

Single-shot, transverse self-wakefield reconstruction from screen images

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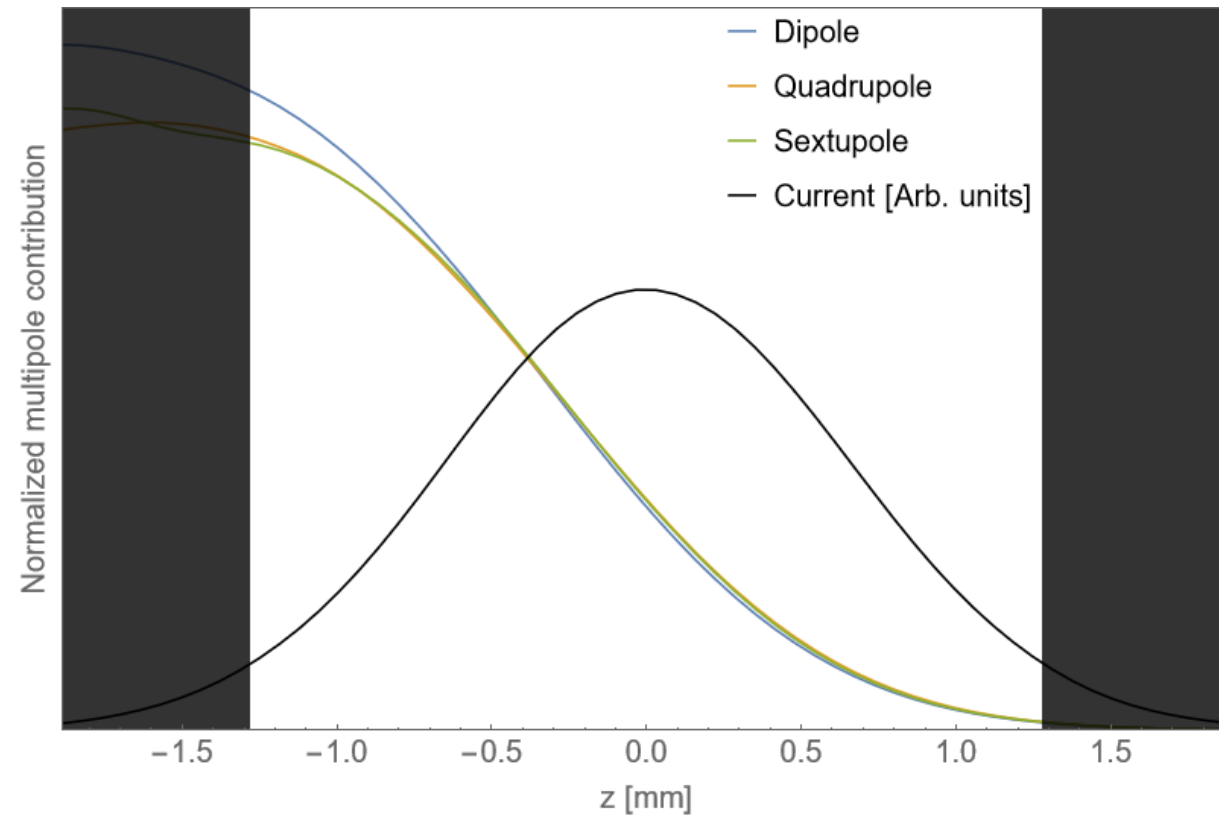
Motivation

- Goal is to understand the transverse self-wakefields
 - Can be both beneficial (streakers) or problematic (BBU)
 - See Walter Lynn's talk "Observation of Skewed Electromagnetic Wakefields in an Asymmetric Structure Driven by Flat Electron Bunches"
- Complements simulation
- May pair with longitudinal diagnostics for greater insight
- May play well with multileaf collimators; giving very granular, real-time control over wakes

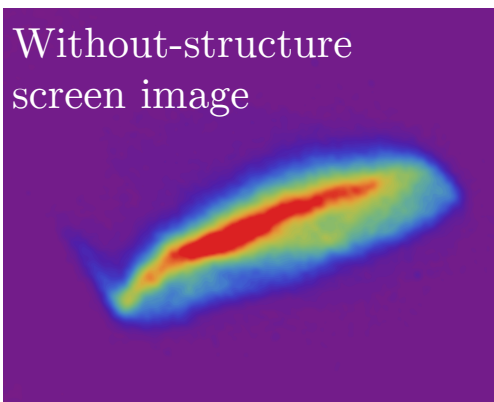
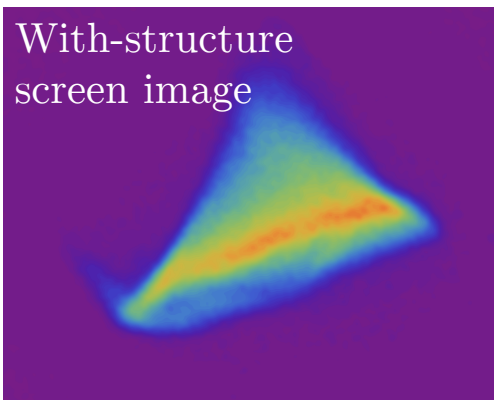


Assumptions

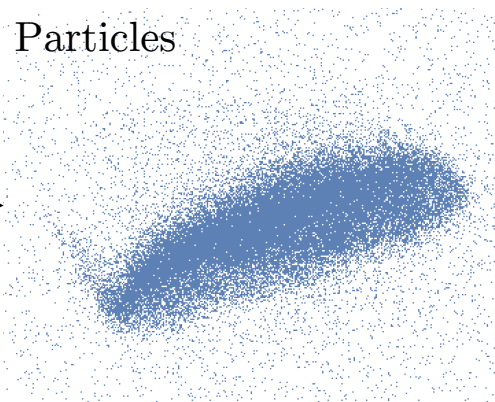
- Thin lens approximation
 - Beam spatial distribution does not appreciably change over interaction
 - Each particle only “feels” one part of the wakefield
- Wakefield is the same, up to multiplicative constant, along ζ
 - Satisfied if beam is short relative to the wavelength of all relevant modes
 - Each mode’s wake response approximated by linear function with different slopes; ratio constant
- Wakefield is well approximated by low order multipoles



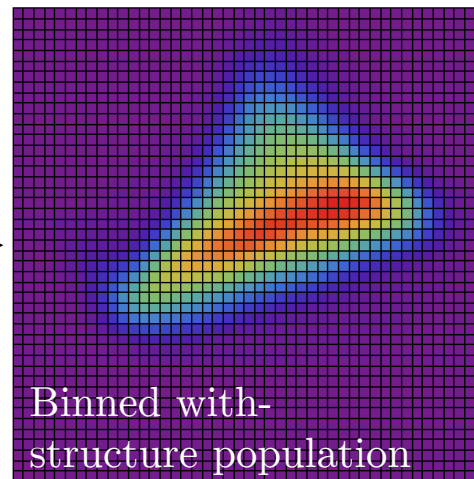
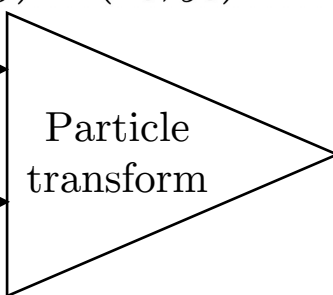
Technique summary



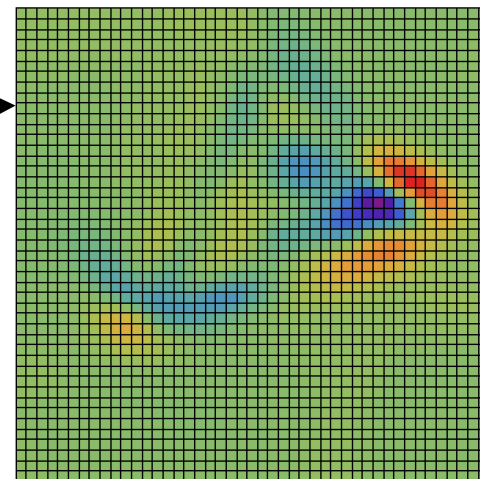
Particles



$$(x, y) + (x_c, y_c) + \kappa \mathbf{W}$$

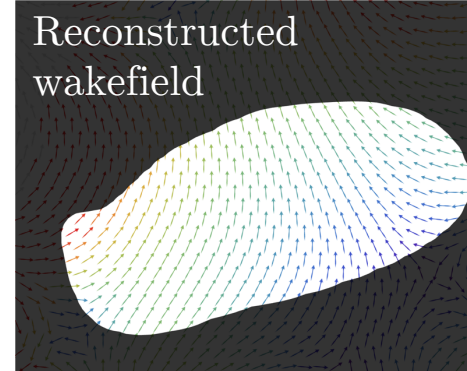


Binwise errors



ϵ_{fit}

Parameter vector



$$\mathbf{W} = - \left\{ \frac{\partial V}{\partial x}, \frac{\partial A}{\partial x} \right\} =$$

$$\{-b_1 - 2a_2y - 2b_2x - 6a_3xy + 3b_3(-x^2 + y^2) + \dots,$$

$$-a_1 - 2a_2x + 2b_2y + 3a_3(-x^2 + y^2) + 6b_3xy + \dots\}$$

Parameter vector: $\{x_c, y_c, \mathcal{K}_1, \mathcal{K}_2, \dots, a_1, b_1, a_2, b_2, \dots\}$

Benchmarking to simulation: setup

Extract exact 3D wakefields, \mathbf{W}_{sim}

Get slice-wise multipole coefficients

$$\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_{\text{slice}}) - \mathbf{W}_{\text{guess}}(x_k, y_k)| \rangle_{95\%}$$

Generate aggregate field

$$\mathbf{c}_{\text{agg}} = \frac{\sum_{\text{slices}} I(z_{\text{slice}}) \mathbf{c}_{\text{slice}}}{\sum_{\text{slices}} I(z_{\text{slice}})}$$

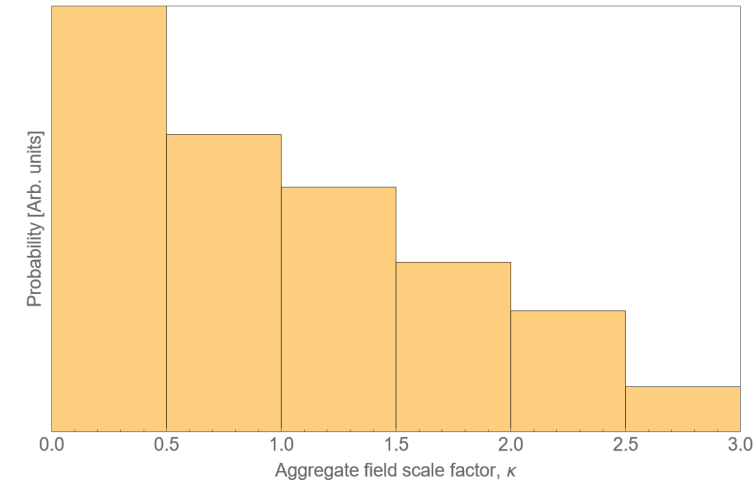
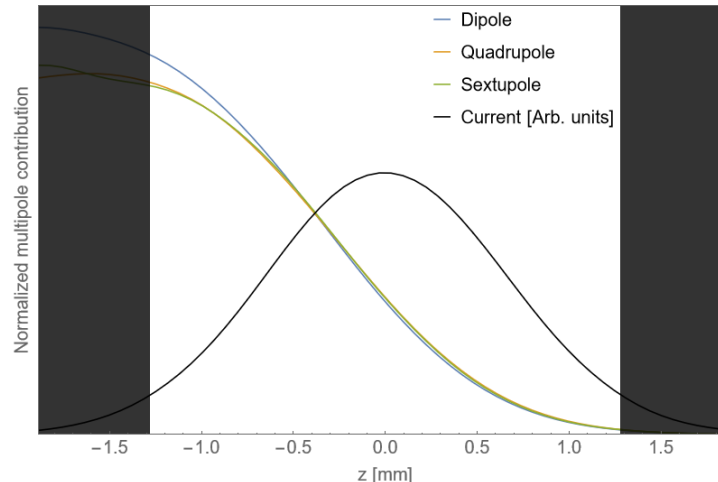
Get $\kappa(z)$ and kick PDF

Quantify with $\epsilon_{\text{slice-wise}}$

$$\epsilon_{\text{slice-wise}} = \frac{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k) - \mathbf{W}_{\text{slice-wise}}(x_k, y_k, z_k)| \rangle_{95\%}}{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k)| \rangle_{95\%}}$$

Quantify with ϵ_{agg}

$$\epsilon_{\text{agg}} = \frac{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k) - \kappa(z_k) \mathbf{W}_{\text{agg}}(x_k, y_k)| \rangle_{95\%}}{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k)| \rangle_{95\%}}$$



500 MeV, 2 nC, 2.2° tilt,
7.7:1 spot size, run past
15 cm long dielectric slab

Benchmarking to simulation: example

Extract exact 3D wakefields, \mathbf{W}_{sim}

Fit to $n=6$

Get slicewise multipole coefficients

$$\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_{\text{slice}}) - \mathbf{W}_{\text{guess}}(x_k, y_k)| \rangle_{95\%}$$

Generate aggregate field

$$\mathbf{c}_{\text{agg}} = \frac{\sum_{\text{slices}} I(z_{\text{slice}}) \mathbf{c}_{\text{slice}}}{\sum_{\text{slices}} I(z_{\text{slice}})}$$

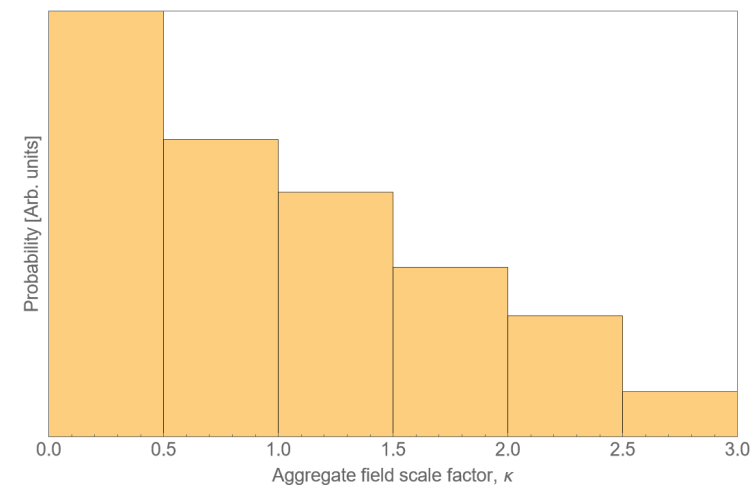
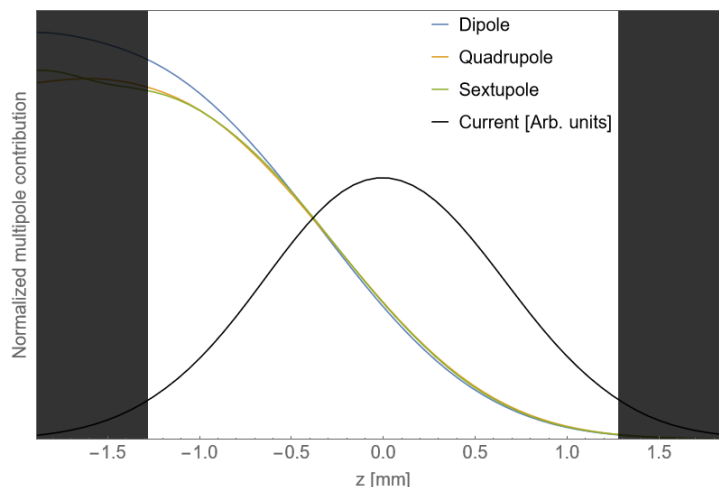
Get $\kappa(z)$ and kick PDF

Quantify with $\epsilon_{\text{slicewise}}$ **1.5%**

$$\epsilon_{\text{slicewise}} = \frac{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k) - \mathbf{W}_{\text{slicewise}}(x_k, y_k, z_k)| \rangle_{95\%}}{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k)| \rangle_{95\%}}$$

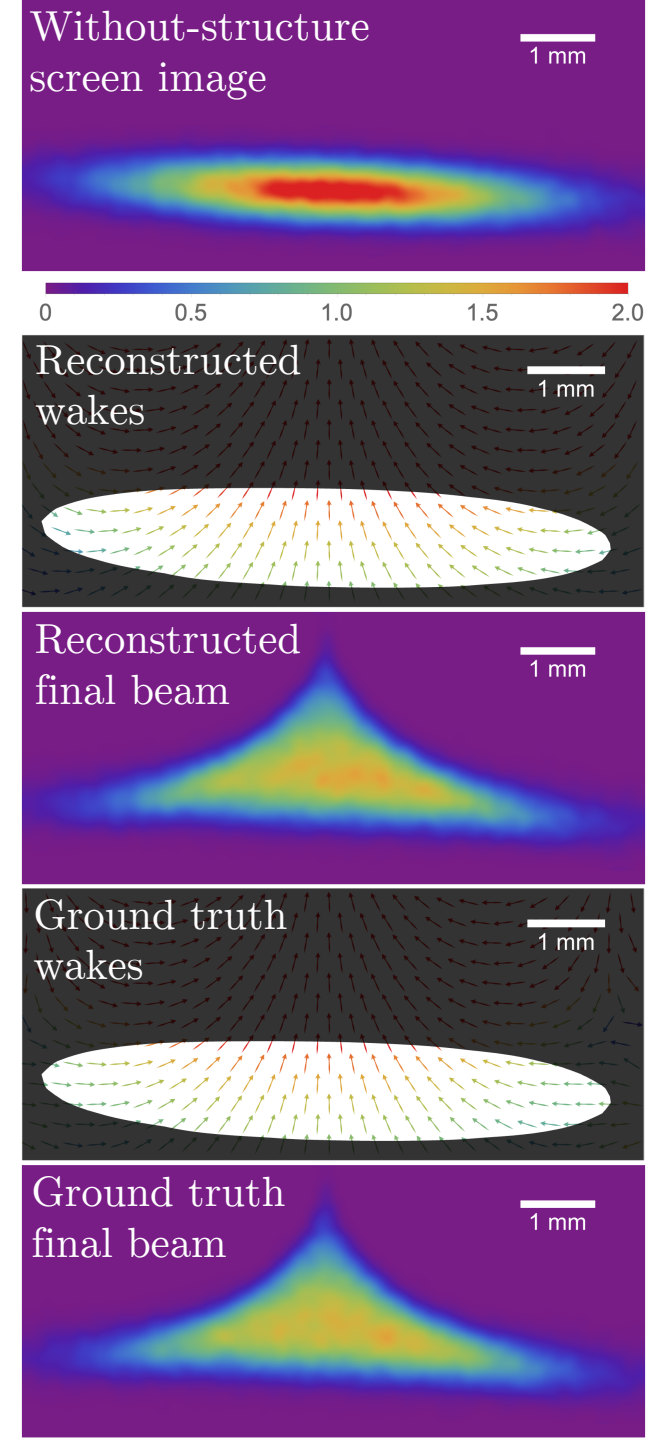
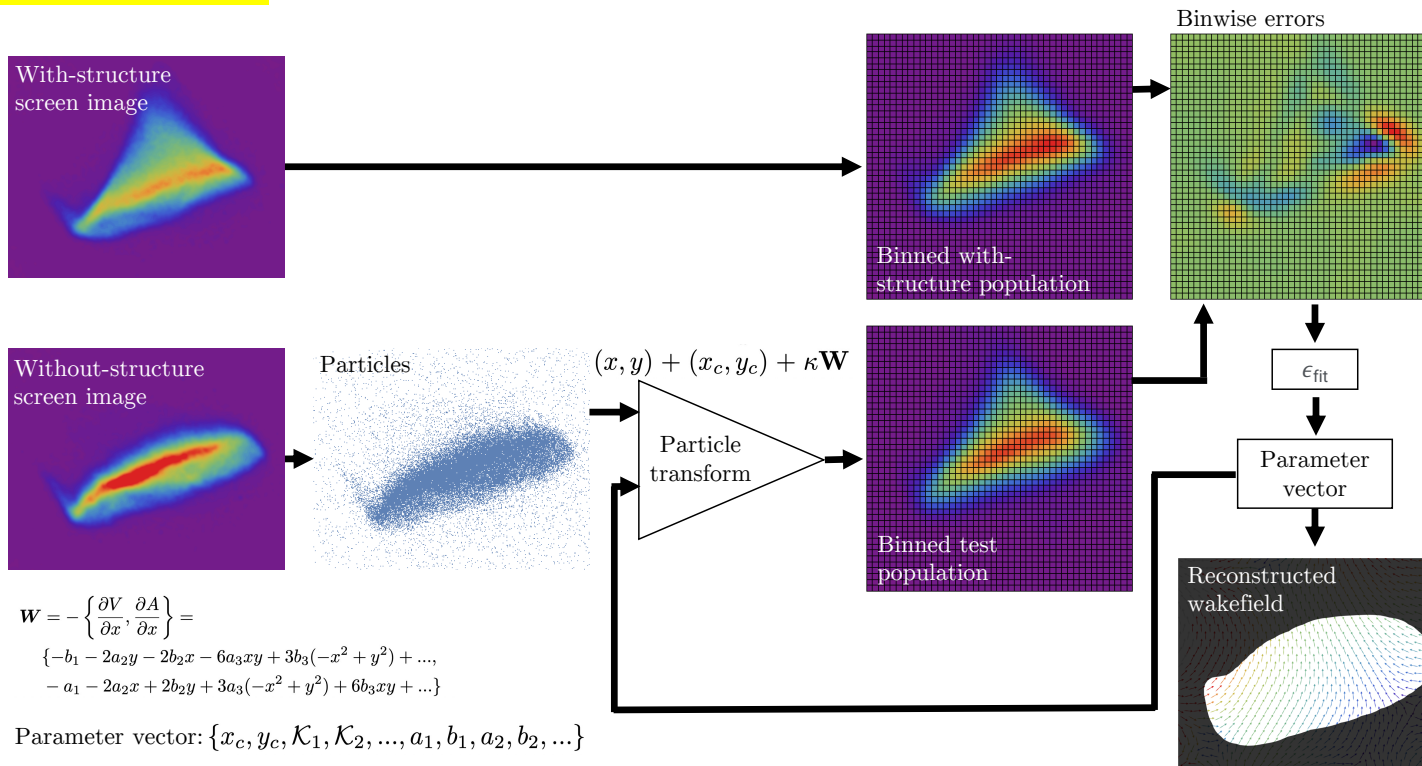
Quantify with ϵ_{agg} **2.3%**

$$\epsilon_{\text{agg}} = \frac{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k) - \kappa(z_k) \mathbf{W}_{\text{agg}}(x_k, y_k)| \rangle_{95\%}}{\langle |\mathbf{W}_{\text{sim}}(x_k, y_k, z_k)| \rangle_{95\%}}$$



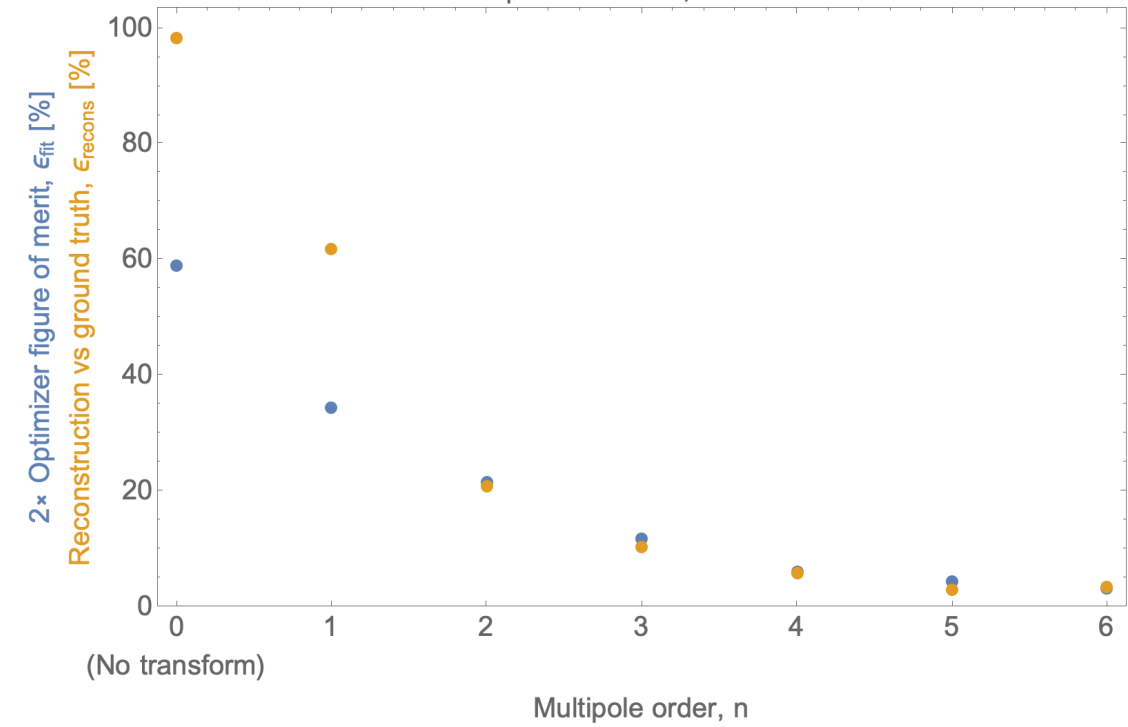
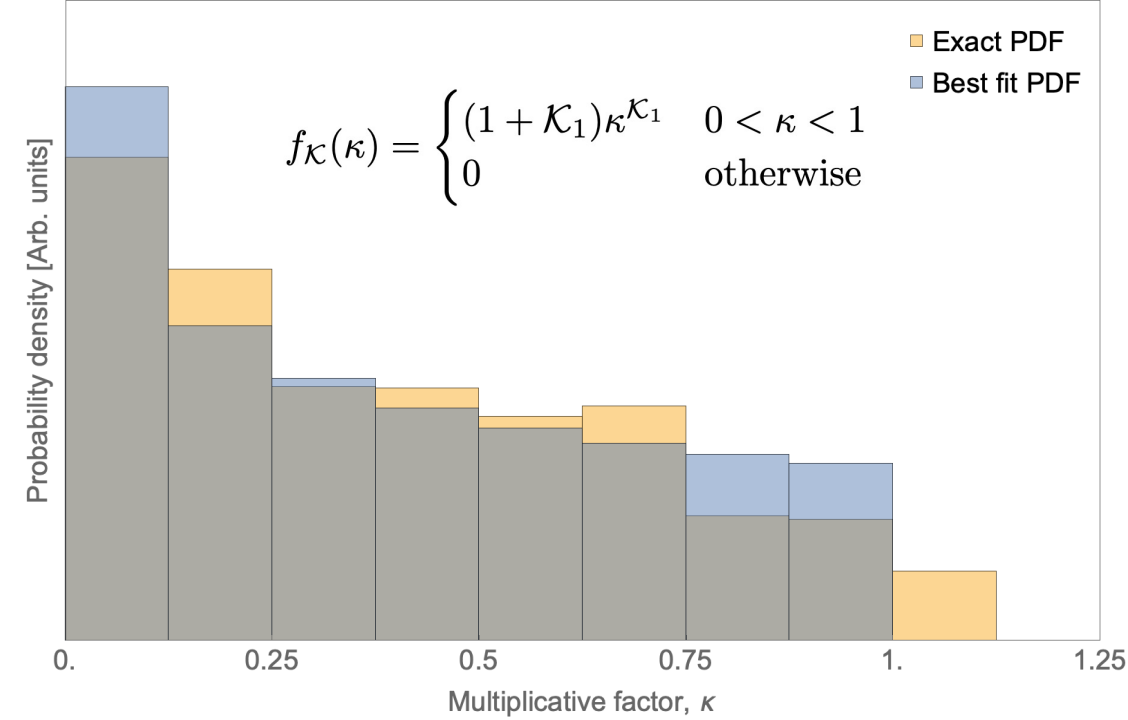
Benchmarking to simulation: reconstruction

- Create virtual screen images
- Reconstruct to $n = 6$, using known κ PDF
 - 12 free parameters
- Optimizer figure of merit: $\epsilon_{\text{fit}} = 1.3\%$
- Compare reconstructed fields to ground truth: $\epsilon_{\text{recons}} = 2.6\%$



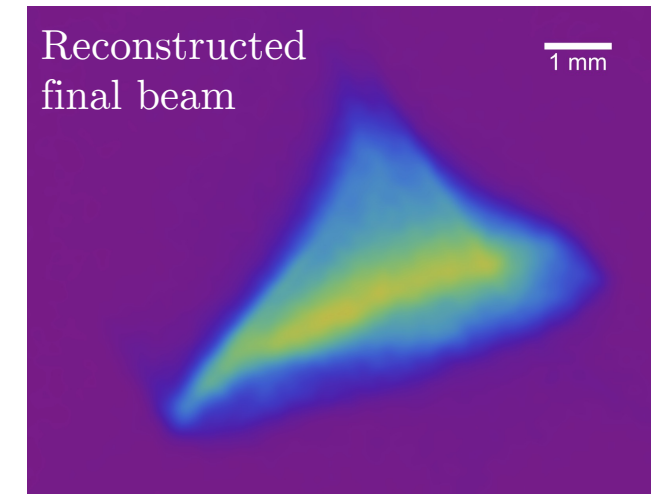
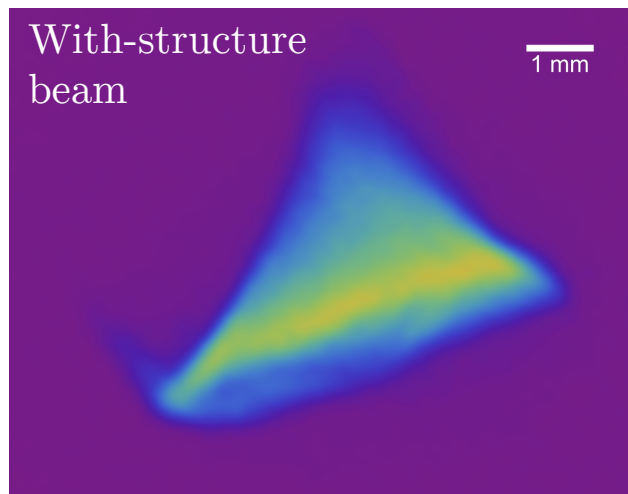
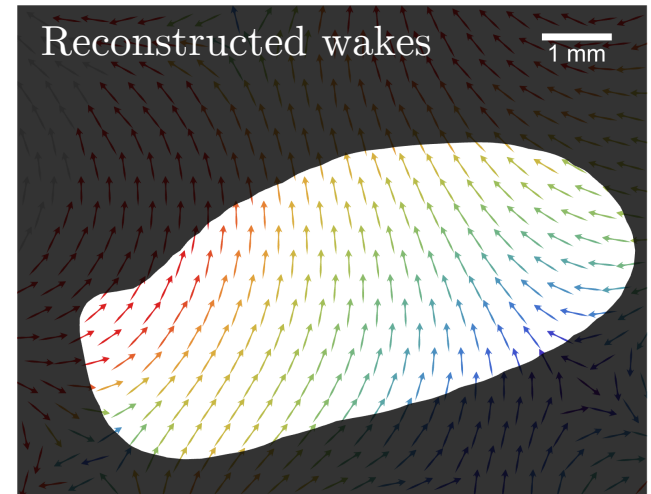
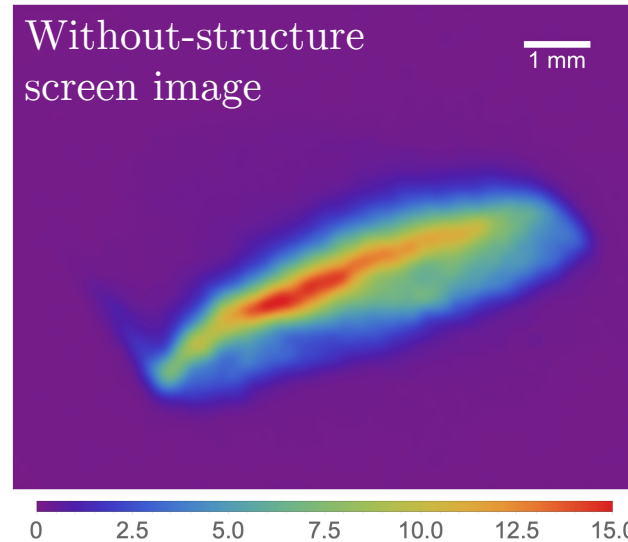
Moving closer to experiment

- Drop known κ PDF, use parameterized distribution
 - Simple but powerful, single parameter
 - ϵ_{fit} from 1.3% to 1.8%, ϵ_{recons} from 2.6% to 3.1%
- Vary n to see how ϵ_{fit} and ϵ_{recons} respond
 - We knew $n=6$ was sufficient, wouldn't know that in experiment before reconstructing



Experimental data reconstruction

- AWA flat beam skew wake data
 - 43 MeV
- Parameterized κ PDF, $n = 6$, x_c and y_c active
 - 15 free parameters
- $\epsilon_{\text{fit}} = 6.0\%$



Conclusion

- Transverse wakefields are important to understand but hard to measure
- Introduced a technique to reconstruct the self-wakefields without additional hardware; a single screen is sufficient
- Relies on assumptions which can be defended
- Benchmarked against simulation data
- Applied to experimental data
- Room to grow

