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Investigating transverse trapping conditions in Beam-Induced Ionization Injection in PWFAs

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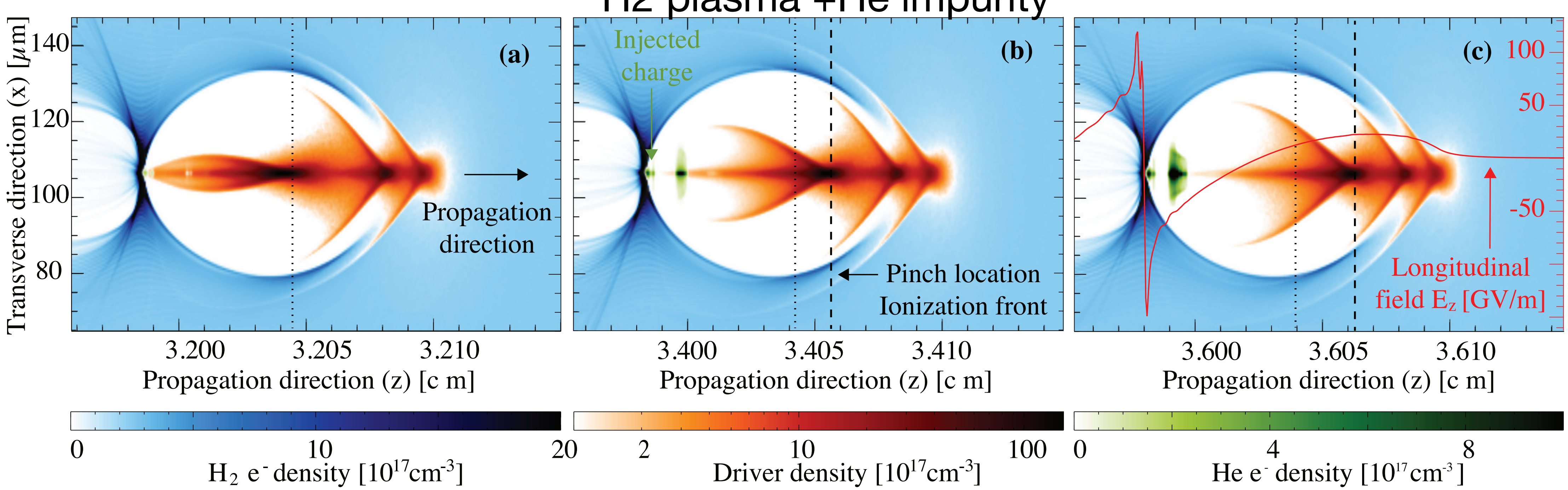
Beam-Induced Ionization injection (B-III)

- Beam-slice envelope is determined by the betatron equation,

$$\frac{d^2\sigma_r(z)}{dz^2} + K_\beta^2\sigma_r(z) - \frac{\epsilon_N^2}{\gamma^2\sigma_r^3(z)} = 0$$

- When the transverse size of a beam slice is reduced to its minimum (known as pinch), its transverse field increases and if it exceeds the ionization threshold of the high-ionization-threshold impurity, electrons are released at the pinch.
- If the trapping condition is satisfied, the ionized electrons can be trapped at the back of the bubble
- Our goal is to get a controllable ultrashort injected electron beam using B-III method

H2 plasma +He impurity



N. Vafaei-Najafabadi, et al., Philos Trans A Math Phys Eng Sci,377(2151) (2019)

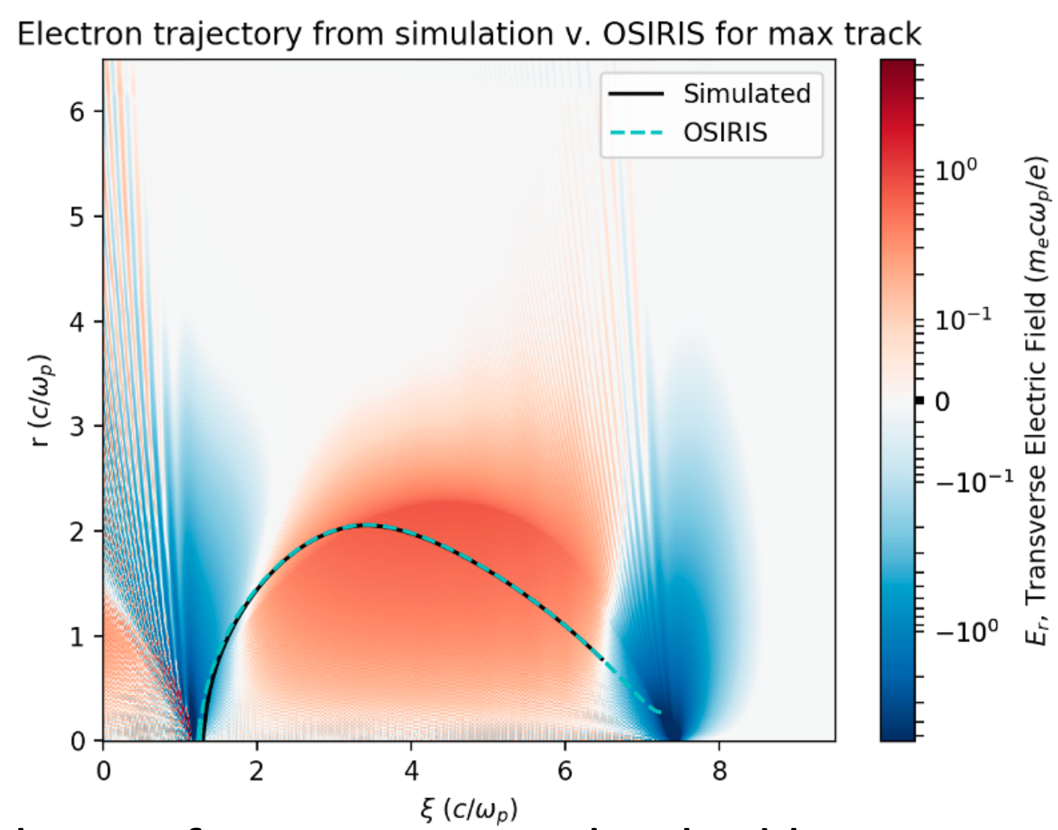
Simulation Tools

- A 2D Particle-In-Cell (PIC) simulation is run using OSIRIS in cylindrical coordinate. In the simulation, a pre-ionized hydrogen plasma is used to form the wakefield.

H_2 Plasma	Density n_0	$5 \times 10^{16} cm^{-3}$
	Ramp length	1 mm
	Plateau length	5 cm
	Ionization status	pre-ionized
Driver	Charge density n_b	$0.4 n_0$
	Energy	10 GeV
	Transverse beam size σ_r	35.55 μm
	Longitudinal beam size σ_z	23.7μm
	Emittance	10 mm mrad

To save the computation resource, simulation code (eTracks) was developed to

- •Track arbitrarily released electron trajectories in the electromagnetic field provided by one frame of the PIC simulation.
- •Determine the ionization region for a certain impurity component/profile using ADK model.
- Calculate the profile of injected electrons.



The performance was checked by comparing eTracks trajectory with OSIRIS trajectory*

*A. C. Farrell. B.S. Thesis (2020).

Beam Envelope Evolution

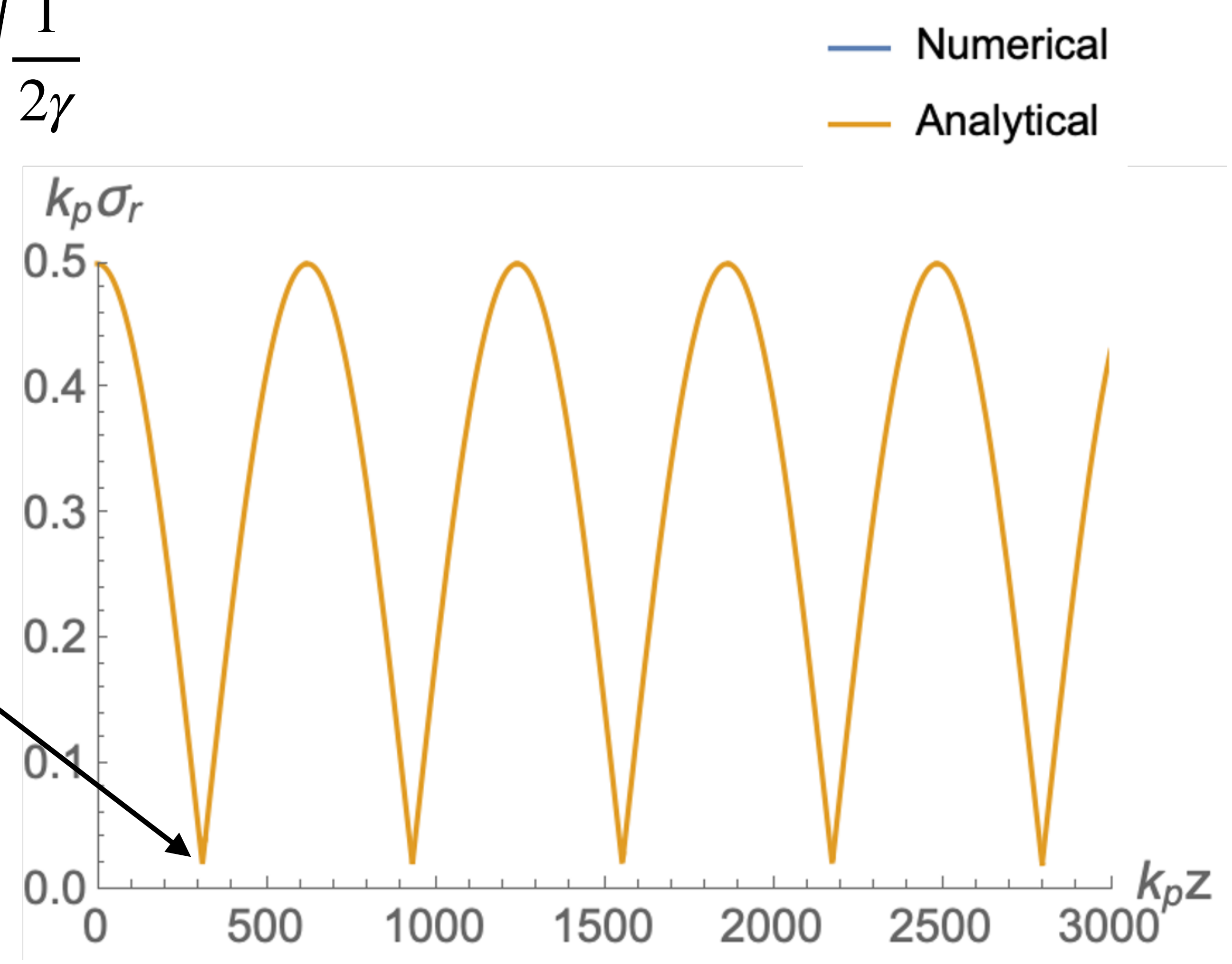
Analytical solution to the beam slice evolution

$$\frac{d^2\sigma_r(z)}{dz^2} + k_\beta^2\sigma_r(z) - \frac{\epsilon_N^2}{\gamma^2\sigma_r^3(z)} = 0 \qquad k_\beta = \sqrt{\frac{1}{2\gamma}}$$

$$\sigma_r(z) = \left| \left(\sigma_{r_0} - \Delta\sigma_r\right) \cos\left(k_\beta z\right) \right| + \Delta\sigma_r$$

$$\Delta\sigma_r = \sqrt{\frac{2}{\gamma}\frac{\epsilon_N c}{\omega_p \sigma_{r_0}}}$$

Pinch size



Propagation distance z

$$k_p\sigma_{r_0} = 0.5, k_p\epsilon_N = 1, \gamma = 10GeV$$

Knowing the pinch evolution allows us to make a very good estimate for the focusing of an electron beam as well as the ionization location/size

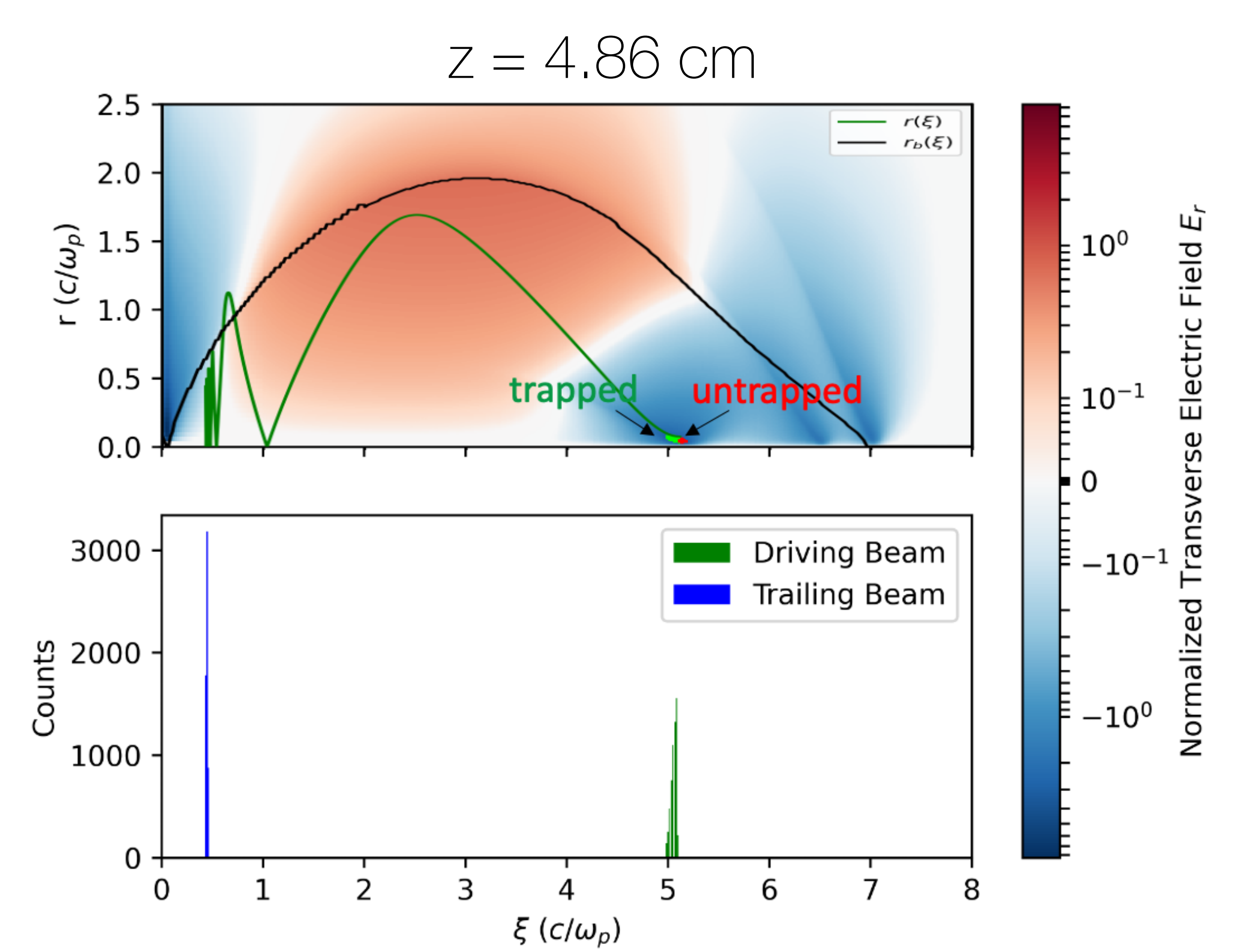
J. Yan, et al., arXiv:2107.00119 (2020)

Beam Injection in the Longitudinal Space

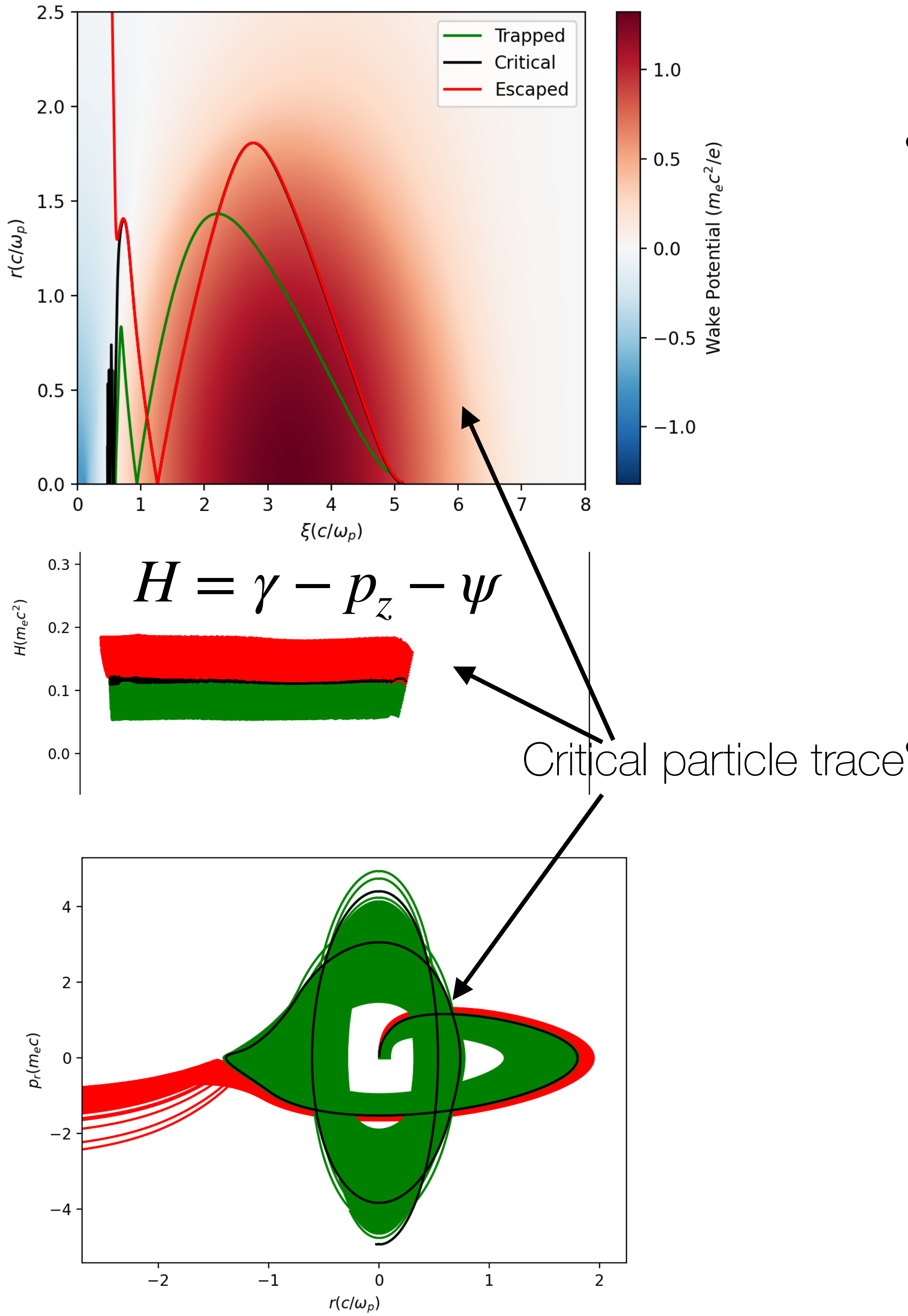
- Due to the ionization control, the injection happens only in a small number of time steps
- A portion of ionized electrons are trapped.
- For trapped electrons, the injection volume is much smaller than the ionization volume leading to ultrashort electron beam
    - **Compression factor** χ

$$\frac{1}{\chi} = \frac{\Delta\xi_f}{\Delta\xi_i} = \frac{\frac{\Delta\xi_f}{\Delta\psi_f}}{\frac{\Delta\xi_i}{\Delta\psi_i}} = -\frac{\partial_\xi\psi_i}{\partial_\xi\psi_f} = -\frac{E_{z_i}}{E_{z_f}}$$

    - The injected beam is further compressed if electrons are ionized in small Ezi



Trapping Mechanism and Critical Particle



- Trapping Condition:
  - Besides $\delta\psi < -1$, a transverse trapping condition also confine the trapping process
  - The transverse phase space shows this "transverse trapping condition" dominates the trapping process
- Critical Particle:
  - The critical particle corresponds to the critical energy. Only particles initialized with lower energies can be trapped

Transverse Motion

Transverse Force with a beam driver in the co-moving frame ($\xi = z - ct$)*

$$F_r = -\frac{r}{2} - \frac{r(1-v_z)}{2}\frac{d^2\psi_0}{d\xi^2} + \frac{(1-v_z)\lambda_0(\xi)}{r}$$

$$H = \gamma - p_z - \psi + \gamma^2 = 1 + p^{2*}$$

$$\rightarrow 1 - v_z = \frac{2(H_0+\psi)^2}{1+p_r^2+(H_0+\psi)^2}$$

Differential equation

$$F(r'', r', r, \xi) = 0$$

$$F(p_r', p_r, r, \xi) = 0$$

$$p_r = -(H_0+\psi)\frac{dr}{d\xi}$$

where $H_0 = 1 - \psi_i$

$$F(r', p_r, r, \xi) = 0$$

W. Lu et al, Physics of Plasmas 13, 056709 (2006)

Transverse Hamiltonian

$$F(p_r', p_r, r, \xi) = 0$$

$$F(r', p_r, r, \xi) = 0$$

⇒

$$p_r' = \frac{r}{4}\frac{p_r^2}{(H_0+\psi)^2} + \frac{r}{4(H_0+\psi)^2} + \frac{r}{2}\left(\frac{1}{2} + \psi_0''\right) - \frac{\lambda_0(\xi)}{r}$$

$$r' = -\frac{1}{(H_0+\psi)}p_r$$

⇓

Hamiltonian Equations

$$p_r' = -\frac{h_r}{\partial r}$$

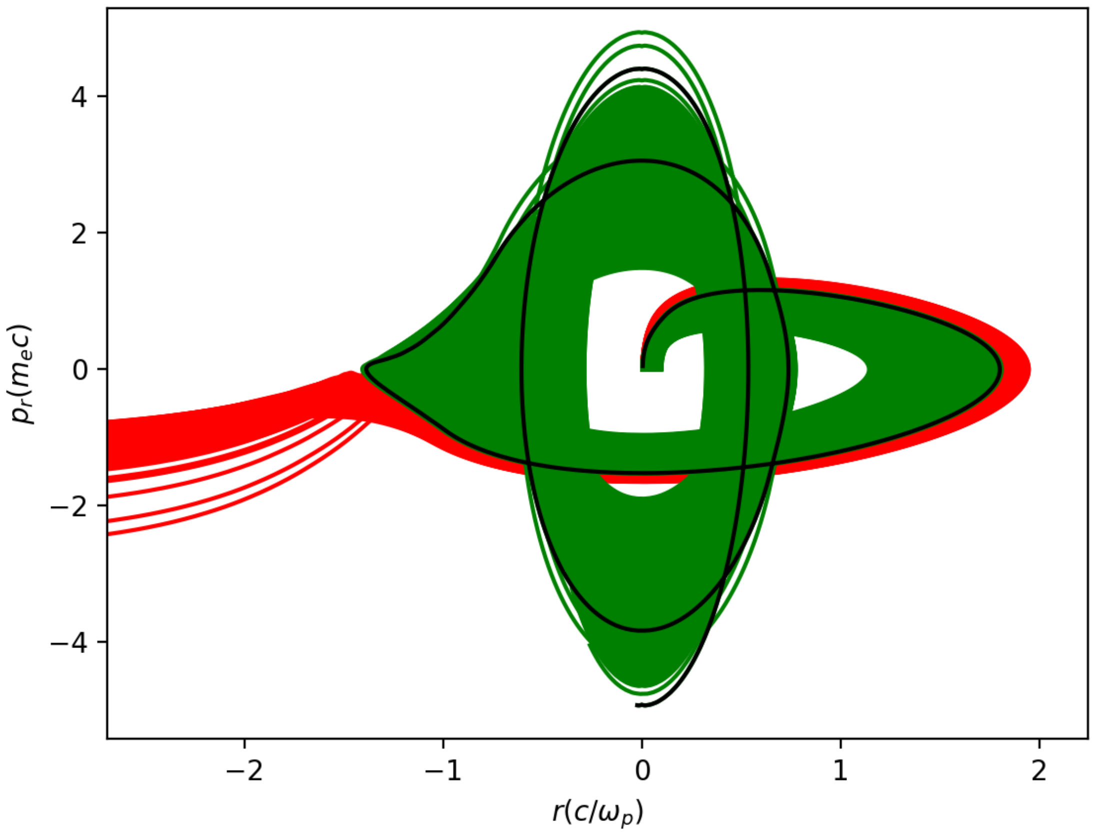
$$r' = \frac{\partial h_r}{\partial p_r}$$

ξ-dependent Transverse Hamiltonian

$$H_r(r, p_r, \xi) = -\frac{1}{2}\frac{1+p_r^2}{H_0+\psi} - \frac{r^2}{4}\left(\frac{1}{2} + \psi_0''\right) + \lambda_0(\xi)\ln(|r|)$$

$$\frac{1}{2}(\gamma + p_z)$$

Transverse phase space description



$$\gamma - p_z = H_0 + \psi = H_0 + \psi_0 - \frac{r^2}{4}$$

High energy limit:

$$p_z \sim \gamma, p_z >> 1, p_z >> p_r$$

$$LHS = \sqrt{1 + p_r^2 + p_z^2} - p_z = p_z(\sqrt{1 + \frac{p_r^2}{p_z^2} + \frac{1}{p_z^2}} - 1)$$

$$\sim \frac{1 + p_r^2}{2\gamma}$$

$$\frac{p_r^2}{2\gamma(H_0 + \psi_0) - 1} + \frac{r^2}{4(H_0 + \psi_0) - \frac{2}{\gamma}} = 1$$

Ellipses in pr-r space at injection

Normalized Emittance

$$\frac{p_r^2}{2\gamma(H_0+\psi_0)-1}+\frac{r^2}{4(H_0+\psi_0)-\frac{2}{\gamma}}=1$$

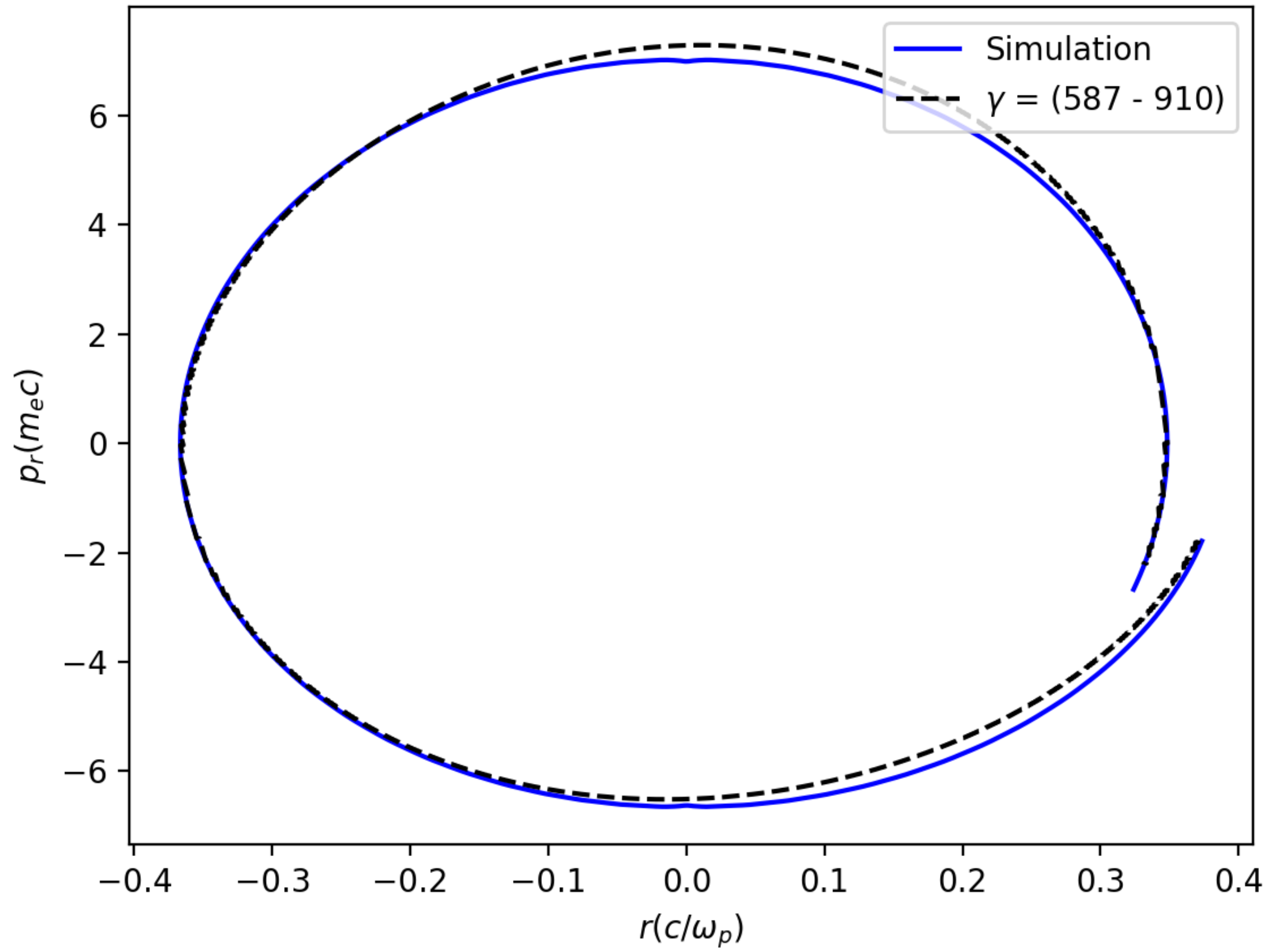
$$H_0 = 1-\psi_{0_i}+\frac{r_i^2}{4}$$

At the trapping position $\psi_0=\psi_{0_f}$

$$H_0+\psi_{0_f}=1+\psi_{0_f}-\psi_{0_i}+\frac{r_i^2}{4}$$

In the beam driver case

$$\epsilon \sim 2\sqrt{2}\pi\sqrt{\gamma}(1+\psi_{0_i}-\psi_{0_i}+\frac{r_i^2}{4}) \sim 8$$



Critical Particle:
One cycle in transverse phase space

Summary

- To get a controllable high-quality injected electron beam using B-III method:
  - ☑ We have developed a simple and accurate tool to track the particle motion inside a given wakefield
  - ☑ We have analytically solved the beam slice oscillation equation in the wake field, which helps us understand the ionization process
  - ☑ We understand how to control the beam quality

- We are working on understand critical transverse trapping condition from the aspects of Hamiltonian formalism.

Acknowledgment

We acknowledge the support by U.S. Department of Energy, Office of High Energy Physics (HEP) program under Award No. DE-SC-0014043 and resources of NERSC facility, operated under Contract No. DE-AC02-5CH11231

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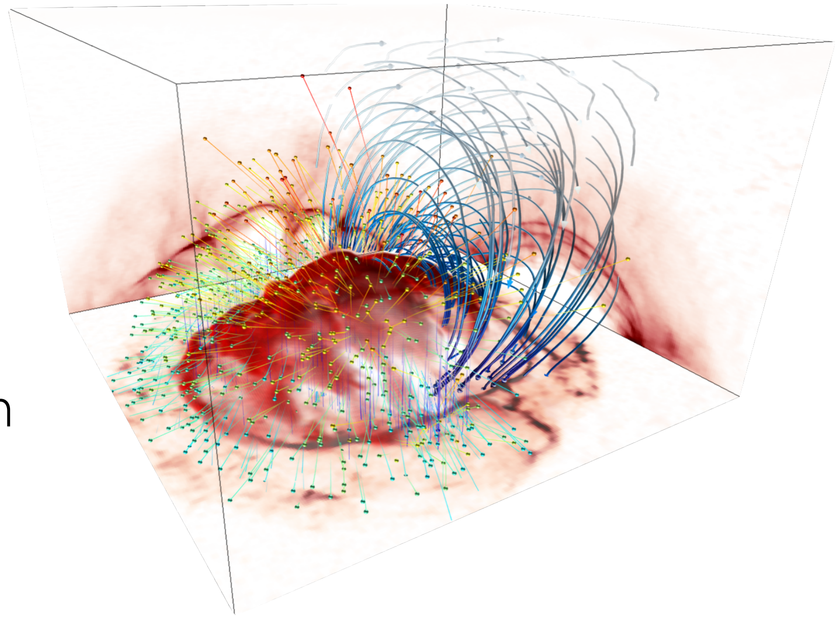
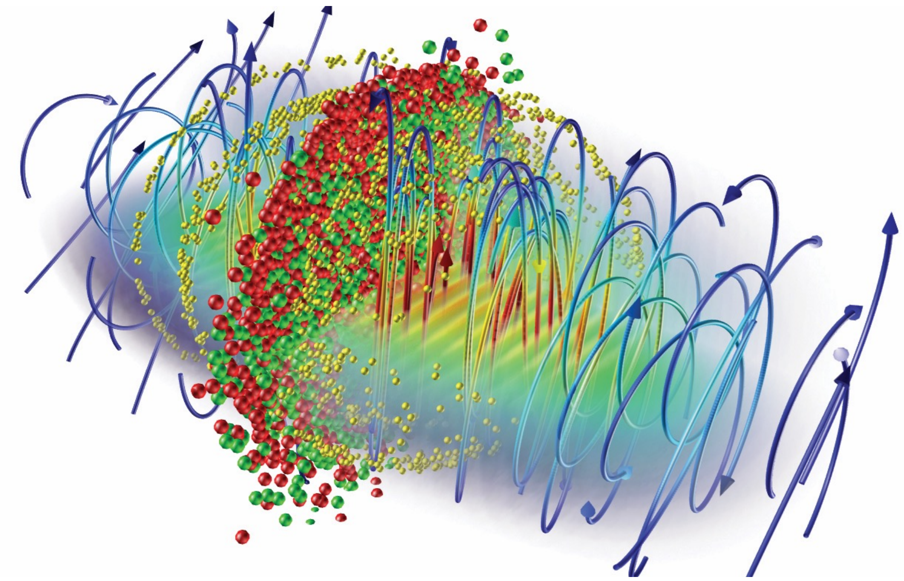
Osiris 4.0

osiris framework

- Massivelly Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium
  - ⇒ UCLA + IST





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code features

- Scalability to ~ 1.6 M cores
- SIMD hardware optimized
- Parallel I/O
- Dynamic Load Balancing
- QED module
- Particle merging
- GPGPU support
- Xeon Phi Support

SeaWulf   Cori NERSC

Thank you!

Backup slides

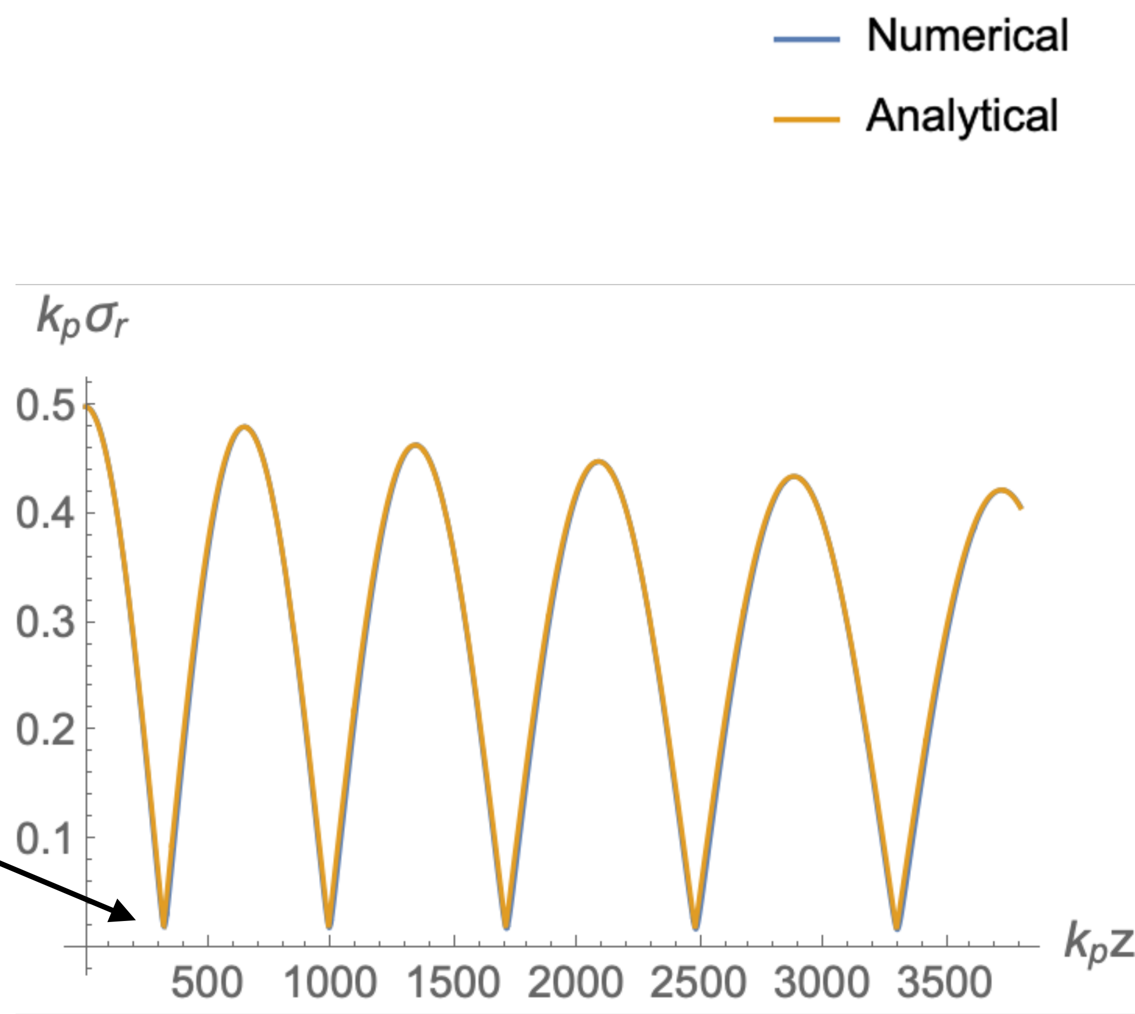
Beam Envelope Evolution

The beam slice evolution with acceleration

$$\frac{d^2\sigma_r(z)}{dz^2} + \frac{\gamma(z)'}{\gamma(z)}\sigma_r'(z) + k_\beta^2\sigma_r(z) - \frac{\epsilon_N^2}{\gamma(z)^2\sigma_r^3(z)} = 0$$

$$\sigma_r(z) = \left| \left[\sigma_{r_0} \left(\frac{\gamma_0}{\gamma(z)} \right)^{1/4} - \Delta\sigma_r(z) \right] \cos\left(\int k_\beta(z)dz \right) \right| + \Delta\sigma_r(z)$$

$$\Delta\sigma_r = \sqrt{\frac{2}{\gamma(z)}} \frac{\epsilon_N c}{\omega_p \sigma_{r_0} \left(\frac{\gamma_0}{\gamma(z)} \right)}$$



Propagation distance z

J. Yan, et al., arXiv:2107.00119 (2020)