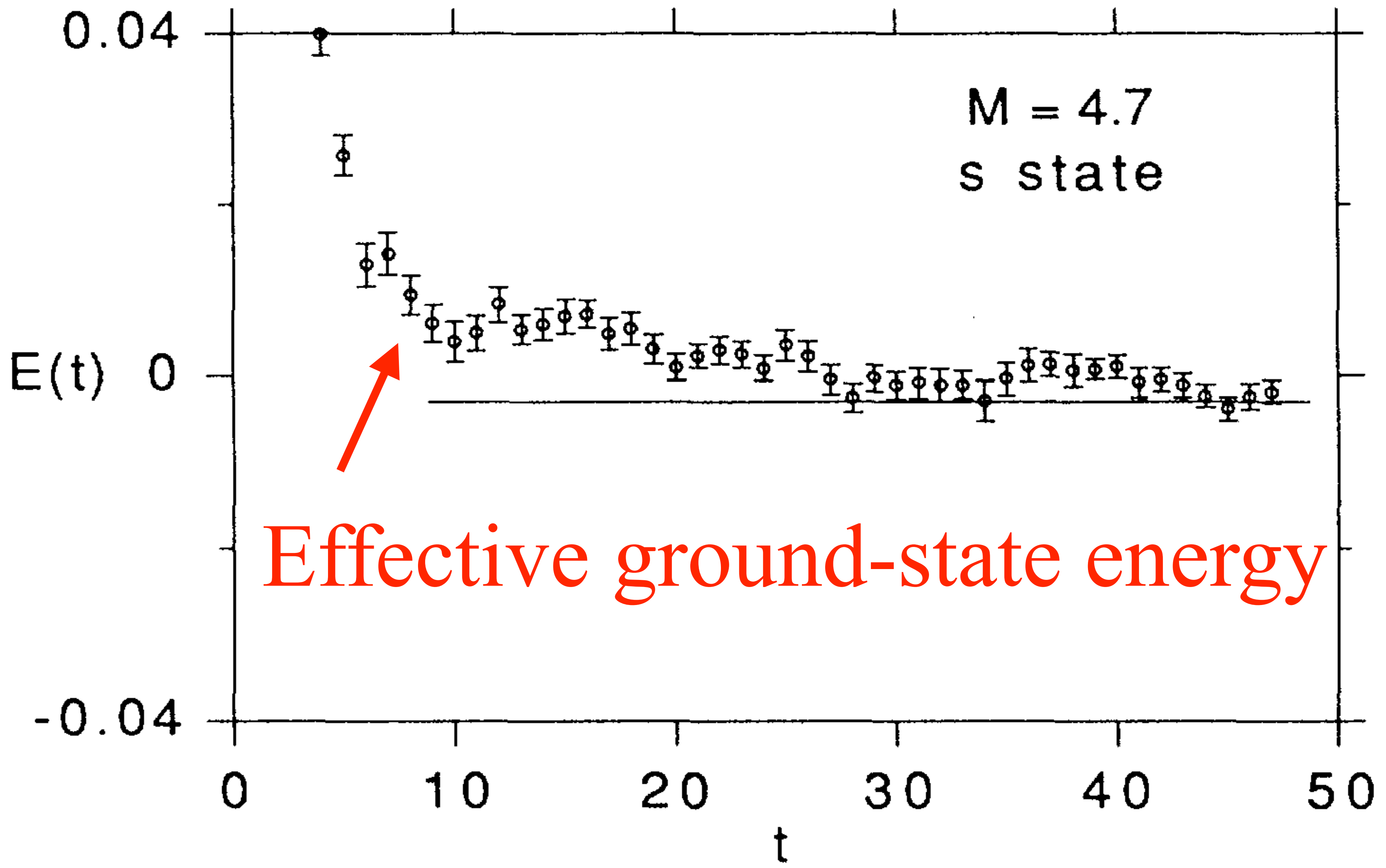
Heavy b, c are nonrelativistic in bound states

$$\mathcal{L}_{\mathrm{NRQCD}} = -\tfrac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \psi^\dagger\left(iD_t + \frac{\mathbf{D}^2}{2M}\right)\psi$$
$$+\psi^\dagger\left(c_1\frac{\mathbf{D}^4}{8M^3} + c_2\frac{g}{2M}\boldsymbol{\sigma}\cdot\mathbf{B}\right)\psi + \psi^\dagger\left(c_3\frac{g}{8M^2}\nabla\cdot\mathbf{E} + c_4\frac{ig}{8M^2}\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})\right)\psi$$
$$+\text{antiquark terms} + \text{quark-antiquark terms} + \cdots \; .$$

M=quark mass, leading mass term removed

Very simple fast propagator calculation on lattice:

$$G_{x,t+1} = U^\dagger_{x,t,\mu=4}\left(1 + \frac{\Delta^{(2)}}{2aM} + \ldots\right)G_{x,t} \longrightarrow$$



Effective ground-state energy

Small statistical errors for relatively low-cost calculation

Many narrow states in bottomonium and charmonium spectra so this allows good test of how well lattice QCD works if lattice systematics can be controlled.

no sea quarks

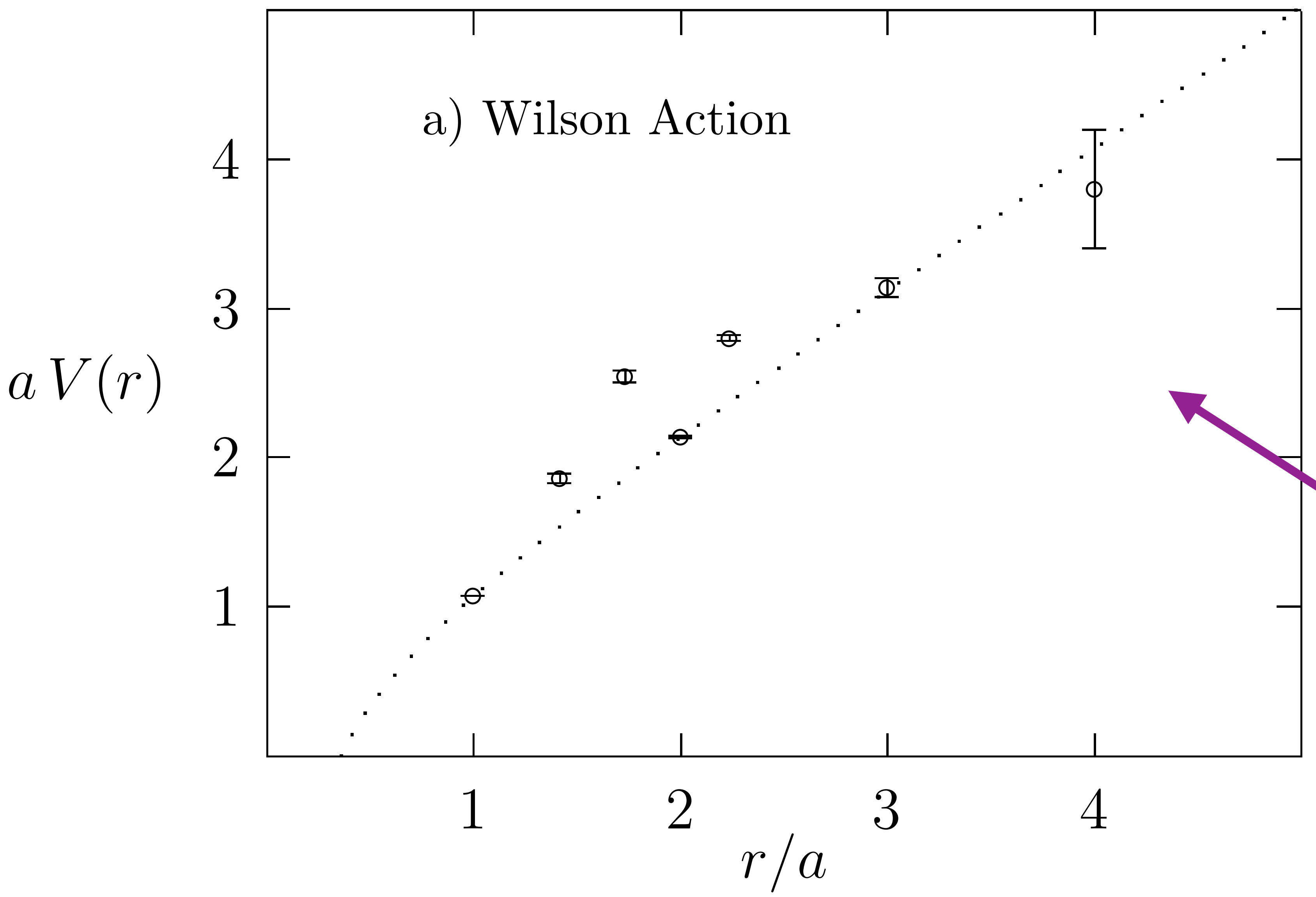
Quenched coarse lattice (beta=5.7, a=0.2 fm)),
no spin-dependence, no tuning of quark mass
1 hour per propagator on SUN workstation

1990s : improvements to lattice QCD and NRQCD

How to achieve accurate results with the computing power available?

Working on coarse lattices requires improving the lattice action with additional terms to remove discretisation effects at tree-level and BEYOND.
'Tadpole-improvement' was critical to doing this.

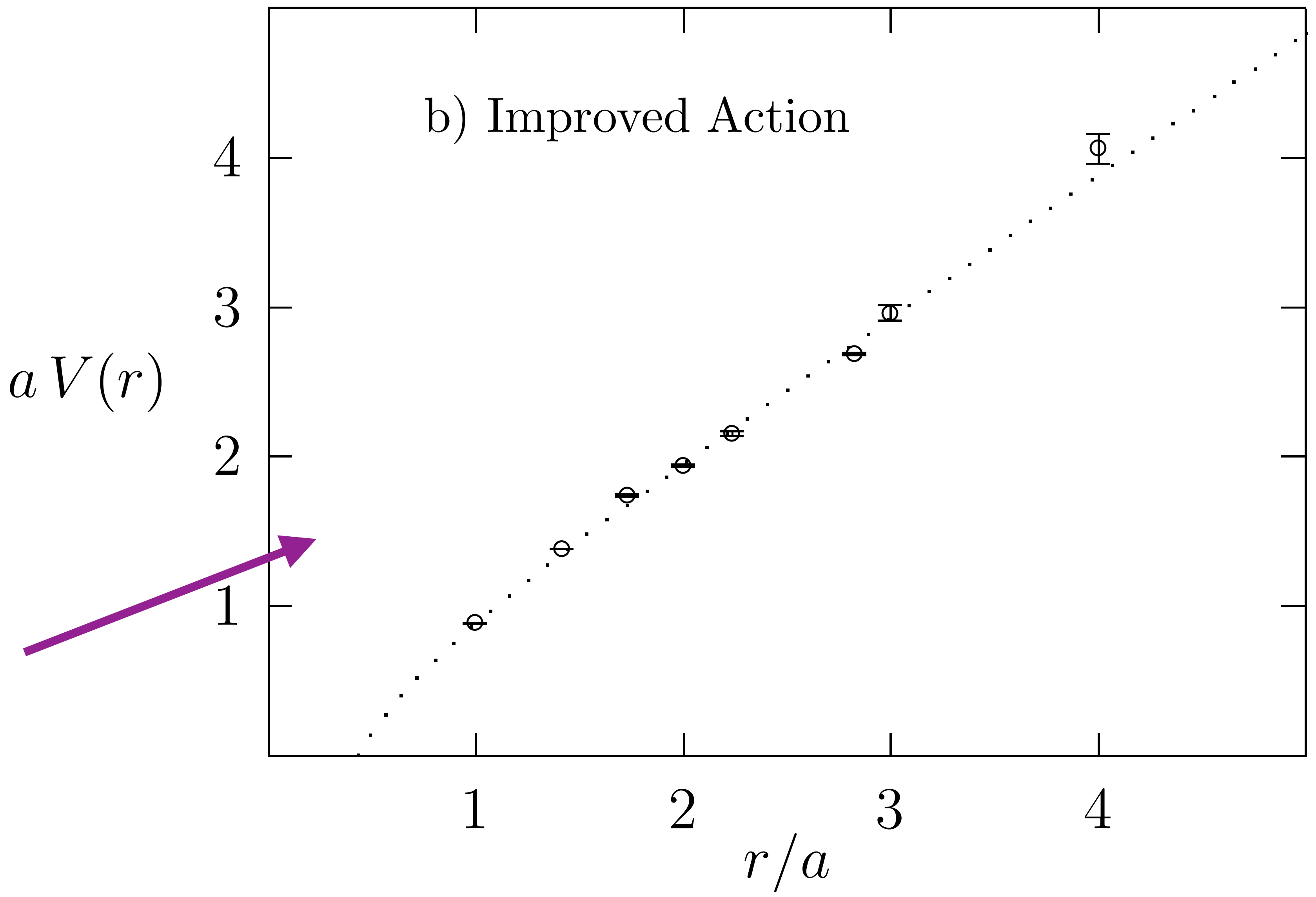
Lepage+Mackenzie, hep-lat/9209022; Alford, Dimm, Lepage, Hockney + Mackenzie, hep-lat/9507010



Pure gluon calculation of potential energy of 2 infinitely heavy quarks: 6^4 lattice with a ~0.4 fm

$O(a^2)$ errors

$O(\alpha_s^2 a^2)$ and $O(a^4)$ errors



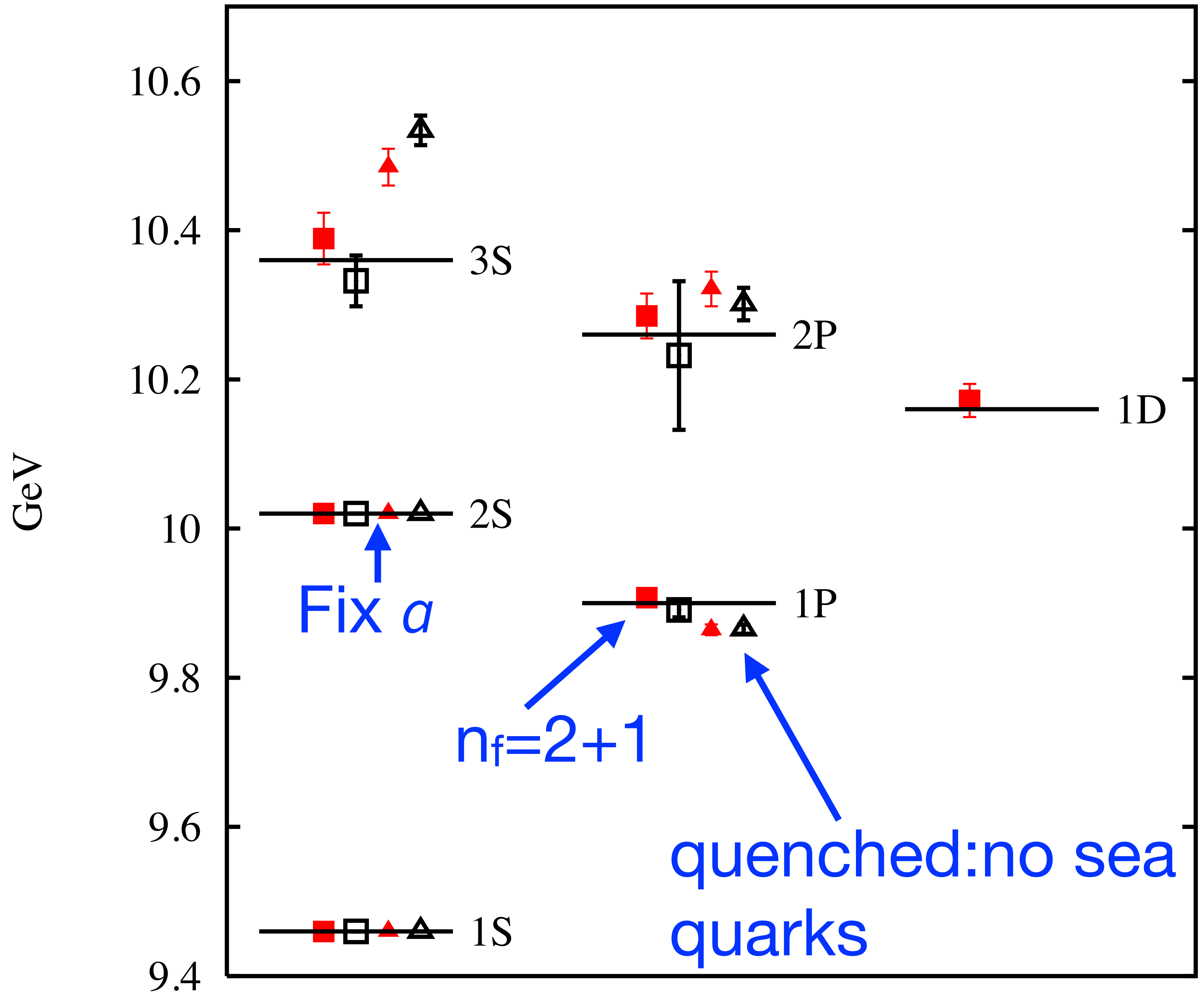
Many improvements to NRQCD: adding higher order relativistic corrections, removing discretisation effects, testing tadpole-improvement, perturbative matching and improvement etc.

e.g. Davies+Thacker, PRD45:915,PRD48,1329; Lepage et al, hep-lat/9205007; Catterall et al, 9211033,9311006; Morningstar, 9301005,9406002; Davies et al, 9406017,9802024; Shakespeare+Trottier, 9802038; Lewis+Woloshyn,9803004; etc. etc.
See also Fermilab approach to clover quarks, hep-lat/9604004

Early 2000s: Improved lattice NRQCD tests lattice QCD

Gray et al, HPQCD, hep-lat/0507013

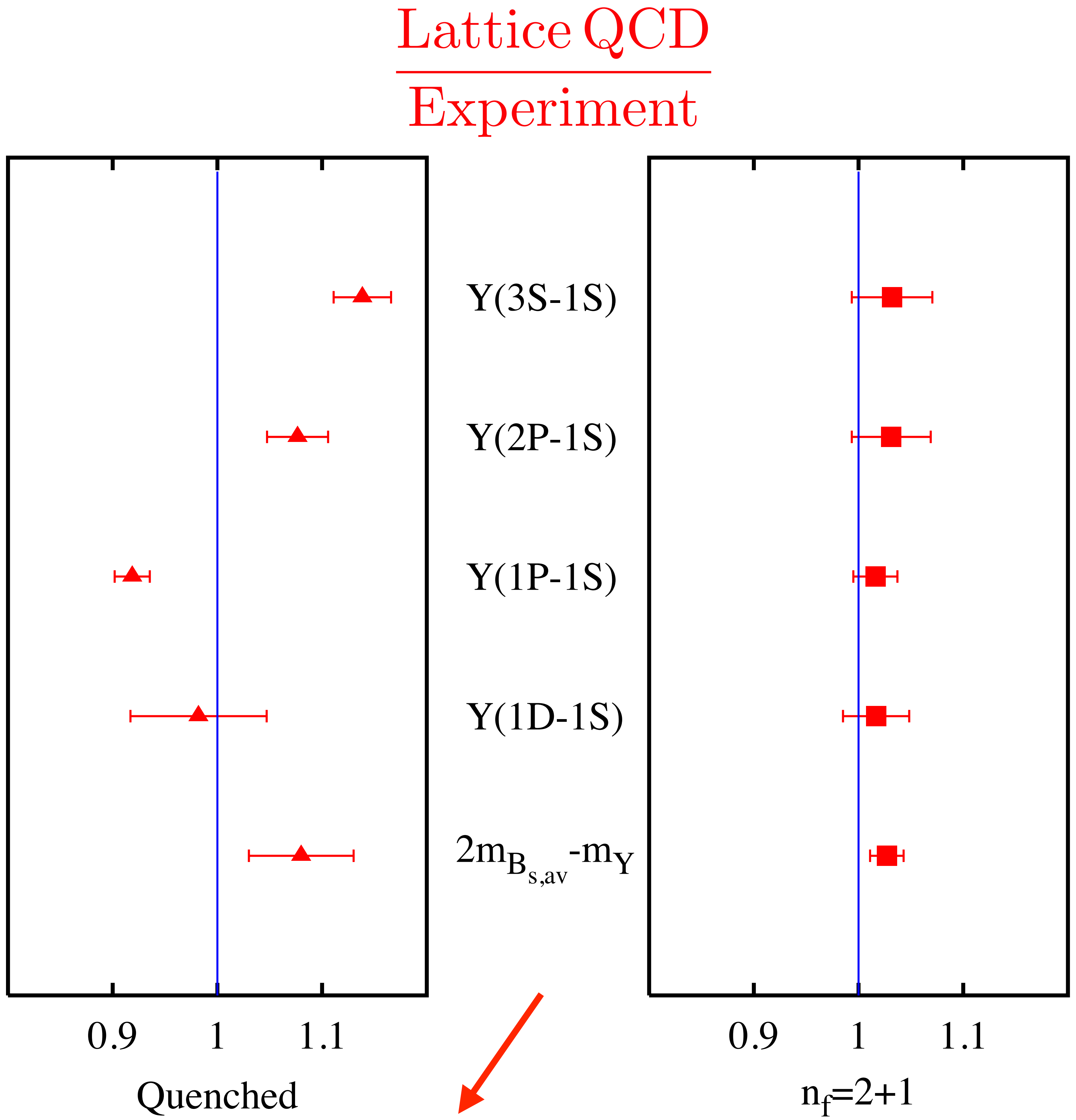
Work on MILC gluon fields with u, d, s, ‘asqtad’ staggered sea quarks with $m_u=m_d=m_l$ and m_l values from $m_l=m_s$ down to $m_l = m_s/5$



Inconsistencies in quenched approx. become clear

First demo. that lattice QCD ‘works’ when sea quarks are included.

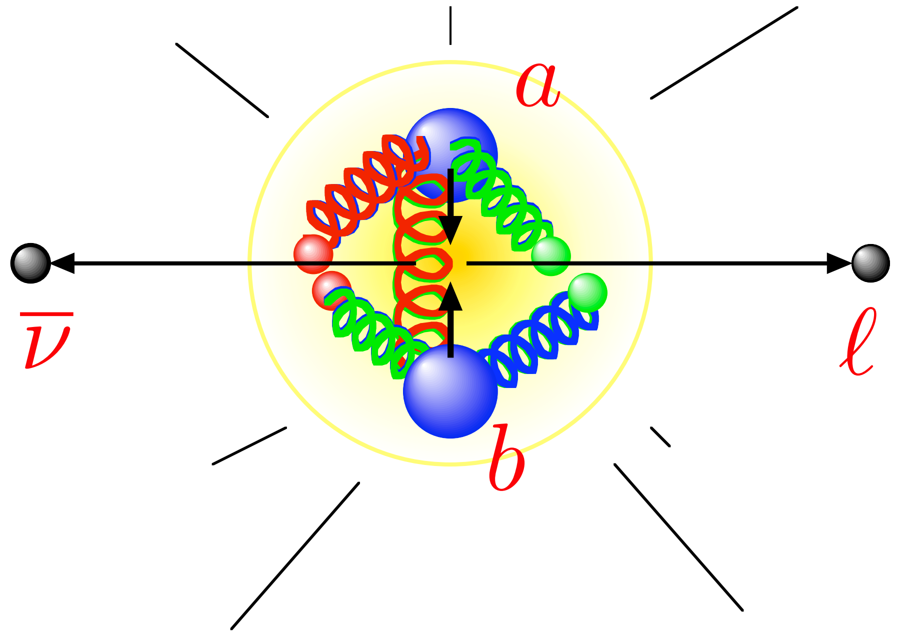
Consistent heavy-onium and heavy-light physics now possible



HPQCD/Fermilab/MILC, High-Precision Lattice QCD confronts experiment, hep-lat/0304004

Heavy-light physics with lattice NRQCD

Key target : weak annihilation amplitude of B and D mesons, parameterised by decay constant, f

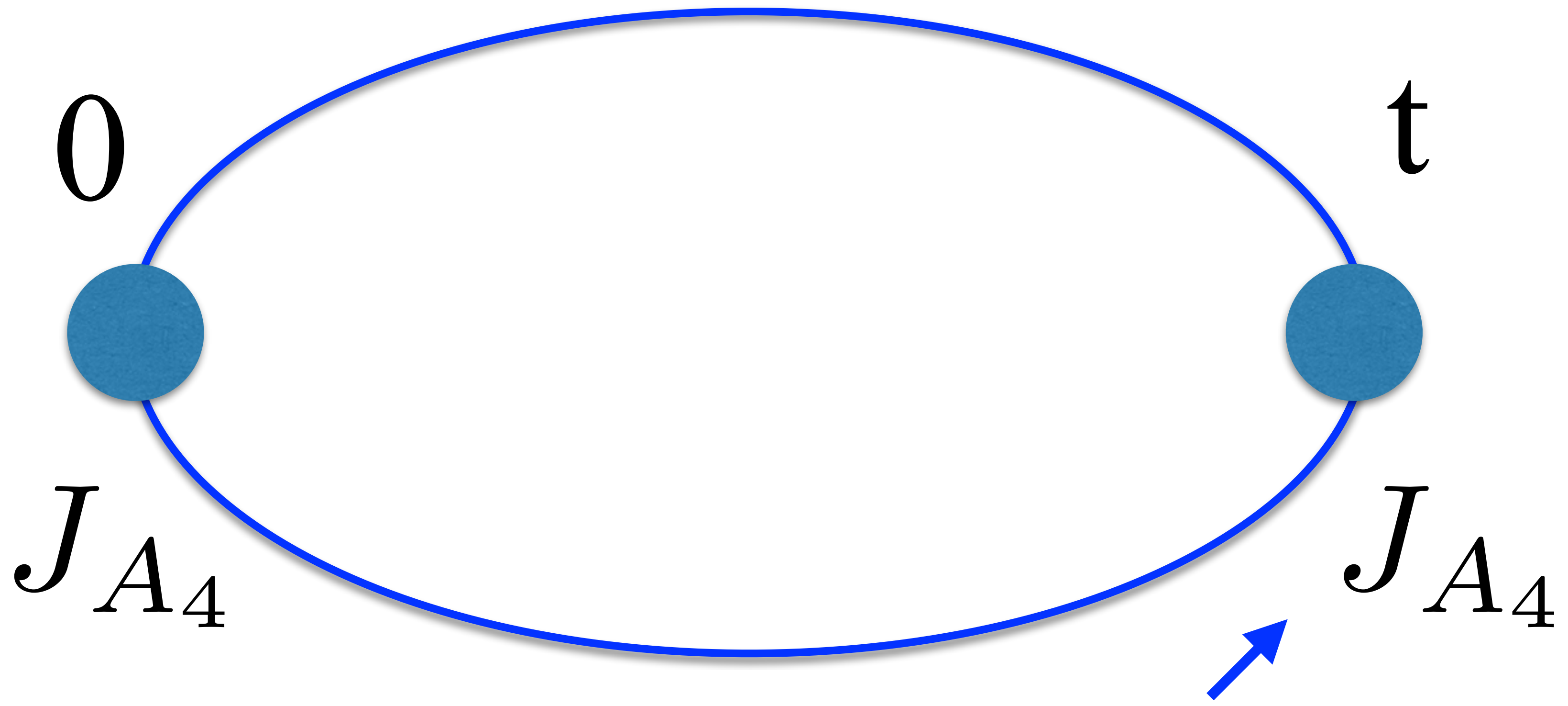


$$\Gamma(P \to \ell\overline{\nu}) \propto V_{ab}^2 f_P^2$$

Expt = CKM x theory(QCD)

$$A_0 = \frac{\langle 0|J_{A_4}|P\rangle}{\sqrt{2M_P}} = \frac{f_P\sqrt{M_P}}{\sqrt{2}}$$

fit amplitude



= weak axial current

1990s: combine NRQCD and light quark propagators on quenched glue using nonrelativistic expansion of weak axial current, with renormalisation calculated at O(α_s). Morningstar+Shigemitsu, hep-lat/9712016,9810047

A lot of work …

2000s: success of MILC unquenched glue

Combine NRQCD and staggered light quark propagators. Straightforward but controversial (!)

$$f_B = 216(9)(19)(4)(6)\ \mathrm{MeV}$$

Stat/fit

renorm

finite-a

rel. corrns

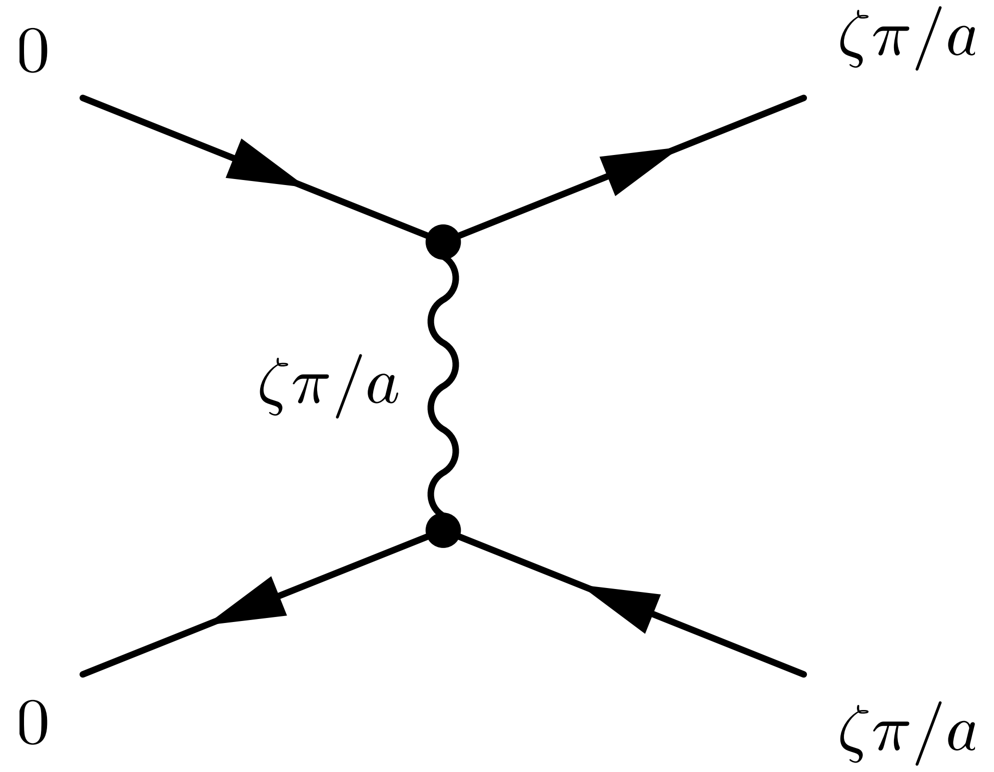
HPQCD: Wingate et al, hep-lat/0211014,0311130; Gray et al, 0507015

Improved staggered quarks - asqtad

Naive discretisation of Dirac equation : $\mathcal{S} = \sum_x \overline{\psi}(x)\,(\gamma\cdot\Delta(U) + m_0)\,\psi(x)$

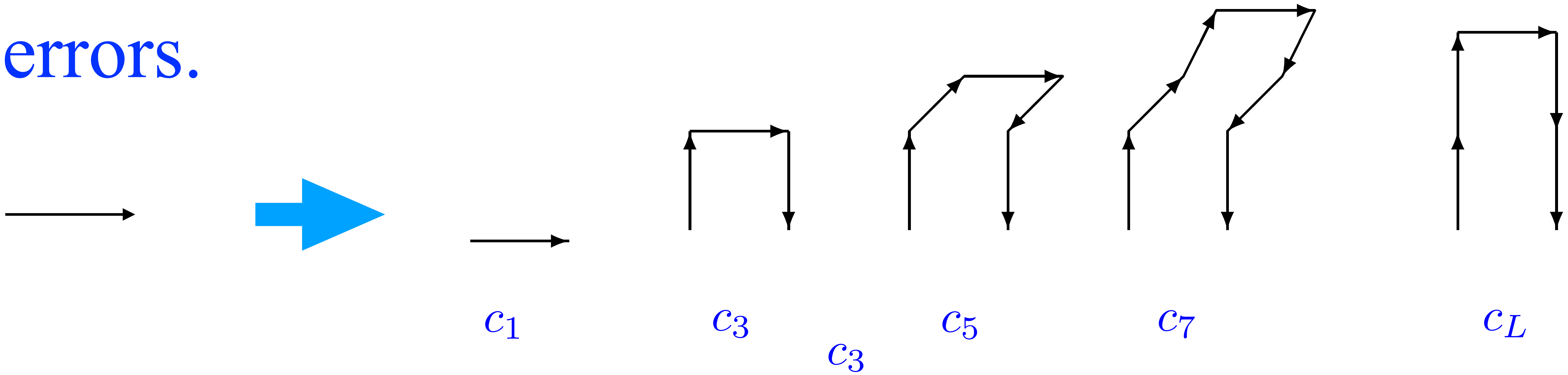
transformed simply to staggered quarks with 1 spin component.

Numerically v. efficient but a remnant of the 'doubling problem' remains in large a^2 errors from 'taste-exchange'.



Reduce by 'smearing' the gluon field in Δ(U); cuts quark coupling to π/a gluons but avoid adding a^2 errors.

Lepage, hep-lat/9809157, Orginos+Toussaint, 9903032



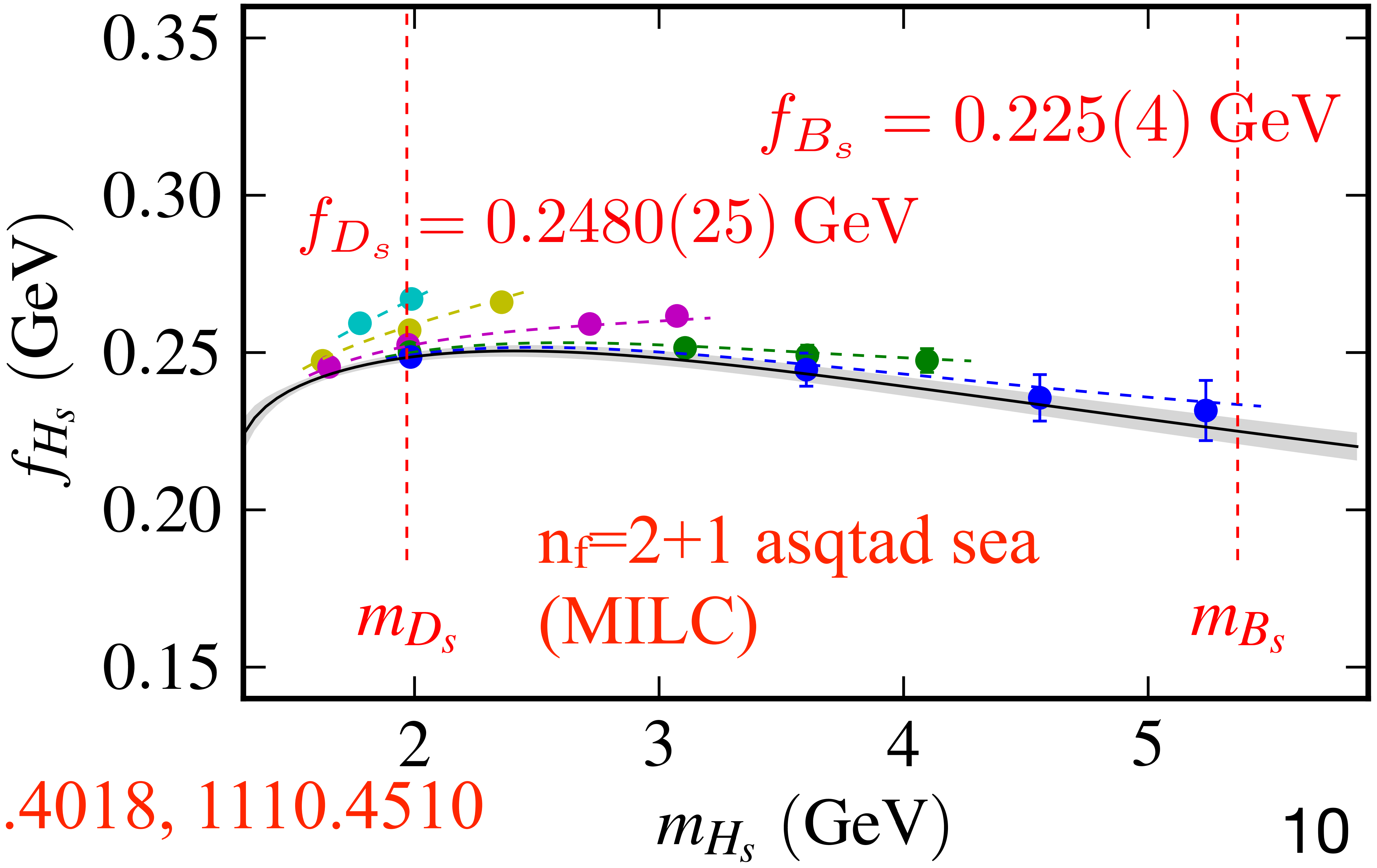
+ 3-link 'Naik' term to improve Δ

= 'asqtad' action

Improving improved staggered quarks - HISQ

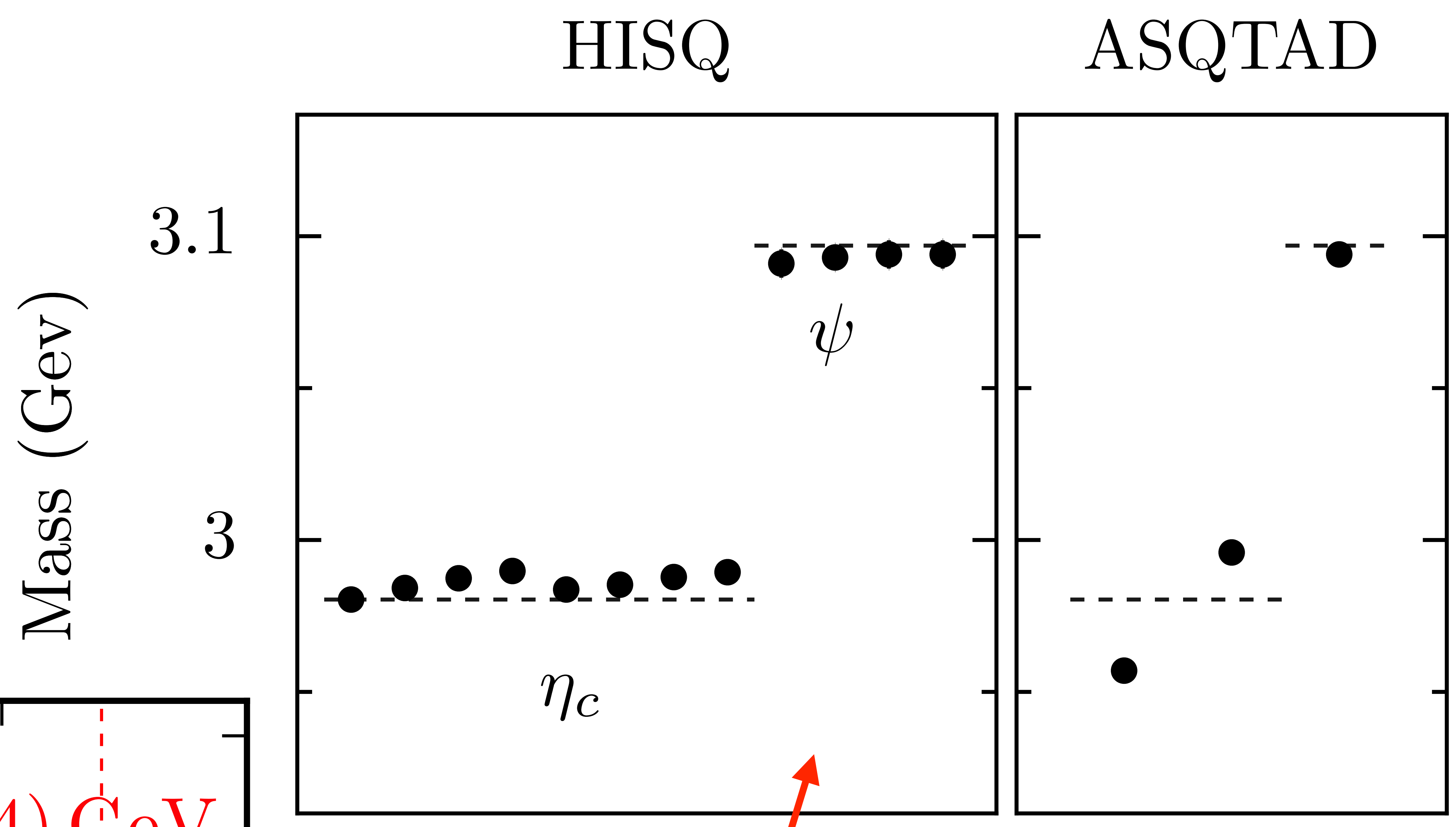
- Reduce taste-exchange effects further by double smearing
- Remove $(am)^4$ errors in energy for heavy quarks. Disc. errors then suppressed by powers of $(v/c)^2$.

Accurate decay constants for heavy-light mesons! (no renormln)



HPQCD, 1008.4018, 1110.4510

HPQCD, hep-lat/0610092



Excellent heavy (and light) quark action - test 'taste-splittings'

Ensembles with HISQ 2+1+1 sea and a^2-improved glue:

MILC, 1004.0342, 1212.4768

Hart, von Hippel, Horgan, 0812.0503

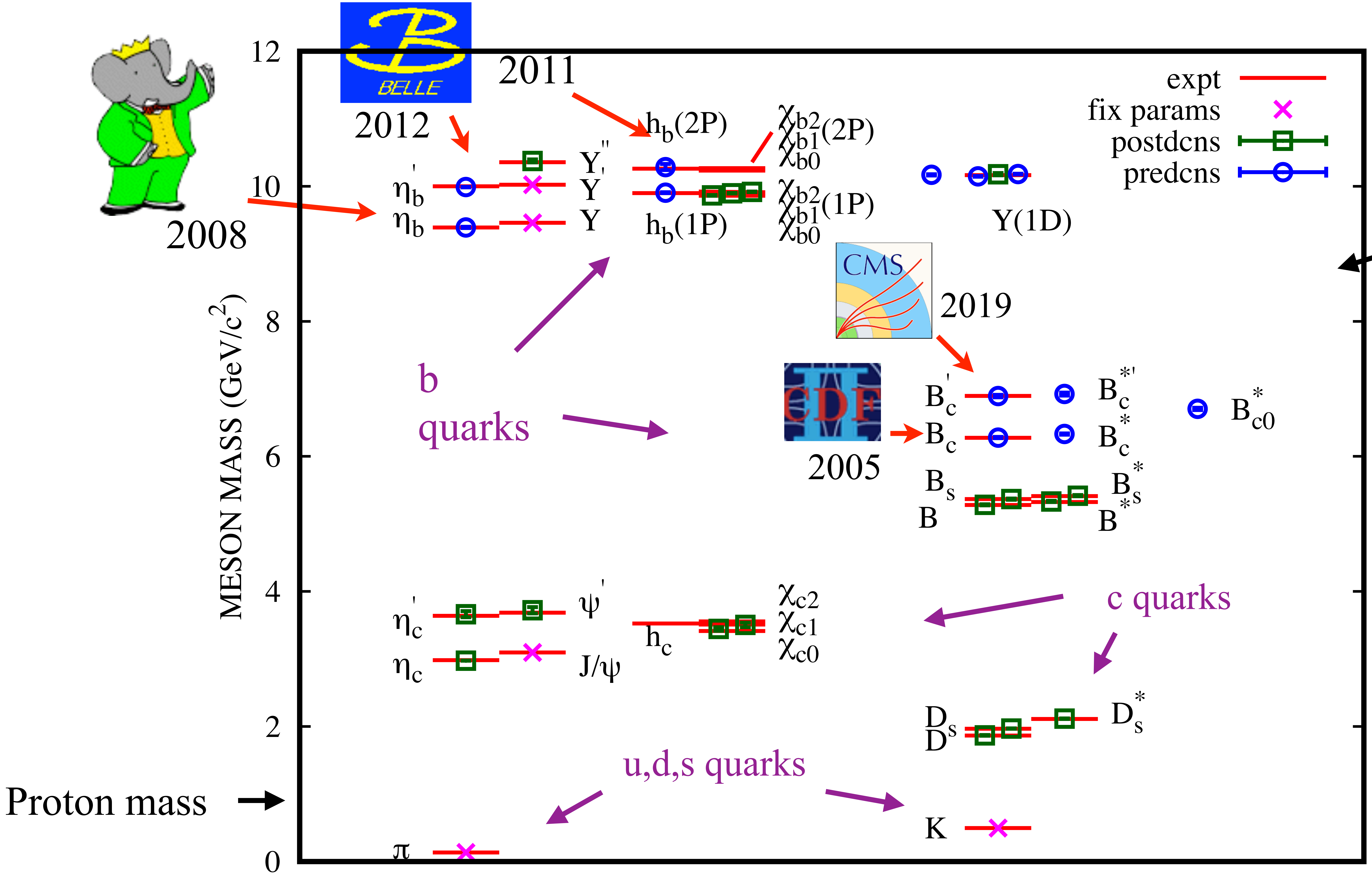
HPQCD - Sept. 2010

Move on to a survey of some RESULTS …

The masses of mesons from lattice QCD

HPQCD

on MILC configs



These are mesons that have relatively long lifetime and a well-determined mass from experiment.

Agreement is good to very high (few MeV) accuracy. Now including QED effects to reduce uncertainties further …

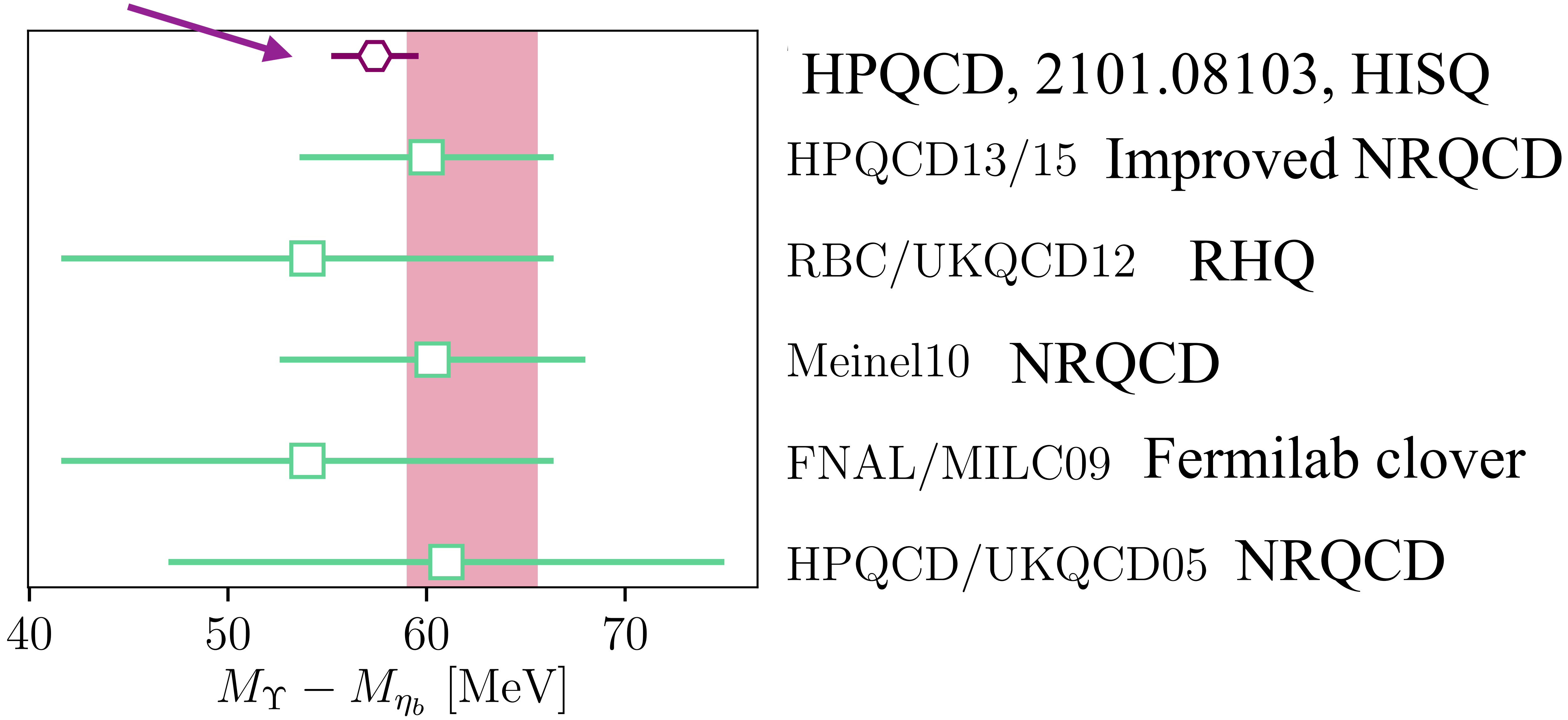
For exotica, see Braaten, Lewis…

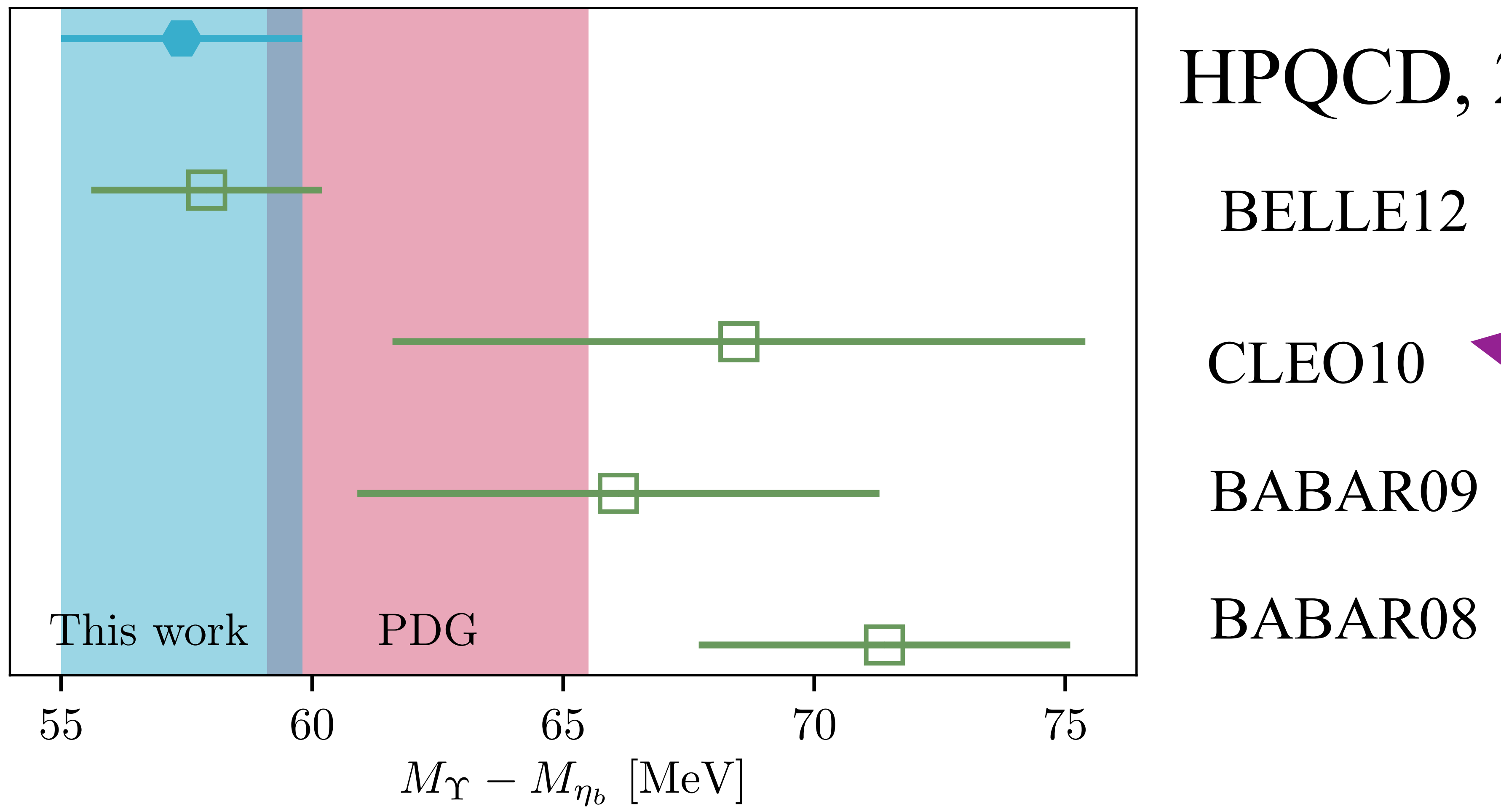
Some important mass splittings from lattice QCD

Bottomonium hyperfine splitting

$M_\Upsilon - M_{\eta_b}$

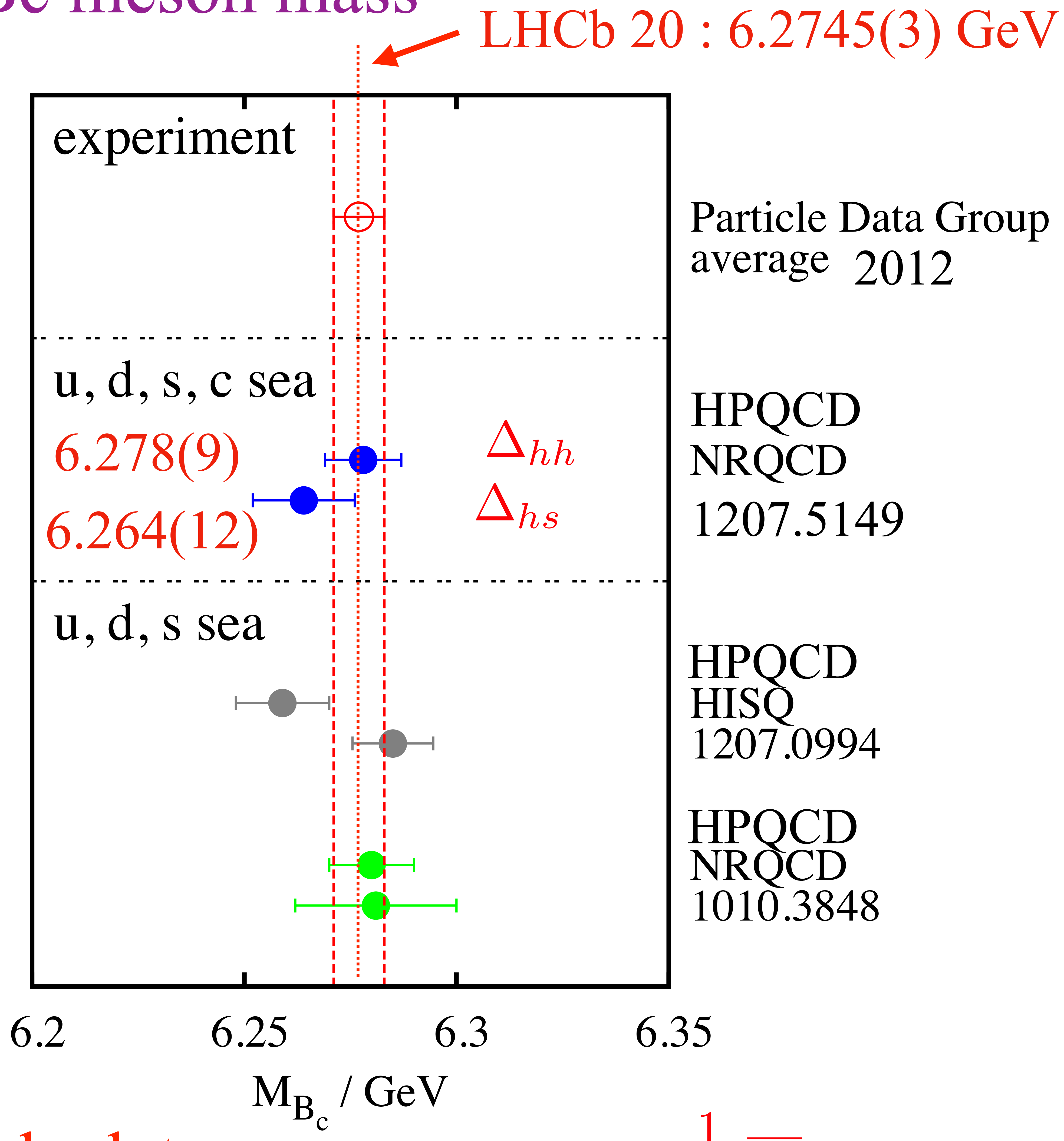
57.5(2.3)(1.0) MeV





Good agreement with expt.

Bc meson mass



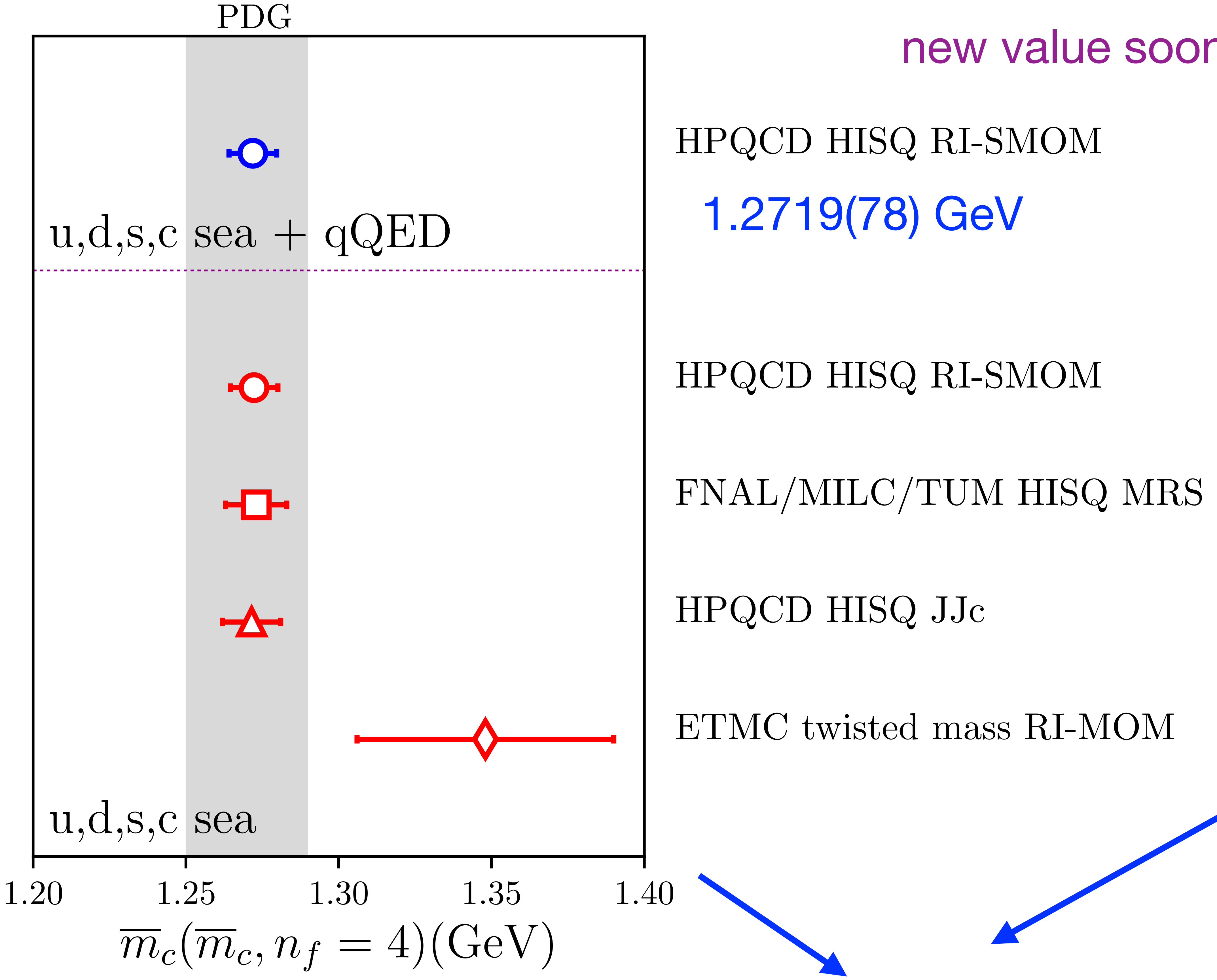
Calculate Splittings:

$$\Delta_{hh} = M_{B_c} - \frac{1}{2}(\overline{M}_{b\bar{b}} + M_{\eta_c})$$

$$\Delta_{hs} = M_{B_c} - M_{B_s} - M_{D_s}$$

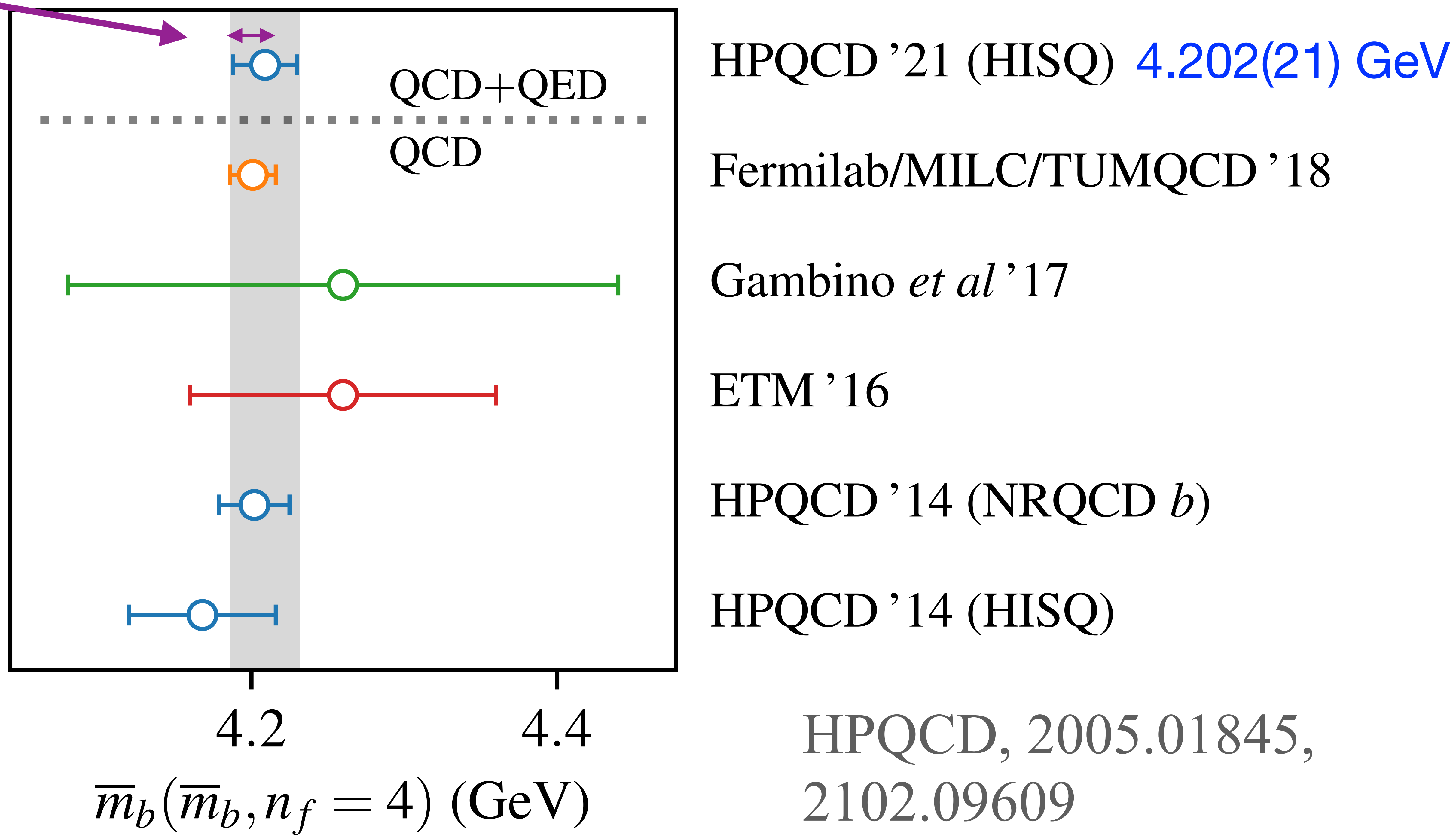
QCD parameters - quark masses

Multiple lattice methods agree well - now including effects from electric charge of valence quarks



1.2719(78) GeV

new value soon!



4.202(21) GeV

HPQCD, 2005.01845, 2102.09609

Ratio more accurate than individual masses:

$$\left.\frac{\overline{m}_b(3\,\mathrm{GeV}, n_f = 4)}{\overline{m}_c(3\,\mathrm{GeV}, n_f = 4)}\right|_{\mathrm{QCD+QED}} = 4.586(12)$$

0.3% accurate

$$\left.\frac{\Gamma(H \to b\bar{b})}{\Gamma(H \to c\bar{c})}\right|_{\mathrm{SM}} = \frac{\overline{m}_b^2(M_H)}{\overline{m}_c^2(M_H)} \frac{(1 + r_b)}{(1 + r_c)}$$

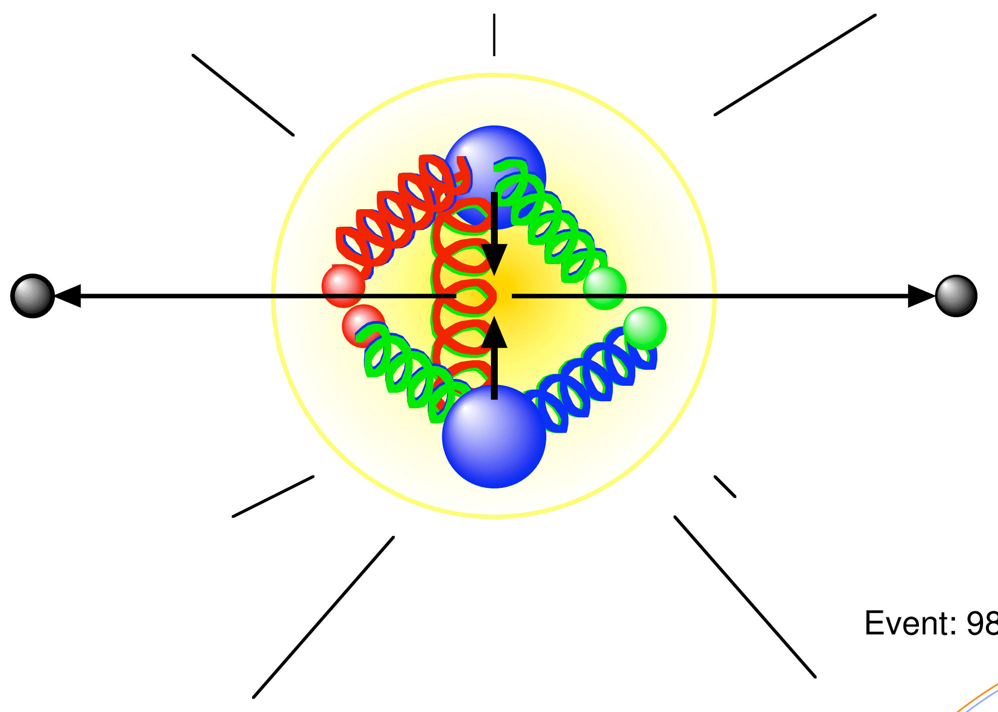
calculable to 0.9%; r_b, r_c give 3% shift

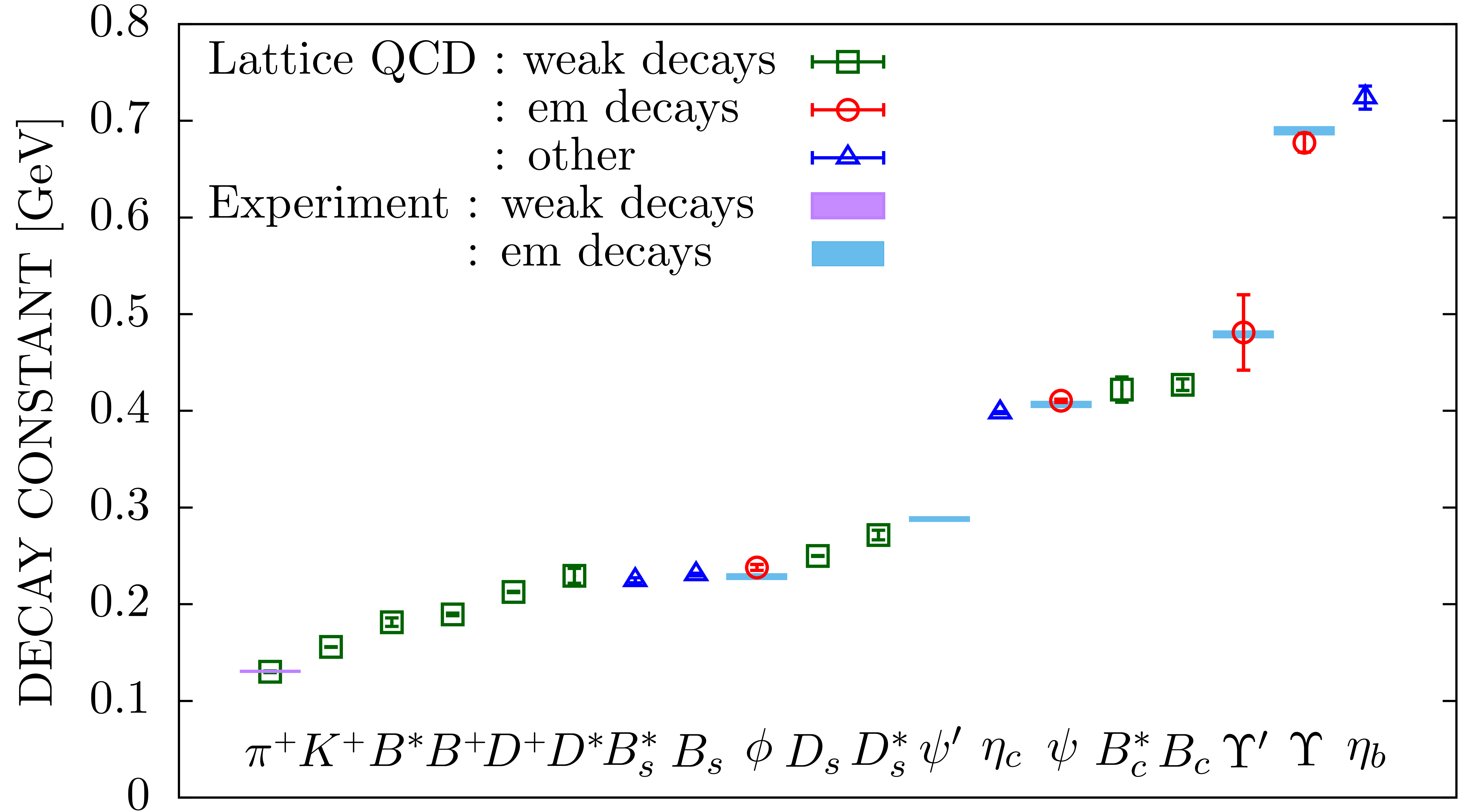
(LHC HiggsWG give 6%)

Meson weak and electromagnetic decay rates

$P \to \ell\overline{\nu}$

$V \to \ell^+\ell^-$

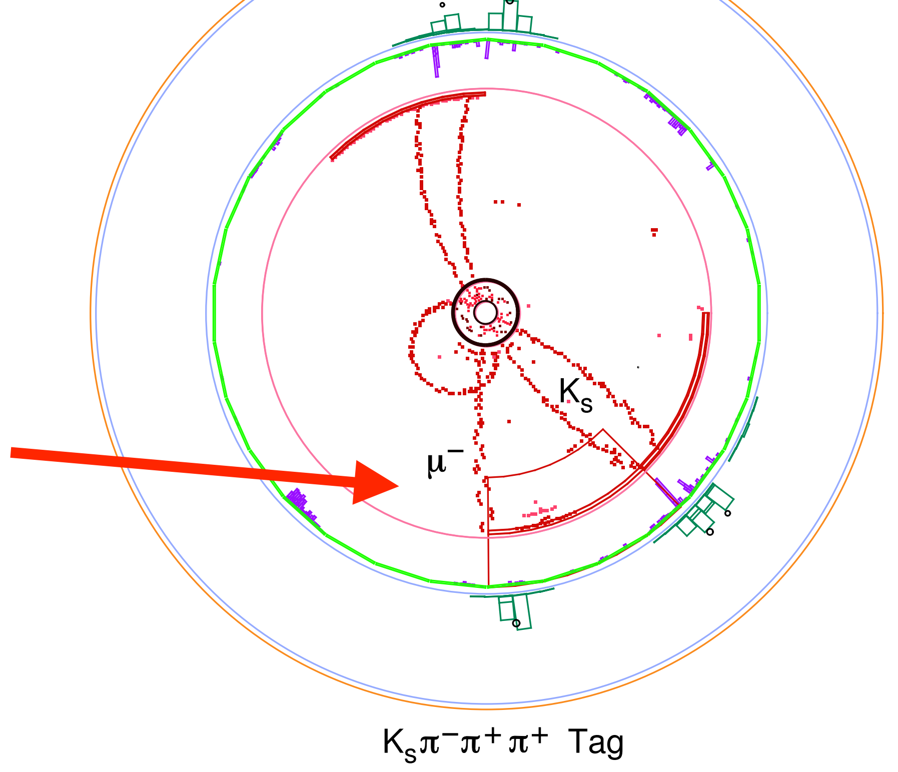




C. Davies in *50yrs of QCD*, 2212.11107

Uncertainty <1% from lattice QCD:
e.g: 0.2% f_K/f_π, 0.2% f_{D_s}, 0.4% f_ψ using HISQ

Fermilab/MILC, 1712.09262; HPQCD, 2005.01845, 2101.08103



CLEO

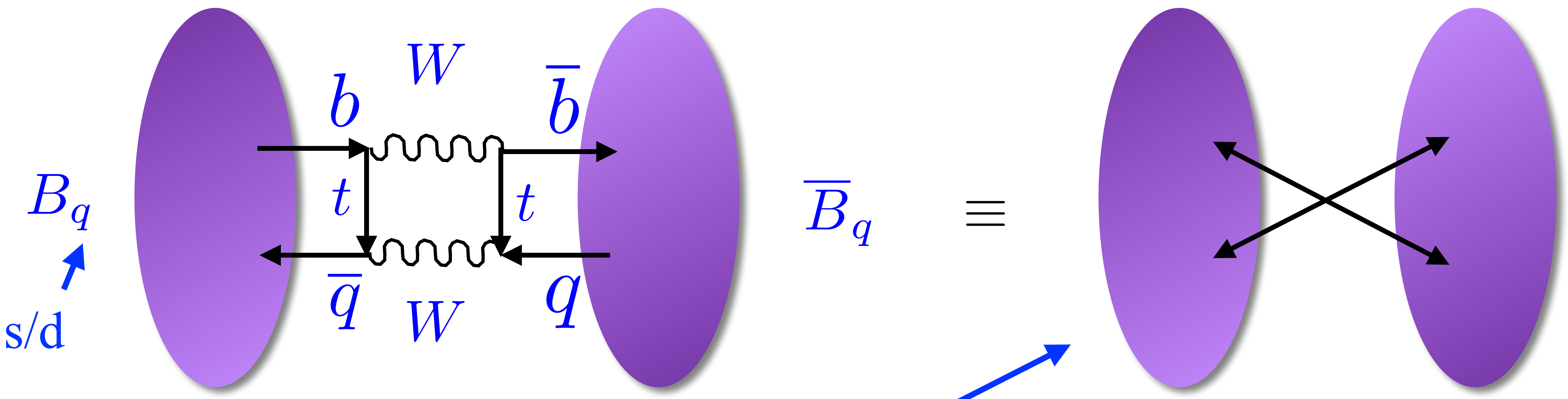
$D^- \to \mu^-\overline{\nu}$

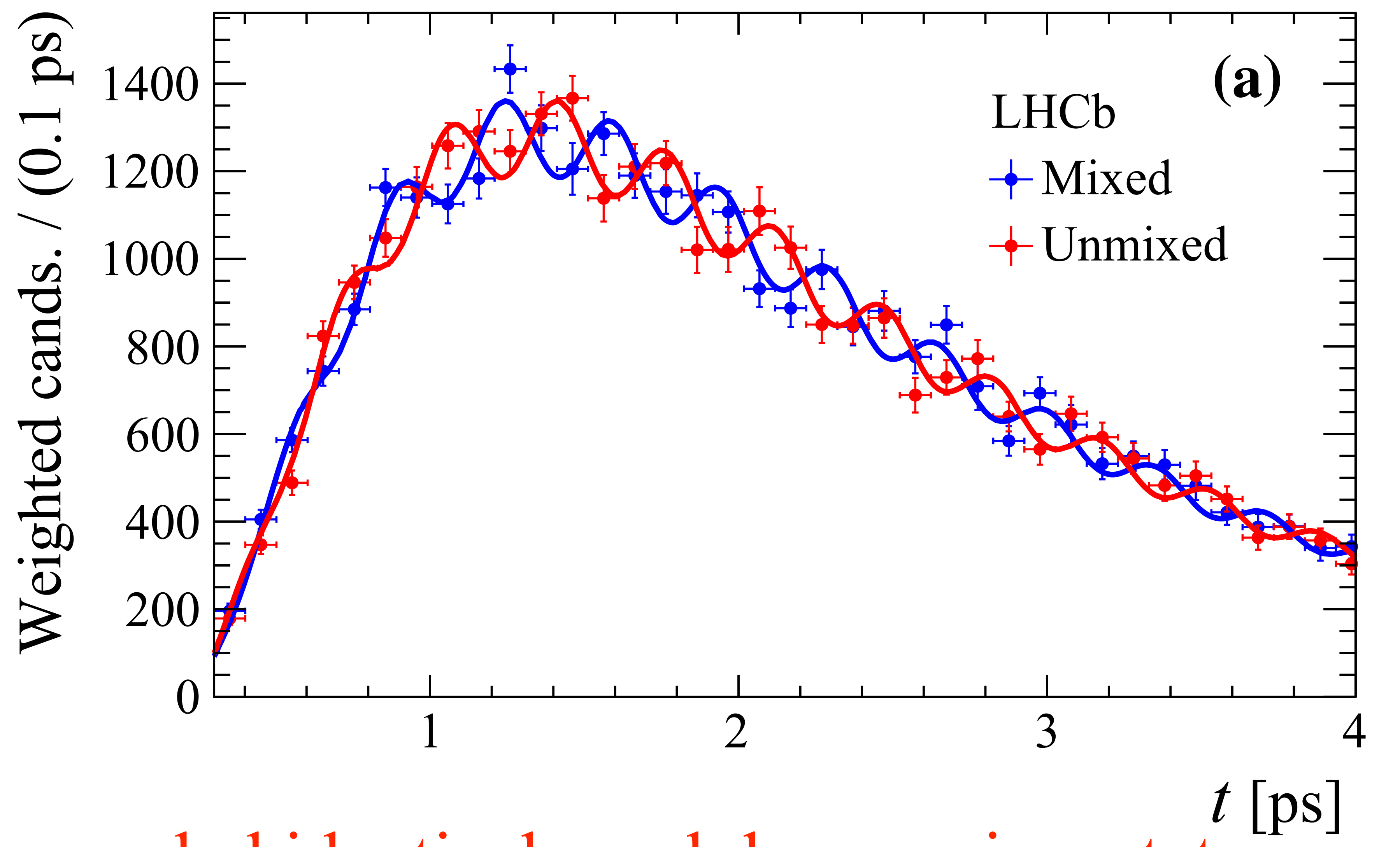
Annihilation rate to **γ** or W determined by hadronic parameter called decay constant, f.

$$\Gamma = f^2 \times (\text{kin. factors}) \times (\text{CKM}^2 \text{ or } e_q^2)$$

Comparison to expt. tests SM and/ or gives CKM elements

B and Bs mixing



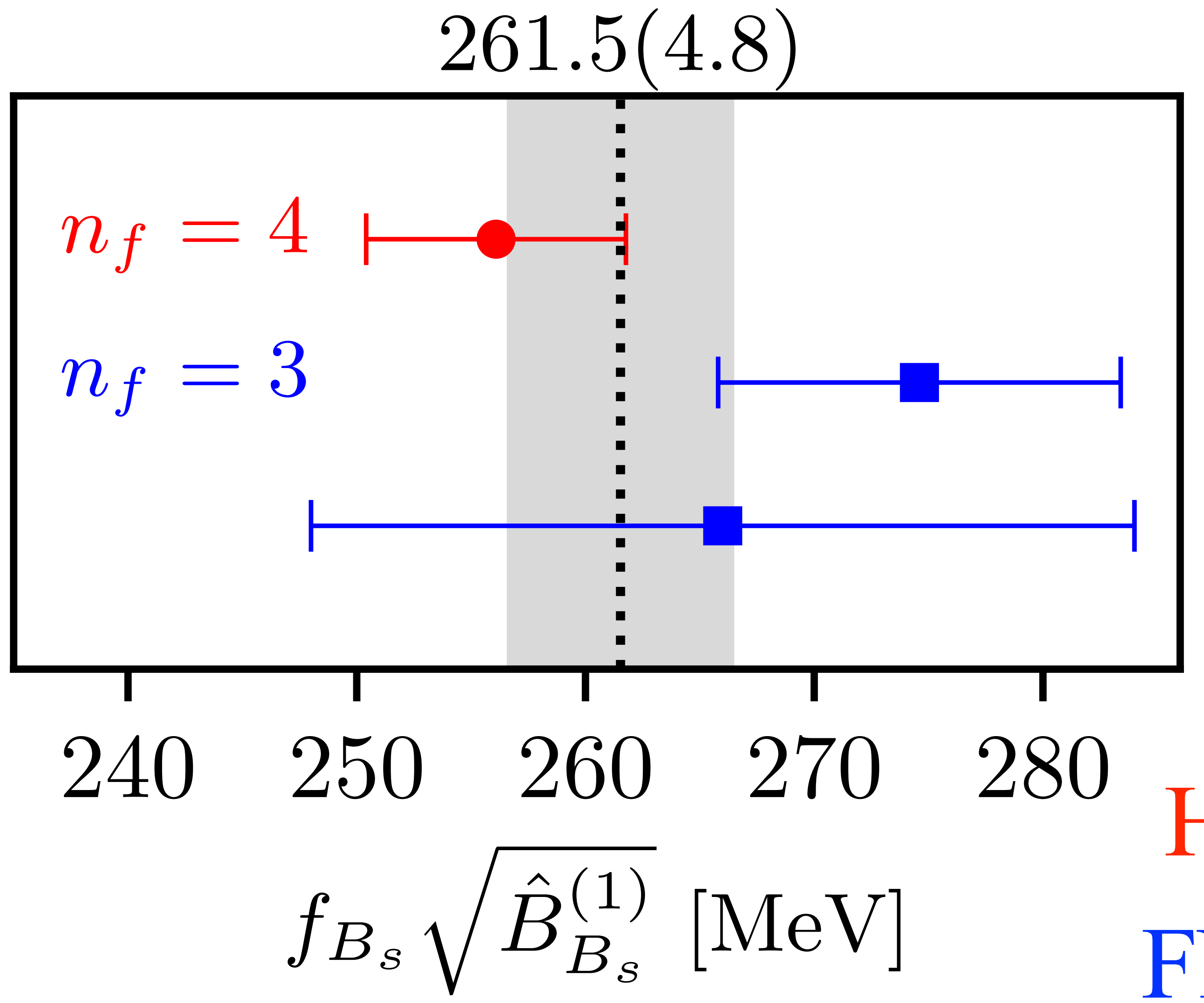


Calculate ME on lattice. Match to continuum at O(α_s)

$$\langle O_1^q\rangle(\mu) = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_{B_q}^{(1)}(\mu)$$

‘bag parameter’

Like weakly-coupled identical pendulums - eigenstates are orthogonal mixtures of flavour states. Flavour states then ‘oscillate’ at f=ΔM of eigenmodes, fn of <O$_1$> and Vts/Vtd.



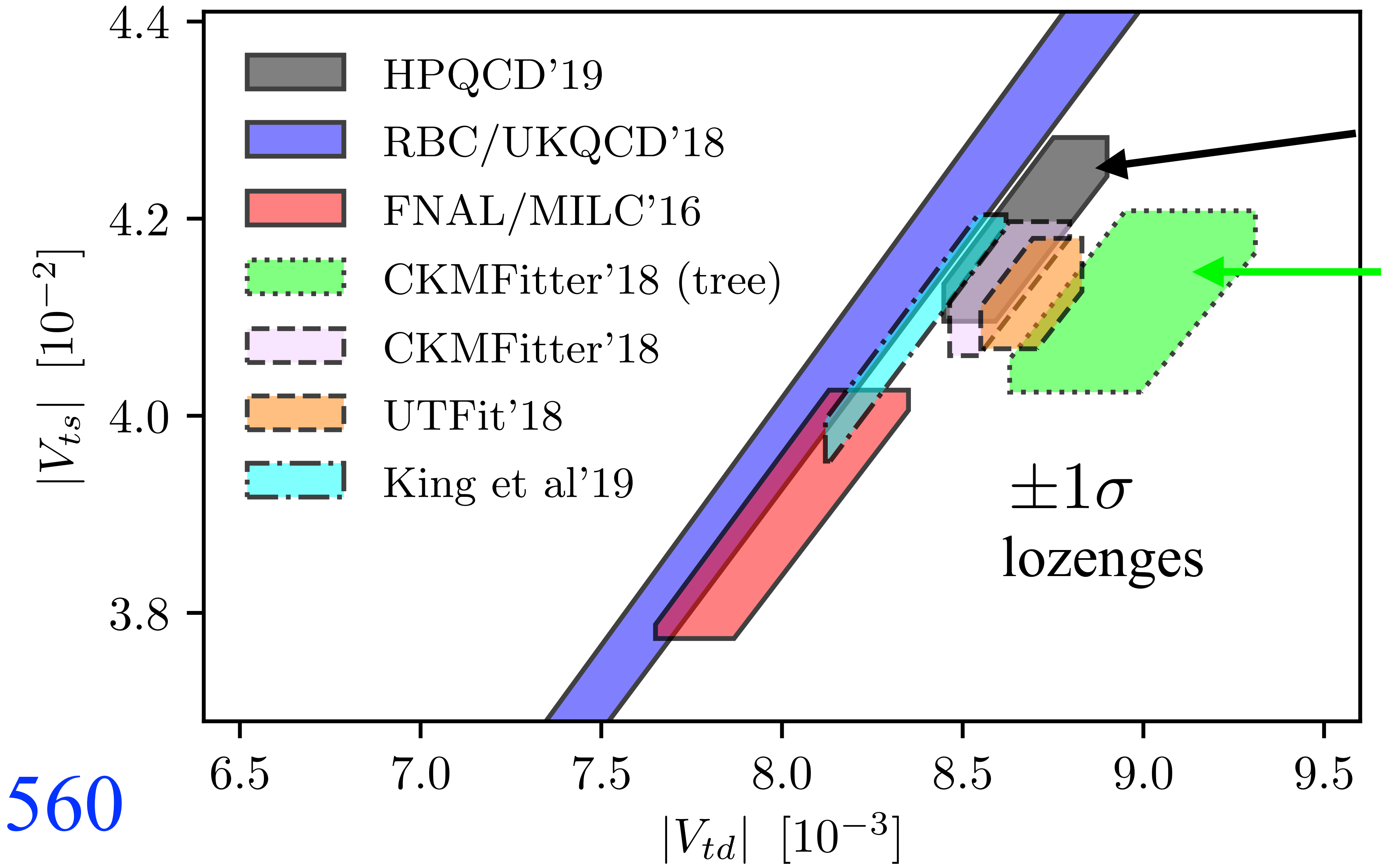
HPQCD’19 NRQCD b

FNAL/MILC’16 clover b

HPQCD’09 NRQCD b

HPQCD,1907.01025

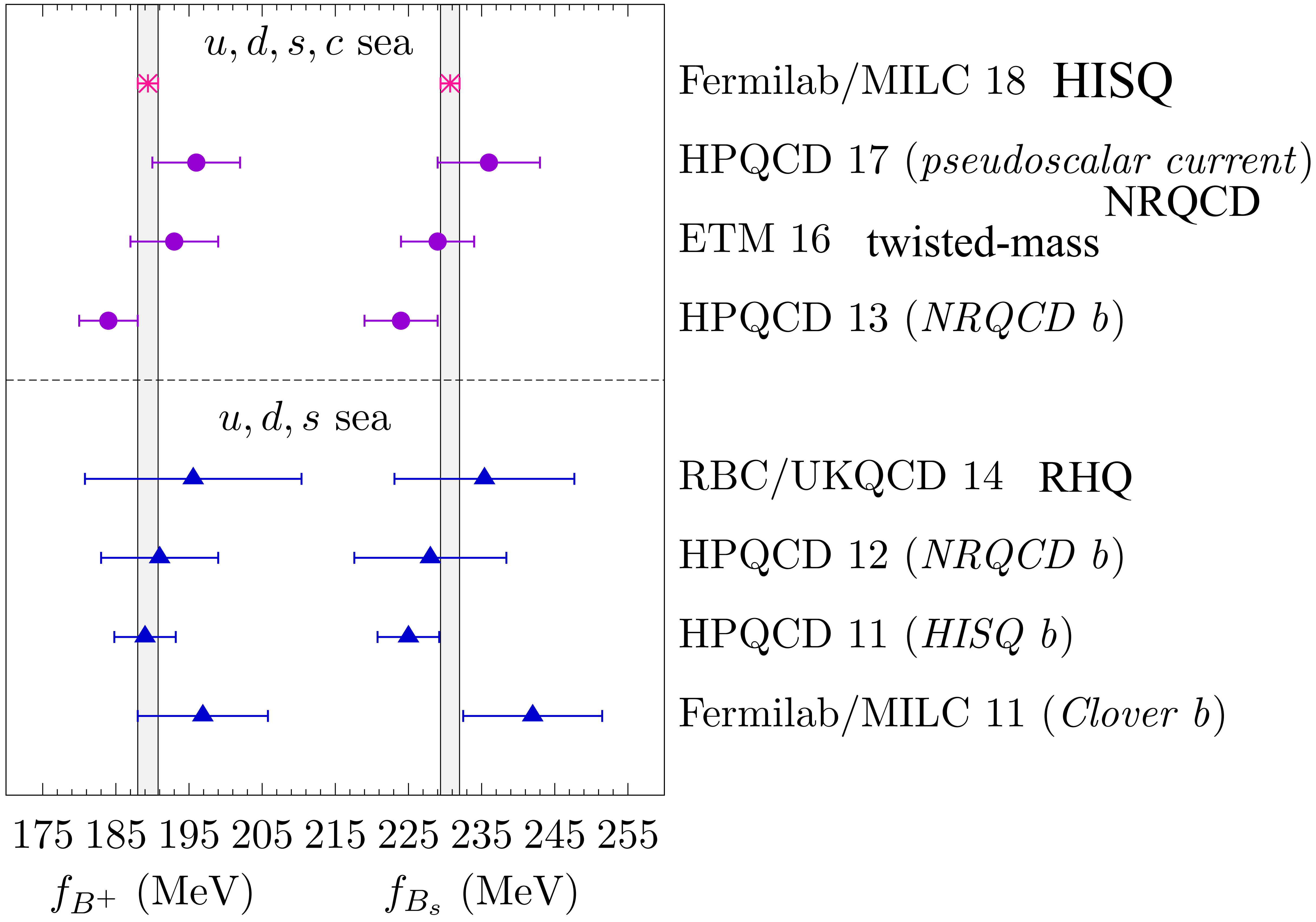
FNAL/MILC,1602.03560



HPQCD - 2.5% errors - consistent at 2σ with CKM unitarity from tree processes (i.e. not inc. ΔM)

pert. renormln of 4q ops dominates error

Meson weak decay rates - vectors and pseudoscalars



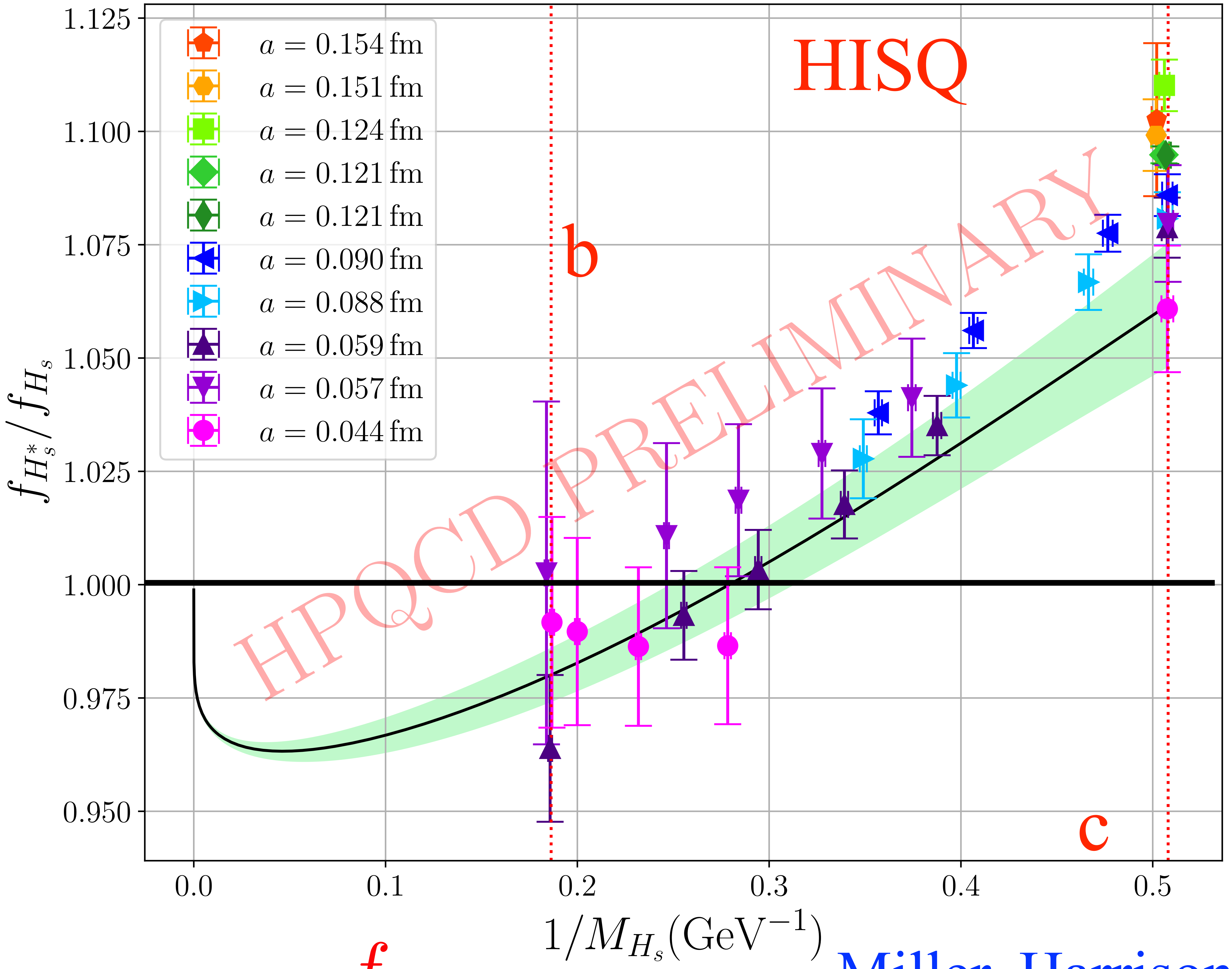
FNAL/MILC,1712.09262

$f_{B_s} = 230.7(1.3)$ MeV

$f_{B^+} = 189.4(1.4)$ MeV

← Uncertainty < 1%

f_V/f_P for heavy-light mesons



$\frac{f_V}{f_P} < 1$ for b, $\frac{f_V}{f_P} > 1$ for c

Miller, Harrison, HPQCD, in prep.; 1503.05762

also true for heavyonium, HPQCD, 2101.08103

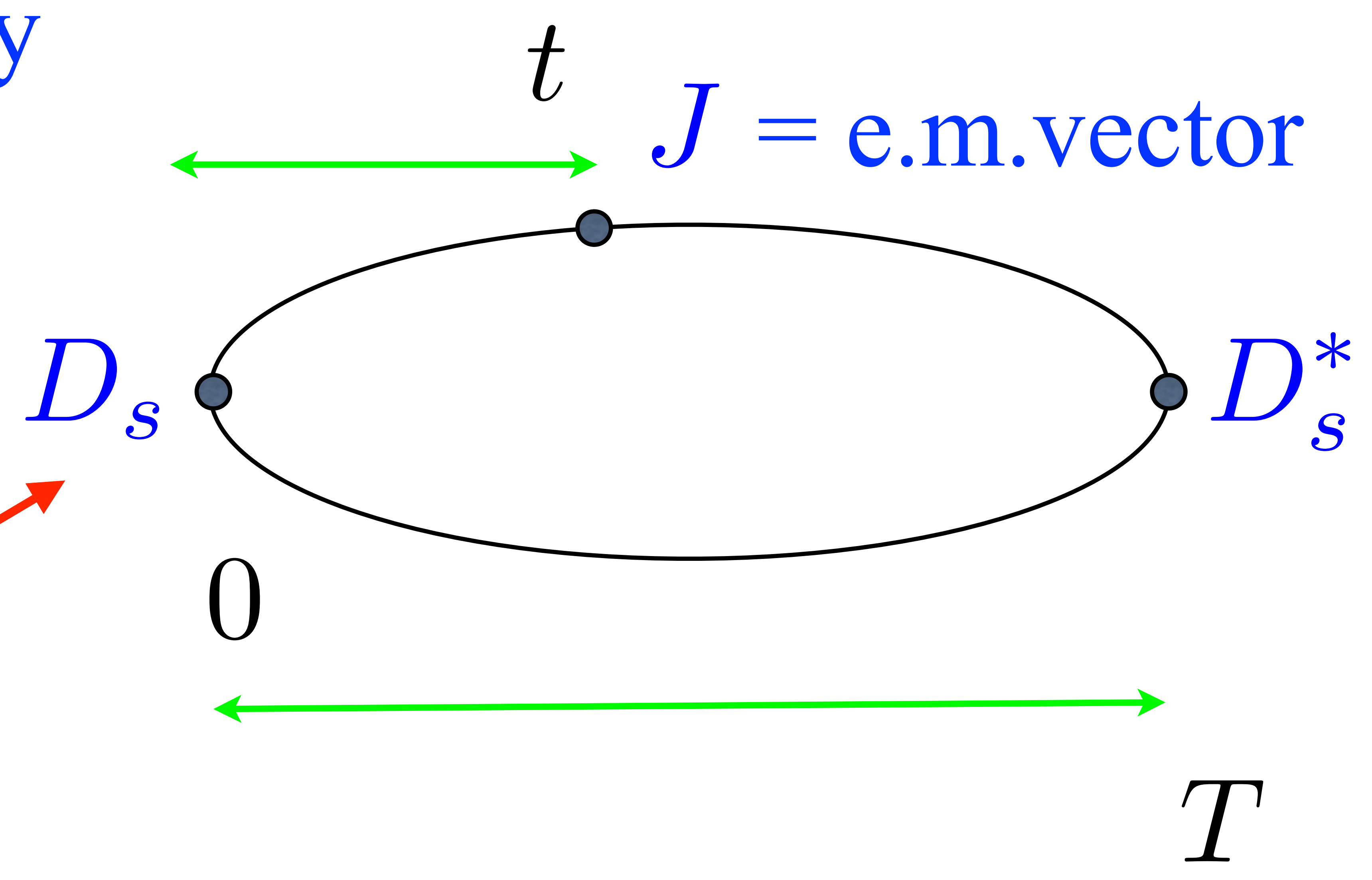
Observing weak decays for vector mesons - the Ds* story

Br(Ds*→ Ds γ) = 93.6(4)%

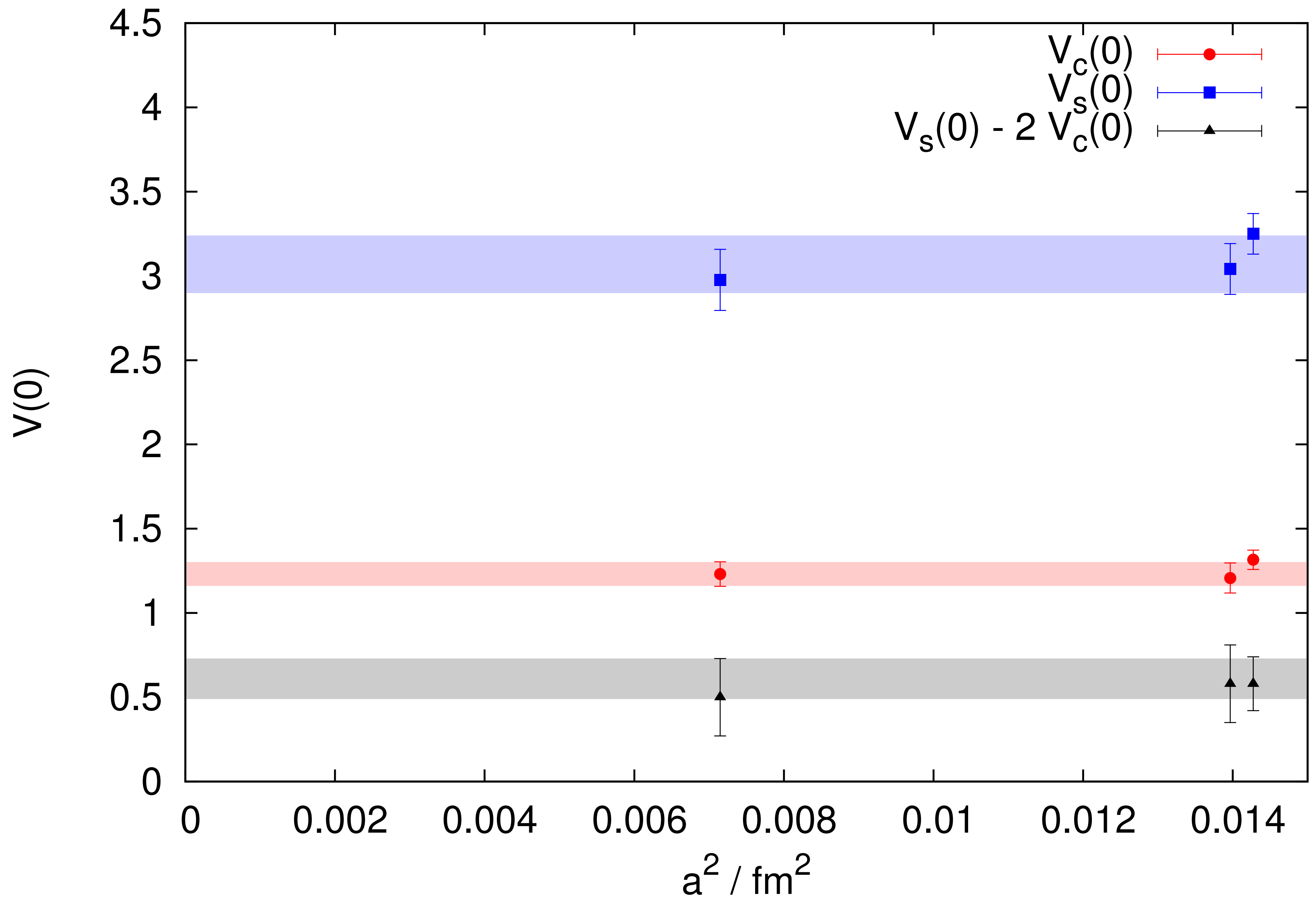
➡ Lattice calculation of Γ(Ds*→ Ds γ) can yield total width to normalise other decay rates

Calculate 3-point correlators for γ emission from c and s

- tune momentum inserted so q²=0 for real photon



HPQCD (HISQ),1312.5264



Combined c/s form factor very small, so total width small

$$\Gamma(D_s^*) = 0.070(28)\,\mathrm{keV} \qquad \tau(D_s^*) = 9.4(3.8) \times 10^{-18}\mathrm{s}$$

$D_s^* \to \ell\nu$ not lepton-mass suppressed, predict

$$\mathcal{B}(D_s^* \to \ell\nu) = 3.4(1.4) \times 10^{-5}$$

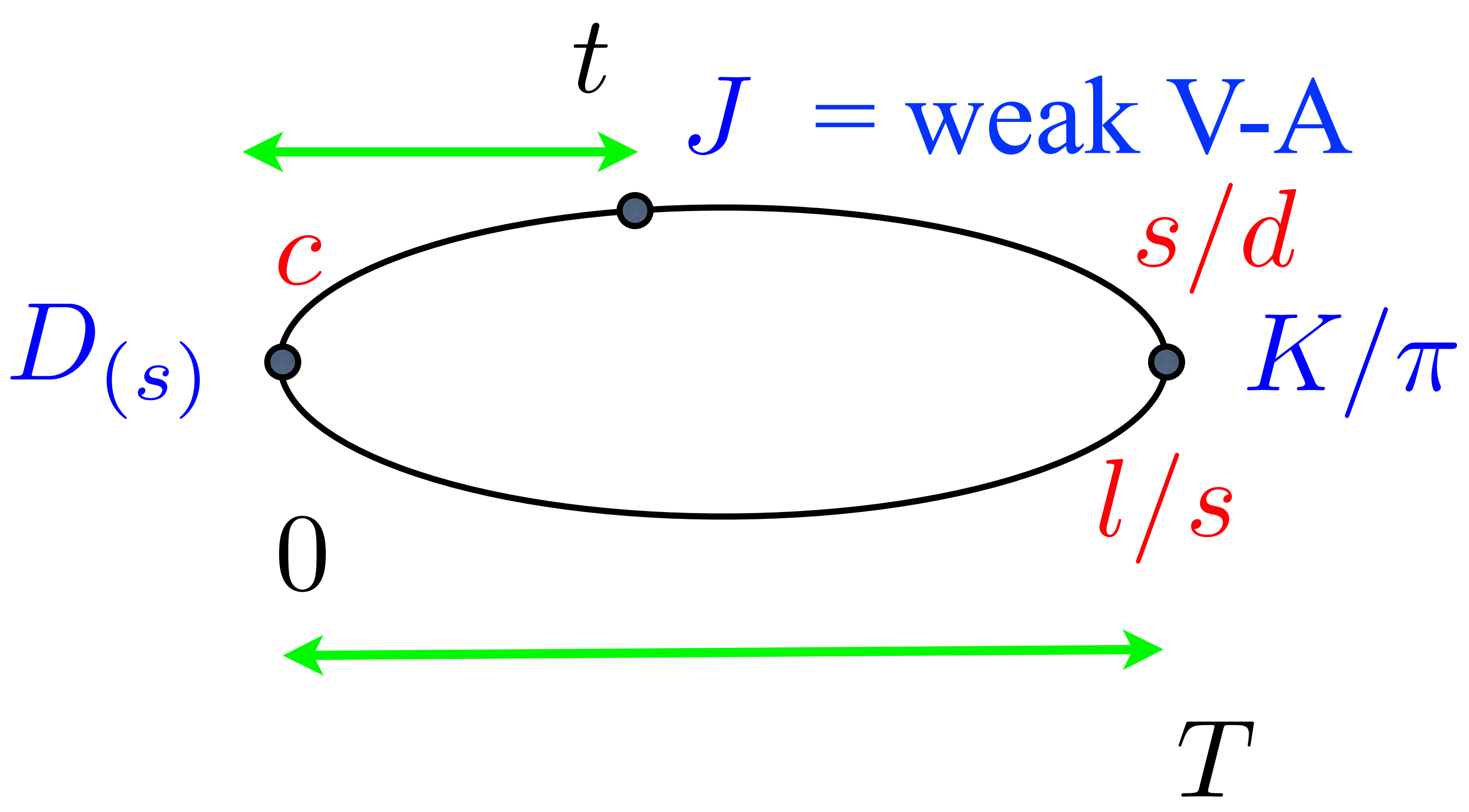
visible?

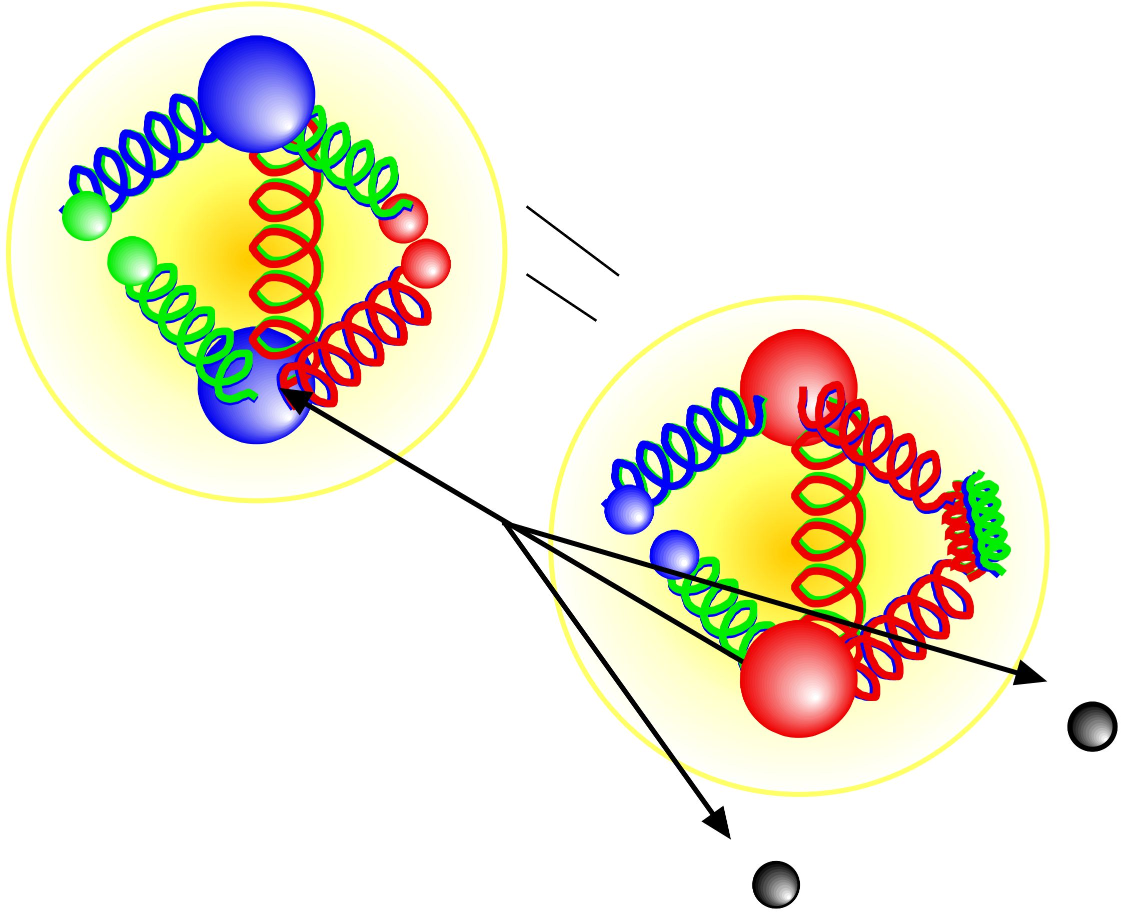
BESIII 2304.12159 find: $2.1^{+1.2}_{-0.9} \pm 0.2 \times 10^{-5}$

With HPQCD Γ this gives : $f_{D_s^*}|V_{cs}| = 208^{+59}_{-45} \pm 43\,\mathrm{MeV}$

Improved lattice + expt. - can test CKM from vector annihilation

D semileptonic decays

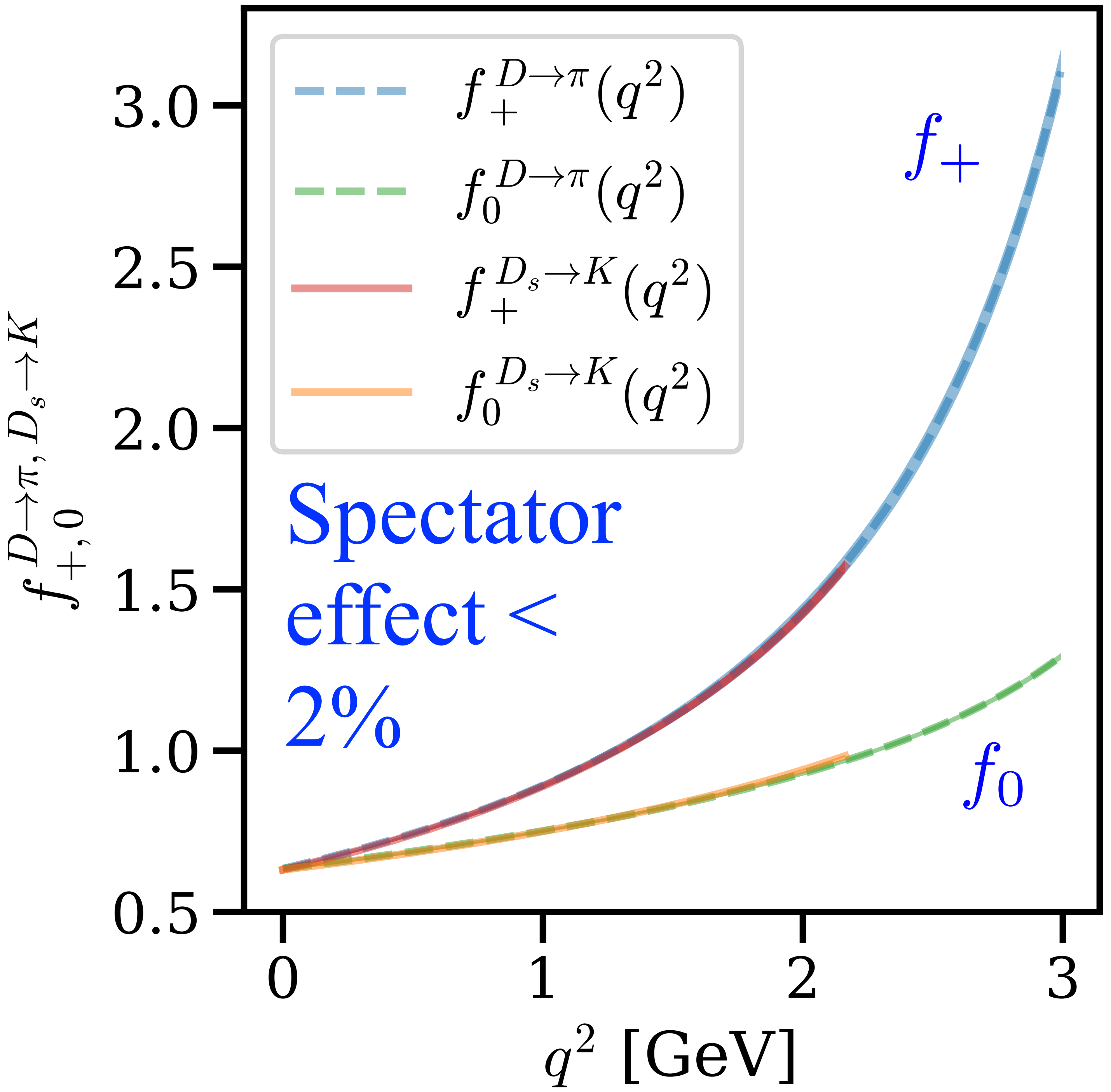




$$\langle K|V^{\mu}|D\rangle = f_+(q^2)\left[p_D^{\mu} + p_K^{\mu} - \frac{M_D^2 - M_K^2}{q^2}q^{\mu}\right] + f_0(q^2)\frac{M_D^2 - M_K^2}{q^2}q^{\mu}$$

f_0 absolutely normalised with HISQ, use to norm. f_+

HPQCD, 1008.4562,1305.1462



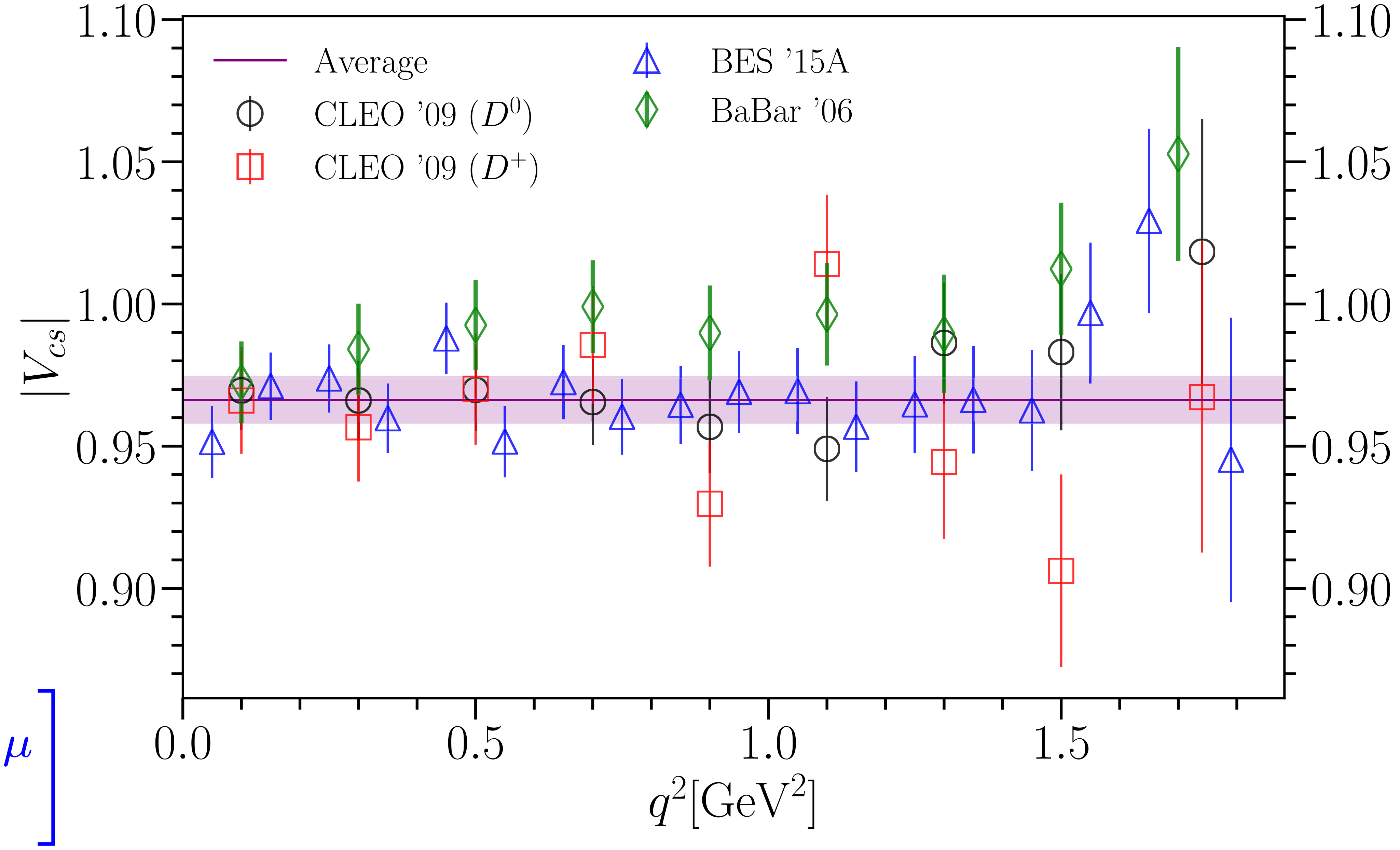
Spectator effect < 2%

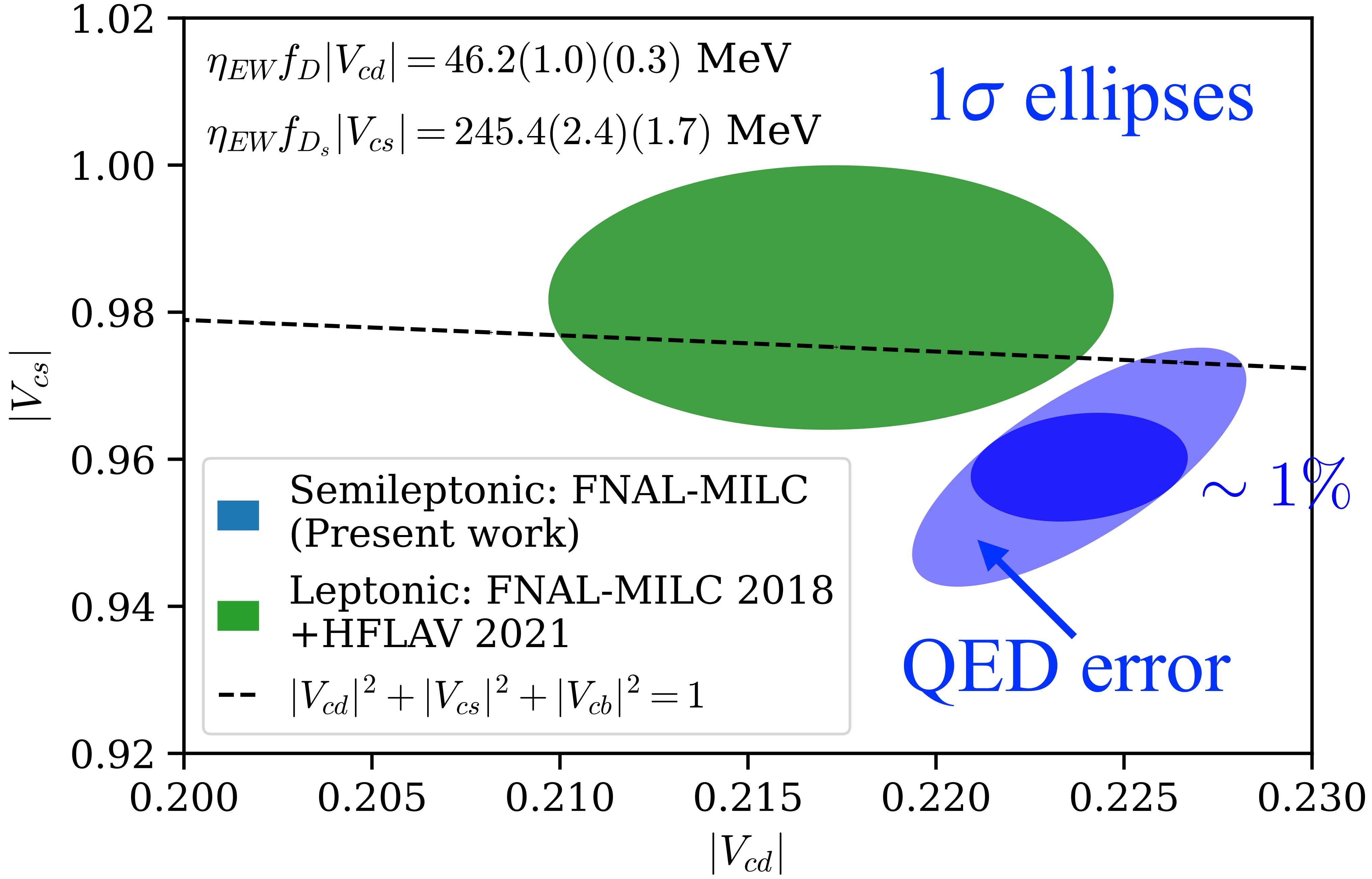
$$\frac{d\Gamma}{dq^2} \sim |V_{cs/d}|^2 \times f^2$$

expt.

lattice

HPQCD (HISQ),2104.09883,D→K





$\eta_{EW} f_D|V_{cd}| = 46.2(1.0)(0.3)$ MeV

$\eta_{EW} f_{D_s}|V_{cs}| = 245.4(2.4)(1.7)$ MeV

1σ ellipses

~ 1%

QED error

- Semileptonic: FNAL-MILC (Present work)
- Leptonic: FNAL-MILC 2018 +HFLAV 2021
- $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$

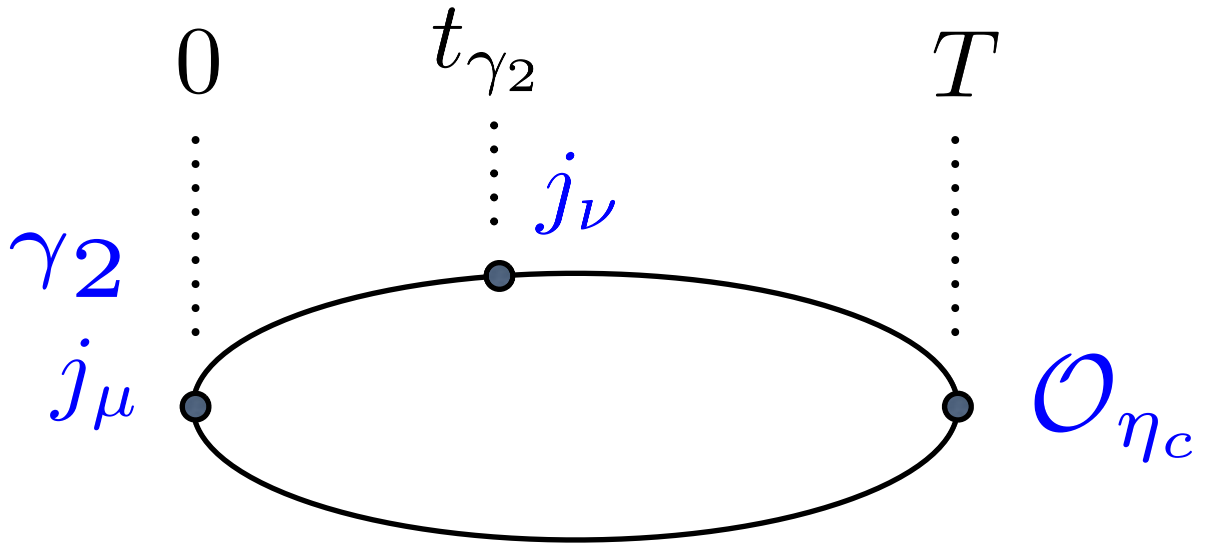
Fermilab/MILC (HISQ),2212.12648,D→K,D→π,D_s→K

Lattice + expt uncertainties now same size

$\eta_c \to \gamma\gamma$

HPQCD, 2305.06231

$$\sum_{t_{\gamma_2}} e^{-\omega t_{\gamma_2}}$$



$$= C(T)$$

$$\downarrow$$

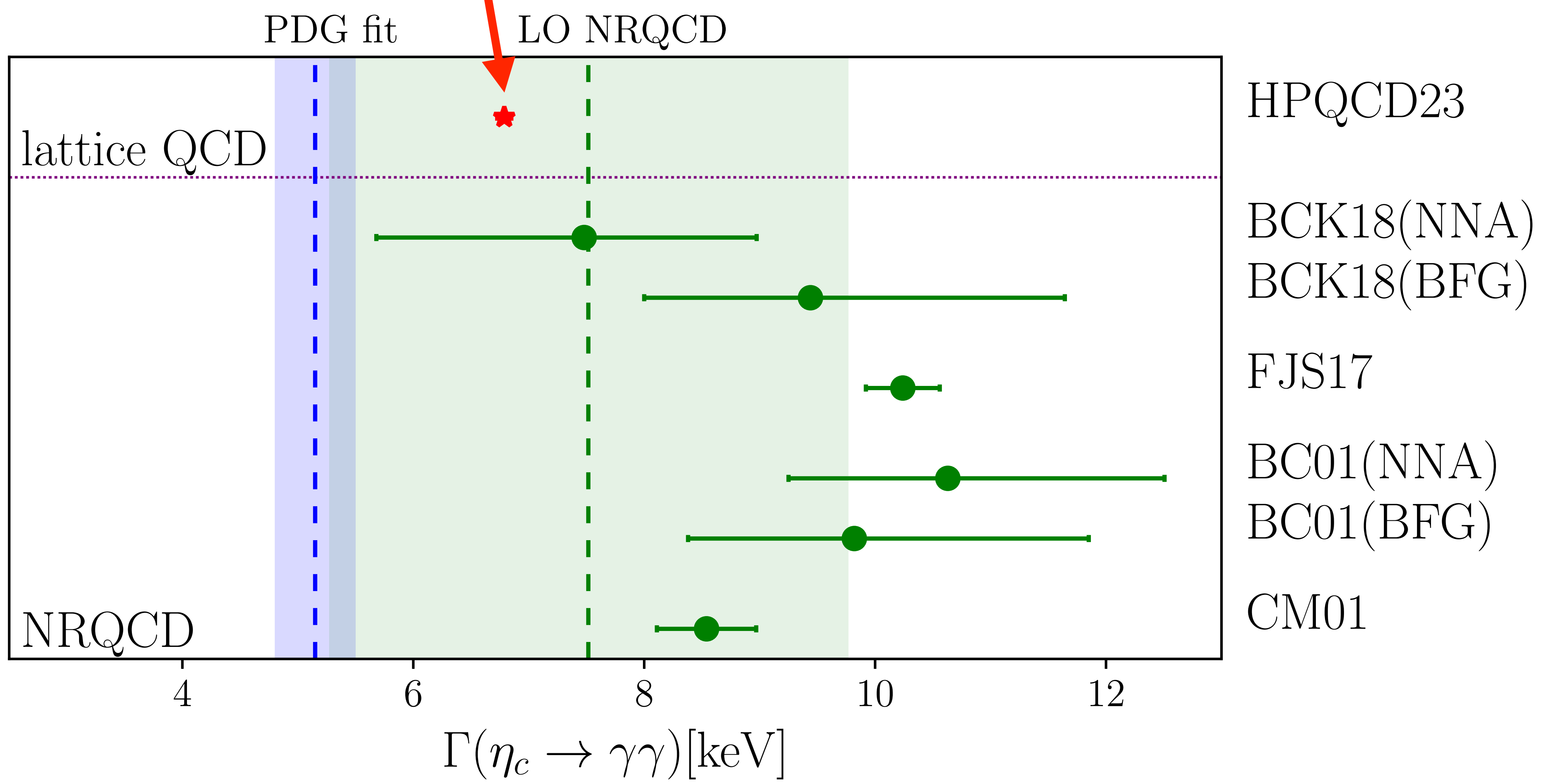
$$F(0,0)$$

in nonrelativistic limit

$$\frac{f}{F(0,0)M^2} = \frac{1}{2}(1+\mathcal{O}(\alpha_s)+\mathcal{O}(v^2/c^2))$$

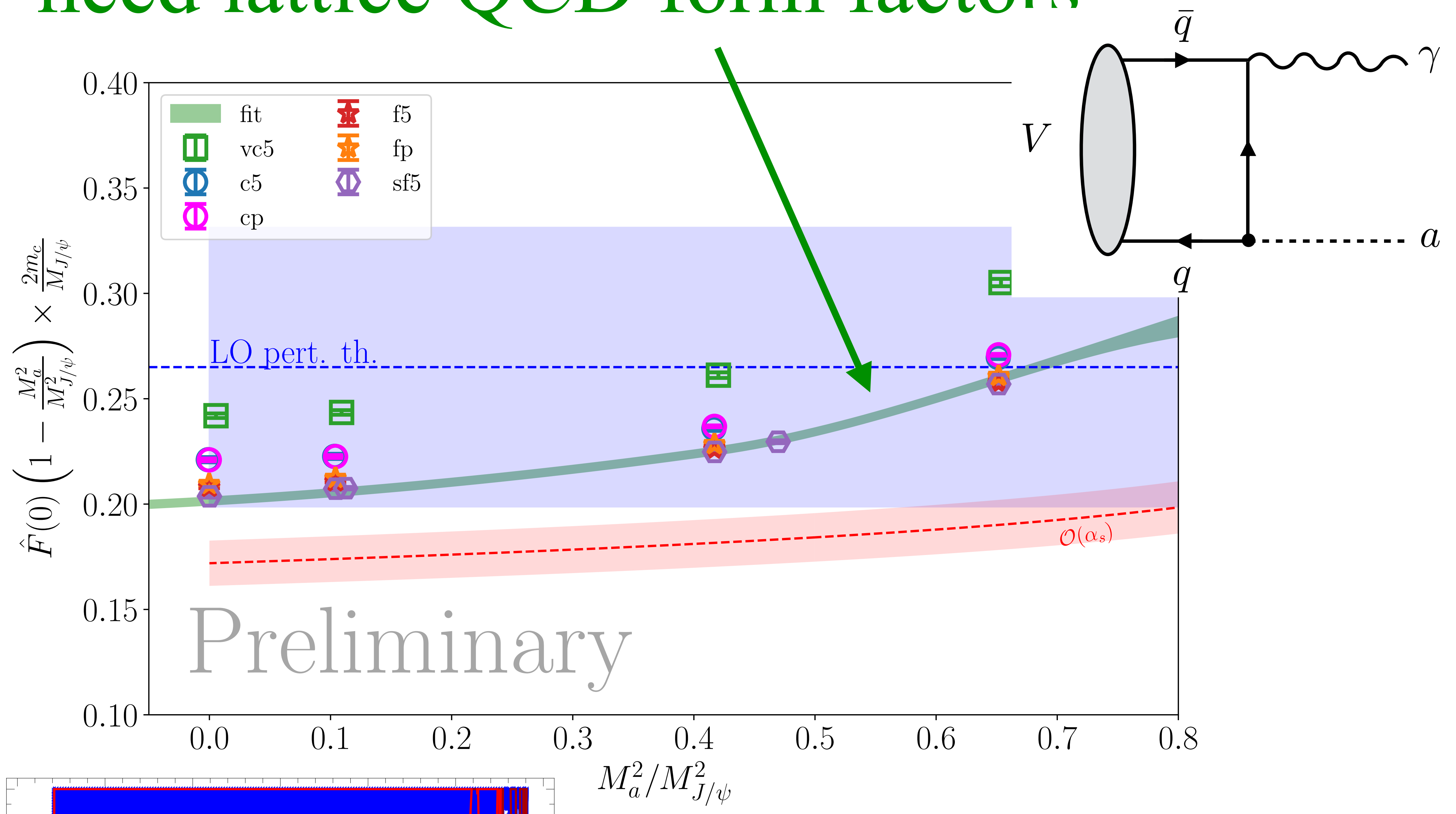
but lattice QCD much better than this

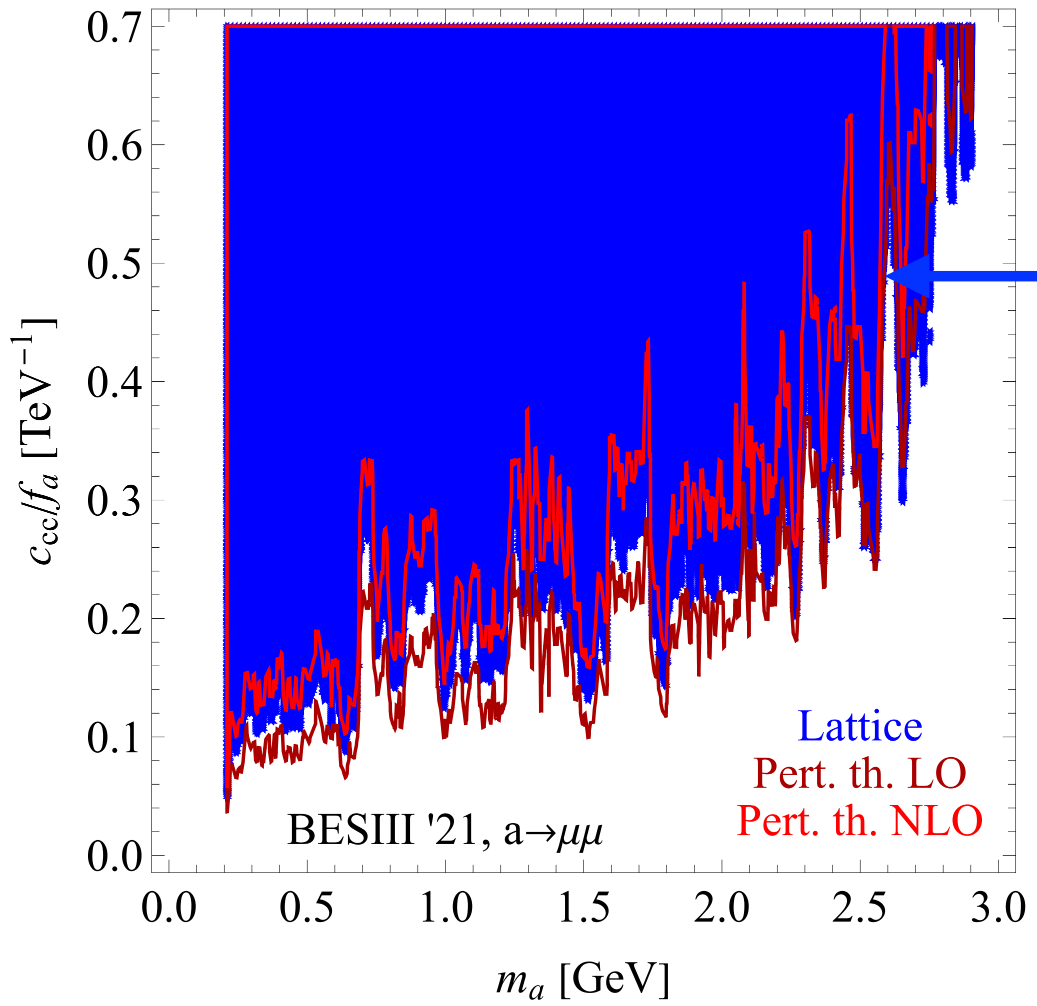
- has 1% uncertainty



$J/\psi \to \gamma a$ ⟵ axion-like particle

accurate constraints from experiment need lattice QCD form factors





constraints on ALP mass/coupling to charm from BESIII search

Colquhoun, CD + GPL + Renner, in prep.

Conclusions

Heavy quark physics is a key success story of lattice QCD.
Dominated by results from HPQCD and Fermilab/MILC using multiple approaches.
HISQ is now the way to go for simple meson weak/em decays

c, b quark masses now 0.5% accurate, ratio 0.3% - LHC need to use the lattice numbers

Decay constants for meson annihilation errors < 1% — issues now experimental.
More work on vector mesons?

Neutral meson mixing - needs improvement (underway?)

Semileptonic D decays in good shape? For B - see Judd Harrison talk

There are lots of other processes that lattice QCD can provide answers on:
radiative decays, two-photon decays, axion searches etc.

Baryons ?????

Lots still to do …