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# ADJOINT OPTIMIZATION OF CIRCULAR LATTICES

L. Dovlatyan, A.K. Einarsson, B. Beaudoin, S. Bernal, I. Haber,
D. Sutter, P. O'Shea and T. M. Antonsen Jr.

Institute for Research in Electronics and Applied Physics
University of Maryland, College Park

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# Adjoint Approach: What it does

- Finds the dependence of system performance on parameters

- Useful for:

  Optimization – gradient descent
  Sensitivity Studies

- Requires identification of a Figure of Merit (FoM)

$F(\mathbf{a})$ where $\mathbf{a}$ is a vector of parameters

- Efficient calculation of $dF(\mathbf{a})/d\mathbf{a}$

Direct evaluation w/N parameters, requires N solutions
Adjoint approach – One or two solutions

# How Does It Work?

$\mathbf{X}$ - State of System $\quad$ $\mathbf{a}$ - Vector of Parameters

System state determined by nonlinear eq.

$$\mathbf{A}(\mathbf{X}) = \mathbf{a}$$

Figure of Merit (FoM)

$$F(\mathbf{X},\mathbf{a})$$

Linearize: $\mathbf{a} \rightarrow \mathbf{a} + \delta\mathbf{a} \quad \mathbf{X} \rightarrow \mathbf{X} + \delta\mathbf{X} \quad F \rightarrow F + \delta F$

Change in FoM

$$\delta F = \delta\mathbf{a} \cdot \frac{\partial F}{\partial \mathbf{a}} + \boxed{\delta\mathbf{X} \cdot \frac{\partial F}{\partial \mathbf{X}}}$$

Where

$$\delta\mathbf{X} \cdot \frac{\partial \mathbf{A}}{\partial \mathbf{X}}(\mathbf{X}) = \delta\mathbf{a}$$

Instead Solve Adjoint

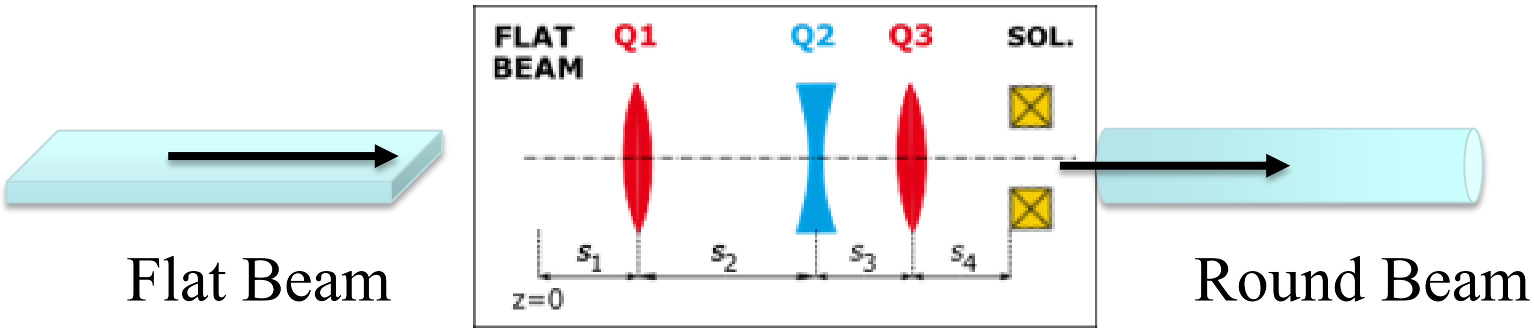
$$\frac{\partial \mathbf{A}}{\partial \mathbf{X}} \cdot \mathbf{Y} = \frac{\partial F}{\partial \mathbf{X}}$$

$$\delta F = \delta\mathbf{a} \cdot \frac{\partial F}{\partial \mathbf{a}} + \boxed{\delta\mathbf{a} \cdot \mathbf{Y}}$$

Single Y handles changes in all **a**

**Optimization of Flat to Round Transformers Using Adjoint Techniques***

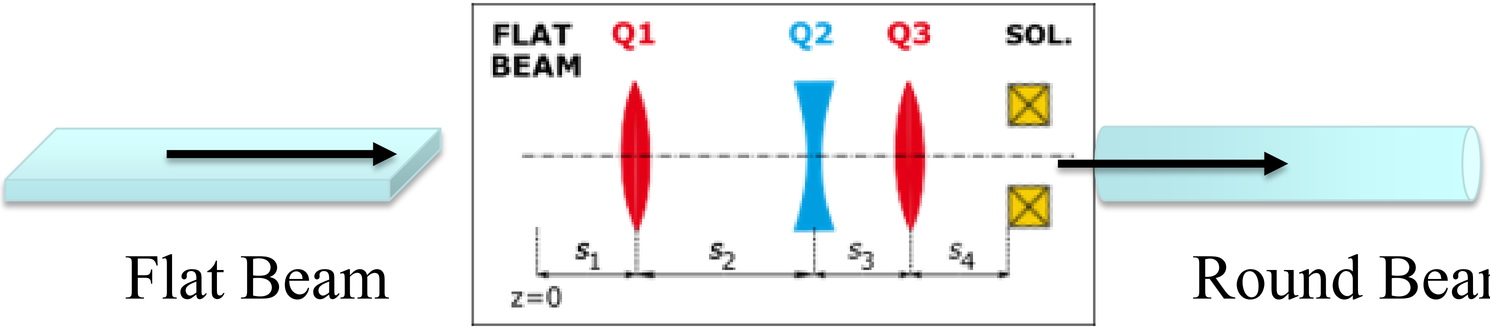
L. Dovlatyan, B. Beaudoin, S. Bernal, I. Haber, D. Sutter and T. M. Antonsen Jr , Phys Rev Accel and Beams V25, 044002 (2022).




Flat to Round and Round to Flat transformers were proposed for cooling of hadron beams. Ya. Derbenev, Adapting optics for high-energy electron cooling, Tech. Rep. UM-HE-98-04-A (1998).

# Steps

1. Derive system of moment equations (include self fields)
2. Linearize (to compute parameter gradient)
3. Find adjoint system
4. Decide on Figures of Merit
5. Optimize by Gradient Descent




Flat Beam

Round Beam

# Moment Equations

The second order moments are averages of the 4 by 4 elements of the Sigma matrix.

10 independent moments

$$\underline{\underline{\Sigma}} = \left[ \left\langle \begin{matrix} xx & xx' & xy & xy' \\ x'x & x'x' & x'y & x'y' \\ yx & yx' & yy & yy' \\ y'x & y'x' & y'y & y'y' \end{matrix} \right\rangle \right] \qquad x' = \frac{dx}{dz}$$

Moments: ***X**(z),* 10 components

Matrices

$$\frac{d}{dz}\mathbf{X} = \dot{\mathbf{X}}(\mathbf{X}, z | \mathbf{a}) = \mathbf{O}_{magnets}(z|\mathbf{a}) \cdot \mathbf{X}(z) + \mathbf{O}_{SpCh}(\mathbf{X}, z) \cdot \mathbf{X}(z)$$

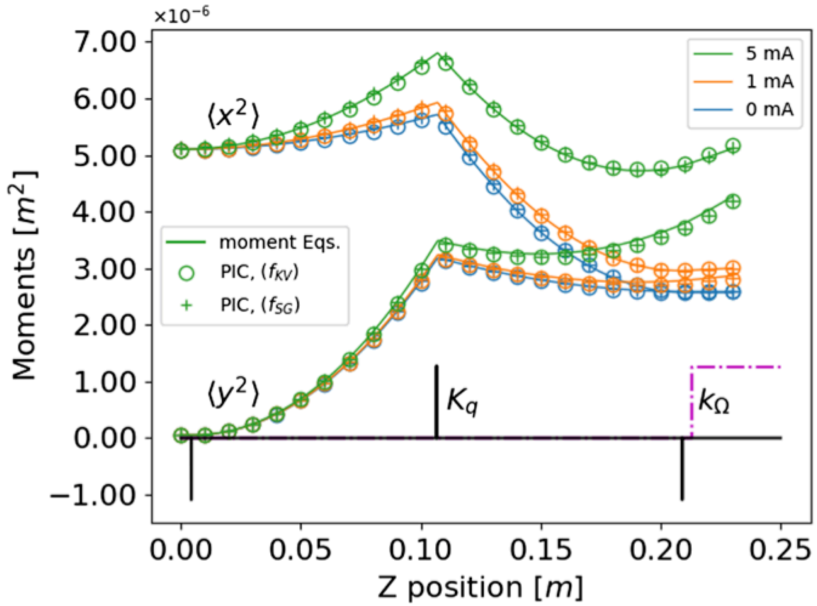
Magnet parameters - **a**

Nonlinear
Assumes K-V

Figure of Merit (FoM): $F(\boldsymbol{X}(z_f))$

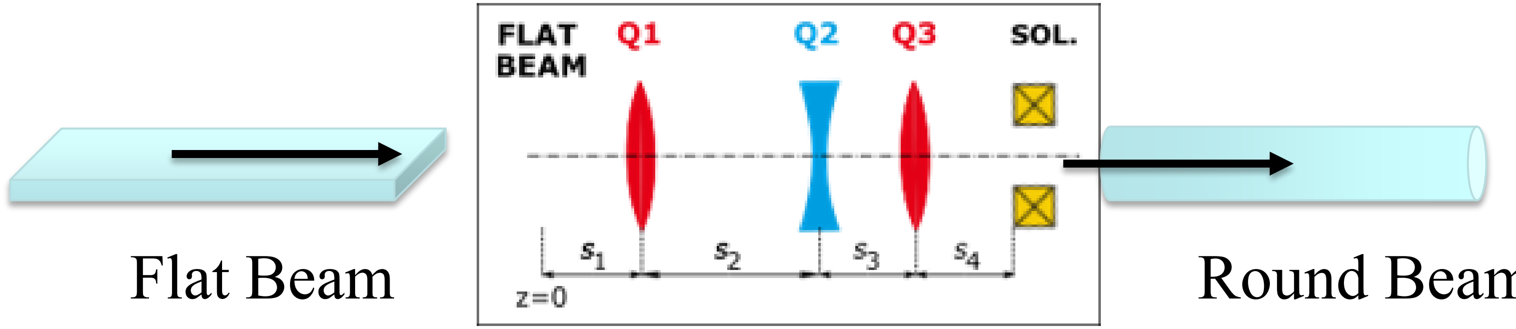
Function of moments at plane $z=z_f$

# Moment-PIC Comparison for FTR




Symbols –PIC (WARP)
o -KV
+ Semi-Gaussian
Lines – Moment Eqs.

A. V. Burov and S. Nagaitsev, Technical Report No. FERMILAB-TM- 2114, 2000



# Linearization to Calculate Gradient in Parameter Space - *a*

Base case: $$\frac{d}{dz}\mathbf{X} = \mathbf{O}_{magnets}(z|\mathbf{a})\cdot\mathbf{X}(z) + \mathbf{O}_{SpCh}(\mathbf{X})\cdot\mathbf{X}(z)$$

$$\mathbf{a} \rightarrow \mathbf{a} + \delta\mathbf{a}$$

Direct variation:

$$\frac{d}{dz}\delta\mathbf{X}^{(X)} = \left[\mathbf{O}_{magnets} + \mathbf{O}_{SpCh}\right]\cdot\delta\mathbf{X}^{(X)} + \left(\delta\mathbf{a}\cdot\frac{\partial}{\partial\mathbf{a}}\mathbf{O}_{magnets} + \delta\mathbf{X}^{(X)}\cdot\frac{\partial}{\partial\mathbf{X}}\mathbf{O}_{SpCh}(\mathbf{X})\right)\cdot\mathbf{X}(z)$$

$$0 = \left.\delta\mathbf{X}^{(X)}\right| \text{ at } z = z_i$$

Change in FoM: $$\delta F = \left.\delta\mathbf{X}^{(X)}\right|_{z_f}\cdot\frac{\partial F}{\partial\mathbf{X}}$$

Adjoint equation: $$\frac{d}{dz}\delta\mathbf{X}^{(Y)} = \left[\mathbf{O}_{magnets} + \mathbf{O}_{SpCh}\right]\cdot\delta\mathbf{X}^{(Y)} + \delta\dot{\mathbf{X}}^{(Y)}$$

↑ TBD

Final condition: $$\left.\delta\mathbf{X}^{(Y)}\right|_{z_f}$$ ← TBD

# Adjoint Magic – Can Show…

User-defined

Adjoint equation: $$\frac{d}{dz}\delta\mathbf{X}^{(Y)} = \left[\mathbf{O}_{magnets} + \mathbf{O}_{SpCh}\right]\cdot\delta\mathbf{X}^{(Y)} + \delta\dot{\mathbf{X}}^{(Y)}$$

Integrated backward in z

F – Figure of Merit

Final condition: $$\left.\delta\mathbf{X}^{(Y)}\right|_{z_f}\cdot\hat{\mathbf{J}} = \frac{\partial F}{\partial \mathbf{X}}$$

$\hat{\mathbf{J}} =$ Matrix of zeros and ones

$$\delta F = \int_{z_i}^{z_f} dz\, \delta\mathbf{X}^{(Y)}\cdot\delta\mathbf{a}\cdot\frac{\partial}{\partial\mathbf{a}}\mathbf{O}_{magnets}\cdot\mathbf{X}(z)$$

Change in FOM

Sensitivity function

Change in Magnet Parameters

Base case

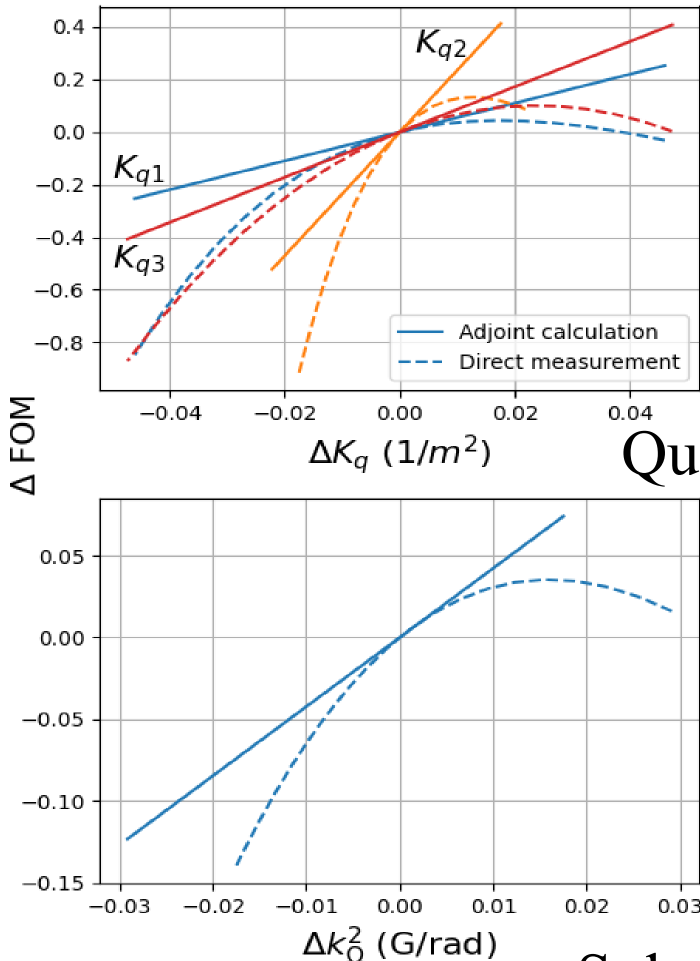
# Figure of Merit and Gradient

$$FoM = \frac{1}{2} \sum_{Terms-i} T_i$$

$T_1$ - beam is round
$T_2$ – radius is locally constant
$T_3$ – velocity space is isotropic
$T_4$ – radial force balance
$T_5$ – rigid rotation

<u>11 Parameters</u>

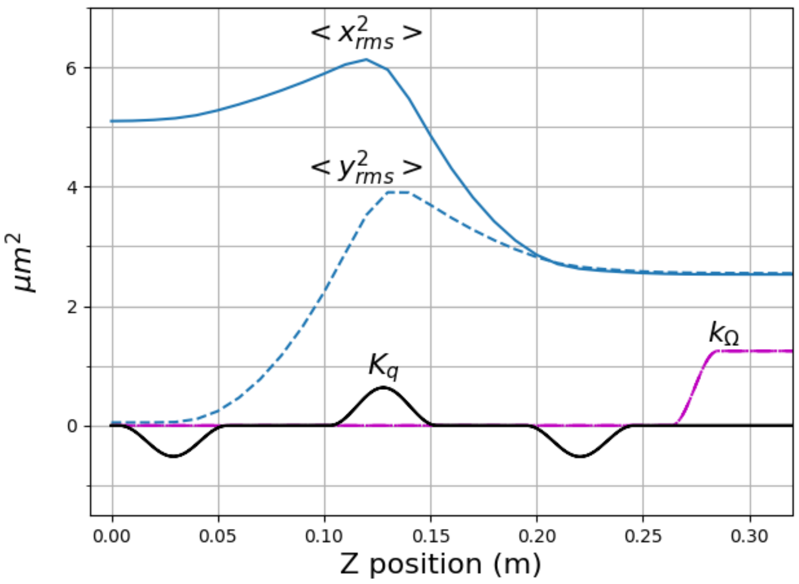
Location of 4 magnets
Strength of 4 magnets
Orientation of 3 Quads



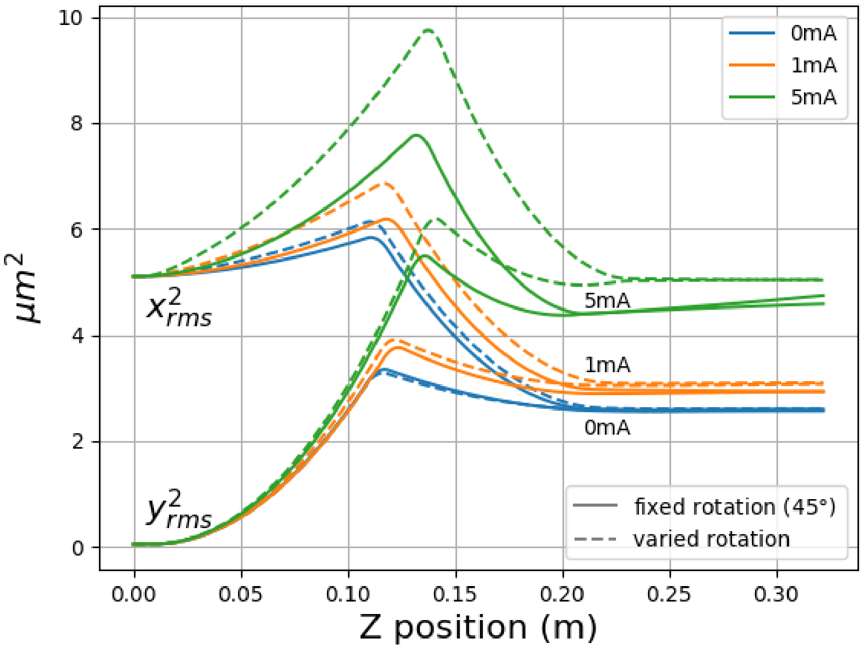

Quad strength

Solenoid strength

# Optimization Results



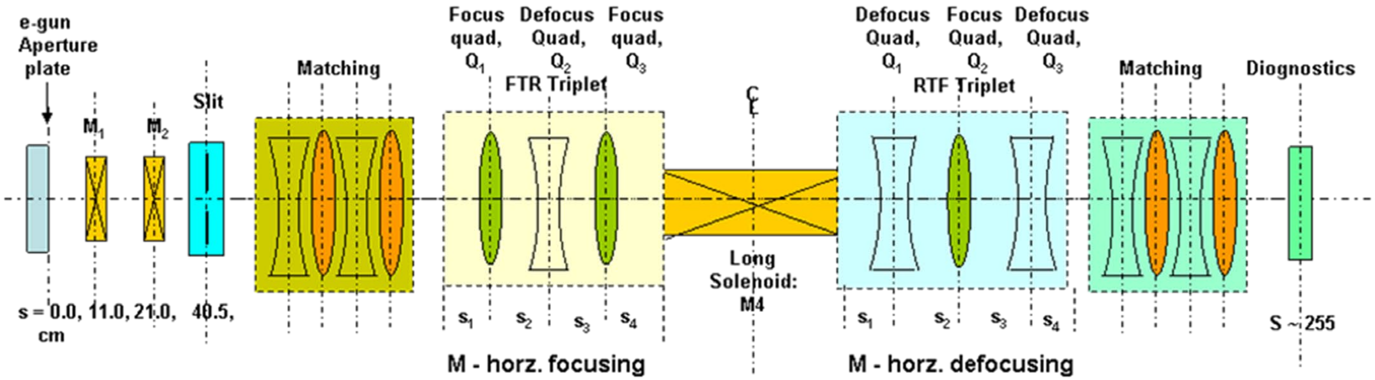

Continuous magnetic field profiles

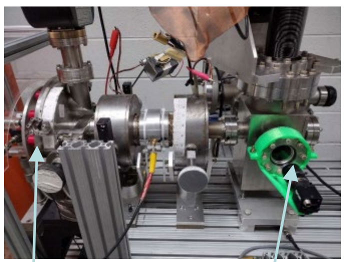



Variable magnet orientations Required for "strong" space charge

Also S. B. Moroch, et al., arXiv: 2102.13567.

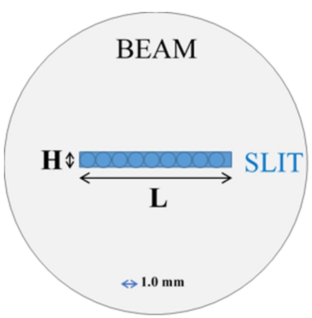
# Demonstration of Flat/Round Transformations of Angular Momentum and Space Charge Dominated Electron Beams




Slit

E-Gun




## Generation of highly asymmetric beams

| SLIT: L × H | BEAM CURRENT | $\tilde{\varepsilon}_x, \tilde{\varepsilon}_y$ | $\tilde{\varepsilon}_x / \tilde{\varepsilon}_y$ |
|---|---|---|---|
| **10 × 0.2 mm** | 0.60 mA | 53 μm, 1.1 μm | 50 |
| **10 × 0.5 mm** | 1.4 mA | 53 μm, 2.7 μm | 20 |
| **10 × 1.0 mm** | 2.9 mA | 53 μm, 5.3 μm | 10 |

# Adjoint Relations for Particle Description

TMA, D. Chernin, J. Petillo, Phys. Plasmas 26, 013109 (2019)

$$\frac{d\mathbf{q}}{dt} = \frac{\partial H}{\partial \mathbf{p}}$$

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{q}}$$

Base Solution + Perturbation

$$\left(\mathbf{x}_j, \mathbf{p}_j\right) \rightarrow \left(\mathbf{x}_j, \mathbf{p}_j\right) + \left(\delta\mathbf{x}_j, \delta\mathbf{p}_j\right)$$

$$\rho(\mathbf{x}) \rightarrow \rho(\mathbf{x}) + \delta\rho(\mathbf{x})$$

$$\Phi_T(\mathbf{x}) \rightarrow \Phi_T(\mathbf{x}) + \delta\Phi_T(\mathbf{x})$$

Two Linearized Solutions

$[\delta x_j(t), \delta P_j(t)]$ true

$[\delta x_j(t), \delta P_j(t)]$ adjoint

$$\frac{v_z}{c}\delta A_z^{(X)}$$ ← Changes in focusing magnets

Sensitivity function

Change in symplectic area

$$\sum_j I_j \left( \delta\mathbf{P}_j \cdot \delta\mathbf{x}_j - \delta\mathbf{P}_j \cdot \delta\mathbf{x}_j \right)\Big|_0^L = -\int d^3x \frac{v_z}{c} \delta A_{zB}^{(X)} \delta\rho^{(y)}$$

# Adjoint Treatment of Particle Equations

Change in symplectic area

$$\sum_j I_j \left( \delta \mathbf{P}_j \cdot \delta \mathbf{x}_j - \delta \mathbf{P}_j \cdot \delta \mathbf{x}_j \right)\Big|_0^L = -\int d^3x \frac{v_z}{c} \delta A_{zB}^{(X)} \delta \rho^{(y)}$$

Pick: $\left( \delta \mathbf{P}_j, \delta \mathbf{x}_j \right) \text{at } z = L$

FoM arb. $F(\boldsymbol{x}, \mathbf{P}, \mathbf{z}_f)$

Integrate backward in z

$$\delta F = \sum_j \left( \frac{\partial F}{\partial \mathbf{x}_j} \cdot \delta \mathbf{x}_j - \delta \mathbf{P}_j \cdot \left( -\frac{\partial F}{\partial \mathbf{P}_j} \right) \right)\Bigg|_L = -\int d^3x \frac{v_z}{c} \delta A_{zB}^{(X)} \delta \rho^{(y)}$$

Change in FoM
Arb. $F(\boldsymbol{x}, \mathbf{p}, \mathbf{z}_f)$

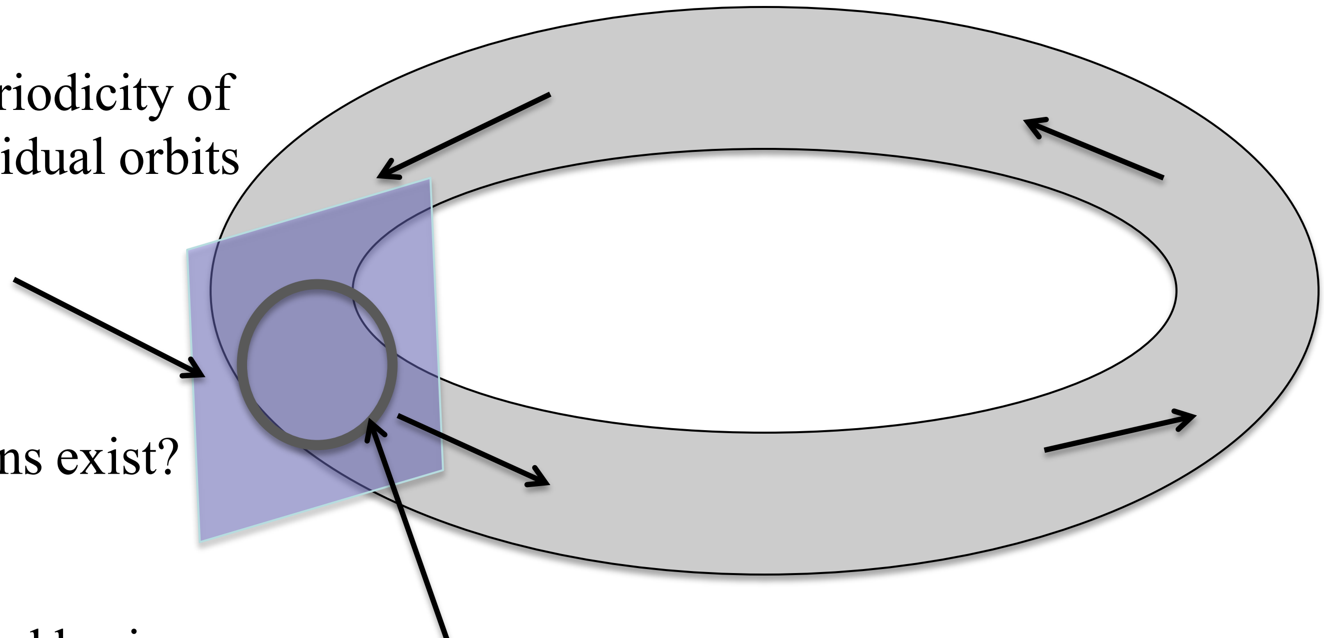
Change in focusing magnets.
Includes multipole variation

Sensitivity

# Circular Accelerators-Periodicity?

Particles return to initial plane.

Desire to maintain periodicity of distribution, not individual orbits




Problems:
Do periodic distributions exist?
Most likely no.

Represent DF in reduced basis.
Make basis coefficients periodic.

Optimize FoM subject to periodicity

**Constrained Optimization**
**"Adjoint with a Chaser"**

# Summary

Adjoint methods are a powerful way to evaluate parameter dependences in many systems involving charged particle dynamics.

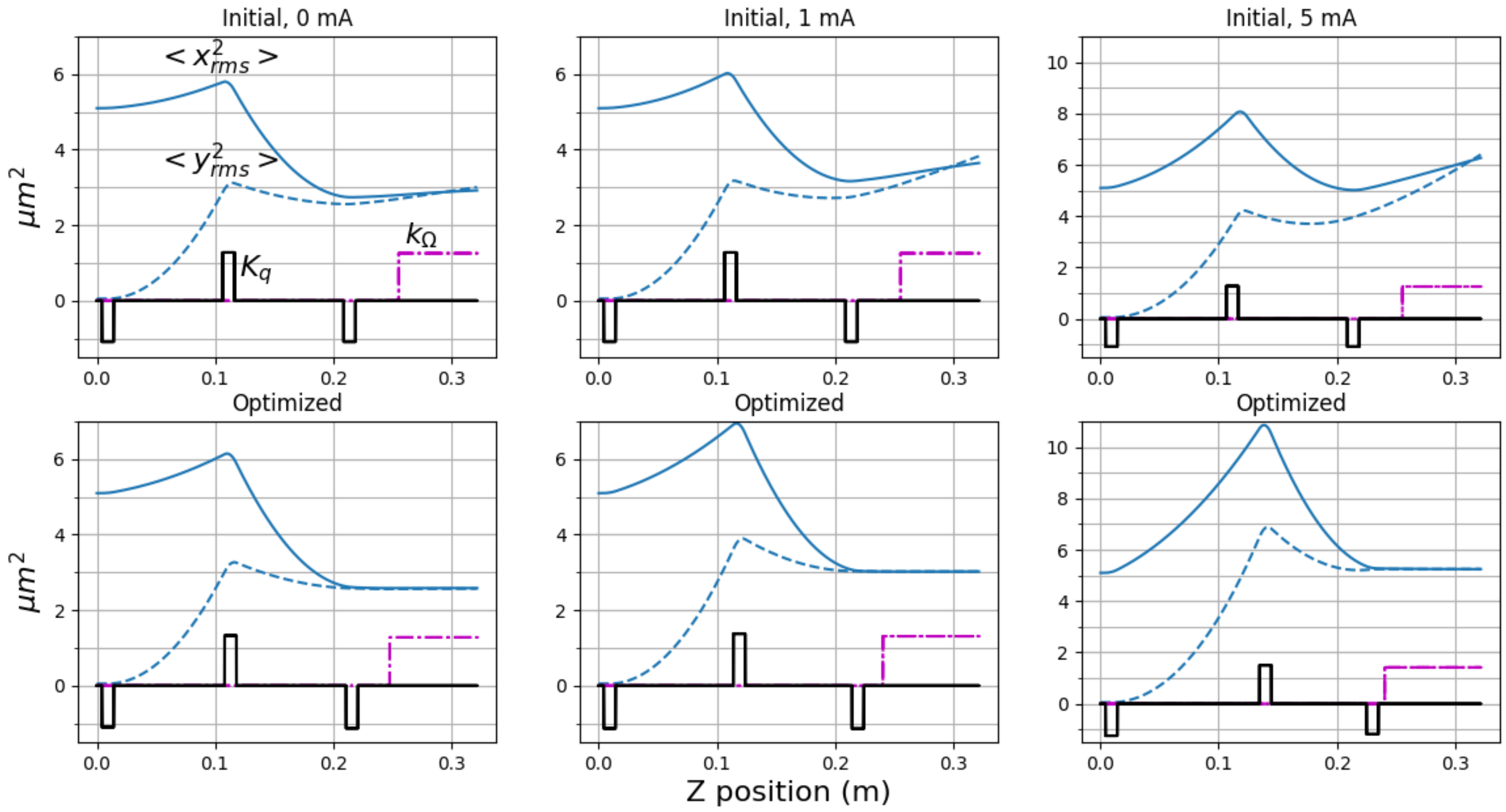
Application to 2nd-moments has been implemented.

Need to develop applications to particle distributions.
Optimize phase space distributions

Constrained optimization for circular accelerators.

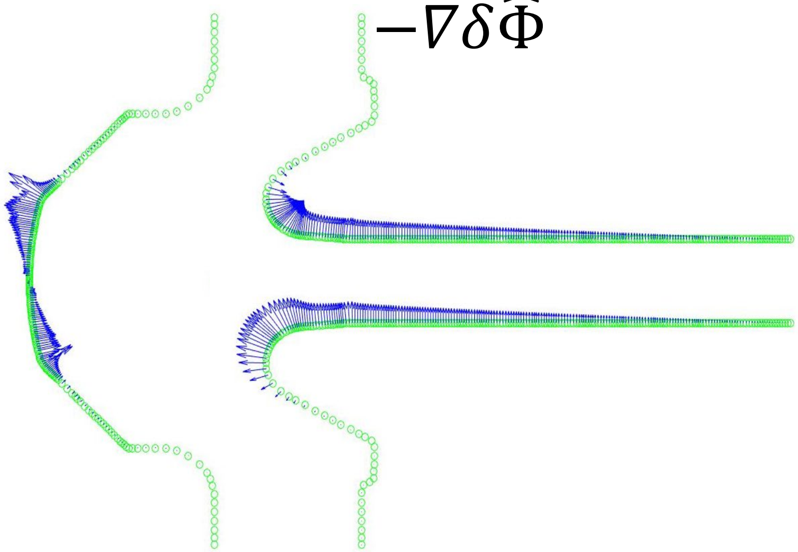
Adjoint with a Chaser?

# Optimization – Space Charge Compensation

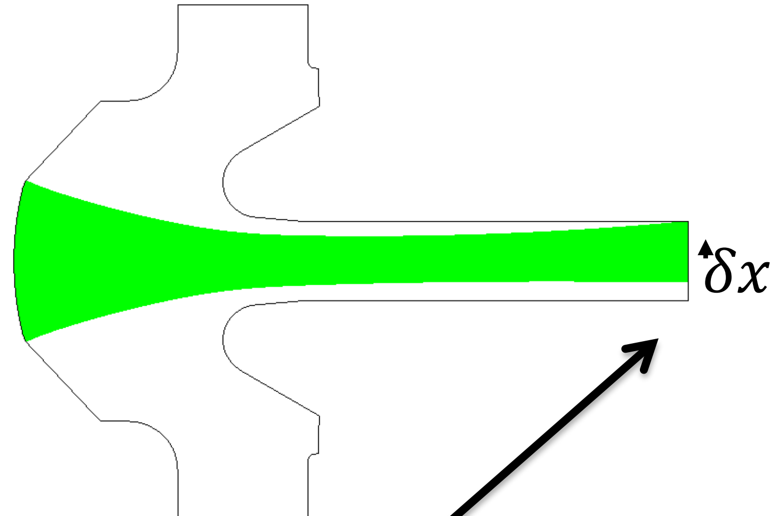


# Vertical Displacement of the Beam

**Vector plot of the 'sensitivity' or Green's function**




**'Direct' MICHELLE Simulation with Perturbed Anode Voltages**




$$\delta x = -\frac{q\varepsilon_0}{\lambda I}\int_S da\mathbf{n}\cdot\delta\Phi\nabla\delta\hat{\Phi}$$

Predicted displacement / Calculated displacement = 0.9969

# Recent Adjoint Approaches

- Beam optics sensitivity function, TMA, D. Chernin, J. Petillo, Phys. Plasmas 26, 013109 (2019); doi: 10.1063/1.5079629
- Stellarator Optimization and Sensitivity, E. Paul, M. Landreman, TMA, *J. Plasma Phys*. (2019), v*ol*. 85, 905850207, *J. Plasma Phys.* (2021), v*ol*. 87, 905870214
- Optimization of Flat to Round Transformers in Particle Accelerators, L. Dovlatyan, B. Beaudoin, S. Bernal, I. Haber, D. Sutter and TMA, Phys Rev Accel and Beams V25, 044002 (2022).
- Adjoint Equations for Beam-Wave Interaction and Optimization of TWT Design, A. Vlasov, TMA, D. Chernin and I. Chernyavskiy, IEEE Trans. Plasma Sci. V. March (2022).

Give a child a hammer and everything becomes a nail.