Simulation of electromagnetic pulses through high-power solid state laser amplifiers

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Outline

- Motivation and scope
- Modeling approach
- Comparison with experimental data
- Initial lessons learned
- Linear canonical transforms (LCT)
- Next steps

Motivation

Workshop on Laser Technology for k-BELLA and Beyond. 2017. eprint: https://www2.lbl.gov/LBL-Programs/atap/Report_Workshop_k-BELLA_laser_tech_final.pdf.

The development and engineering of complex state-of-the-art laser systems capable of delivering new levels of performance requires sophisticated computational design tools and predictive models. As existing and emerging missions and applications continue to drive advances in the technology, they are pushing—and in some cases exceeding—the existing state of the art in laser modeling capability.

Better positioning our capabilities to support the next generation of high-intensity (large bandwidth) systems and/or high-average power systems will require additional code development/integration, as well as the incorporation of additional laser physics. Some of the needed physics includes, but is not limited to:

– *Spatiotemporal coupling effects in diffractive grating compressor.*

– *Effects of thermal stress induced birefringence in high average power amplifiers.*

– *Transient thermal effects on gain; these are beyond our current capability (can only do steady-state or cold).*

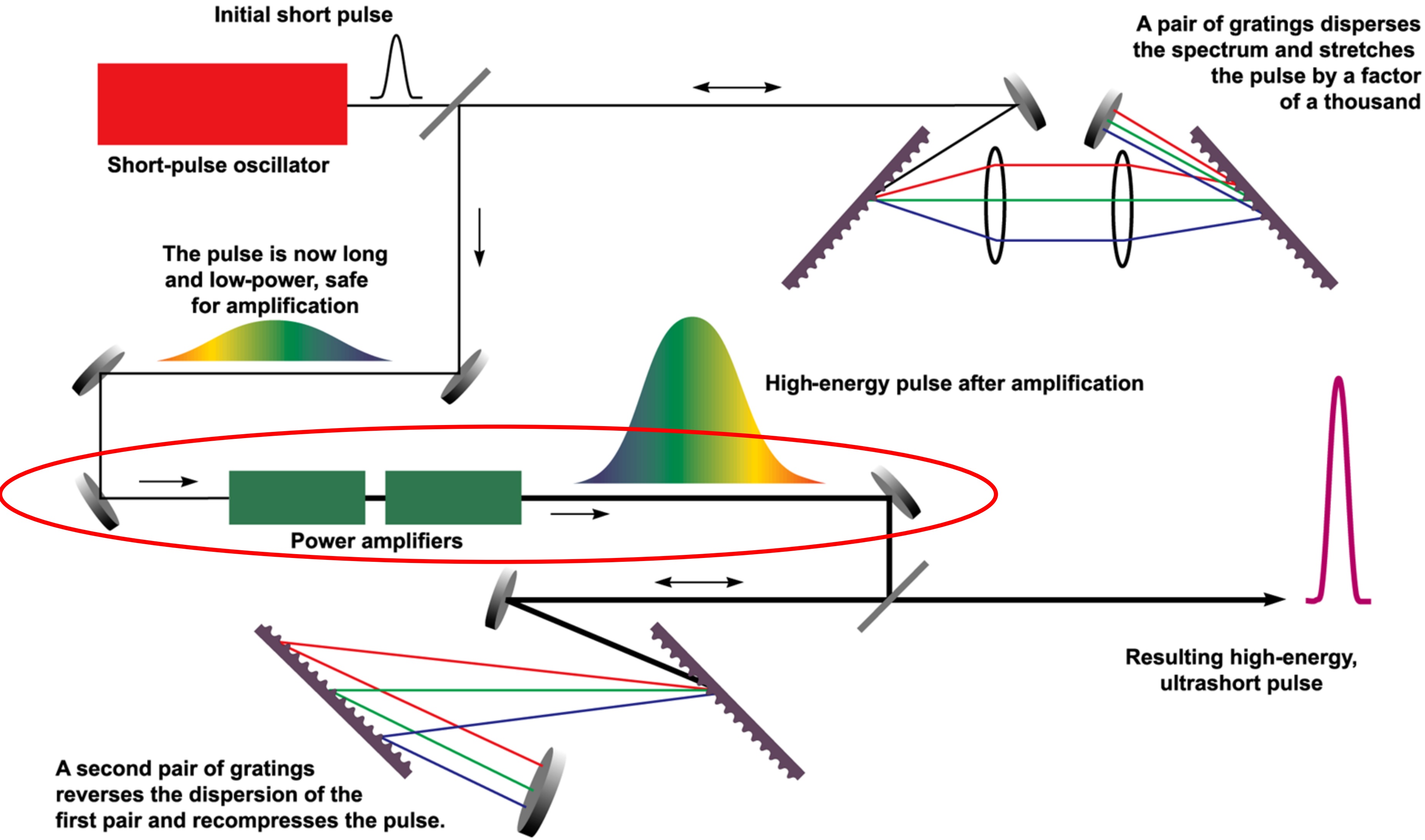
– *Statistical variations in component properties (system optimization, e.g. flaw distribution, diode array pump uniformity).*

– *Fully integrated amplifier models including temperature-dependent physical properties (thermal conductivity, birefringence. . . .) and laser properties (cross-section, lower level absorption, population inversion).*

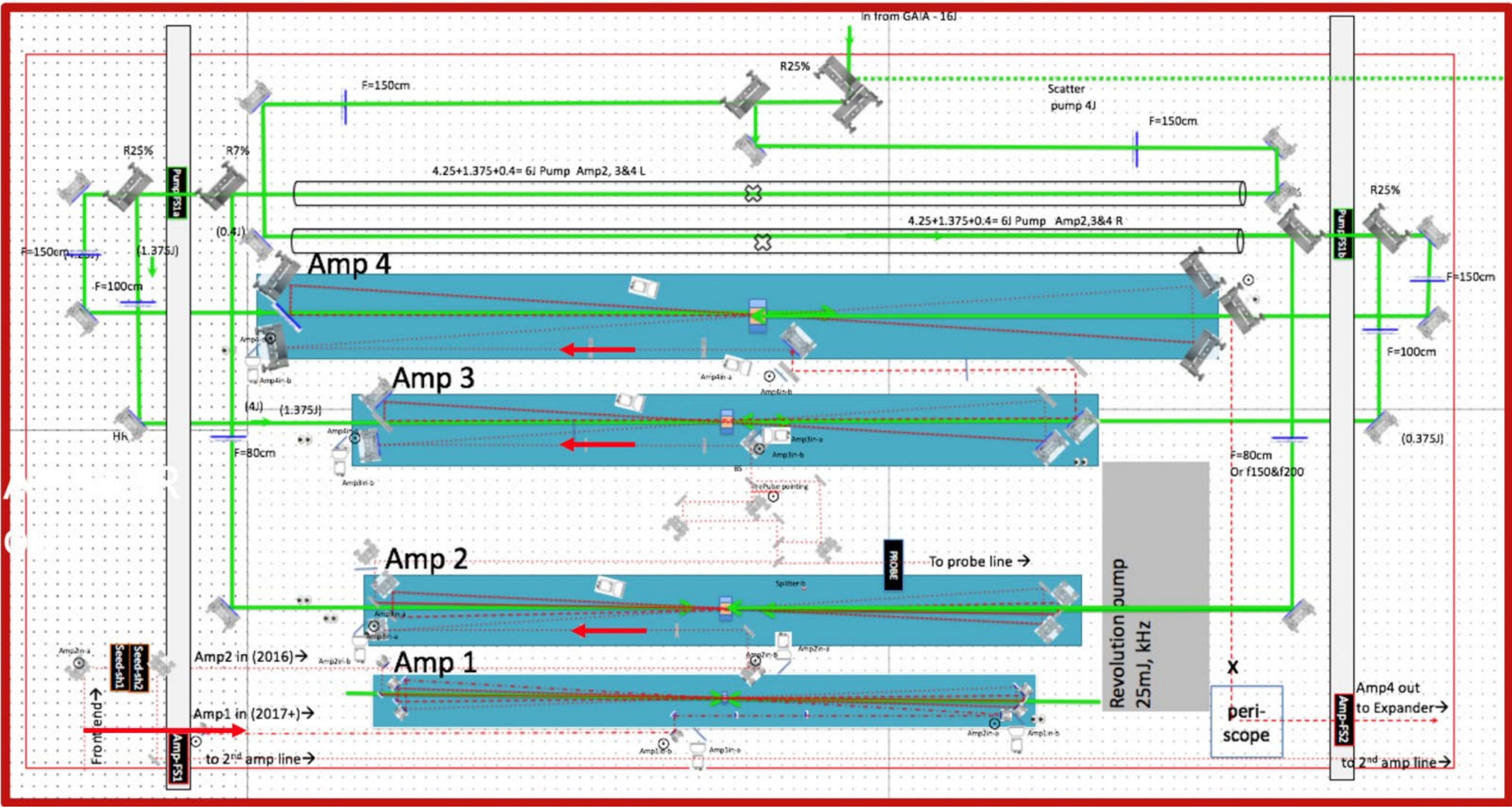
– *Light induced damage mechanisms in optical materials.*

– *Comprehensive databases for materials and material properties.*

Scope – simulating crystal CPA amplifiers



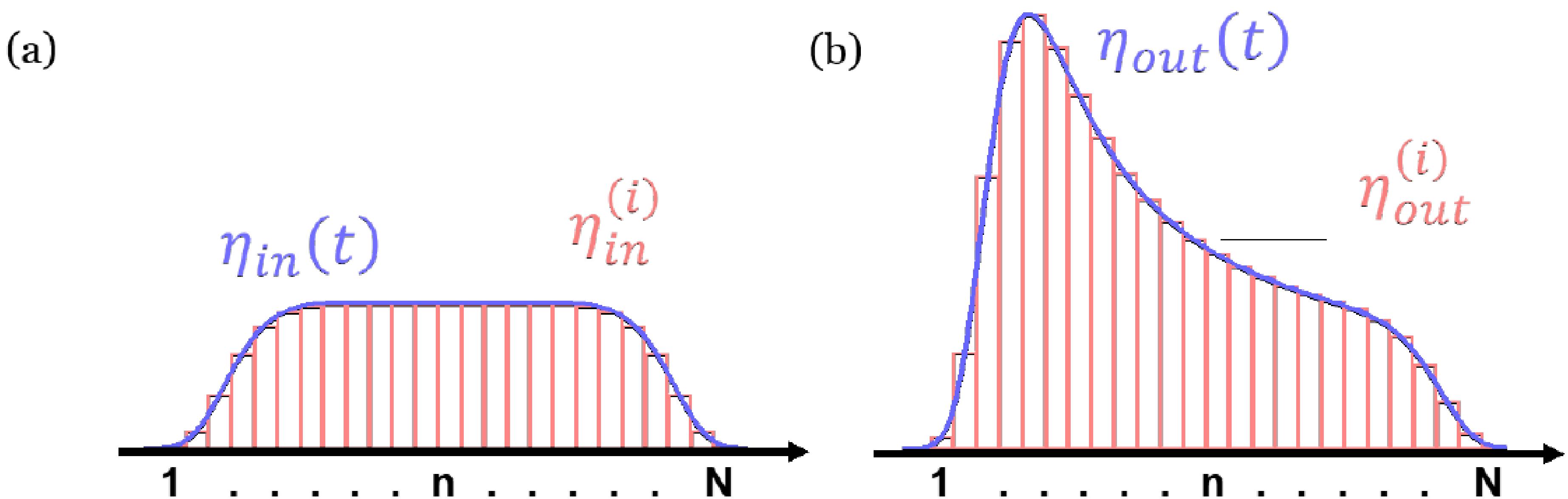
100 TW Ti:Sapphire amplifier layout



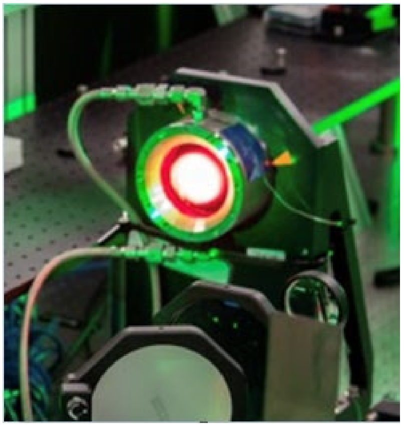
Example 100 TW laser amplifier layout, courtesy of Berkely Lab's BELLA Center

Modeling approach – slice the laser pulse

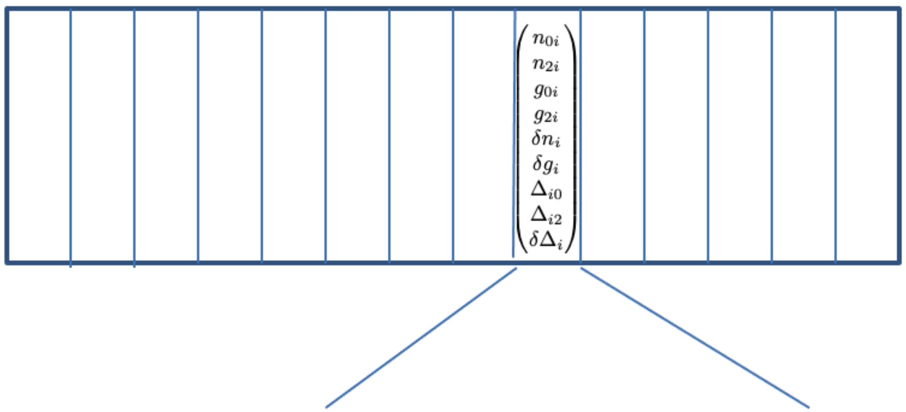
- An operator splitting approach is used
  - simpler and faster than solving 3D PDEs
  - laser pulse slices interact with crystal slices
  - 2D Cartesian wavefront is the fundamental object
  - use open source SRW code for physical optics
    - potentially to be replaced by linear canonical transforms
  - GUI to be implemented via **Sirepo.com**



Modeling approach – slice the crystal



- Fundamental interaction is a laser pulse propagating through a crystal slice
  - Frantz-Nodvik approximation works well
  - 2D Cartesian mesh for transverse variations
  - quadratic radial variation is useful approximation
    - **nonlinear effects can be included perturbatively**



Index of refraction

$$n_i(r) = n_{0i} - n_{2i} r^2 + \delta n_i(x, y)$$

Gain values

$$g_i(r) = g_{0i} - g_{2i} r^2 + \delta g_i(x, y)$$

Population inversion density

$$\Delta_i(r) = \Delta_{i0} - \frac{1}{2}\Delta_{i2} r^2 + \delta\Delta_i(x, y)$$

$i=0 \ldots N_{sl}-1$

<u>Operator splitting algorithm</u>

- Apply complex LCT for first half
- Apply gain factor and non-linear kicks (g_0, $\boldsymbol{\delta}$n and $\boldsymbol{\delta}$g)
- Apply complex LCT for second half
- Update g_i and Δ_i based on Frantz-Nodvik eqns with wavefront intensity

Leveraging Synchrotron Radiation Workshop (SRW)[1]

- Prop. wavefronts via linear canonical transforms (LCT)[2]
  - we use the decomposition of Pei and Huang[3]
  - this recasts a standard ABCD matrix into 3 simpler matrices
    - each of which SRW can use to transform wavefront

- We are developing a Python library for LCTs[4]
  - enables propagation via more general ABCD matrices

[1] O. Chubar, SRW: Synchrotron Radiation Workshop, https://github.com/ochubar/srw

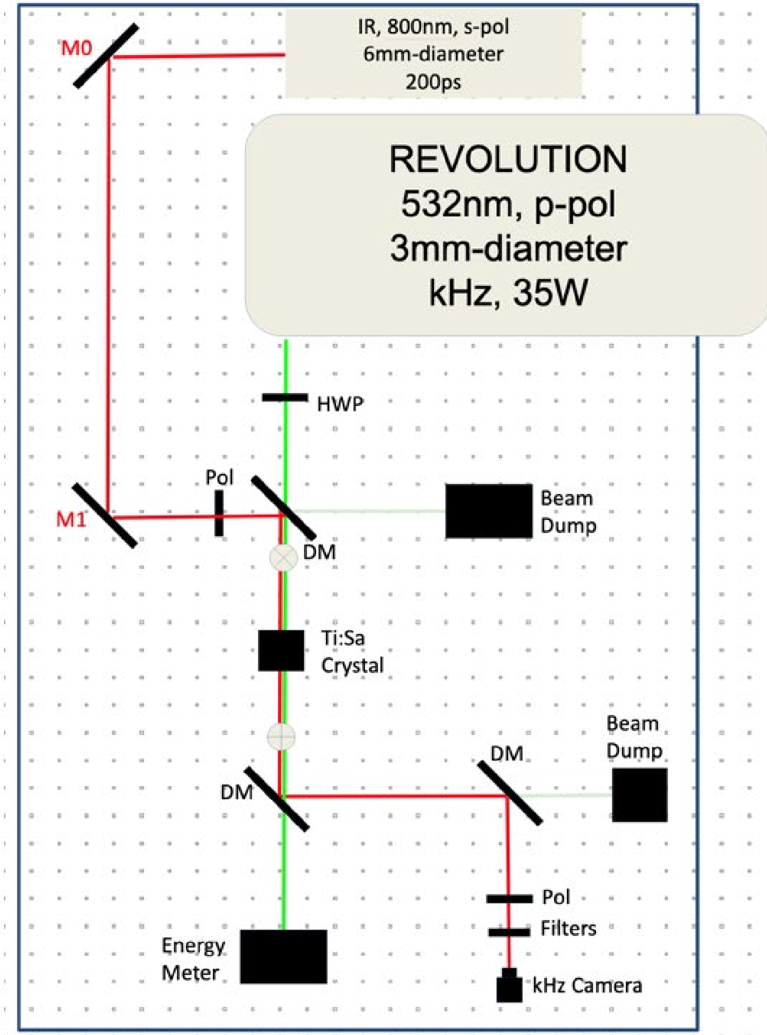
O. Chubar and R. Celestre, "Memory and CPU efficient computation of the Fresnel free-space propagator in Fourier optics simulations," *Optics Express*, **V.27** (2019); doi:10.1364/OE.27.028750

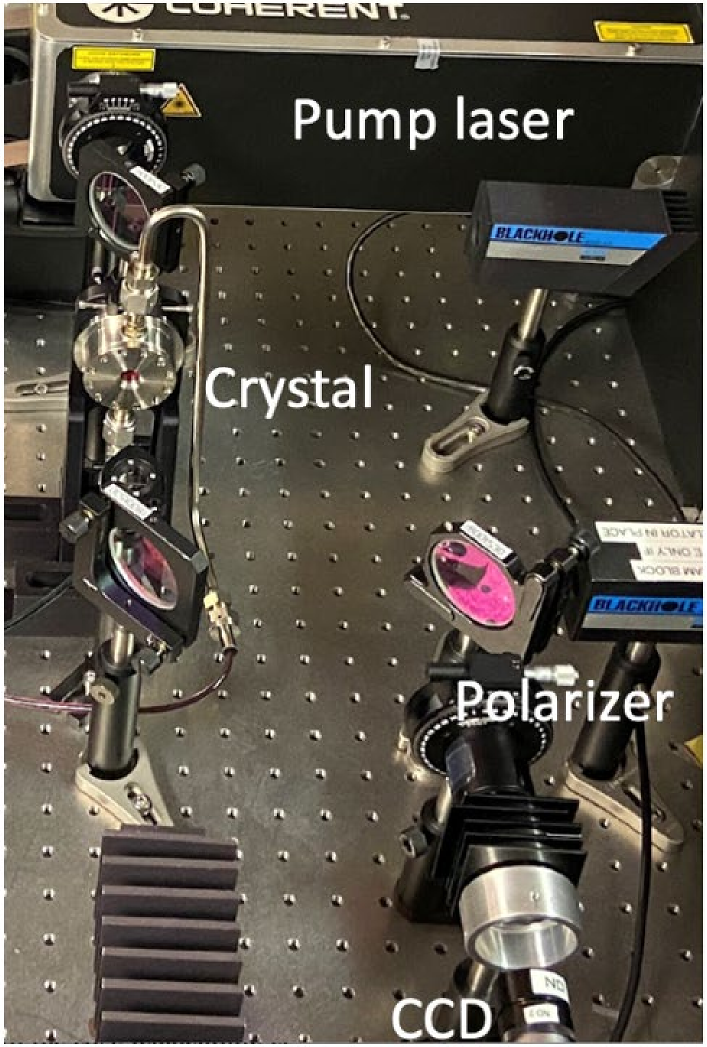
[2] J. Healy *et al.*, *Linear Canonical Transforms: Theory and Applications*, **V.198** (2016); doi:10.1007/978-1-4939-3028-9

[3] S.-C. Pei & S.-G. Huang, "Two-dimensional nonseparable discrete linear canonical transform based on CM-CC-CM-CC decomposition," J. Opt. Soc. Am. A, **V.33** (2016); doi:10.1364/JOSAA.33.000214

[4] B. Nash *et al.*, "Linear Canonical Transform Library for Fast Coherent X-Ray Wavefront Propagation," Intern. Part. Accel. Conf. (2022); doi:10.18429/JACoW-IPAC2022-THPOPT068

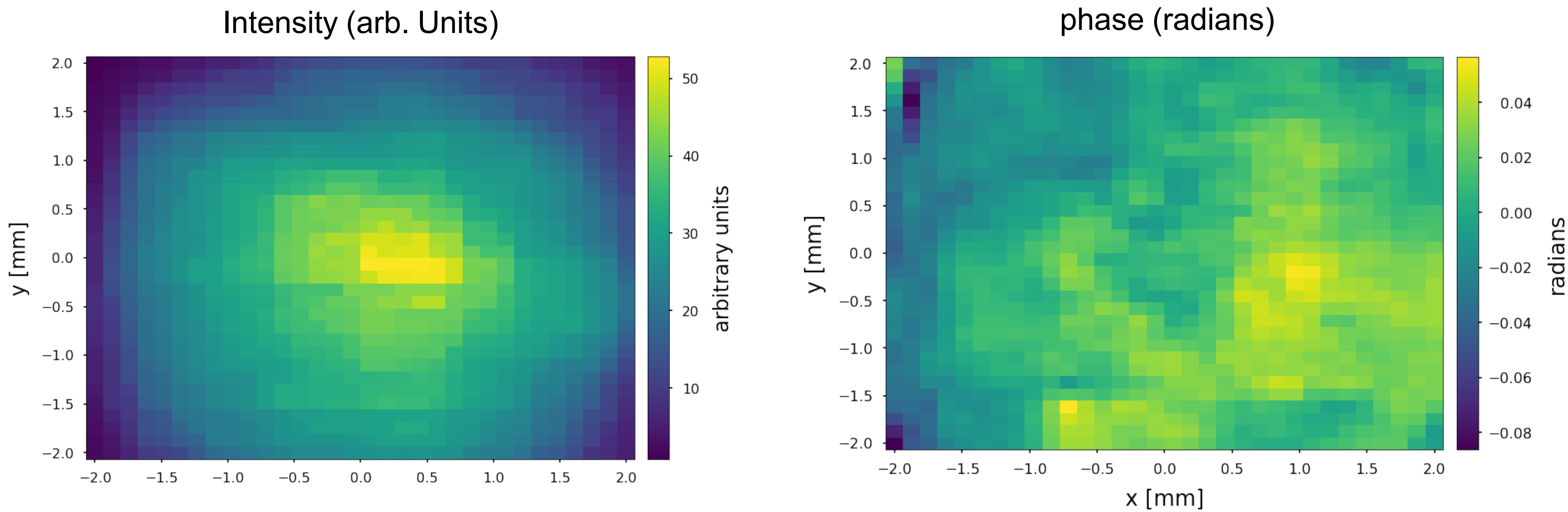
Experimental setup





Experiments conducted at the LBNL BELLA Center

WFS images taken as initial condition for simulations

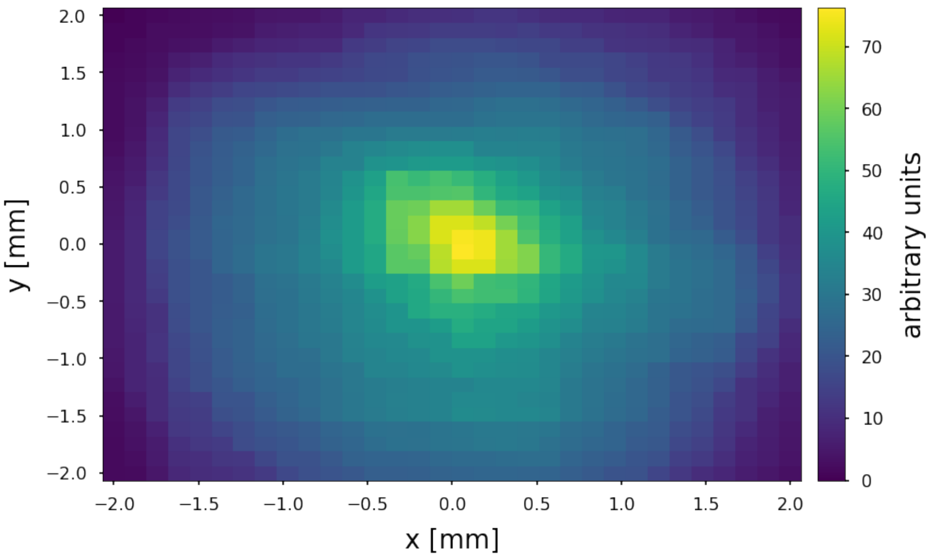


- Wavefront sensor (WFS) images
  - taken in the absence of crystal pumping (no thermal effects)
  - 32x40 pixels is typical WFS resolution

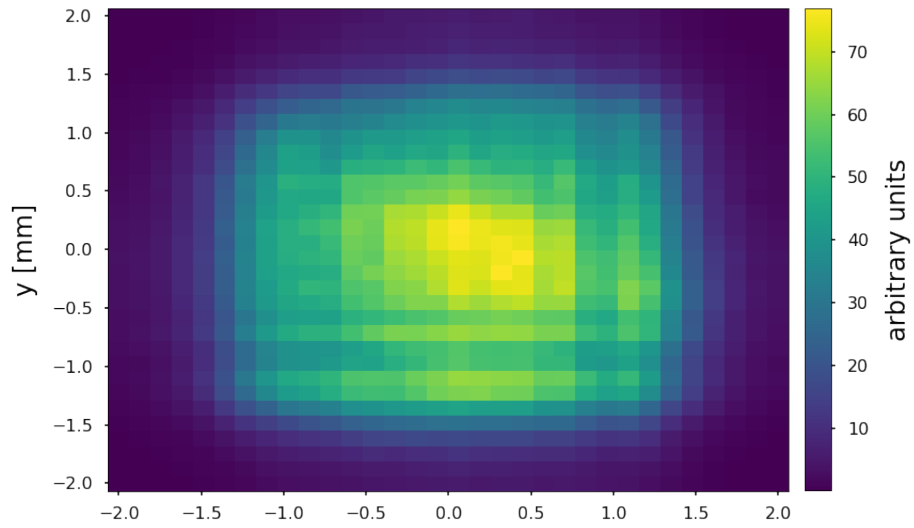
- SRW wavefronts are instantiated – identical to the data
  - truncated to 32x32 mesh to accommodate FFT algorithms

Comparing simulations with data from a warm crystal

Experiment: Intensity (arb. Units)



Simulation: Intensity (arb. Units)



- WFS image (left), taken with crystal pumped at 1 kHz
  - thermal lensing increases peak intensity from 50 to 70
  - pulse timing chosen to prevent laser amplification effects
  - seed pulse wider than pump pulse (not typical for amplifier)
- Simulation assumes linear focusing, which is not correct
  - absence of focusing at large radii can be included in simulations
  - focusing force is increased in simulation to match peak intensity

Initial lessons learned

- Initial experiment/simulation validation effort is underway
  - the simulated laser pulses are instantiated from WFS image pairs
    - **the images: intensity & phase**
    - **32x40 pixel wavefronts (truncated to 32x32) propagate well with SRW**
- dn/dT (aka n_2) inferred from thermal modeling of Ti:Sapph crystal
  - simulations done with the open source FEniCS finite-element solver

[5] FEniCS, https://fenicsproject.org

[6] M.A. Rincon *et al*., "A nonlinear heat equation with temperature-dependent parameters," Math. Phys. Electron. J. **12** (2006); http://www.maia.ub.es/mpej/Vol/12/5.pdf

[7] D.T. Abell *et al*., "Thermal Modeling and Benchmarking of Crystalline Laser Amplifiers," Intern. Part. Accel. Conf. (2022); doi:10.18429/JACoW-IPAC2022-THPOTK062

- Thermal focusing seen in experiments are stronger than simulation
  - the discrepancy is not yet understood
  - two possibilities to consider – both to be addressed with FEniCS
    - **nonlinear effects due to temperature dependence of physical quantities**
    - **outward bulging of the crystal faces could yield focusing effects**

Definition of the 1D Linear Canonical Transform

The *Linear Canonical Transform*, or *LCT*, of a function $f\,(\mathrm{u})$ is defined by the rule (convention here is *not* unique)

$$\mathcal{L}_M[f](v) = \mathrm{e}^{-\mathrm{i}\pi/4}\sqrt{\beta}\int_{-\infty}^{\infty} f(u)\,\mathrm{e}^{\mathrm{i}\pi(\alpha v^2 - 2\beta u v + \gamma u^2)}\,\mathrm{d}u$$

phase factor "requires some care" (KB Wolf)

The properties of the *particular* LCT are determined by the associated 2×2 symplectic matrix M that describes the ray optics of an arbitrary beamline.

det M = 1

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \gamma/\beta & 1/\beta \\ \alpha\gamma/\beta - \beta & \alpha/\beta \end{pmatrix}$$

The LCT obeys the very important *group property*

$$\mathcal{L}_{M_2} \circ \mathcal{L}_{M_1} = \mathcal{L}_{M_2 \cdot M_1}$$

- Other integral transforms are special cases of the general LCT:
  - Fourier & fractional Fourier, Fresnel, chirp multiplication & scaling
- An LCT can be composed of simpler LCTs
  - decompose matrices, then compose the corresponding transforms

Definition of the 2D LCT

The 2D LCT of a function $f(\vec{u})$ is defined by the rule (as in 1D, the convention here is *not* unique)

$$\mathcal{L}_M[f](\vec{u}) = \frac{1}{\sqrt{\det \mathrm{i}B}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left[\mathrm{i}\pi p(\vec{u},\vec{v})\right] f(\vec{v})\, \mathrm{d}^2\vec{v}$$

$$p(\vec{u},\vec{v}) = \vec{u}^{\,\mathrm{tr}} D B^{-1} \vec{u} - 2\vec{v}^{\,\mathrm{tr}} B^{-1} \vec{u} + \vec{v}^{\,\mathrm{tr}} B^{-1} A \vec{v}$$

The properties of the *particular* LCT are determined by the associated 4×4 real symplectic matrix

$$A^{\mathrm{tr}}C = C^{\mathrm{tr}}A, \qquad B^{\mathrm{tr}}D = D^{\mathrm{tr}}B, \qquad A^{\mathrm{tr}}D - C^{\mathrm{tr}}B = I$$
$$AB^{\mathrm{tr}} = BA^{\mathrm{tr}}, \qquad CD^{\mathrm{tr}} = DC^{\mathrm{tr}}, \qquad AD^{\mathrm{tr}} - BC^{\mathrm{tr}} = I$$

Here the superscript tr denotes matrix transposition, and I denotes the 2×2 identity matrix. The fact that the submatrices obey these particular relations tells us that the matrix M acts on phase-space variables in the order (q_1, q_2, p_1, p_2). See, for example, [Dragt, 2020, §3.2]. This means that if one extracts the ray optical matrix M with the phase-space variables given in some other order, *e.g.* (q_1, p_1, q_2, p_2), then one must make sure to appropriately permute the entries of M.

The 2D LCT also obeys the *group property*

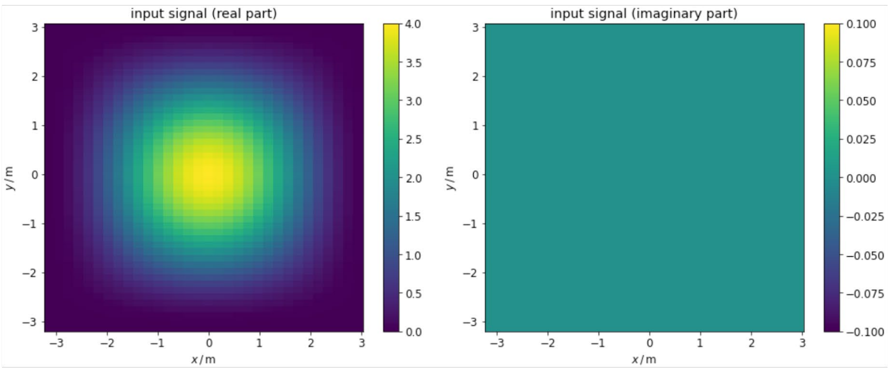
$$\mathcal{L}_{M_2} \circ \mathcal{L}_{M_1} = \mathcal{L}_{M_2 \cdot M_1}$$

with the same implications for developing fast computational methods for evaluating LCTs in two degrees of freedom.

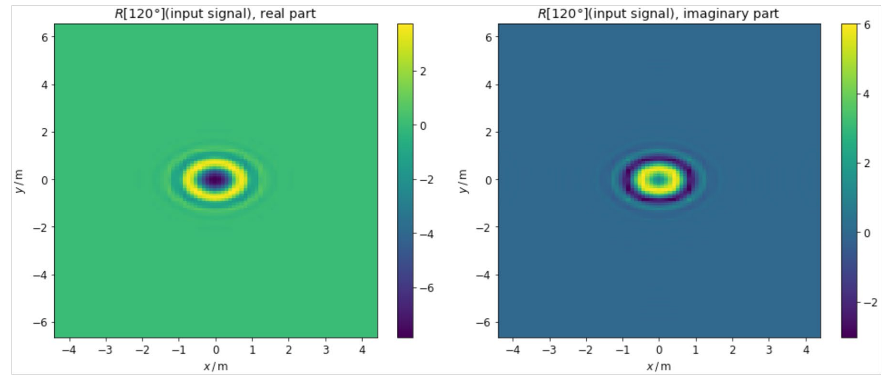
A.J. Dragt, *Lie Methods for Nonlinear Dynamics, with Applications to Accelerator Physics* (Nov. 2020); https://physics.umd.edu/dsat/

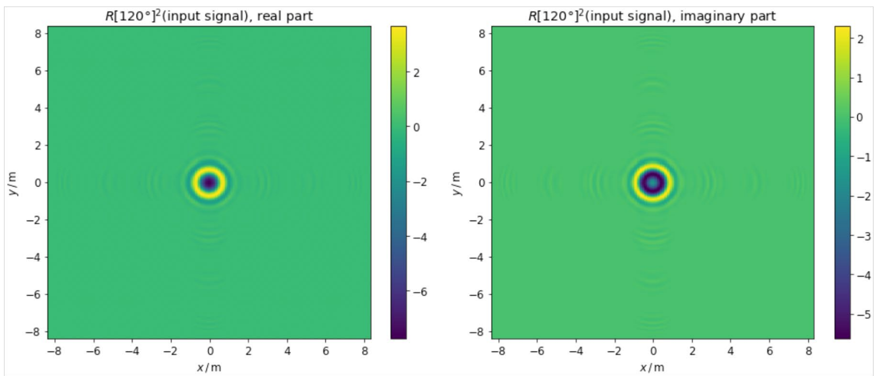
Testing the separable 2D LCT

[Initial beam]

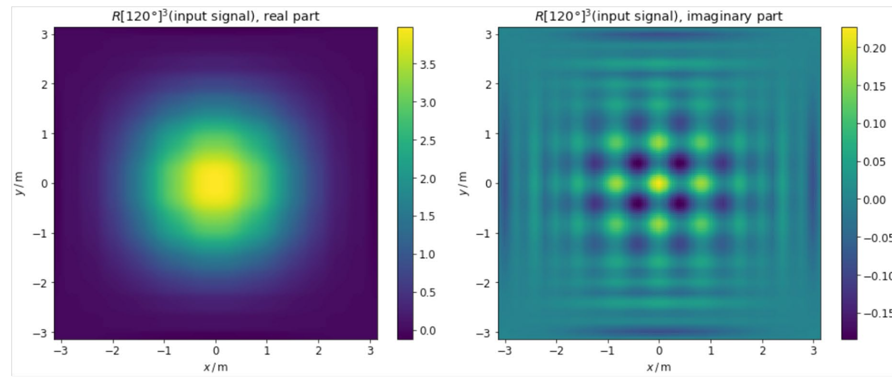


[LCT with $M_x = M_y = R(120°)$]





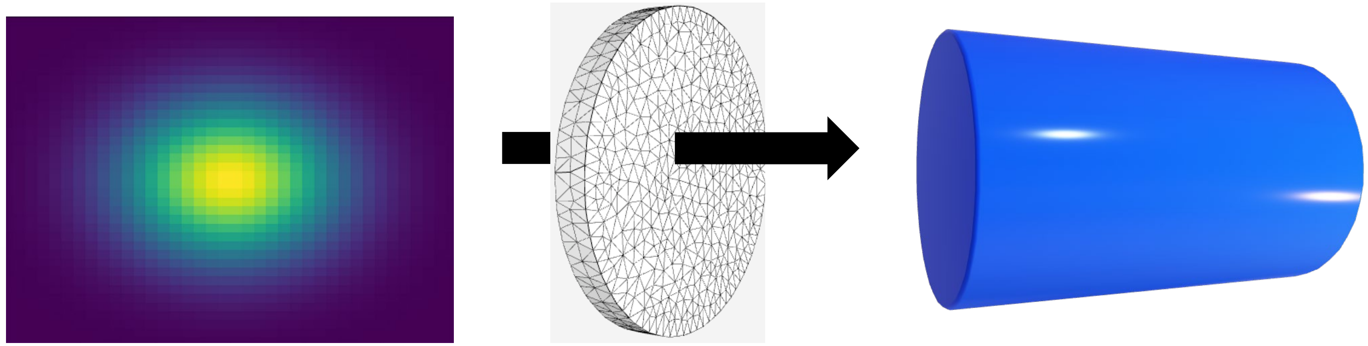
[LCT with $M_x = M_y = R(120°)]^2$



[LCT with $M_x = M_y = R(120°)]^3$

- Application of three 120 deg transformations:
  - approximately recovers the original electric fields

Gaussian duct, for quadratic variation of n_0

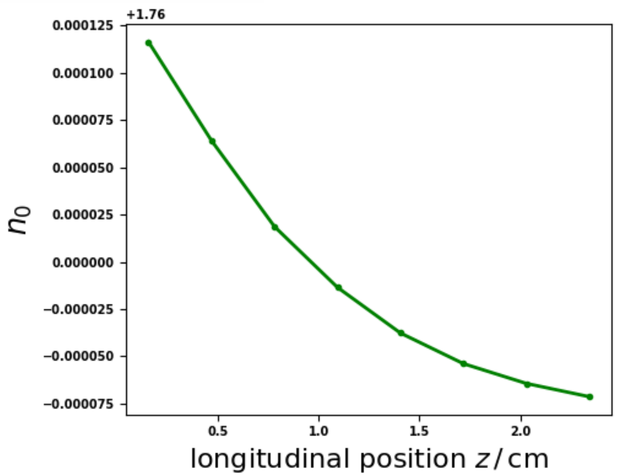


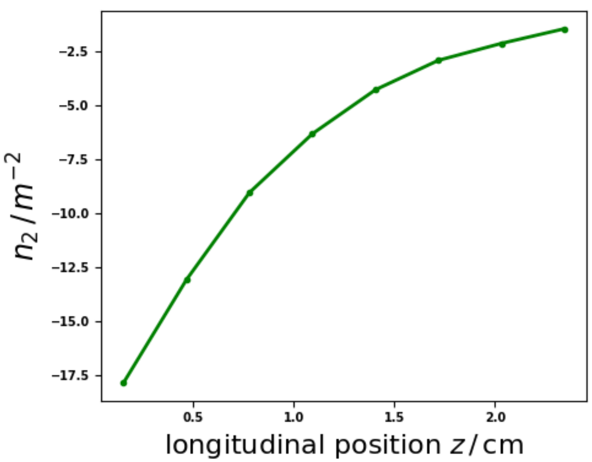
Gaussian duct:

$$\begin{pmatrix} \cos\gamma z & (n_0\gamma)^{-1}\sin\gamma z \\ -(n_0\gamma)\sin\gamma z & \cos\gamma z \end{pmatrix}$$

where the radially varying refractive index is

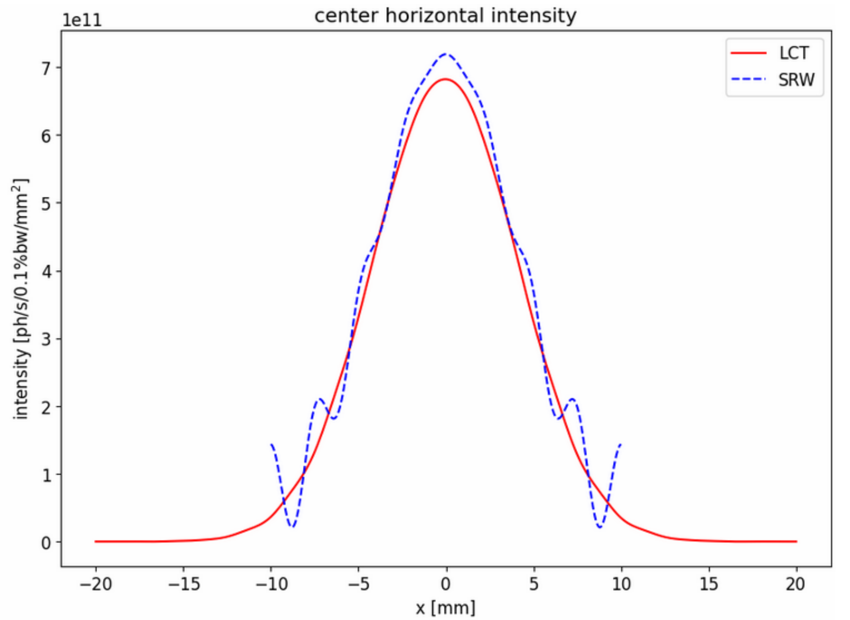
$$n(r) = n_0 - \frac{1}{2}n_2 r^2, \quad \gamma^2 = n_2 / n_0$$





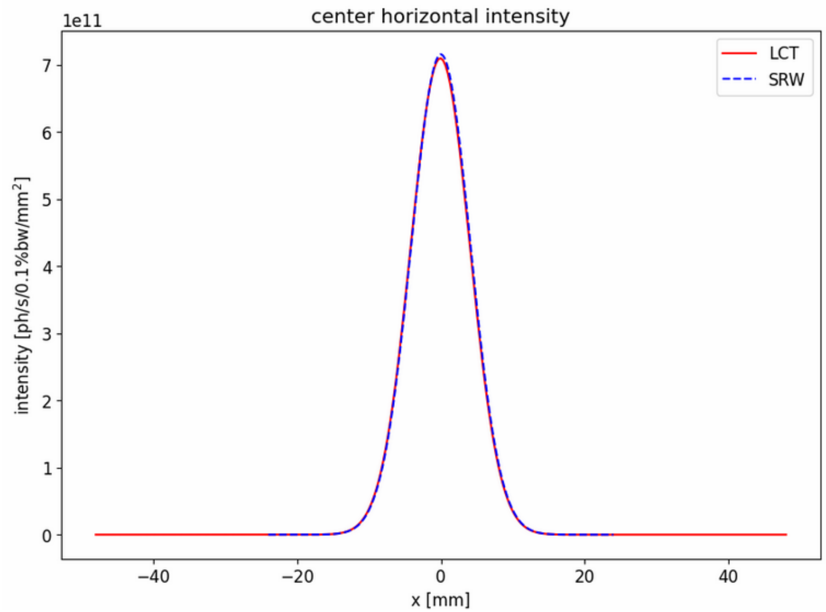
Gaussian duct, comparing **SRW vs LCT**

n_{sigma}: 10
mesh: 300 x 300
LCT calc time: 0.47 s
SRW calc time: 0.013 s



	Initial wavefront	SRW wavefront	LCT wavefront
Integrated energy:	75.6595989	75.659585	73.826219

n_{sigma}: 24
mesh: 300 x 300 points
LCT calc time: 0.45 s
SRW calc time: 0.013 s



	Initial wavefront	SRW wavefront	LCT wavefront
Integrated energy:	75.6595989	75.6595881	75.2317080

Next steps: include amplification effects (1)

- 1D Frantz-Nodvik applied in each cell of the 2D mesh
  - will soon be integrated with the propagation code:

The Frantz-Nodvik (FN) equations describe propagation of a pulse across gain material composed of two-level atoms:

$$\frac{\partial n}{\partial t} + c\frac{\partial n}{\partial z} = \sigma c n \Delta; \qquad \frac{\partial \Delta}{\partial t} = -\gamma \sigma c n \Delta$$

Here, z and t denote, respectively, distance along the beam axis and the time; $n(z,t)$ denotes photon number density in the medium; and $\Delta(z,t)$ denotes the "population inversion", $N_2 - N_1$, giving the difference between the number density of atoms in the excited state as compared to those in the ground state. In addition, c denotes the speed of light in the medium, σ the resonance absorption cross-section, and γ a factor related to the relative degeneracy of the ground and excited states.

Next steps: include amplification effects (2)

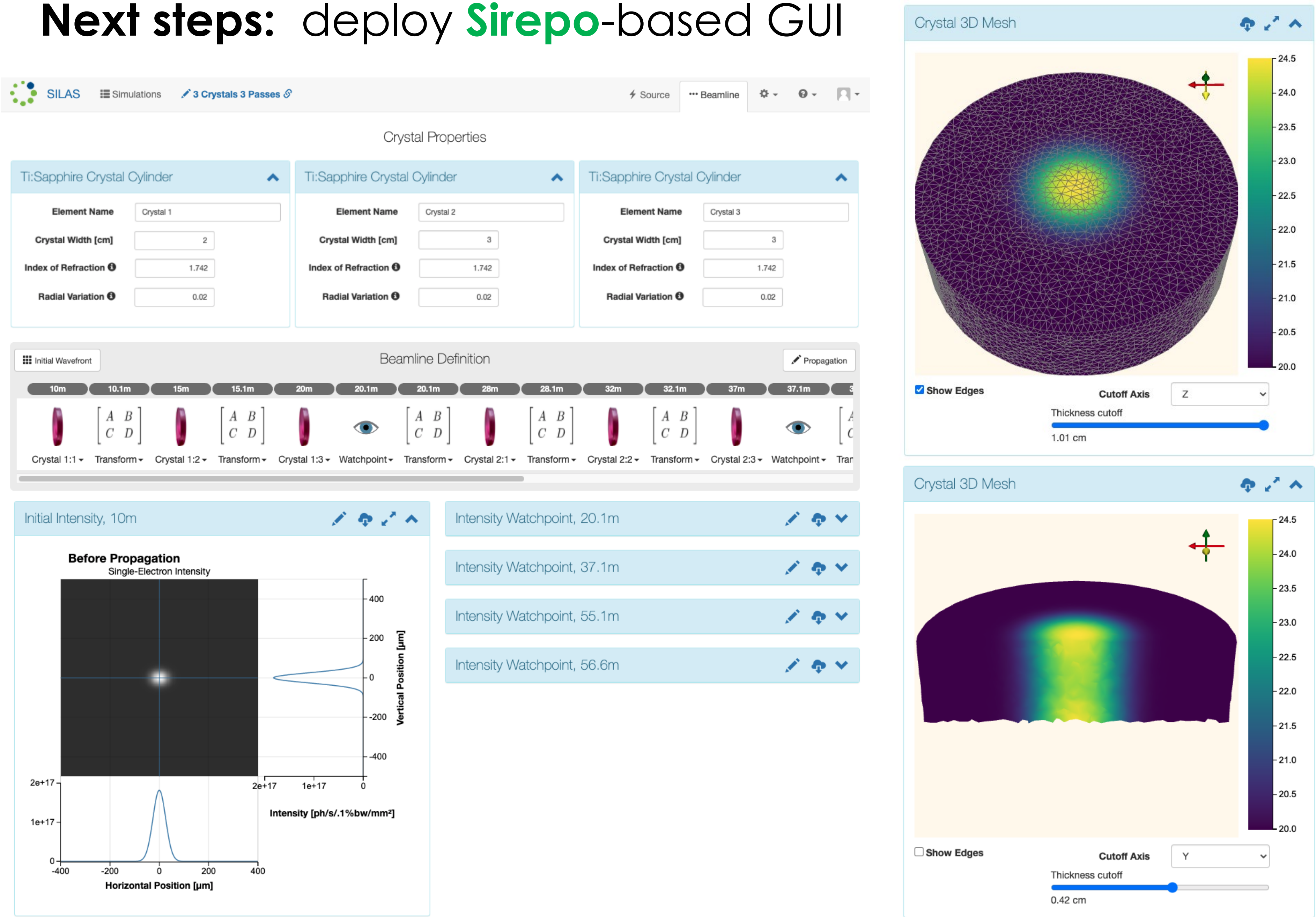
The very simplest case is that of a square laser pulse of duration τ and a uniform photon density n_0 incident on a crystal of length L and uniform population inversion Δ_0. Define η as the total number of incident photons per unit area: $\eta = n_0 c\tau$.

$$n_{\mathrm{sq}}(z,t) = \frac{n_0}{1 - [1 - \exp(-\sigma\Delta_0 z)]\exp(-\gamma\sigma\eta(t - z/c)/\tau)}.$$

A photon that enters at time t exits at time $t + L/c$. Hence the photon density at the exit of the crystal is given by $(z,t) = (L, t + L/c)$. The energy gain G_E is the ratio of the total number of photons exiting the crystal to the total number, η, that entered the crystal.

$$G_E = \frac{1}{\gamma\sigma\eta}\ln\{1 + \exp(\sigma\Delta_0 L)[\exp(\gamma\sigma\eta) - 1]\}$$

Next steps: deploy Sirepo-based GUI



Summary

- High-power short-pulse lasers are moving toward high rep rate; e.g., 1 PW at 1 KHz
- There is a need for widely-available modeling software
  - developing a Python library, https://github.com/radiasoft/rslaser/
  - operator splitting (slicing pulse & crystals) allows for 2D simulations
    - Fourier optics, together with laser amplification and thermal effects
    - Fourier optics is enabled via LCTs, initially by leveraging the SRW code
    - thermal modeling via the FEniCS code; available in Jupyter server of **Sirepo.com**
  - **Sirepo.com** UI is being developed to support these capabilities
- Experiments at the BELLA Center enable detailed comparisons
  - the experimentally observed thermal focusing is stronger than expected
  - possible explanations are being explored
- Future work: add amplification & compare with 100 TW amplifier

Thanks!

Questions?

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under Award Numbers DE-SC0020931 and DE-AC02-05CH11231.

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