Dominance of the seed from a tightly-focused electron bunch over the self-modulation of a long proton bunch in an over-dense plasma

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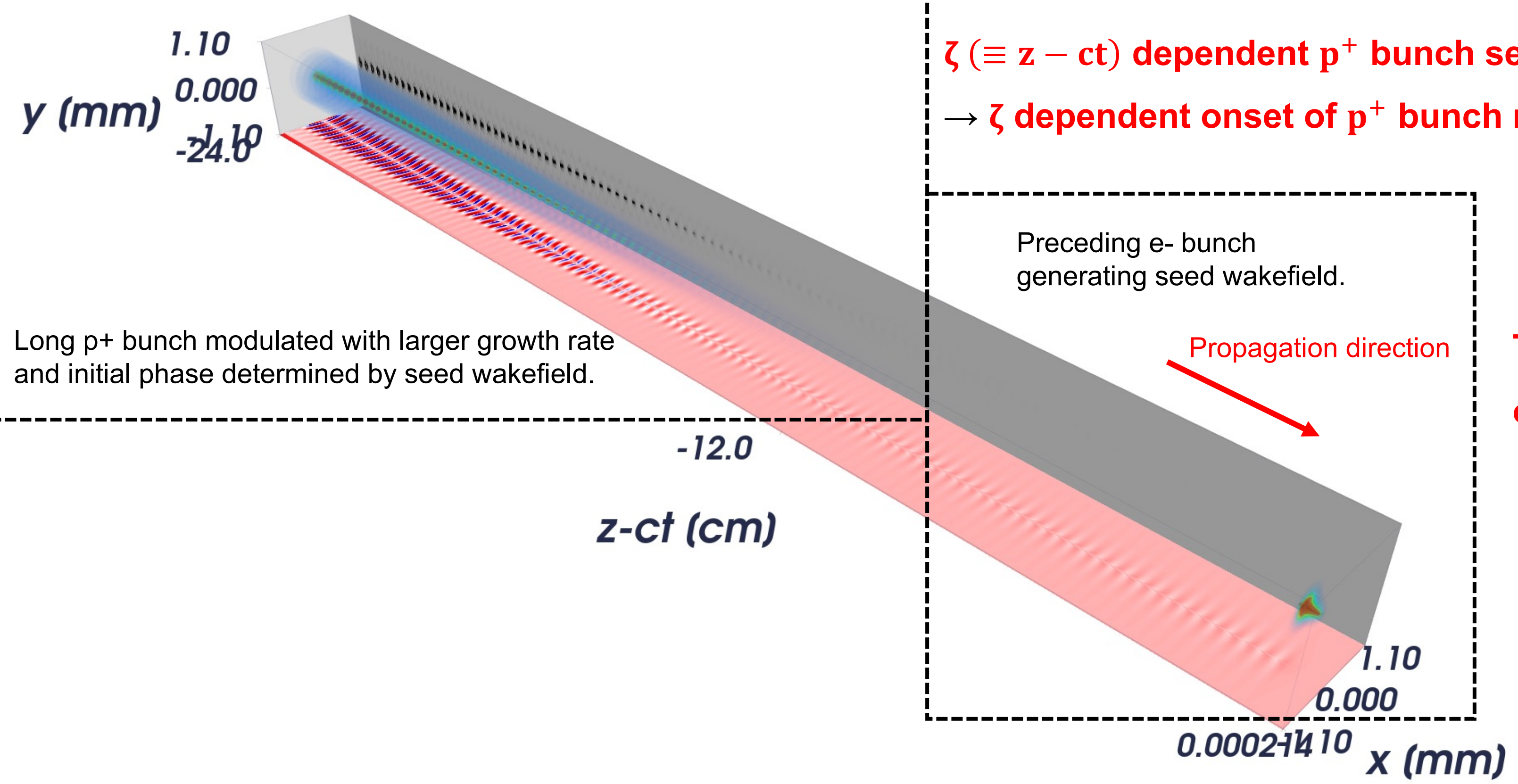
A preceding e- bunch seeds self-modulation of long p+ bunch

- **Electron bunch preceding a long proton bunch generates wakefield for seeding the proton bunch modulation for PWFA.**
- Seed and long proton bunch parameters decoupled.
- More degrees of freedom with new aspects of physics.
- Experimental result of e- bunch seeding reported in [L. Verra et al., PRL 129, 024802 (2022)].
- We analytically and numerically (using FBPIC*) studied the features of SSM for high-energy p+ and low-energy e- seeds.



Long p+ bunch modulated with larger growth rate and initial phase determined by seed wakefield.

Preceding e- bunch generating seed wakefield.

ζ ($\equiv z - ct$) dependent p^+ bunch self-fields and noise + "quasi static" seed

$\rightarrow$ ζ dependent onset of p^+ bunch modulation

Tightly-focused e^- bunch makes onset of p^+ bunch modulation ζ independent!

*R. Lehe, M. Kirchen, I.A. Andriyash, B.B. Godfrey, and J.-L. Vay, Comput. Phys. Comm. 203, 66-82 (2016).

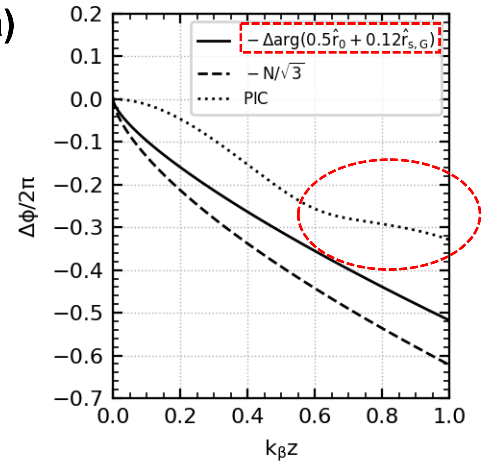
In this work, we will show

- We obtained asymptotic solutions of long p+ bunch modulation amplitude by modifying eqs in [C. B. Schroeder et al., Phys. Plasmas 20, 056704 (2013)] for
    1. radially-matched high-energy proton seed (comparable to ionization front seeding or case of unexpected bump of long p+ bunch) and
    2. (experimentally proved) tightly-focused low-energy electron seed.

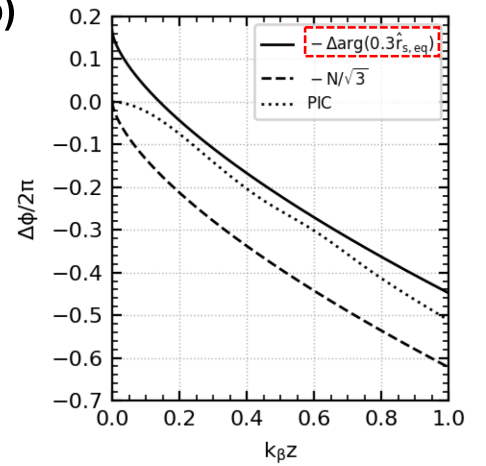
- **(a)** Proton seed causes anomalous phase behavior, which comes from
    1. mixed mode (self-modulation + seeded modulation) or
    2. onset timing difference at each p+ bunch slice.

- **(b)** Electron seed simultaneously drives a single mode modulation, suppressing the anomalous phase behavior.

(a)



(b)



$\mathbf{k_\beta}$: Betatron wavenumber of p+ bunch particle

Physical parameters

Physical parameters are set to follow AWAKE Run 2a experiment in the limit for testing analytical approach.

- Over-dense plasma ($n_b \ll n_{pe}$).
- Thin beams ($r_b \ll 1/k_{pe}$).
- Long p+ bunch initially at radial equilibrium.
- p+ and e- seeds.

Plasma

1. Pre-ionized plasma number density $\mathbf{n_{pe} = 1.0\times10^{14}\ cm^{-3}}$ with $\lambda_{pe} \approx 3.3\ \mathrm{mm}$.
2. Plasma radius $k_{pe}R \approx 1.88\ (1\ \mathrm{mm})$

Seed bunch

1. Bunch rms sizes $\mathbf{r_{s0} = 0.4/k_{pe}\ (\approx 213\ \mu m)}$ and $L_s = 1.4/k_{pe}\ (\approx 744\ \mu\mathrm{m})$ for Gaussian profile.
2. Bunch density $\mathbf{n_s = 1.8\times10^{12}\ cm^{-3}}$ $(|Q_s| \approx 150\ \mathrm{pC})$
3. p+ bunch relativistic gamma $\boldsymbol{\gamma_{s,p+} = 426}$, $\Delta\gamma_s = 0.$
4. e- bunch relativistic gamma from experimental setup $\boldsymbol{\gamma_{s,e-} = 35.2}$, $\Delta\gamma_s = 0.$
5. Normalized emittance from experimental setup $\epsilon_{r,n,s} = 1\ \mu\mathrm{m}$

Proton bunch

1. Bunch rms sizes $\mathbf{r_{p+,0} = 0.4/k_{pe}\ (\approx 213\ \mu m)}$ and $L_{p+} \approx 113/k_{pe}\ (6\ \mathrm{cm})$ for Gaussian profile.
2. Bunch density $\mathbf{n_{p+} = 2.4\times10^{12}\ cm^{-3}\ (Q_{p+} \approx 16\ nC)}$
3. p+ bunch relativistic gamma $\gamma_{p+} = 426$, $\Delta\gamma_{p+} = 1.49.$
4. Normalized emittance for initial equilibrium $\boldsymbol{\epsilon_{r,n,p+} = 0.4\gamma_{p+}k_\beta r_{p+}^2 \approx 1.8\ \mu m}$

p+ bunch modulation seeded by

1. radially-matched high-energy proton seed
2. tightly-focused low-energy electron seed

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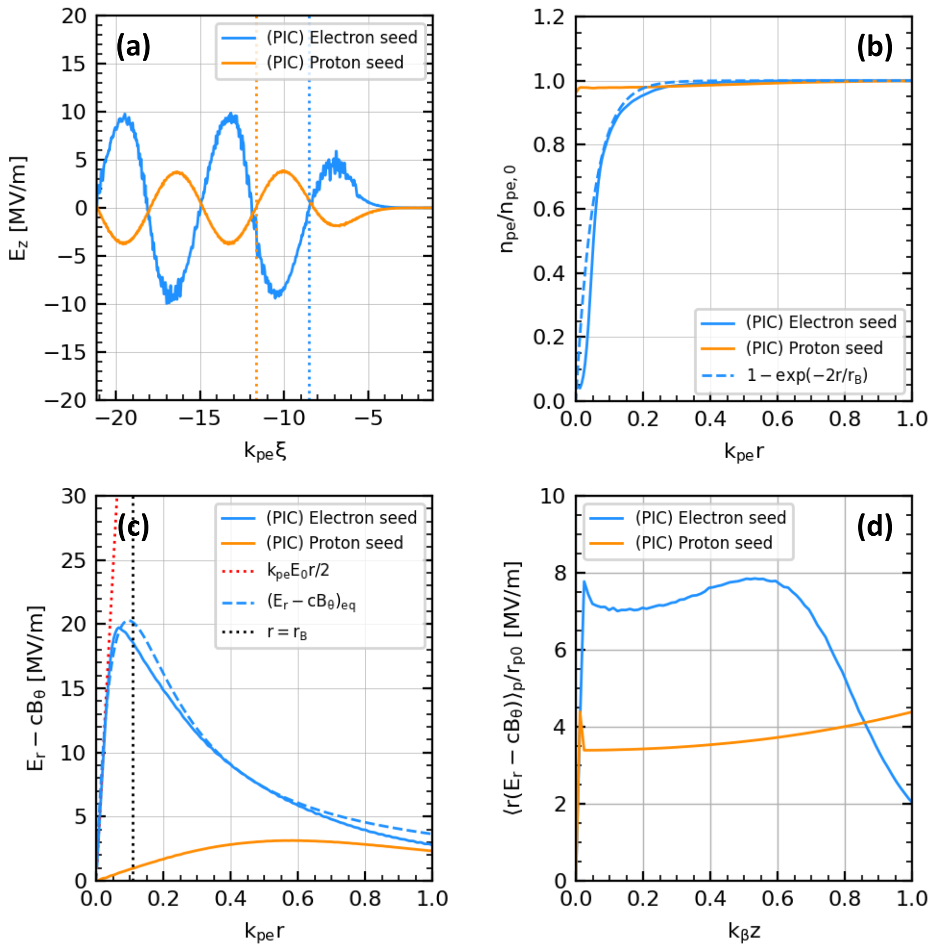
AWAKE

Radially-matched high-energy p+ seed

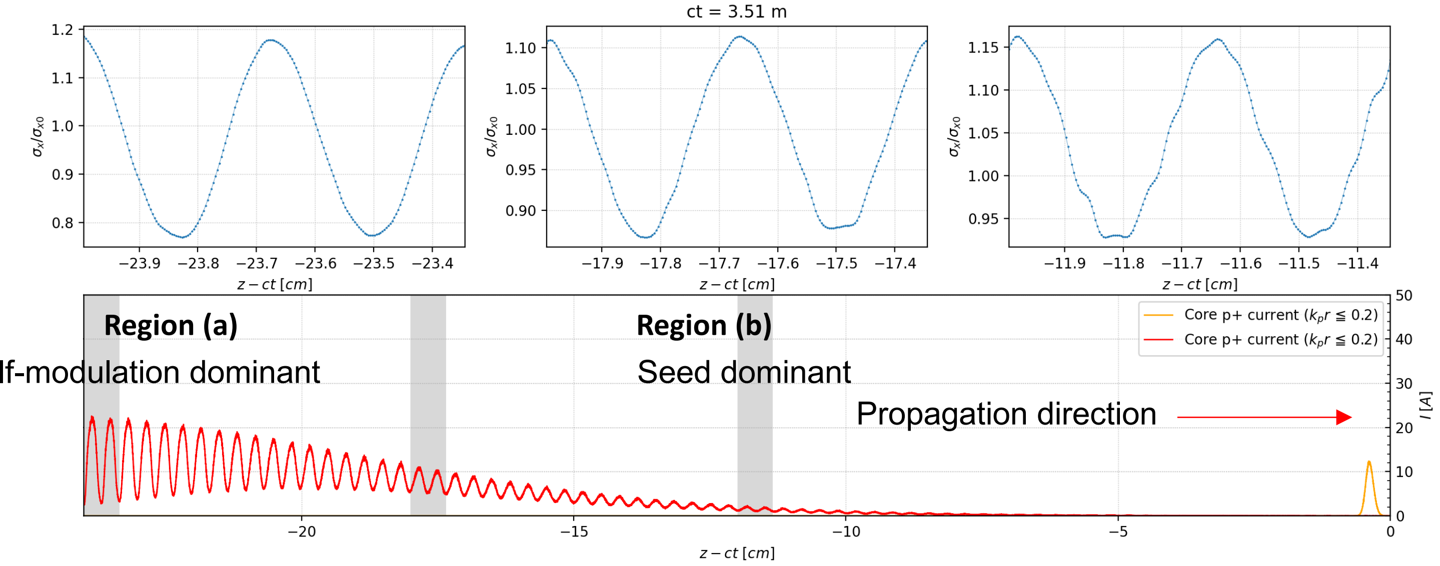


- Orange curves: proton seed.
- High-energy proton seed with Gaussian profile.
- **(b)** Small perturbation of plasma electrons.
- **(c)** Small wakefield amplitude.
- **(d)** Averaged radial wakefield increasing as hyperbolic cosine.

p+ bunch modulation with radially-matched high-energy p+ seed

p+ bunch center

p+ bunch front



Envelope modulation

Current modulation near axis

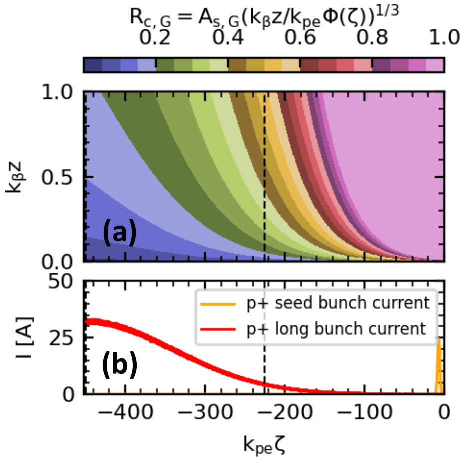
Modulation amplitude seeded by radially-matched high-energy proton seed

Self-modulation

Externally seeded modulation

$$\hat{r} \approx \left(\frac{3^{\frac{1}{2}}}{8\pi}\right)^{\frac{1}{2}}\left[\delta r + r_{p0}\sum_{\ell=1}\left\{\exp\left(-i\frac{\pi}{6}\right)A_{s,G}\left(\frac{k_\beta z}{k_{pe}\Phi(\zeta)}\right)^{\frac{1}{3}}\right\}^{2\ell}\right]\exp\left(N + i\frac{N}{\sqrt{3}} - i\frac{\pi}{12}\right)N^{-\frac{1}{2}} \equiv r_{p0}(\hat{r}_0 + \hat{r}_{s,G})$$

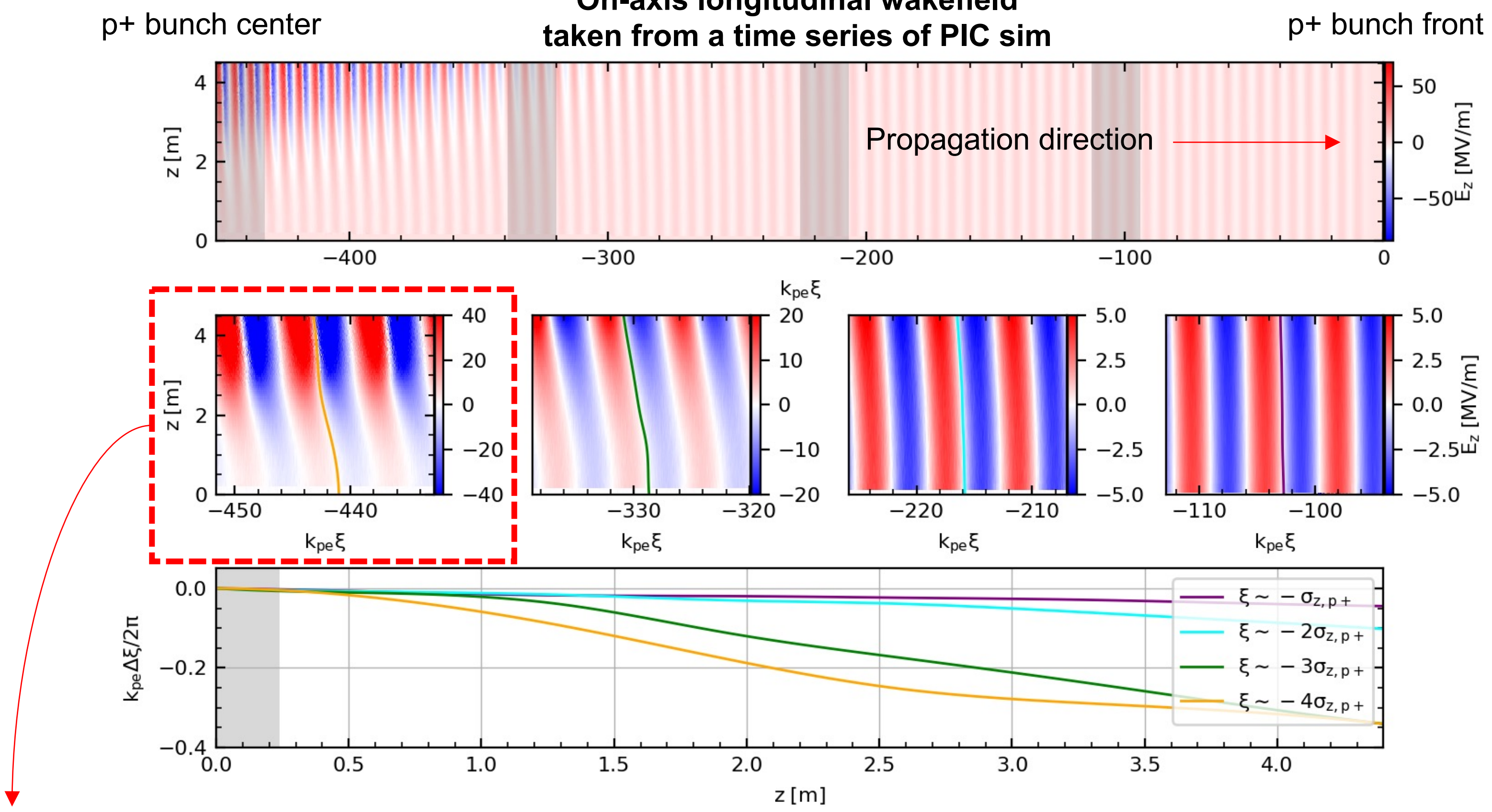
with $A_{s,G} = \left[\left(\frac{n_{s0}}{n_{p0}}\right)Z(\zeta)\right]^{\frac{1}{2}}$, $\Phi(\zeta) = \int_\zeta^\infty f_\parallel(\zeta')|d\zeta'|$, and $N = \frac{3^{\frac{3}{2}}}{4}\left(k_\beta^2 z^2 k_{pe}\Phi(\zeta)\right)^{\frac{1}{3}}$.



- Seed bunch radial dynamics time scale comparable to p+ bunch modulation.
- Seed effect dependent on bunch densities.
- **(a)** Series solution converges at $R_{c,G} < 1$.

Wakefield phase evolution

On-axis longitudinal wakefield taken from a time series of PIC sim

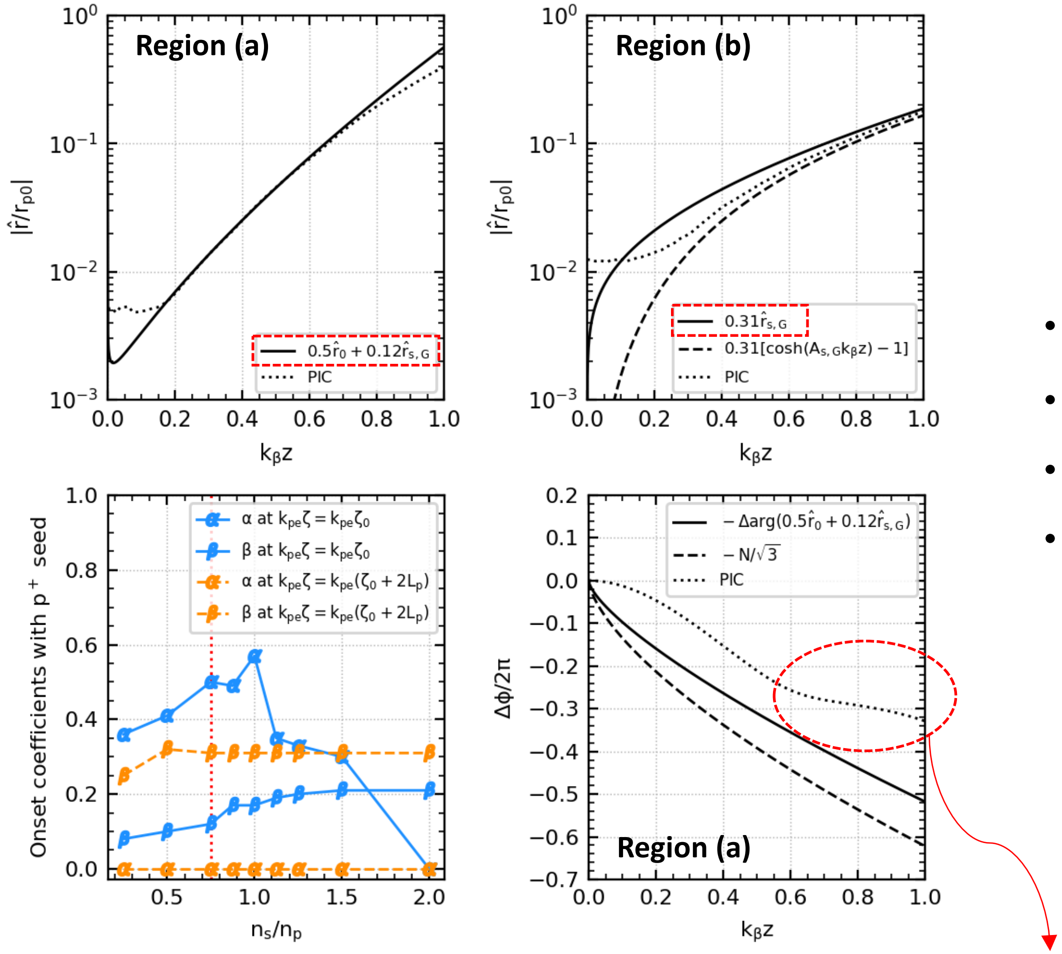


Trend of phase behavior comparable to analytical expression?

Mode of long p+ bunch modulation appears as function of ζ

Region (a): $\zeta \sim \zeta_0$

Region (b): $\zeta \sim \zeta_0 + 2\sigma$



- Normalized modulation amplitude $\hat{\mathbf{r}}/\mathbf{r_{p0}} \equiv \boldsymbol{\alpha}\hat{\mathbf{r}}_\mathbf{0} + \boldsymbol{\beta}\hat{\mathbf{r}}_{\mathbf{s,G}}$.
- α, β: onset coefficients determined by aligning $\hat{r}/r_{p0}$ with PIC sim.
- Modes polarized at p+ bunch front and center.
- Self-modulation $\hat{r}_0$ resonant at $n_s/n_p = 1$.

Anomalous phase behavior coming from onset timing difference along ζ.

p+ bunch modulation seeded by

1. radially-matched high-energy proton seed
2. tightly-focused low-energy electron seed

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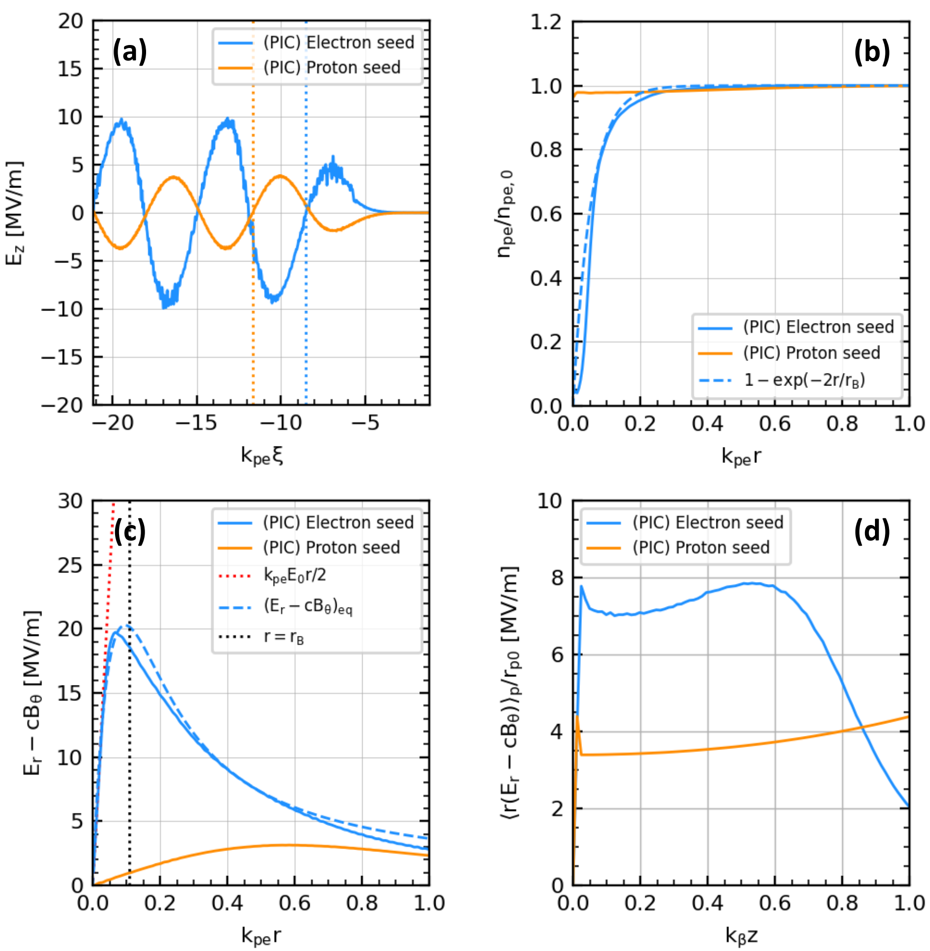
Tightly-focused low-energy e- seed



- Blue curves: electron seed.
- Unknown e- bunch radial profile after evolution.
- **(b)** Perturbed plasma electron radial profile as function of $r_B = 2\left(n_{e0}/n_{pe}\right)^{1/2} r_{s0}$.
- **(c)** Radial wakfield near axis close to $k_{pe}E_0x/2$, where $E_0 \equiv k_{pe}m_ec^2/e$.
- **(c)** Seed wakefield at e- bunch radial equilibrium $(E_r - cB_\theta)_{s,eq}$ (current dependent)

$$\approx E_0\left[\sqrt{2\pi}k_{pe}L_s \exp\left(-\frac{k_{pe}^2L_s^2}{2}\right)\cos(k_{pe}\zeta)\right]\left[\frac{k_{pe}r_B^2}{4r}\left\{1-\exp\left(-\frac{2r}{r_B}\right)\left(1+\frac{2r}{r_B}\right)\right\}\right]$$

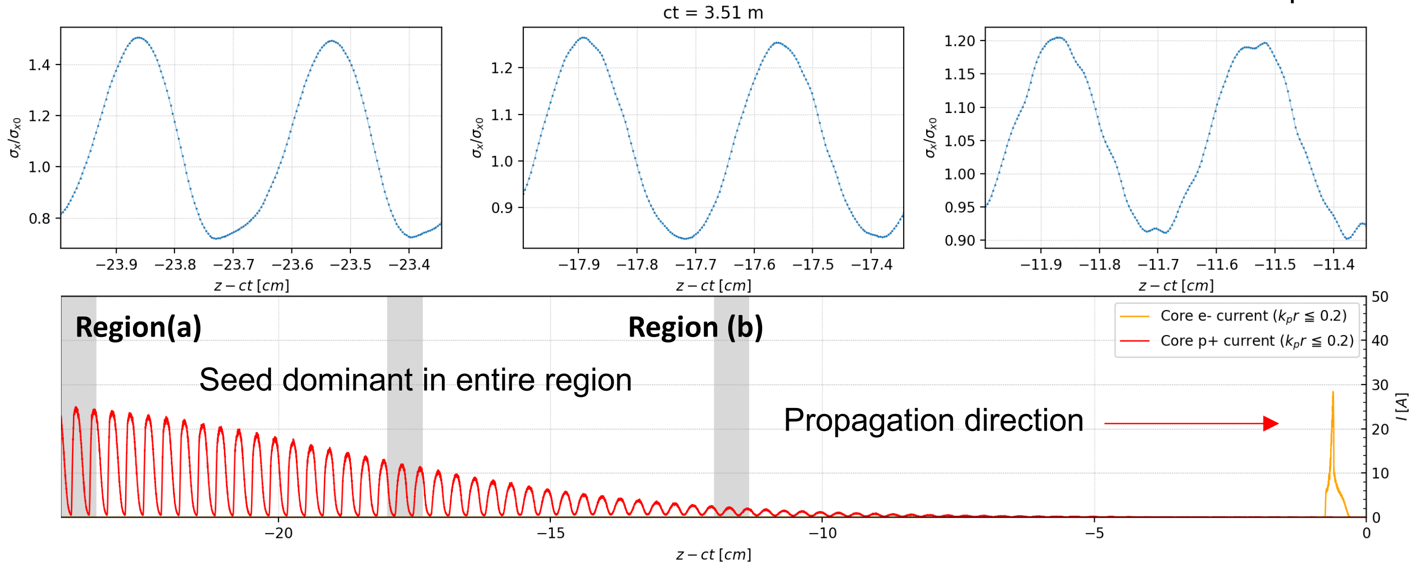
$$\equiv E_0 Z(\zeta)\left[\frac{k_{pe}r_B^2}{4r}\left\{1-\exp\left(-\frac{2r}{r_B}\right)\left(1+\frac{2r}{r_B}\right)\right\}\right].$$

- **(d)** Averaged radial wakefield assumed to be constant in $k_\beta z$ in analytical approach.

p+ bunch modulation with tightly-focused low-energy e- seed

p+ bunch center

p+ bunch front



Envelope modulation

Current modulation near axis

Modulation amplitude seeded by tightly-focused low-energy electron seed

Self-modulation

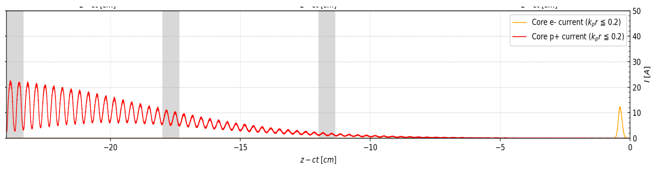
Externally seeded modulation

$$\hat{r} \approx \left(\frac{3^{\frac{1}{2}}}{8\pi}\right)^{\frac{1}{2}}\left[\delta r + r_{p0}\sum_{\ell=1}\left\{\exp\left(-i\frac{\pi}{6}\right)1.2A_{s,eq}\left(\frac{k_\beta z}{k_{pe}\Phi(\zeta)}\right)^{\frac{1}{3}}\right\}^{2\ell}\right]\exp\left(N + i\frac{N}{\sqrt{3}} - i\frac{\pi}{12}\right)N^{-\frac{1}{2}} \equiv r_{p0}\left(\hat{r}_0 + \hat{r}_{s,eq}\right)$$

with $A_{s,eq} = \left[\left(\frac{I_{s0}}{I_{p0}}\right)|Z(\zeta)|\right]^{\frac{1}{2}}$, $\Phi(\zeta) = \int_\zeta^\infty f_\parallel(\zeta')|d\zeta'|$, and $N = \frac{3^{\frac{3}{2}}}{4}\left(k_\beta^2 z^2 k_{pe}\Phi(\zeta)\right)^{\frac{1}{3}}$.

- Seed bunch reaching radial equilibrium before bunch modulation starts.
- Seed effect dependent on bunch currents.

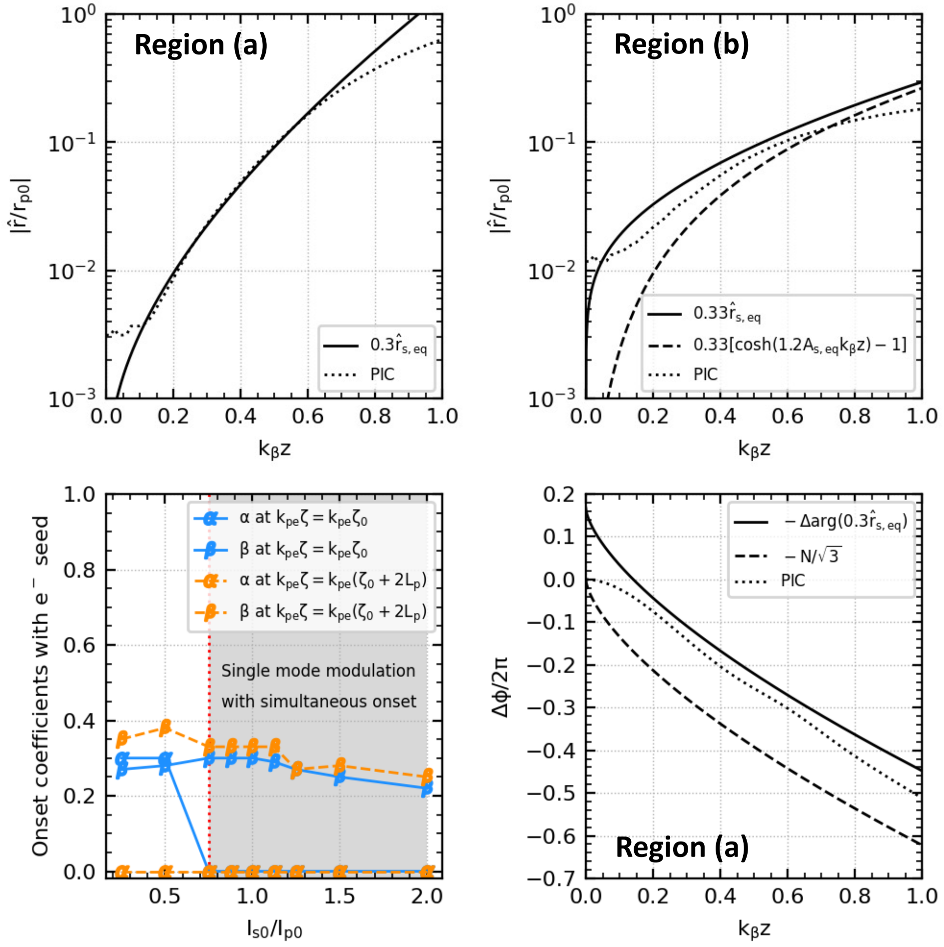
p+ seeded current modulation (p. 7)



e- seed simultaneously drives a single mode modulation

Region (a): $\zeta \sim \zeta_0$

Region (b): $\zeta \sim \zeta_0 + 2\sigma$



- Normalized modulation amplitude $\hat{\mathbf{r}}/\mathbf{r}_{\mathbf{p0}} \equiv \boldsymbol{\alpha}\hat{\mathbf{r}}_{\mathbf{0}} + \boldsymbol{\beta}\hat{\mathbf{r}}_{\mathbf{s,eq}}$.
- α, β: onset coefficients determined by aligning $\hat{r}/r_{p0}$ with PIC sim.
- No α, approximately same β at both regions **(a)** and **(b)**.
- Single mode modulation with simultaneous onset.

Phase behavior comparable to analytical expectation.

Conclusion

- We obtained asymptotic solutions of long p+ bunch modulation amplitude by modifying eqs in [C. B. Schroeder et al., Phys. Plasmas 20, 056704 (2013)] for
    1. radially-matched high-energy proton seed (comparable to ionization front seeding or case of unexpected bump of long p+ bunch) and
    2. (experimentally proved) tightly-focused low-energy electron seed.

- Proton seed causes anomalous phase behavior, which comes from
    1. mixed mode (self-modulation + seeded modulation) or
    2. onset timing difference at each p+ bunch slice.

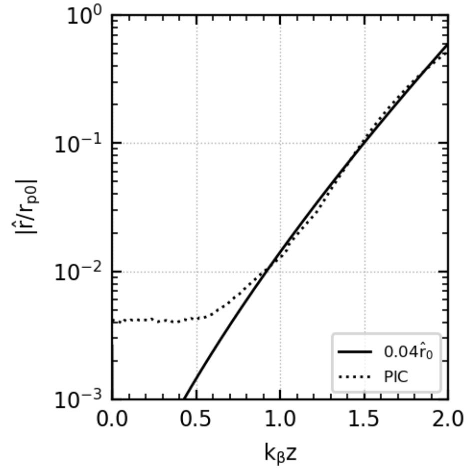
- Electron seed simultaneously drives a single mode modulation, suppressing the anomalous phase behavior.

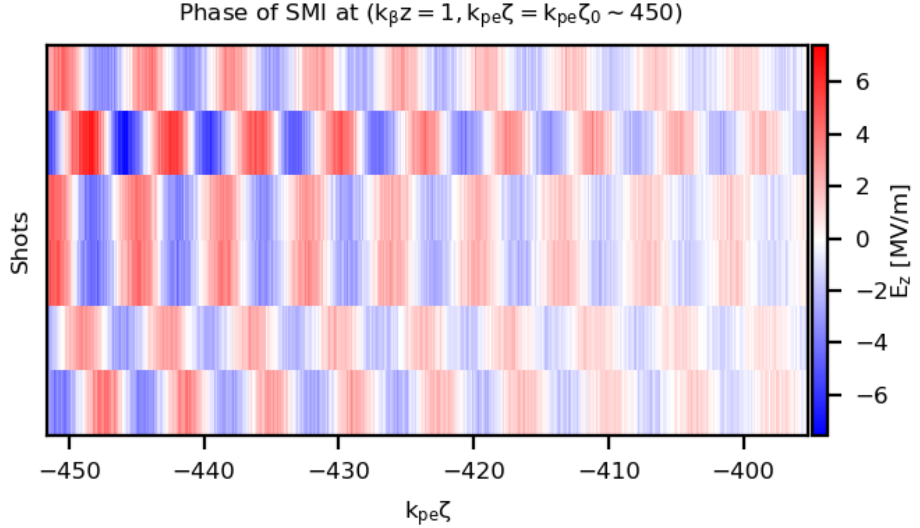
- The anomalous phase shift can be critical when introducing the plasma density step or injecting the witness bunch, thus should be carefully handled.

Thank you!

(no seed) Self-modulation instability of long p+ bunch

$\alpha \approx 0.04$ at $k_{pe}\zeta = \sim 450$





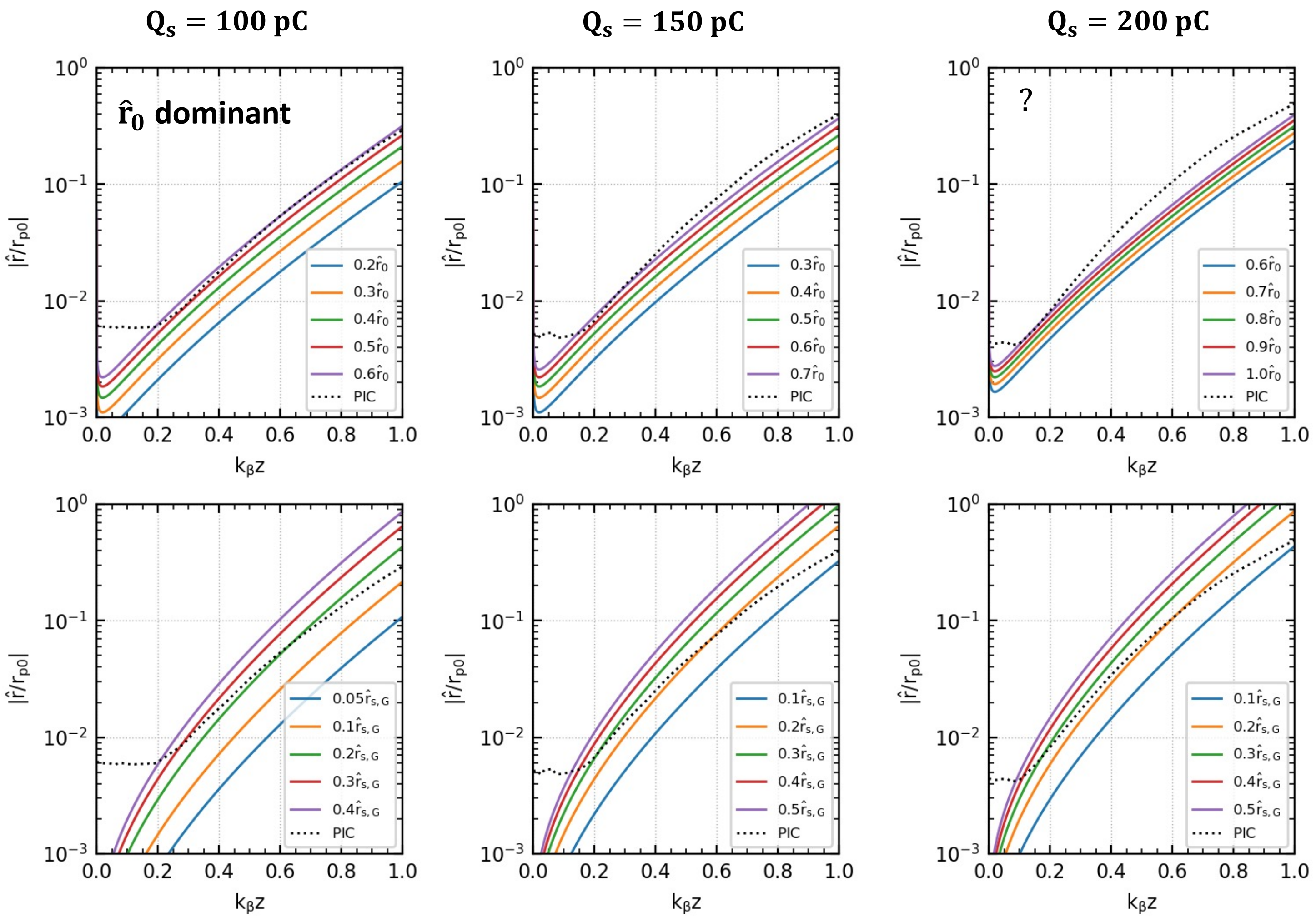
This figure means that wakefield from bunch front cut by simulation window ($L_w = 4L_{p+}$) comparable to p+ bunch noise field (1 simulation bunch particle represents 1000 physical particles), which is very small.

Growth rate of each mode of p+ seeded modulation at $\zeta = \zeta_0$

Growth rate with low density p+ seed not explained by a single mode modulation.

Self-modulation $\hat{r}_0$

Externally seeded modulation $\hat{r}_{s,G}$



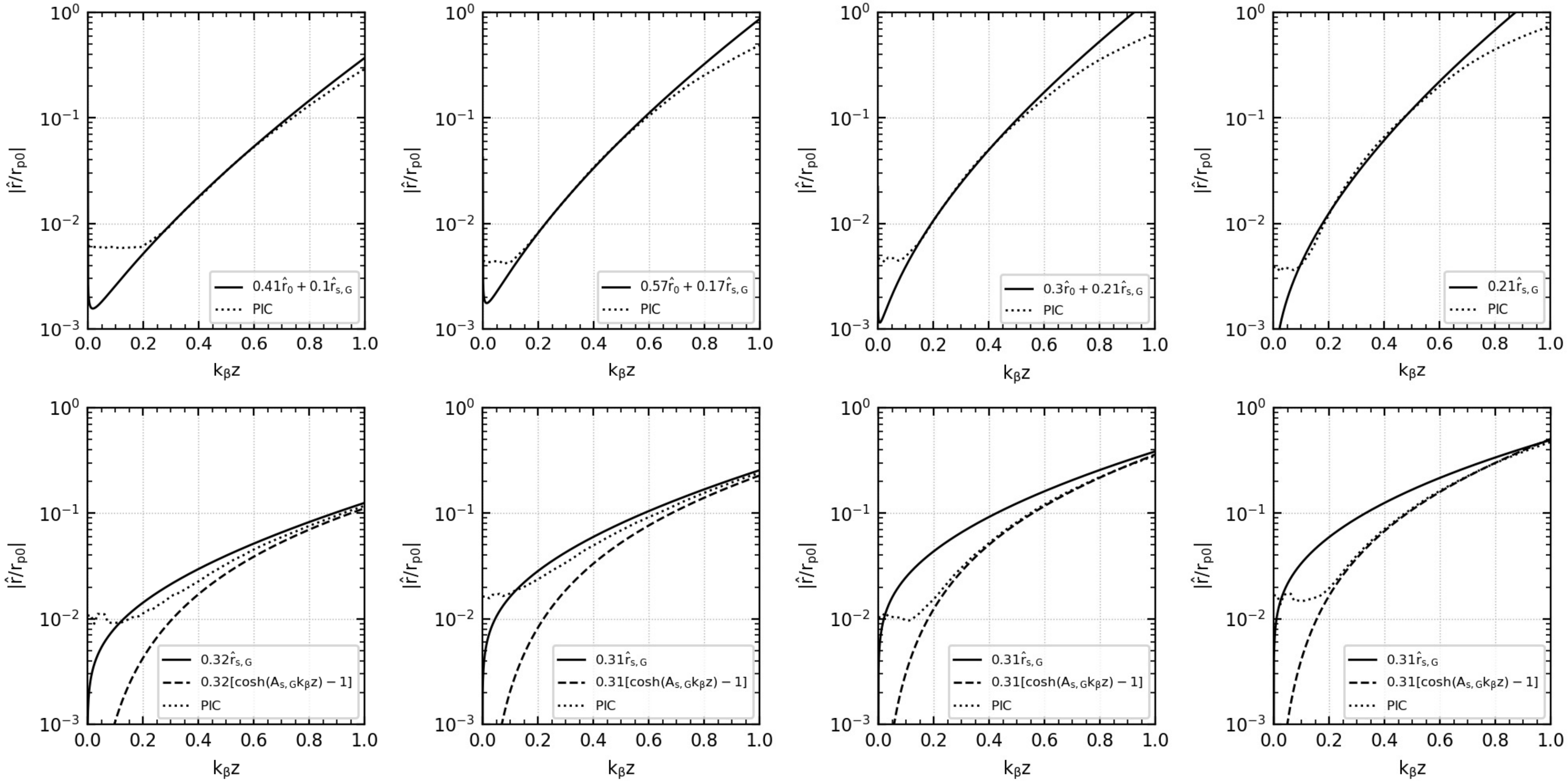
Set $\hat{\mathbf{r}}/\mathbf{r}_{\mathbf{p0}} \equiv \alpha\hat{\mathbf{r}}_0 + \beta\hat{\mathbf{r}}_{s,G}$, where α and β are additional weights for introducing onset timing.

Amplitudes of p+ seeded modulation at $\zeta = \zeta_0$ and $\zeta_0 + 2L_p$

$Q_s = 100$ pC | $Q_s = 200$ pC | $Q_s = 300$ pC | $Q_s = 400$ pC

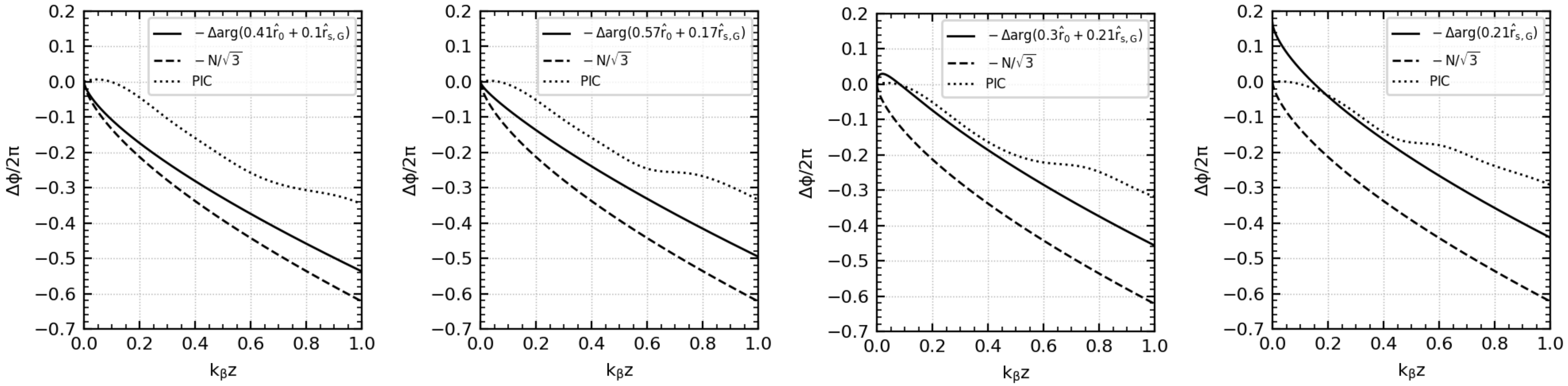
At $\zeta = \zeta_0$

At $\zeta = \zeta_0 + 2L_p$



Anomalous phase shifts of p+ seeded modulation at $\zeta = \zeta_0$

$Q_s = 100$ pC　　$Q_s = 200$ pC　　$Q_s = 300$ pC　　$Q_s = 400$ pC



Onset of anomalous phase behavior found earlier with larger seed bunch charge (or density).