

Single-Shot Reconstruction of Electron Beam Phase-Space in a Laser Wakefield Accelerator



Y. Ma, M.J.V. Streeter, F. Albert, N. Bourgeois, S. Cipiccia, J.M. Cole, S.J.D. Dann, E. Gerstmayr, I.G. Gonzalez, A. Higginbotham, A.E. Hussein, D.A. Jaroszynski, K. Falk, B. Kettle, K. Krushelnick, N. Lemos, N.C. Lopes, C. Lumsden, O. Lundh, S.P.D. Mangles, K. Miller, W. Mori, Z. Najmudin, Qian Qian, P.P. Rajeev, M. Shahzad, M. Smid, D. Seipt, R. Spesyvtsev, D.R. Symes, G. Vieux, L. Willingale, J. C. Wood and A.G.R. Thomas



Acknowledgements

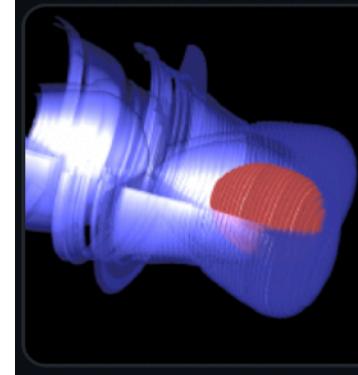
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Acknowledgements

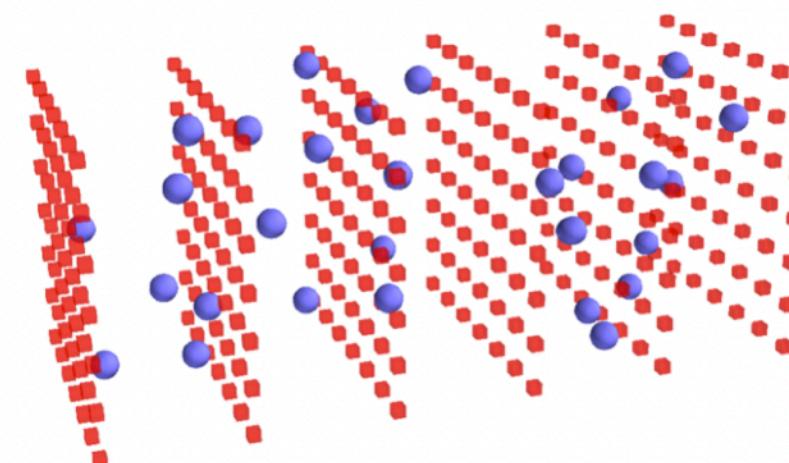


fbpic
A spectral, quasi-3D Particle-In-Cell code, for CPU and GPU
<http://fbpic.github.io>

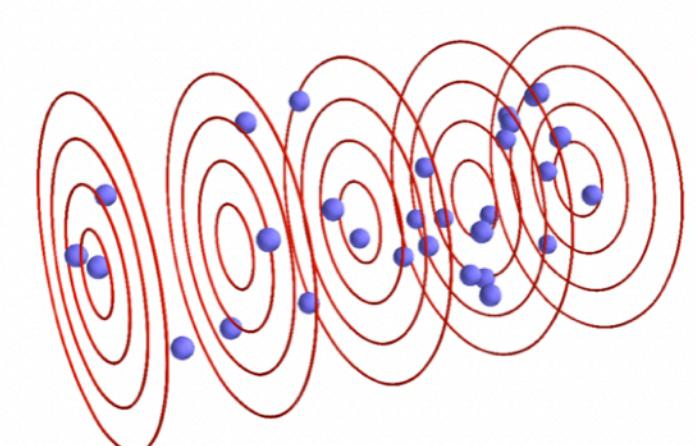
The distinctive features of FBPIC

Cylindrical grid with azimuthal decomposition

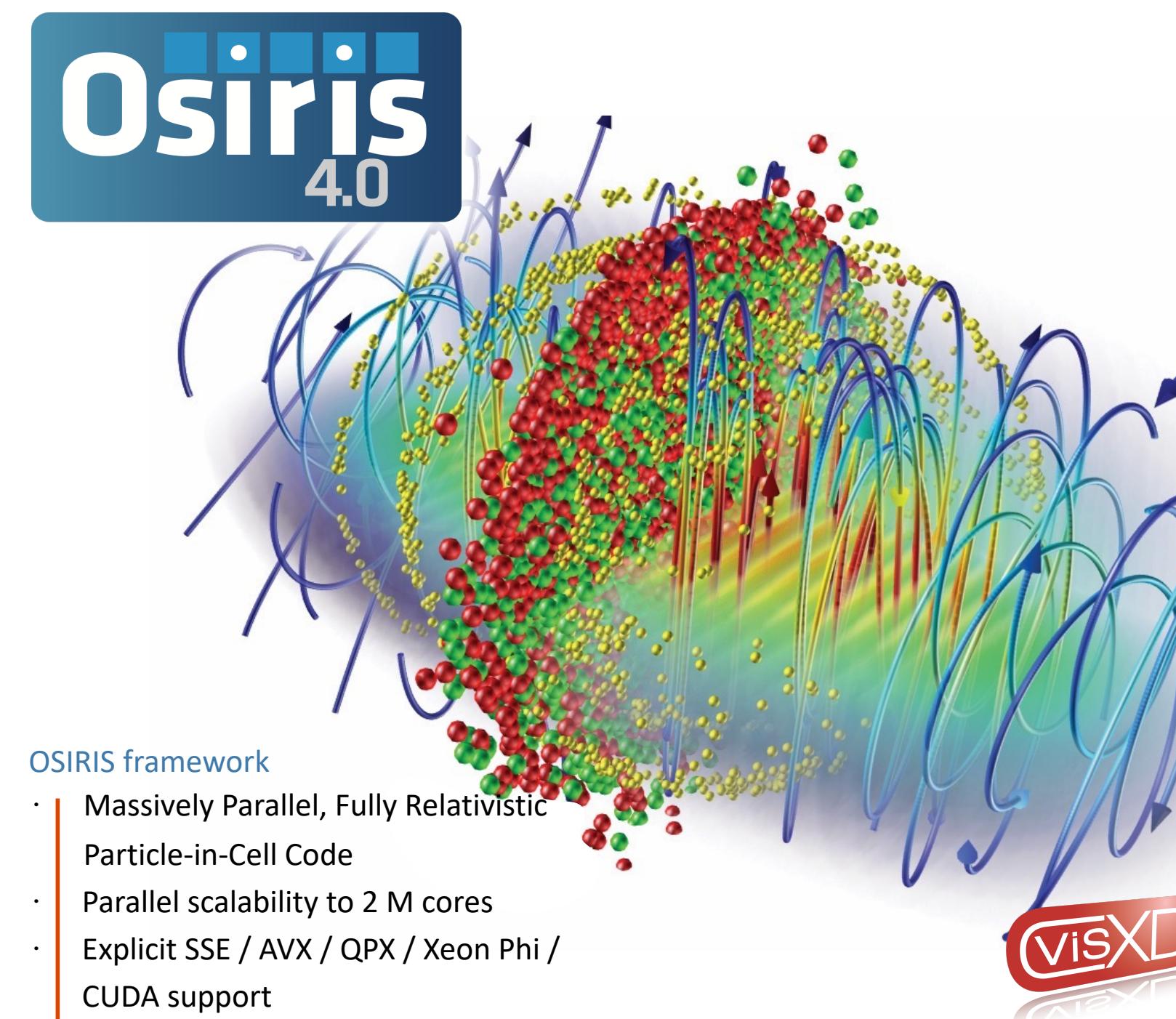
In the standard PIC algorithm, the fields are represented on a **3D Cartesian grid**. This is very generic, but also very computational expensive. For physical situations that have close-to-cylindrical symmetry, it is more efficient to use a **cylindrical grid** for the fields. This is represented below, with macroparticles in blue and the grid for the fields in red.



3D Cartesian grid



Cylindrical grid (schematic)



Committed to open science

Open-access model

- 40+ research groups worldwide are using OSIRIS
- 300+ publications in leading scientific journals
- Large developer and user community
- Detailed documentation and sample inputs files available

Using OSIRIS 4.0

- The code can be used freely by research institutions after signing an MoU
- Find out more at:

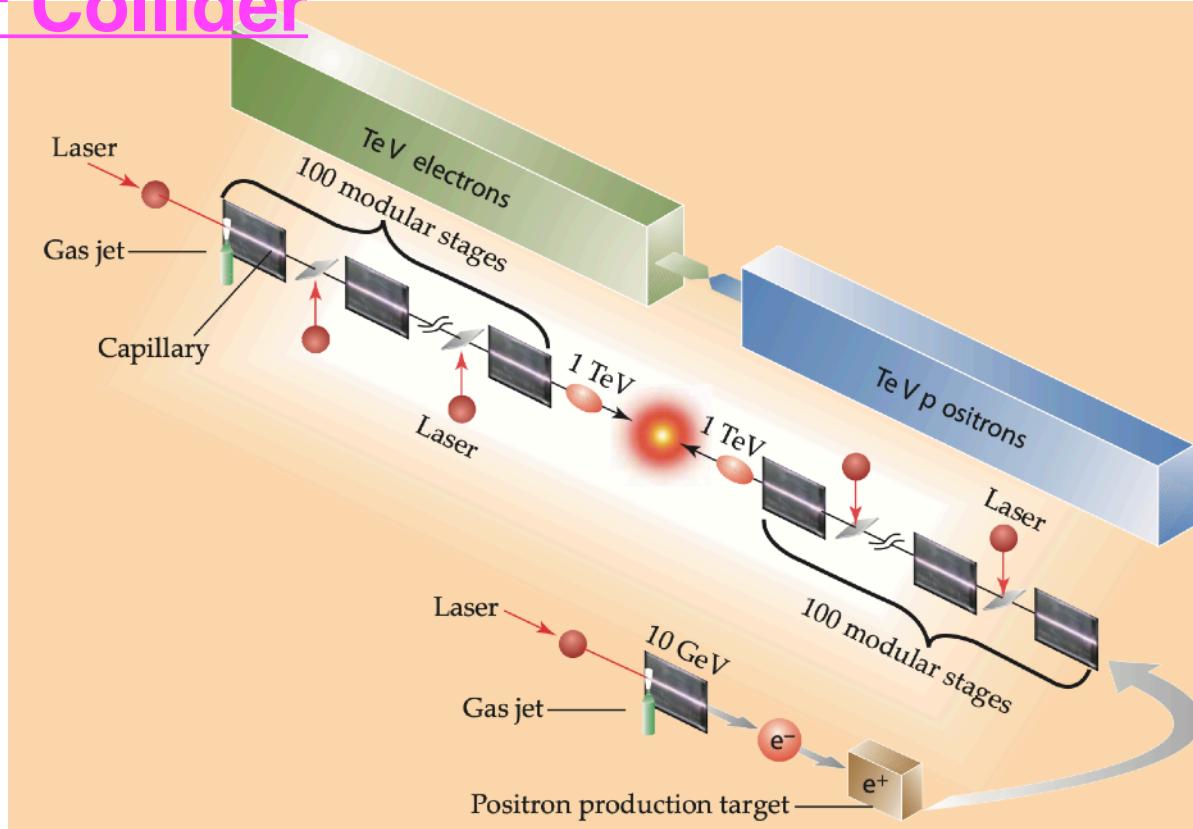
<http://epp.tecnico.ulisboa.pt/osiris>



Ricardo Fonseca: ricardo.fonseca@tecnico.ulisboa.pt

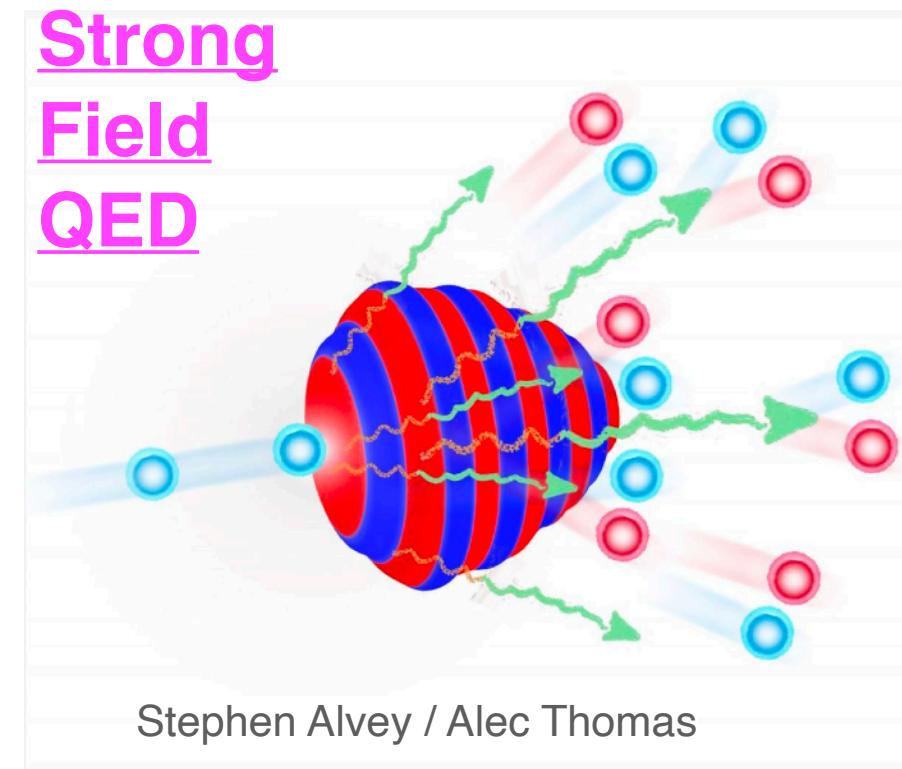
Applications of plasma-based wakefield accelerators

- e+e- Collider



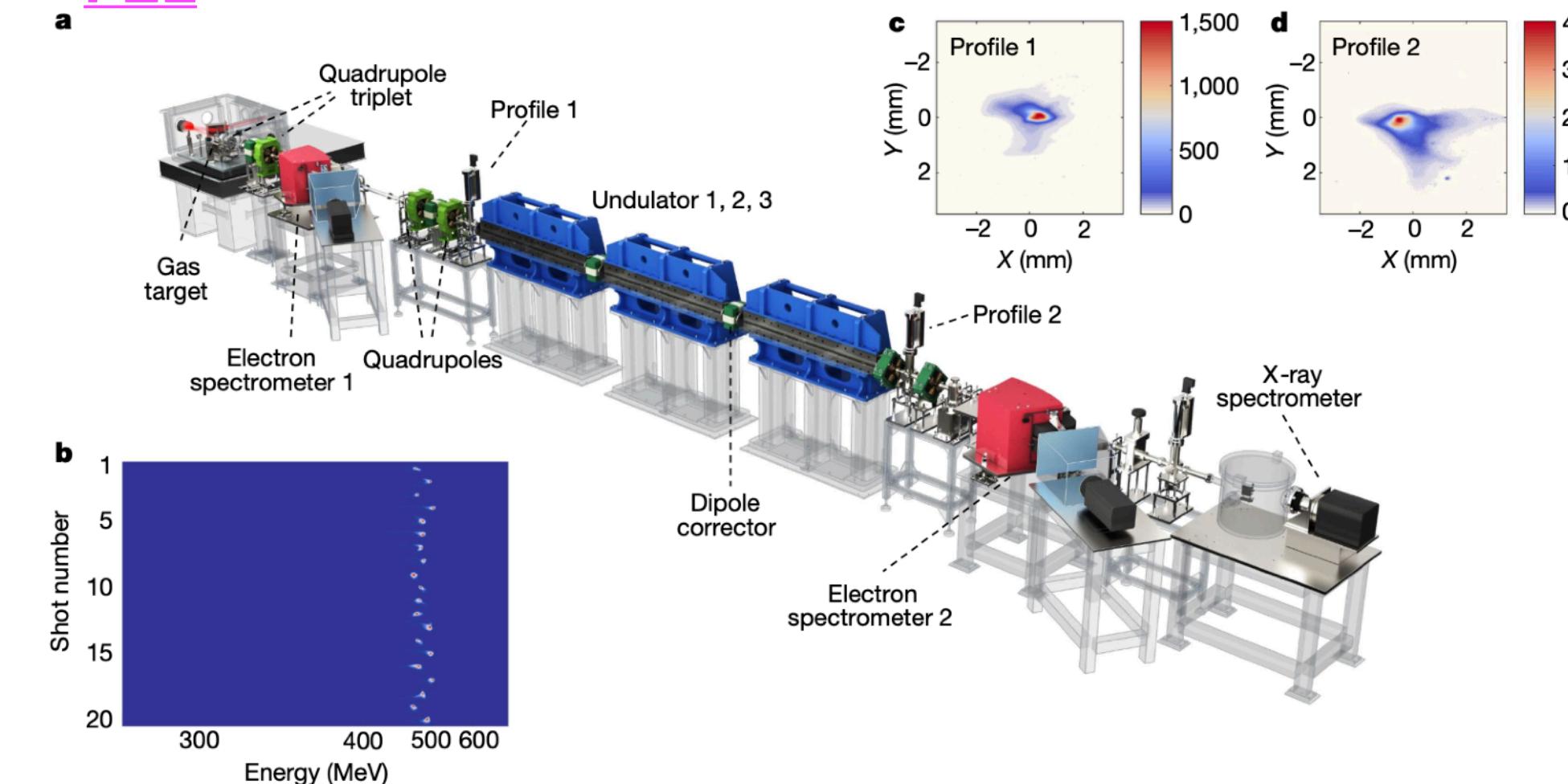
W. Leemans, E. Esarey, Physics Today 62 (3) (2009) 44–49

- Strong Field QED



J. M. Cole et al., PRX 8, 011020 (2018)
K. Poder et al., PRX 8, 031004 (2018)

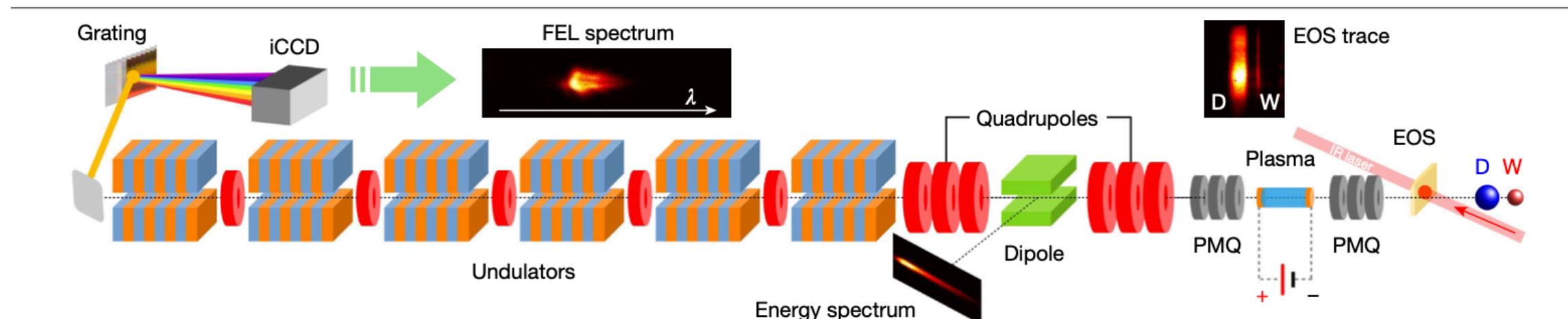
- FEL



W. T. Wang et al., Nature 595, 516–520 (2021).

- Slice Energy spread
- Emittance
- Brightness
- Beam charge
- Pulse length
- Source size
- Stability
- Dark current
-

Downer et al.: *Diagnostics for plasma-based electron accelerators*, Rev. Mod. Phys., Vol. 90, No. 3, 2018.



R. Pompili et al., Nature 605, 659–662 (2022)

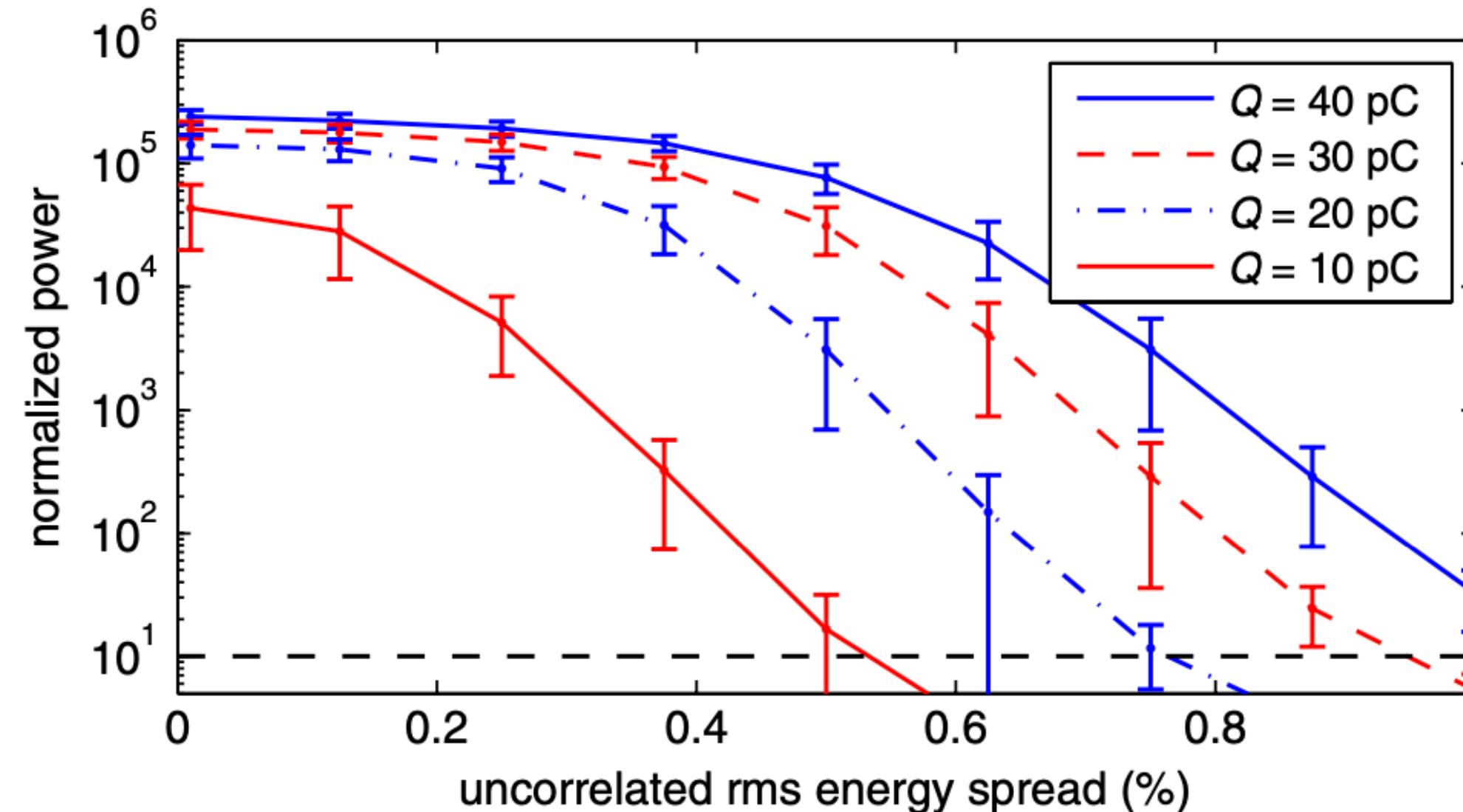
Diagnostics for plasma-based electron accelerators

Slice energy spread for XFEL

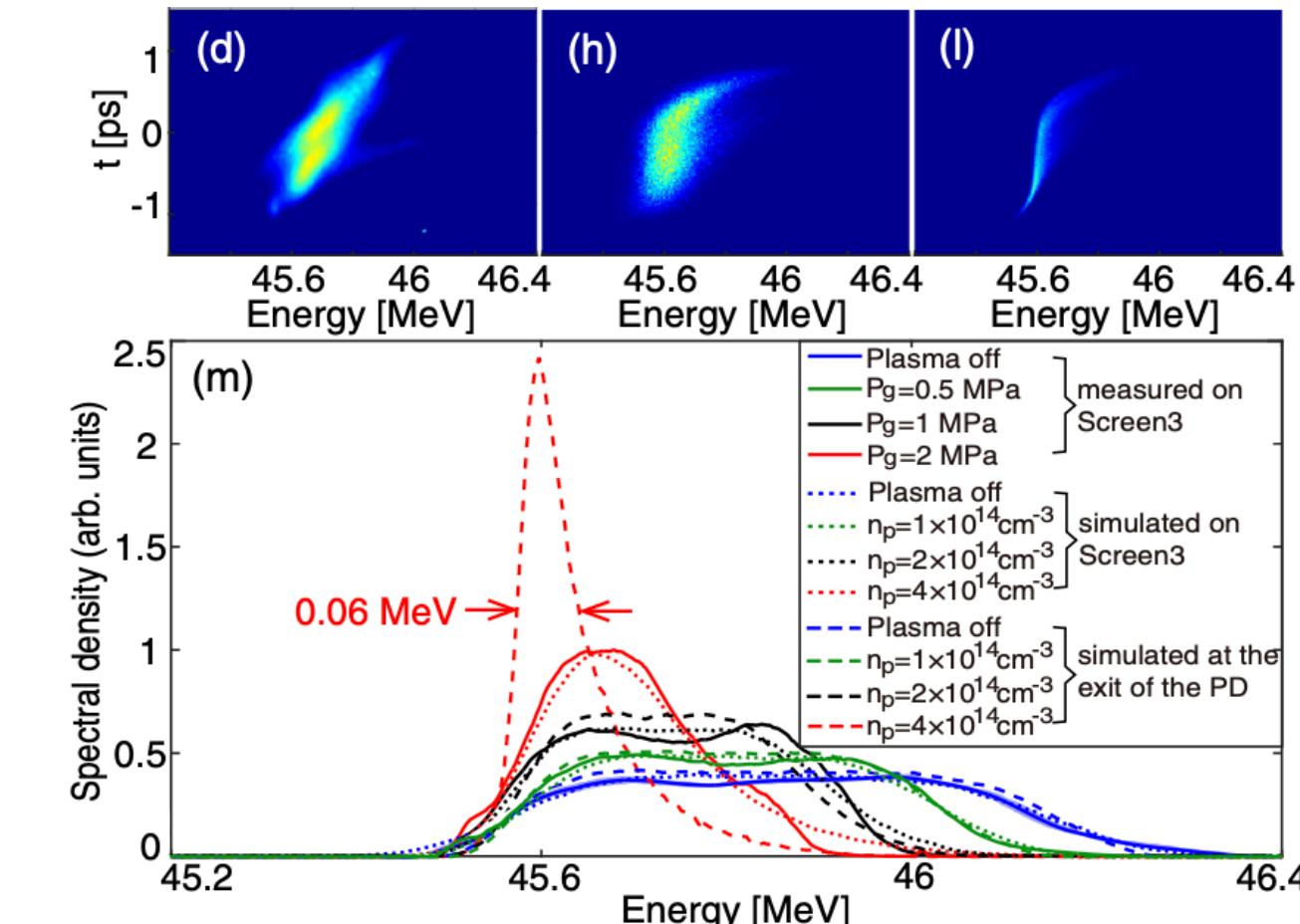
$$\sigma_\delta \ll \rho, \quad \rho = \left[\frac{1}{16} \frac{I_0}{I_A} \frac{K_0^2 [J]^2}{\gamma_0^3 \sigma_\perp^2 k_u^2} \right]^{1/3}$$

FEL parameter, ~0.1%

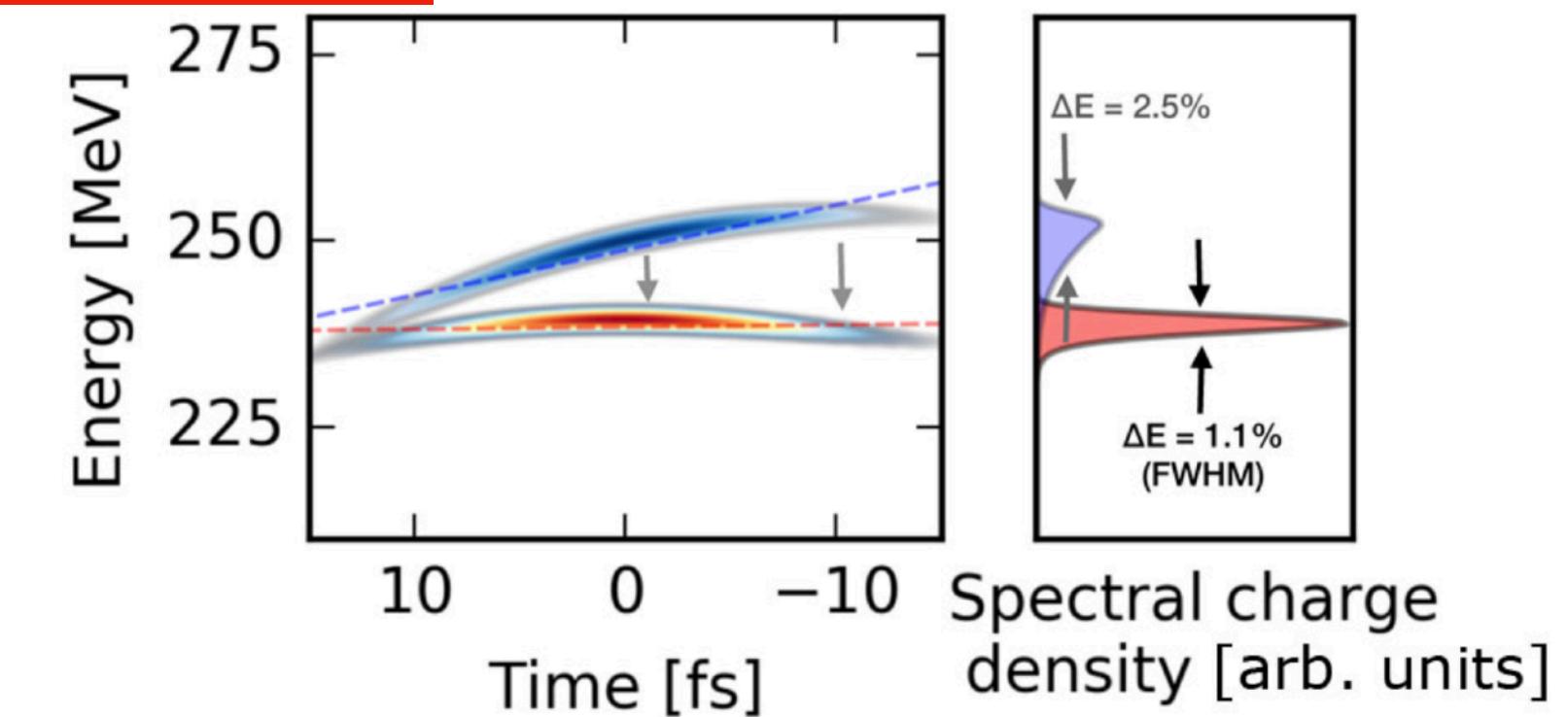
W. Wang et al., *Nature* 595, 516–520 (2021)



Maier et al., PRX 2, 031019 (2012)



de-chirping



L. Ke et al., PRL 126, 214801 (2021).

S. Jalas et al., PRL126, 104801 (2021).

Y. P. Wu et al., PRL122, 204804 (2019).

A. Döpp, PRL121, 074802 (2018).

Diagnostics for plasma-based electron accelerators

Goal:

Diagnose as much as possible in single-shot

Slice energy spread

Coupled motion of electrons in laser driven plasma wakefields and oscillations in the laser fields

Hamiltonian in wake coordinates:

$$H = \sqrt{1 + (\mathbf{P}_\perp - \mathbf{a}_\perp(\mathbf{x}_\perp, \xi, t))^2 + p_\parallel^2} - p_\parallel v_p - \psi(\mathbf{x}_\perp, \xi) \quad 1$$

$$\frac{dP_i}{dt} = \frac{(\mathbf{P}_\perp - \mathbf{a}_\perp)}{\gamma} \cdot \frac{\partial \mathbf{a}_\perp}{\partial x_i} + \frac{\partial \psi}{\partial x_i} \quad 2$$

$$\left(\frac{d^2}{d\xi^2} + 2\Gamma \frac{d}{d\xi} + \kappa_\beta^2 \right) \mathbf{x}_\perp = -\frac{i k_\parallel \mathbf{a}_{\perp 0}(\zeta)}{2\eta} e^{i k_\parallel \zeta} + c.c. \quad 3$$

Driven oscillator

$$\mathbf{x}_{\perp s} = \frac{\mathbf{a}_{\perp 0}(\zeta)}{k_\parallel \eta \mathcal{Z}} \sin(k_\parallel \zeta) \quad 4$$

$$\mathbf{p}_{\perp s} = \frac{\mathbf{a}_{\perp 0}(\zeta)}{\mathcal{Z}} \cos(k_\parallel \zeta)$$

Steady state solutions

Laser

$$\mathbf{x}_{\perp t} = \mathbf{x}_1 \cos \kappa_\beta \zeta + \mathbf{x}_2 \sin \kappa_\beta \zeta \quad 5$$

$$\mathbf{p}_{\perp t} = \mathbf{p}_1 \cos \kappa_\beta \zeta + \mathbf{p}_2 \sin \kappa_\beta \zeta$$

Transient solutions

Wakefield

$$\mathbf{x}_\perp = \mathbf{x}_{\perp s} + \mathbf{x}_{\perp t} \quad 6$$

$$\mathbf{p}_\perp = \mathbf{p}_{\perp s} + \mathbf{p}_{\perp t}$$

General solutions

Coupled motion of electrons in laser driven plasma wakefields and oscillations in the laser fields

$$\gamma - P_{\parallel} v_p - \psi \simeq C_1$$

7

conservation of Hamiltonian

$$p_{\parallel} = \gamma_p^2 (C_1 + \psi) \left(v_p + \sqrt{1 - \frac{1 + \mathbf{p}_{\perp}^2}{\gamma_p^2 (C_1 + \psi)^2}} \right)$$

8

$$p_{\parallel} \simeq p_{\parallel 0} - \frac{1}{2} \alpha^2 \gamma_p^2 \mathbf{x}_{\perp}^2 - \frac{\mathbf{p}_{\perp}^2}{2(C_1 + \psi_0(\xi))}$$

9

$$p_{\parallel 0} \simeq 2\gamma_p^2 (C_1 + \psi_0(\xi))$$

$$p_{\parallel} \simeq p_{\parallel 0} - \frac{\gamma_p^2}{p_{\parallel 0}} \left(\frac{1}{2} \alpha^2 p_{\parallel 0} \mathbf{x}_{\perp}^2 + \mathbf{p}_{\perp}^2 \right)$$

10

Wakefield acceleration

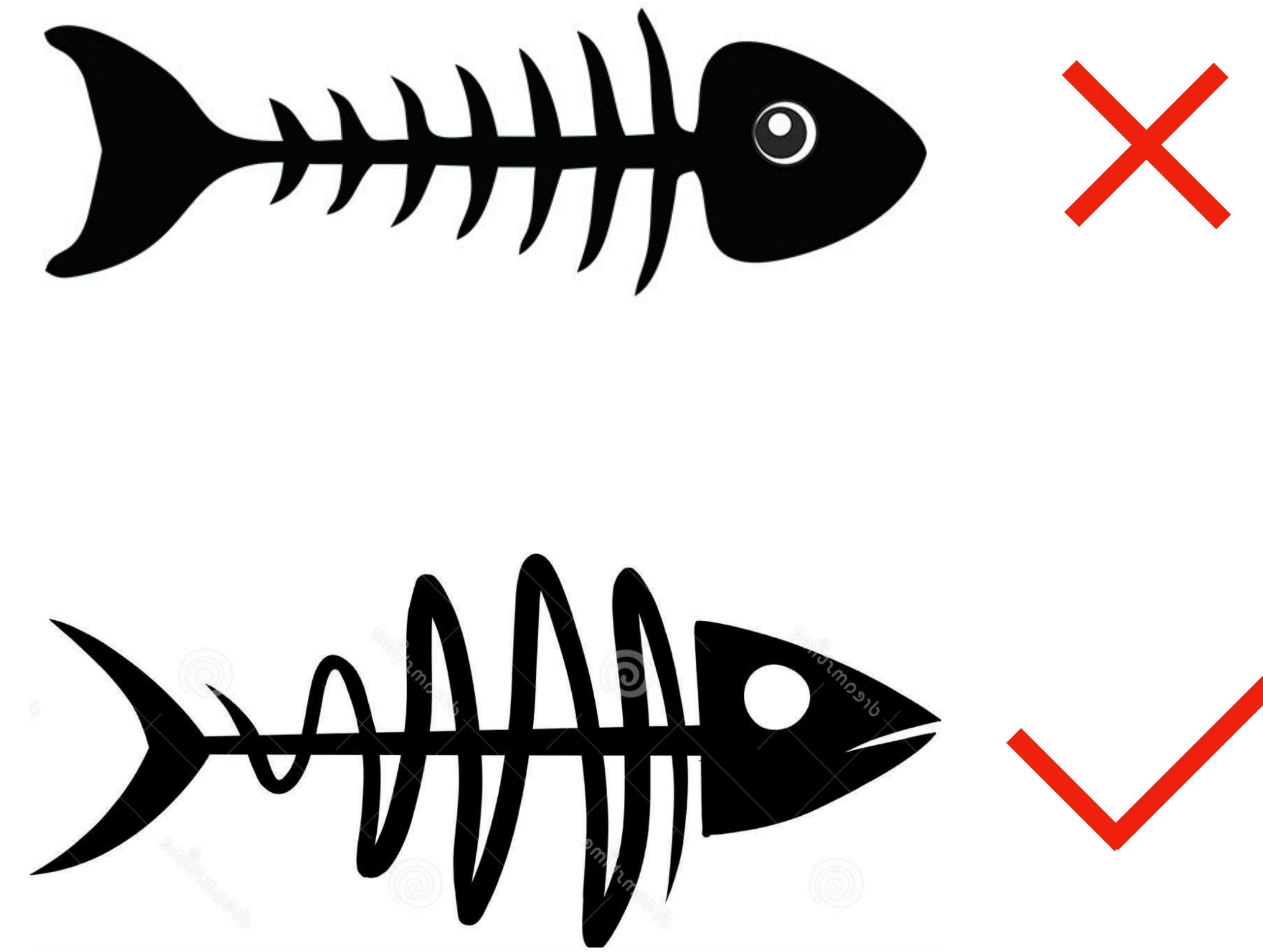
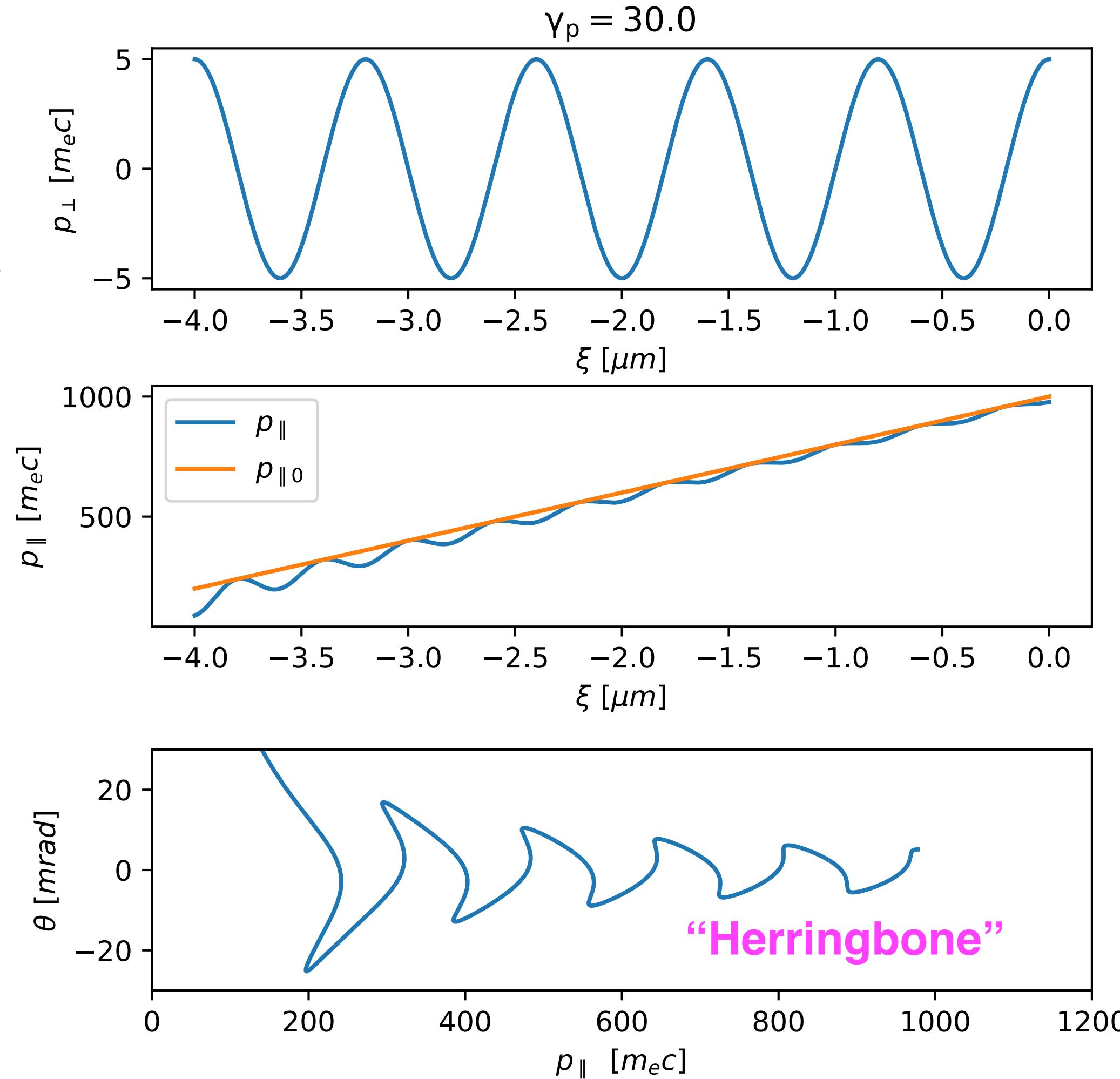
Transverse modulation

$$\mathbf{x}_{\perp} = \mathbf{x}_{\perp s} + \mathbf{x}_{\perp t}$$
$$\mathbf{p}_{\perp} = \mathbf{p}_{\perp s} + \mathbf{p}_{\perp t}$$

Laser Wakefield

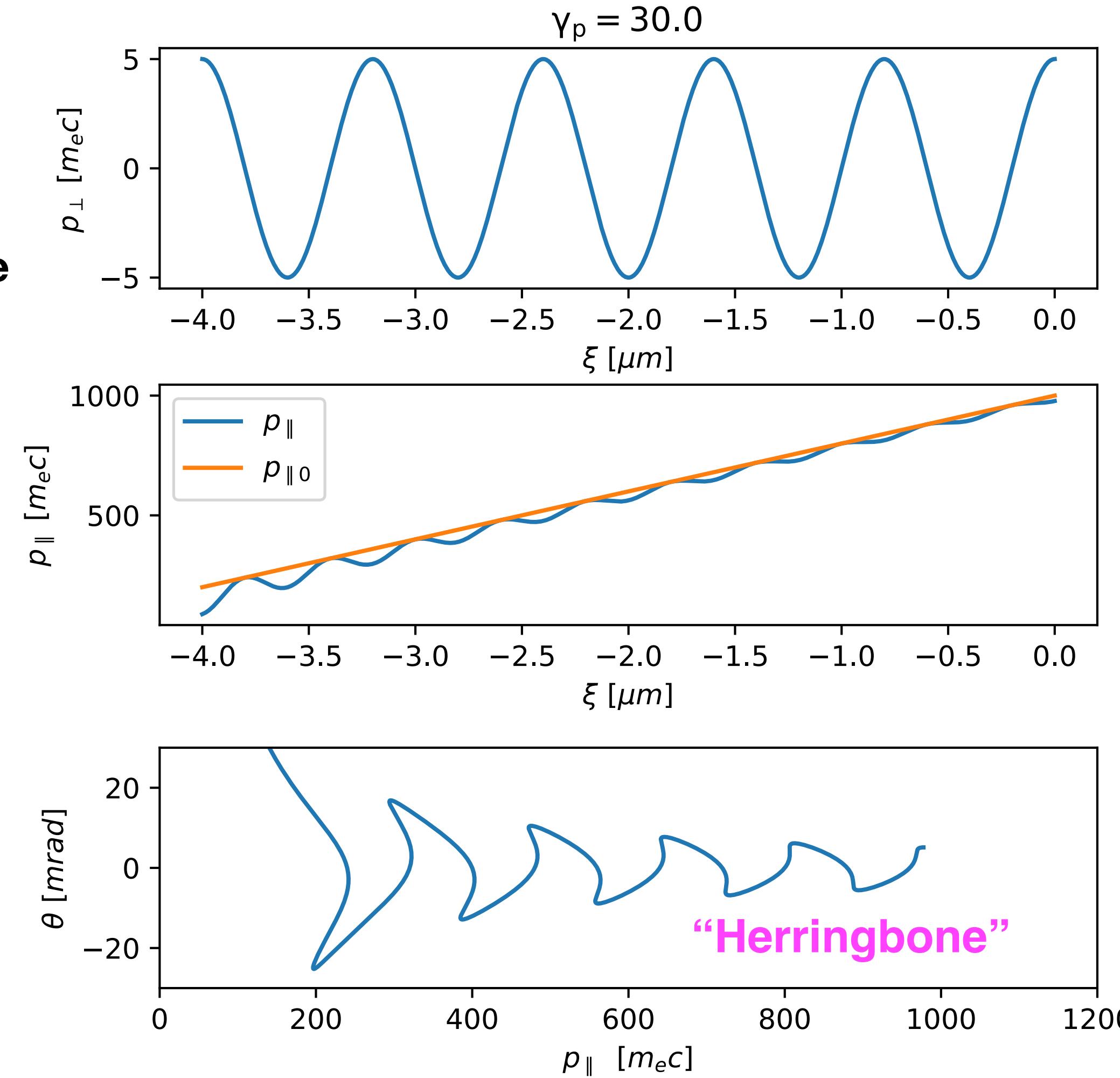
Coupled motion of electrons in laser driven plasma wakefields and oscillations in the laser fields

Steady-state trajectory

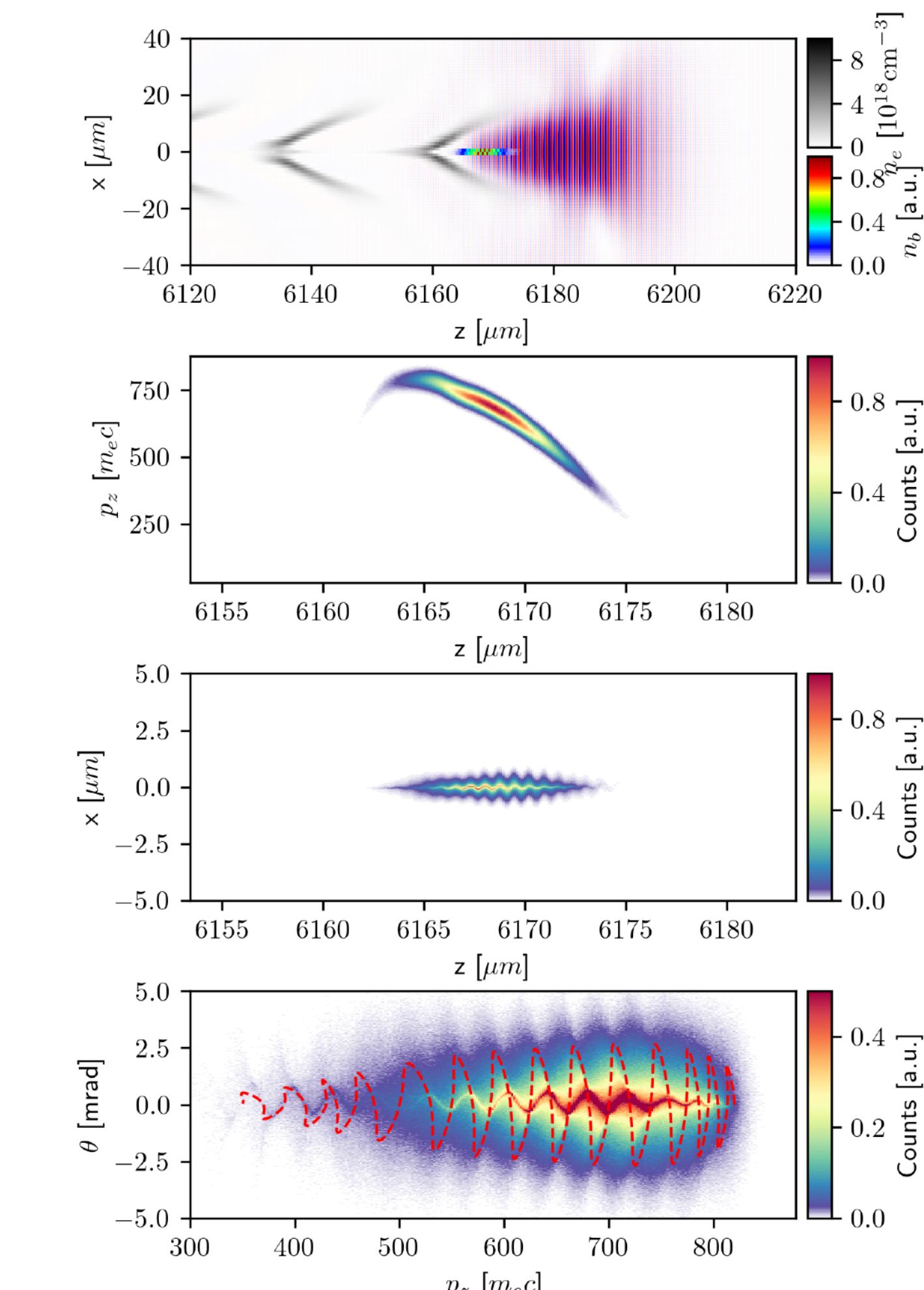


Coupled motion of electrons in laser driven plasma wakefields and oscillations in the laser fields

Steady-state trajectory



FBPIC simulation



“Herringbone” observed in LWFA experiments and theoretical fitting

Gemini TA3 at RAL, CLF (UK)

$E: 6.3 \pm 0.6\text{J}$

Pulse duration: $45 \pm 4\text{ fs}$

Spot size: $40(\pm 2) * 50(\pm 2)\text{ }\mu\text{m}$

$I_0 = 4.6(\pm 0.8)\text{e}18\text{ W/cm}^2$

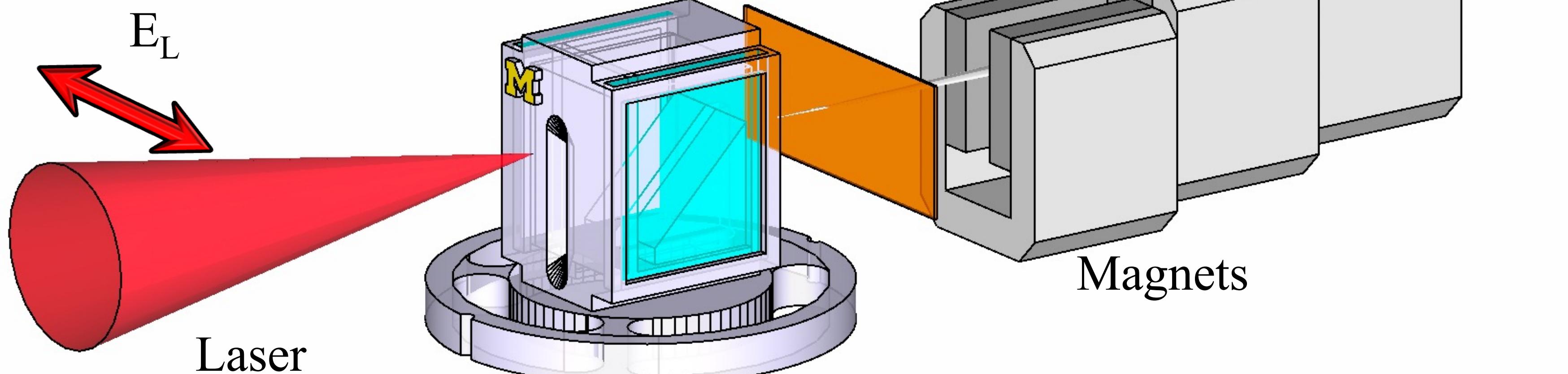
$a_0 = 1.46 \pm 0.12$

See also:

- M. Streeter et al., PRAB 25, 101302 (2022) ;
- A. Hussein et al., Scientific Reports (2019) 9:3249;
- B. Kettle et al., PRL 123, 254801 (2019);
- R. Spesyvtsev et al., Proc. SPIE 11036 2019

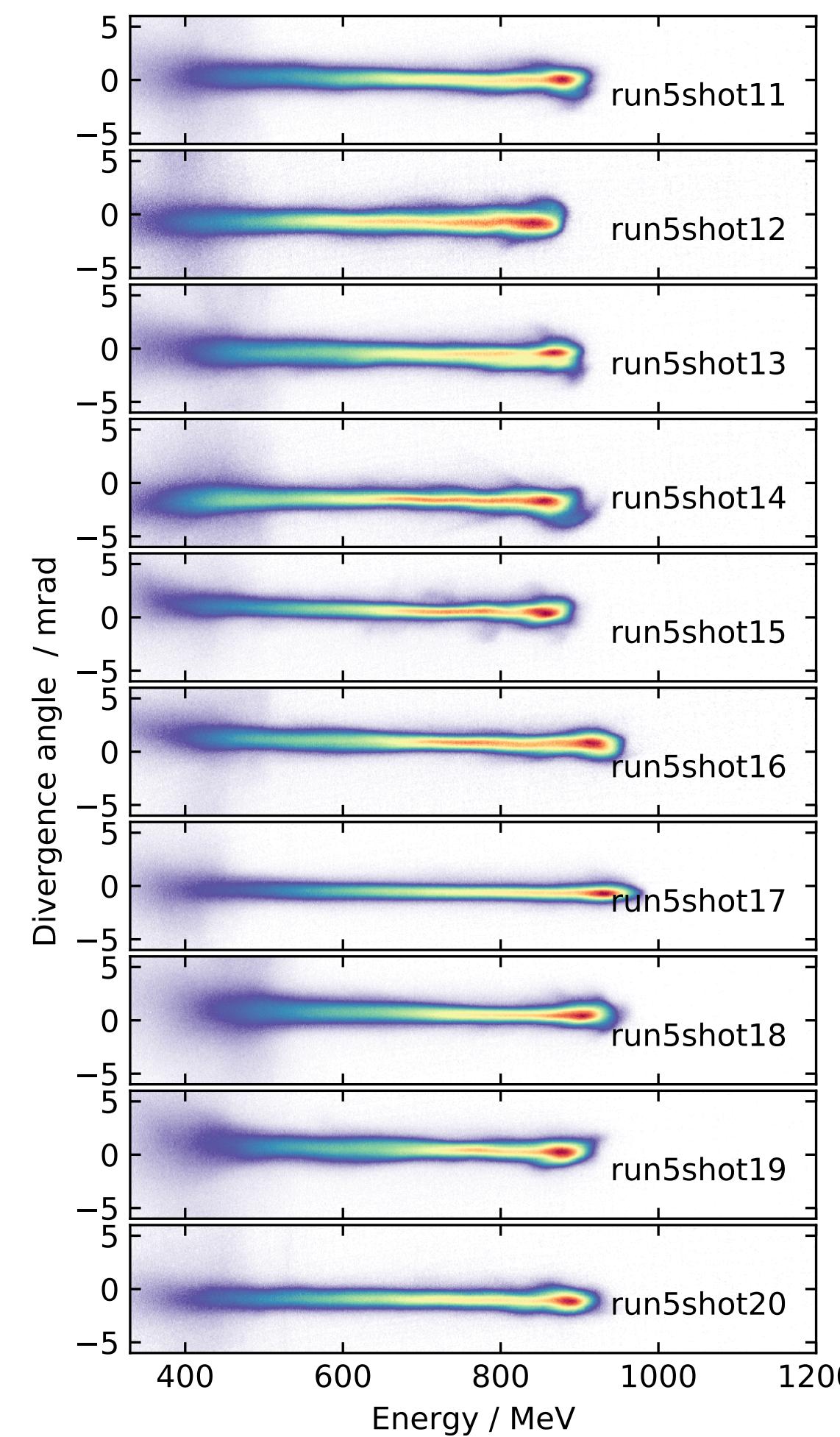
Laser polarization perpendicular to
magnet deflection plane
Modulation of the electron spectral
can be observed

Lanex screen

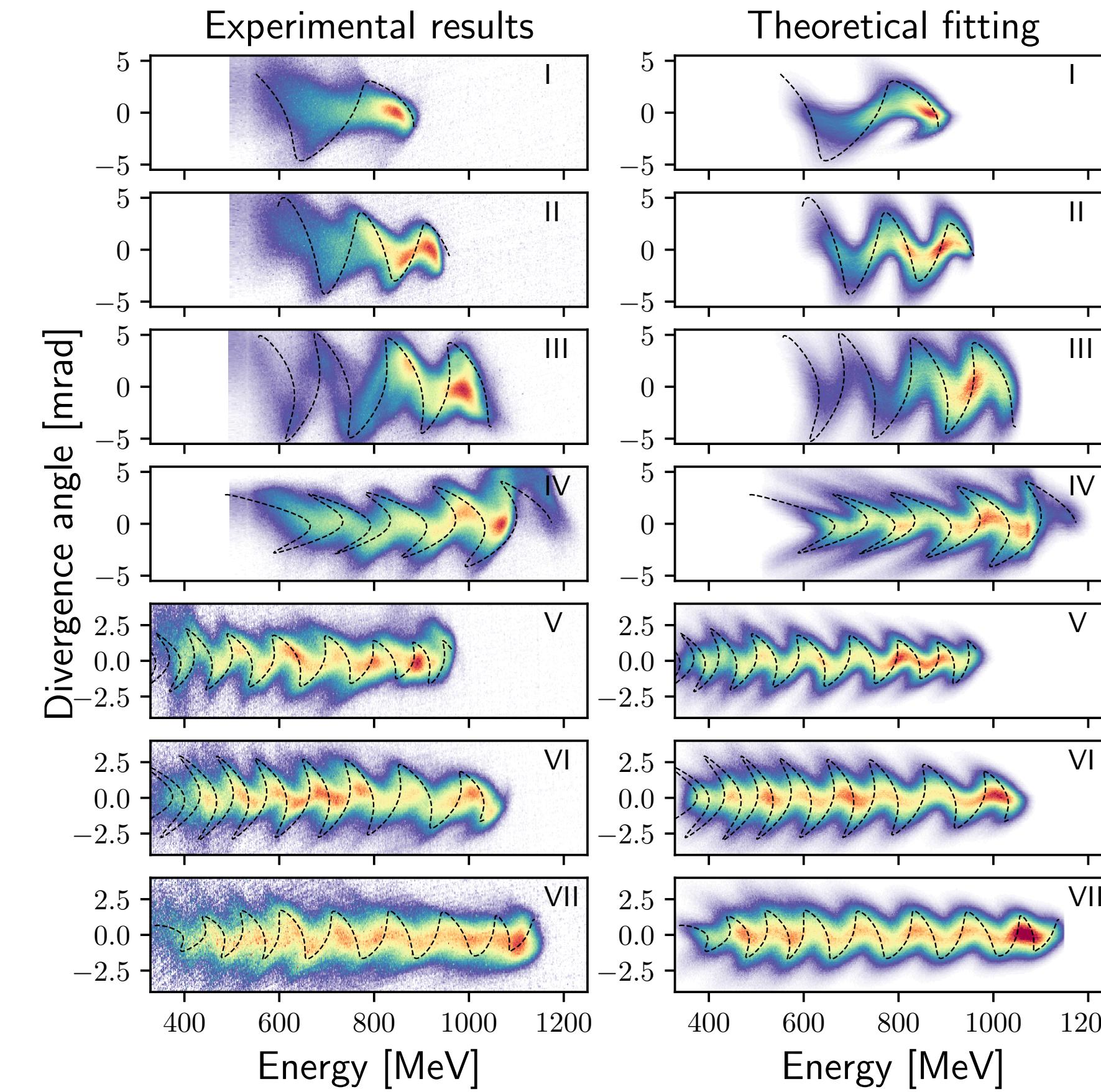


“Herringbone” observed in LWFA experiments and theoretical fitting

Typical spectral without modulations



“Herringbone”

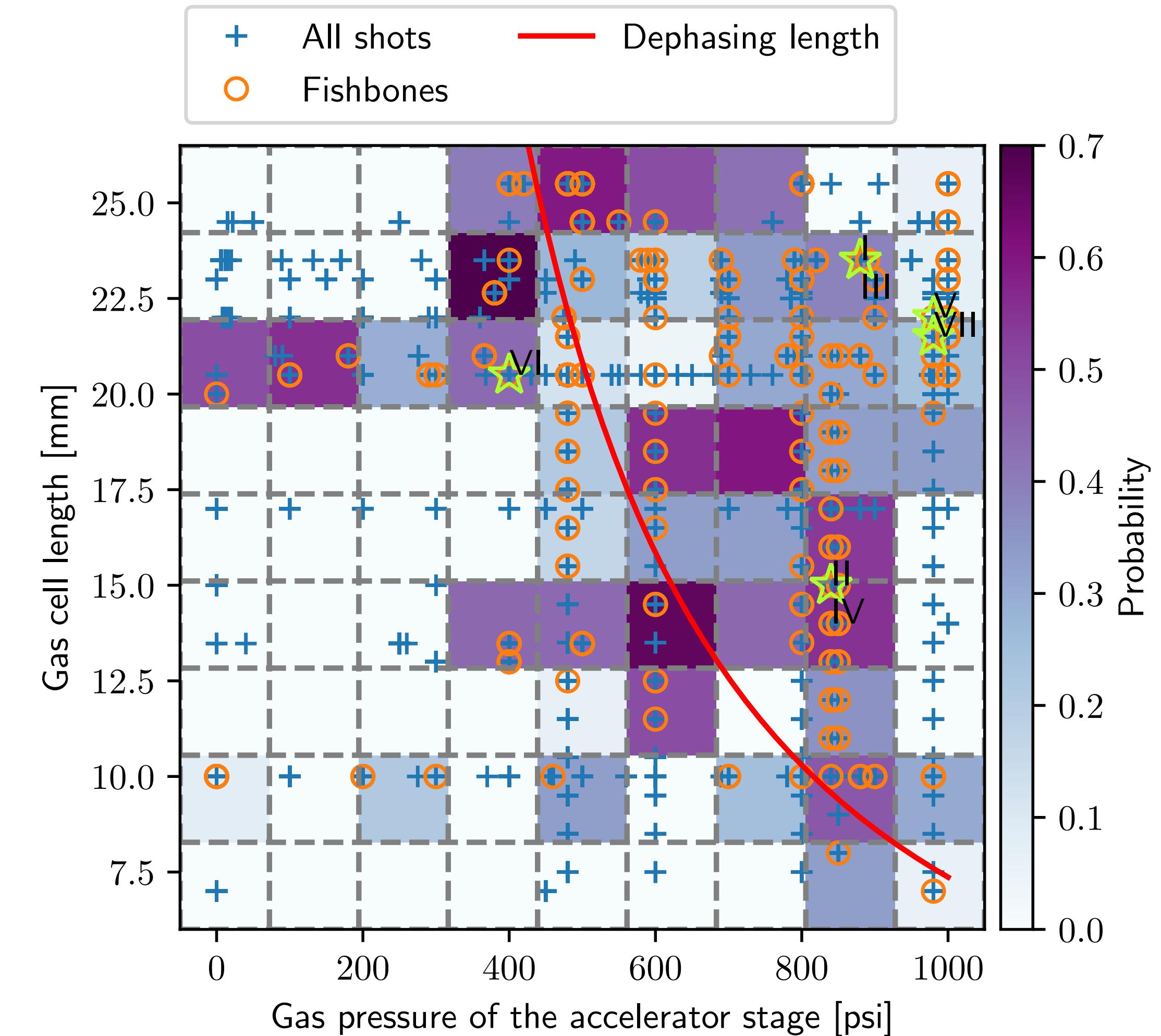


Fitted with theoretical model with some guessed parameters which can tell a lot!

Coupled motion of electrons in laser driven plasma wakefields and oscillations in the laser fields

Long plasma length & high plasma density

Beyond dephasing – interaction between electron and the laser driver



* each data point represents multiple shots at identical condition

Spectral reconstruction

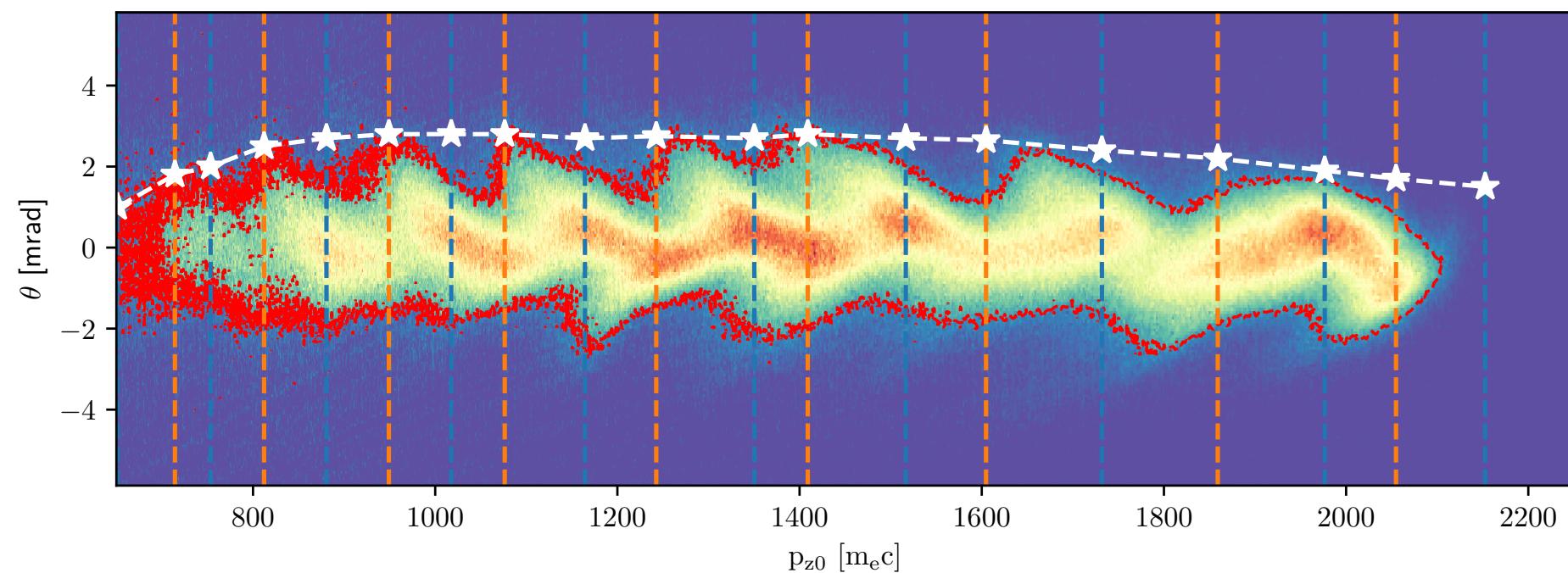
$$p_{\parallel} \simeq p_{\parallel 0} - \frac{\gamma_p^2}{p_{\parallel 0}} \left(\frac{1}{2} \alpha^2 p_{\parallel 0} \mathbf{x}_{\perp}^2 + \mathbf{p}_{\perp}^2 \right)$$

$$p_x(\zeta) = \hat{p}_{xs}(\zeta) a_0 \cos(k_z \zeta + \Omega) + \hat{p}_{xt}(\zeta) p_{xt} \quad p_{xt} = \sigma_{p_{xt}} \cdot \mathcal{N}$$

$$\mathbf{x}_{\perp} = \mathbf{x}_{\perp s} + \mathbf{x}_{\perp t} \quad \mathbf{p}_{\perp} = \mathbf{p}_{\perp s} + \mathbf{p}_{\perp t}$$

transient solutions represented by standard normal distributions \mathcal{N}

$$p_z = (p_{z0} + \sigma_{\Delta p_z}) - \frac{\gamma_p^2}{p_{z0}} \left[\frac{1}{2} \alpha^2 p_{z0} \left(\frac{\hat{x}_s a_0}{k_z \eta Z} \sin(k_z \zeta + \Omega) + \hat{x}_t \sigma_{xt} \mathcal{N} \right)^2 + \left(\frac{\hat{p}_{xs} a_0}{Z} \cos(k_z \zeta + \Omega) + \hat{p}_{xt} \sigma_{p_{xt}} \mathcal{N} \right)^2 \right]$$



1. Extracted from experimental spectrum
 - Longitudinal energy distribution (chirp)
 - Temporal beam charge profile
 - Transverse momentum envelope (steady state)
 - Transverse momentum width (transient)

γ_p , α , a_0 , $\sigma_{p_{xt}}$, σ_{x_t} , Ω , and $\sigma_{\Delta p_z}$

2. Guessed parameters (all single value):

- γ_p (plasma density)
- α (wake strength)
- a_0 (laser intensity *)
- $\sigma_{p_{xt}}$ Transient momentum
- σ_{x_t} Transient real space
- Ω Phase
- $\sigma_{\Delta p_z}$ Slice energy spread

* (eta, Z are not independent parameter)

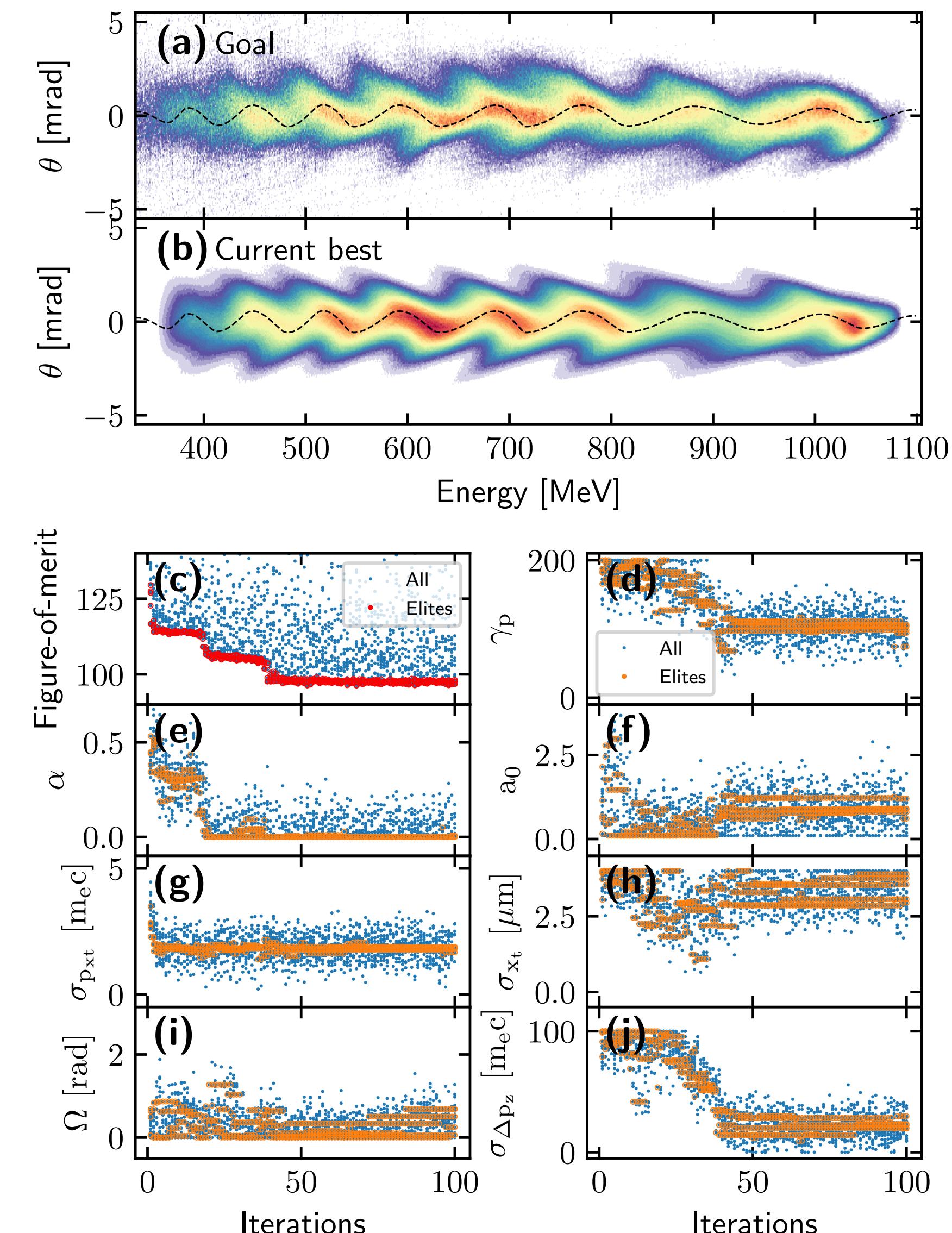
Spectral reconstruction: genetic algorithm

Problem of multi-parameter optimization

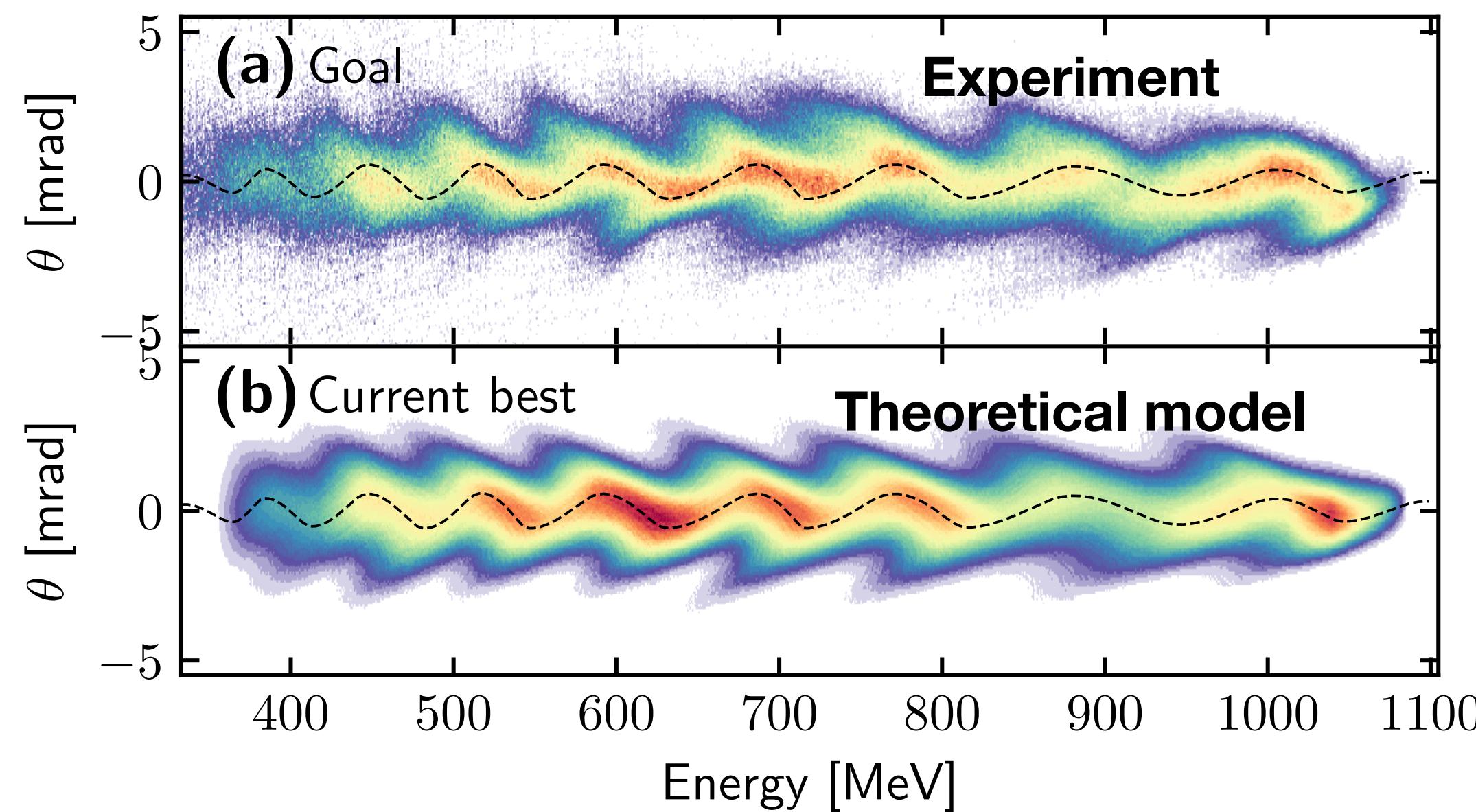
Goal:
experimental spectrum

Genes:
 γ_p , α , a_0 , $\sigma_{p_{xt}}$, σ_{x_t} , Ω , and $\sigma_{\Delta p_z}$
(With extracted longitudinal & transverse
momentum and temporal charge profile)

Figure-of-merit:
Difference between “guessed
spectrum” and exp spectrum

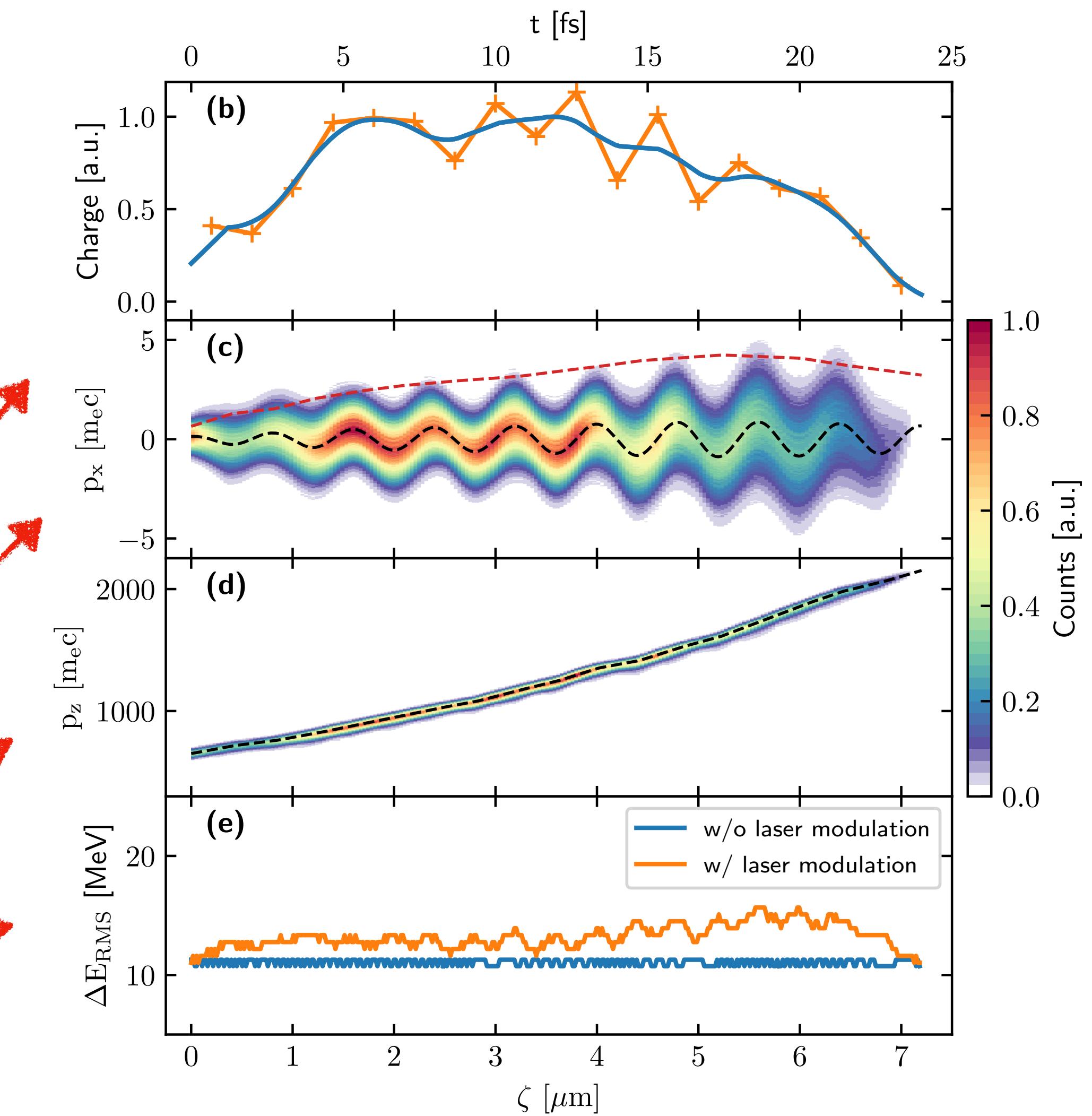


Information retrieved:



Electron beam:

- Temporal profile (also gives pulse duration)
- Transverse momentum
- Longitudinal momentum (energy chirp)
- Slice energy spread

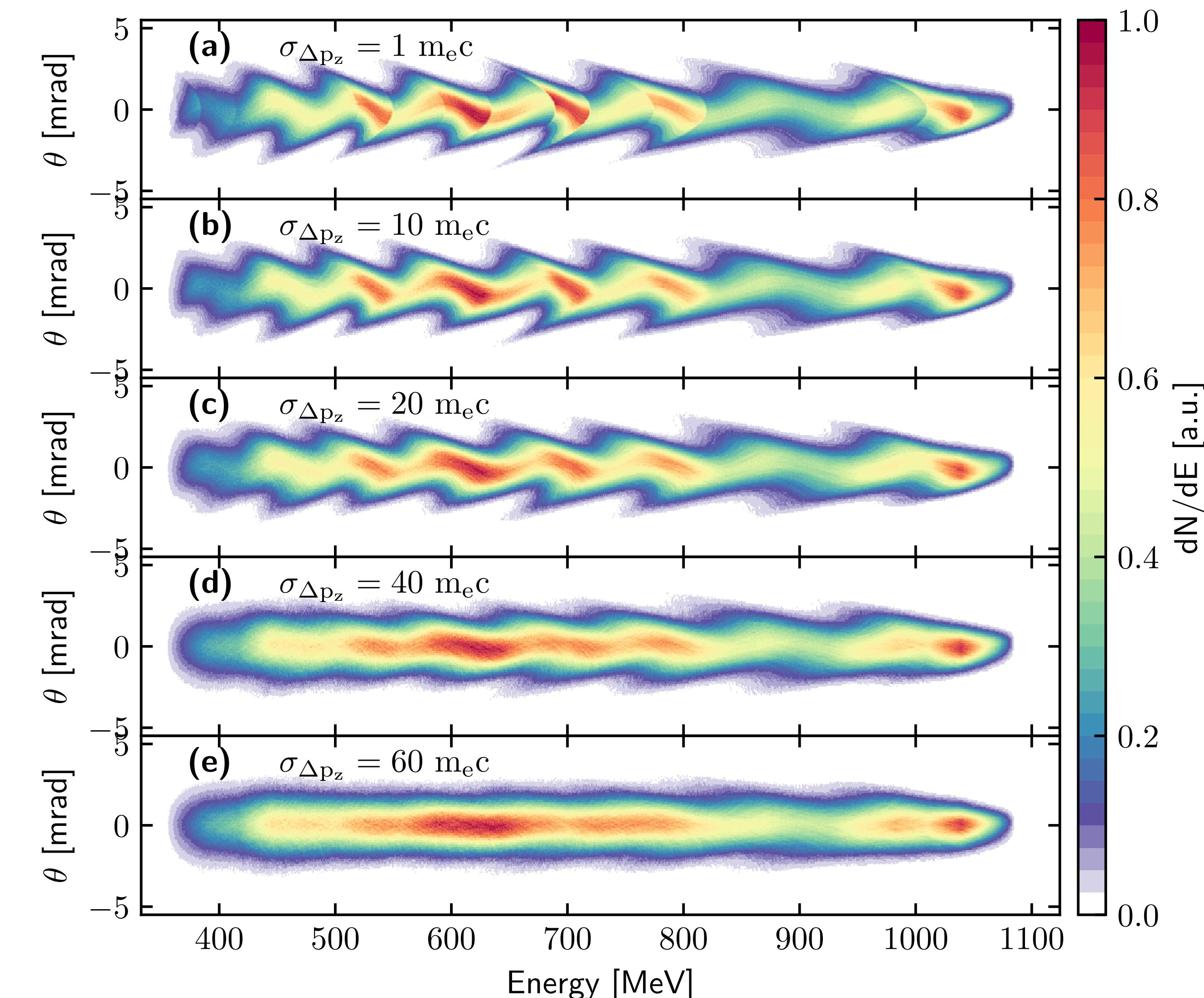


Slice energy spread of 11 MeV
relative energy spread of 1.0%-3.2%

Slice energy spread effects

Best fitting

Upper bound of
the slice energy
spread



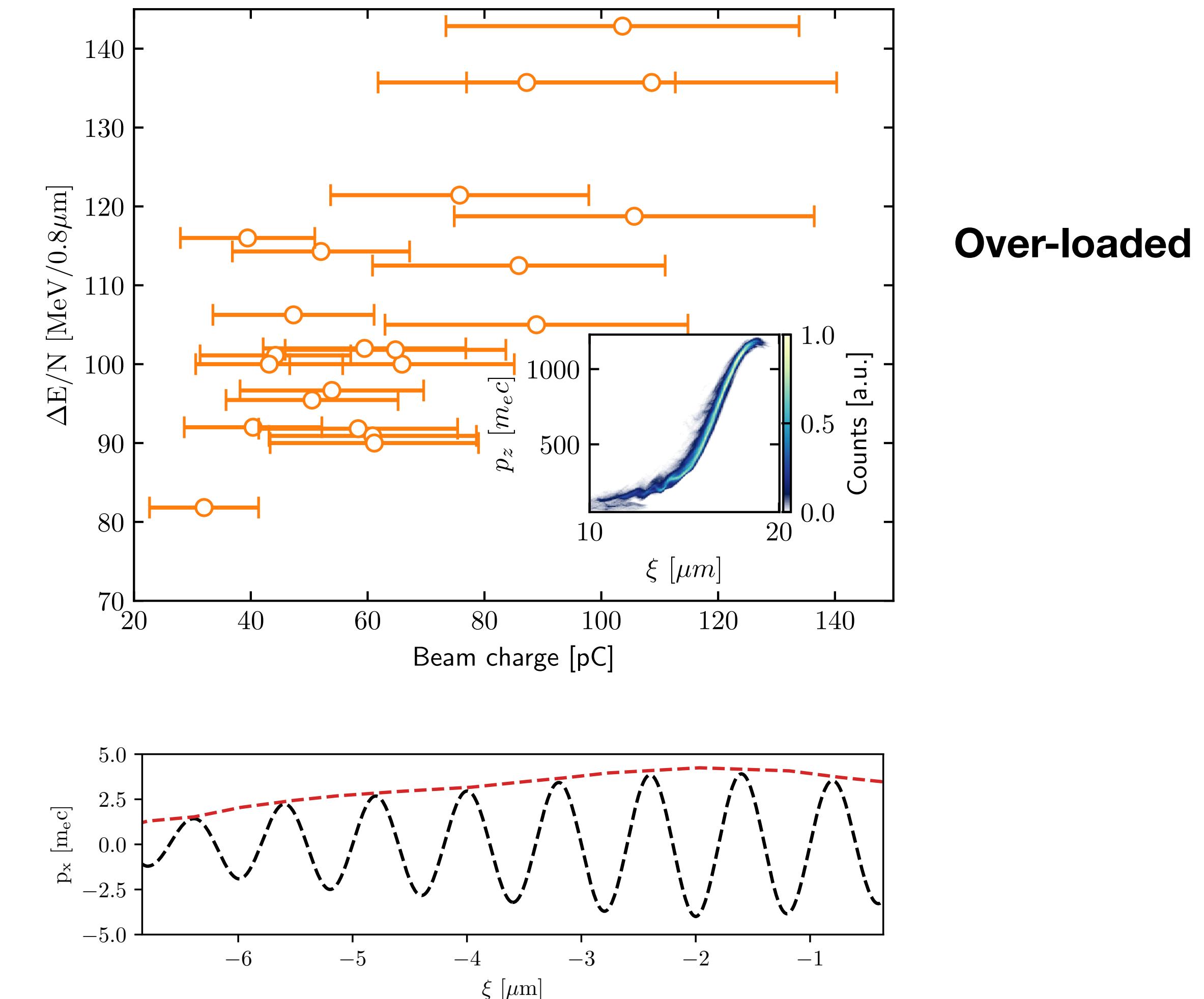
Information retrieved:

Wakefield:

1. $\gamma_p \rightarrow$ plasma density
 - $\gamma_p \sim 20-80 \Rightarrow n_p \sim 5e18 - 3e17 \text{ cm}^{-3}$
2. $\alpha^2 \ll 1 \Rightarrow$ quasi-linear wakefield
 - $\alpha = 1 \Rightarrow$ “blowout” regime
3. Beamloading \rightarrow chirp direction

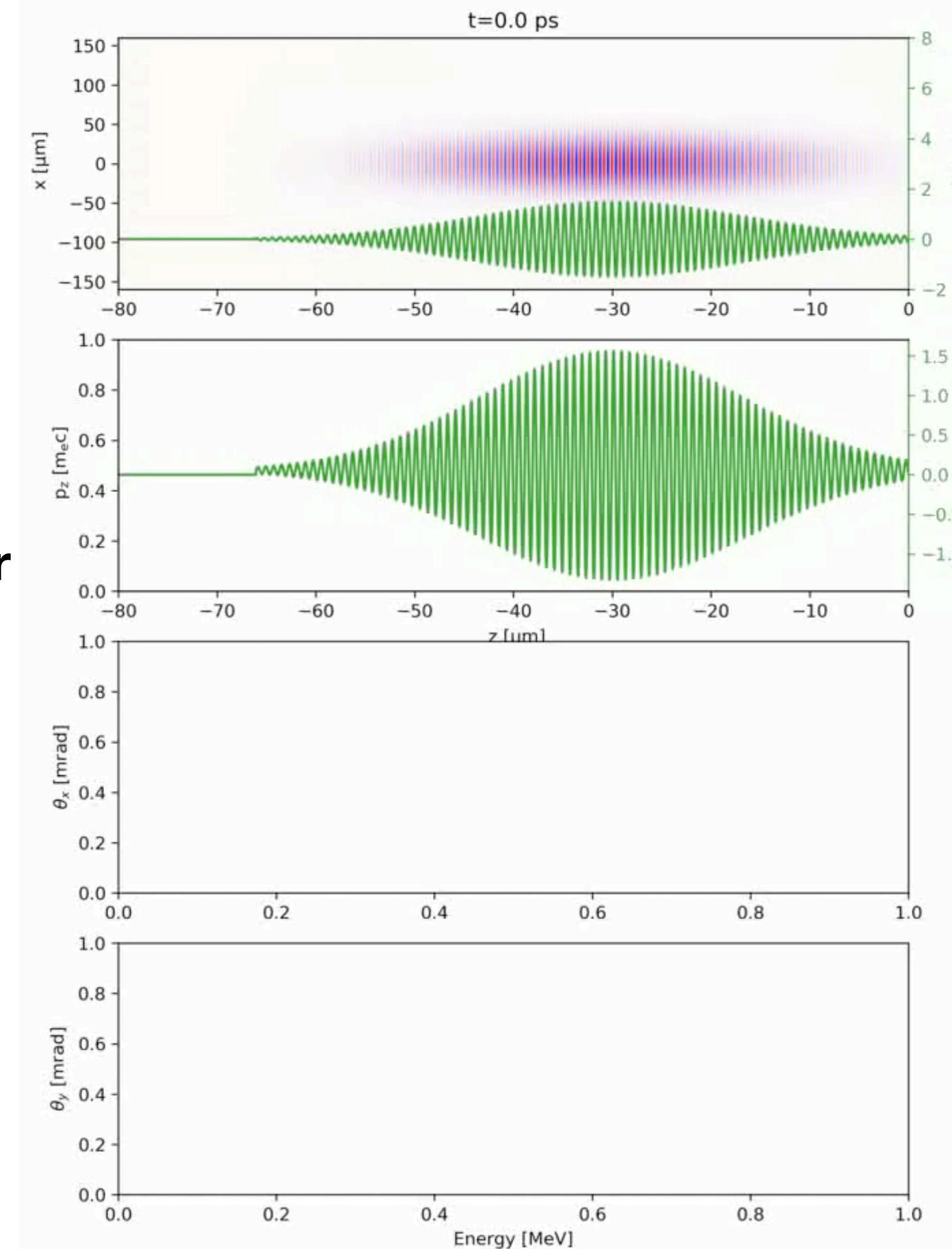
Laser

- Pulse shape at electron position,
 $p_x = a_0 m_e c$
- Electron beam at rear or front part
of the laser beam based on the
envelope



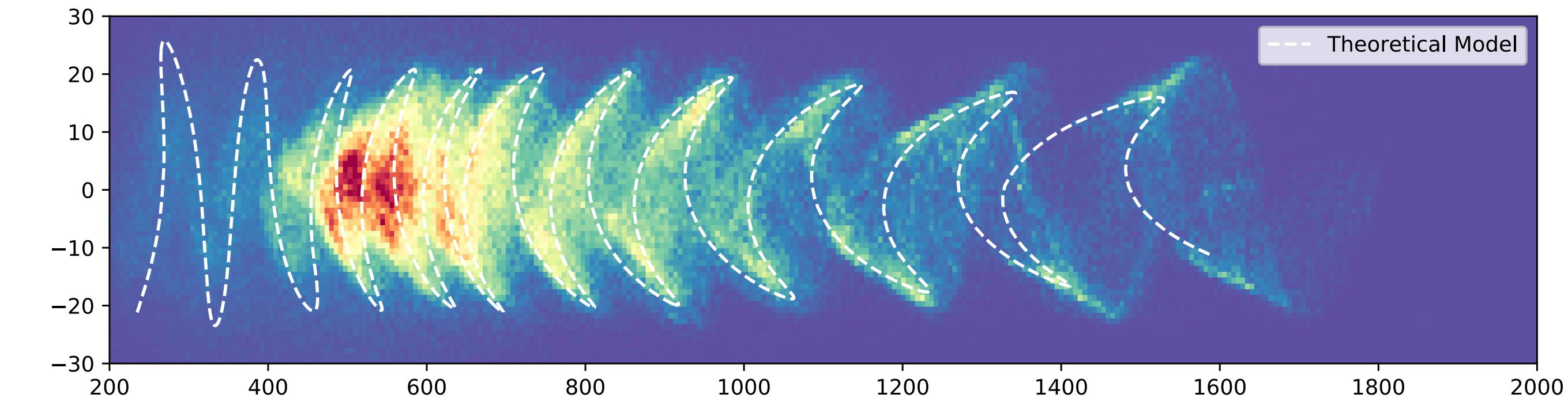
Discussion

$$p_z = (p_{z0} + \sigma_{\Delta p_z}) - \frac{\gamma_p^2}{p_{z0}} \left[\frac{1}{2} \alpha^2 p_{z0} \left(\frac{\hat{x}_s a_0}{k_z \eta \mathcal{Z}} \sin(k_z \zeta + \Omega) + \hat{x}_t \sigma_{x_t} \mathcal{N} \right)^2 + \left(\frac{\hat{p}_{xs} a_0}{\mathcal{Z}} \cos(k_z \zeta + \Omega) + \hat{p}_{xt} \sigma_{p_{xt}} \mathcal{N} \right)^2 \right]$$



Herringbone in backward direction

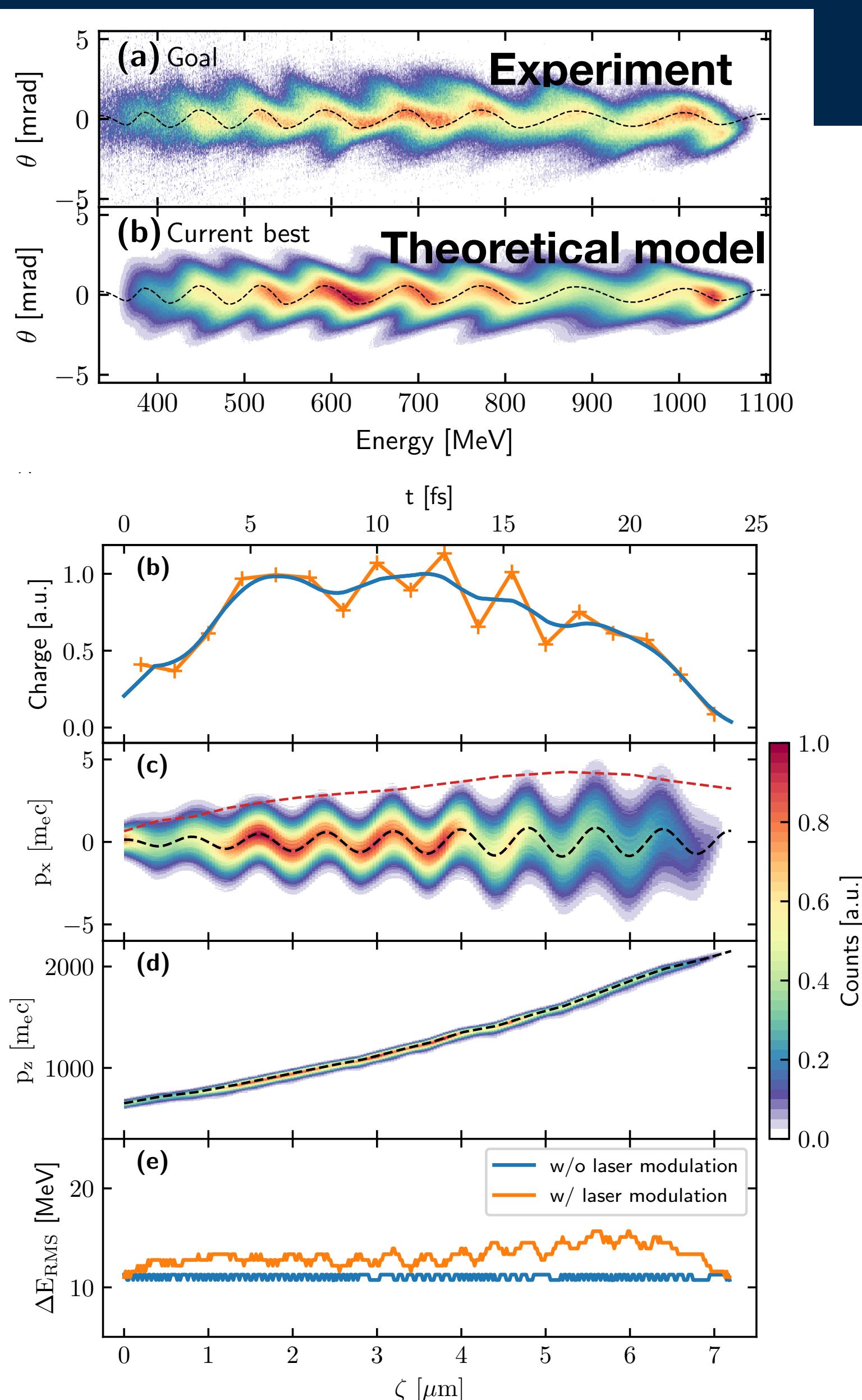
$\alpha \sim 1$, nonlinear wakefield.



The possibility of retrieval of transverse phase space

Summary

- Theoretical model describes coupled motion of electrons in laser driven plasma wakefields and oscillations in the laser fields
- Experimental observation of modulated electron spectral which can be fitted with the theory model
- Reconstruction of the electron beam characteristics including: longitudinal momentum distribution (energy chirp), transverse momentum distribution, temporal profile (pulse duration), slice energy spread. (All at a single shot!)



**Thank you for your
attention!**