Theory of Rare Kaon and Hyperon Decays

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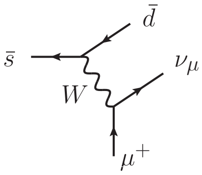
Leptonic and semileptonic K decays

- E.g. leptonic $K^+ \to \mu^+ \nu_\mu$
  - $\langle 0 | j^\mu | K^+ \rangle \sim i p^\mu f_K$
  - only axial current $j^\mu = \bar{u} \gamma^\mu \gamma_5 s$ contributes

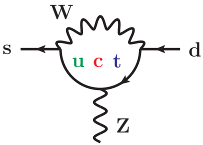
- E.g. semileptonic $K^+ \to \pi^0 \mu^+ \nu_\mu$ ("$K_{\ell 3}$")
  - $\langle \pi^0 | j^\mu | K^+ \rangle \sim f_+(q^2)(p + p')^\mu + f_-(q^2)(p - p')^\mu$
  - only vector current $j^\mu = \bar{u} \gamma^\mu s$ contributes

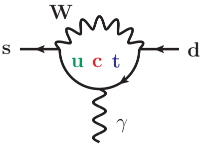
- E.g. hyperon $\Lambda^0 \to p^+ \mu^- \bar{\nu}_\mu$
  - $\langle p^+ | j^\mu | \Lambda^0 \rangle$
  - both vector and axialvector currents contribute

Weak interaction vs "chiral dynamics"



Rare K decays – GIM and Inami Lim





$$X_0(x) \xrightarrow{x\to\infty} x$$

$$X_0(x) \xrightarrow{x\to 0} x \log x$$

$$D_0(x) \xrightarrow{x\to\infty} \log x$$

$$D_0(x) \xrightarrow{x\to 0} \log x$$

Matrix Elements – Isospin

$K^+ \to \pi^+ \nu\bar{\nu}$		$K^+ \to \pi^0 \ell^+ \nu_\ell$
$\langle \pi^+ \vert (\bar{s}d)_{V-A} \vert K^+ \rangle$	$=$	$\sqrt{2} \times \langle \pi^0 \vert (\bar{s}u)_{V-A} \vert K^+ \rangle$
Unknown		Well measured

- similar for other modes like $K^+ \to \pi^+\pi^0\nu\bar{\nu}$, $\Lambda \to n\nu\bar{\nu}$, as well as neutral modes
- Higher-order corrections can be included

Which NP sensitivity can we expect?

In 5 years of running of the "HIKE" experiment with $\mathbf{1 \times 10^{19}}$ POT/year:

- $\mathbf{5 \times 10^{14}}$ $\boldsymbol{K_S}$ decays
- $\mathbf{1 \times 10^{14}}$ $\boldsymbol{K_L}$ decays
- $\mathbf{2 \times 10^{14}}$ $\boldsymbol{\Lambda}$ decays
- For background-free channels, sensitivity $\mathcal{O}(\mathbf{10^{-13} - 10^{-14}})$
- $\mathcal{O}(\mathbf{100})$ events for rare $\boldsymbol{K_L}$ decays
- Backgrounds can be very large
  ($\boldsymbol{S/B} \sim \mathbf{0.2 - 0.4}$ for $\boldsymbol{K_L \to \pi^0 \ell^+ \ell^-}$ modes)

I will express all sensitivities as bounds on a NP scale $\boldsymbol{\Lambda}$!

No asymmetries, ratios, no light NP . . .

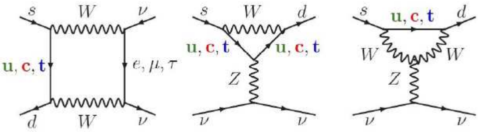
Outline

- $K \to \pi\nu\bar{\nu}$
- $\Lambda \to n\nu\bar{\nu}$
- $K \to \pi\ell^+\ell^-$
- Generic NP

$$K \to \pi \nu \bar{\nu}$$

$K \to \pi\nu\bar{\nu}$ decays: a theoretically clean environment

- FCNC loop processes: s→d coupling and highest CKM suppression



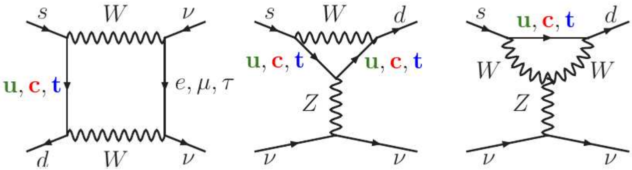
- Very clean theoretically: Short distance contribution. No hadronic uncertainties.

- SM predictions [Buras et al. JHEP 11 (2015) 33]

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = (8.39 \pm 0.30)\cdot 10^{-11}\left(\frac{|V_{cb}|}{0.0407}\right)^{2.8}\left(\frac{\gamma}{73.2°}\right)^{0.74} = (0.84 \pm 0.10)\cdot 10^{-10}$$

$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = (3.36 \pm 0.05)\cdot 10^{-11}\left(\frac{|V_{ub}|}{0.00388}\right)^{2}\left(\frac{|V_{cb}|}{0.0407}\right)^{2}\left(\frac{\sin\gamma}{\sin 73.2°}\right)^{2} = (0.34 \pm 0.06)\cdot 10^{-10}$$

$K^+ \to \pi^+ \nu\bar{\nu}$: Introduction

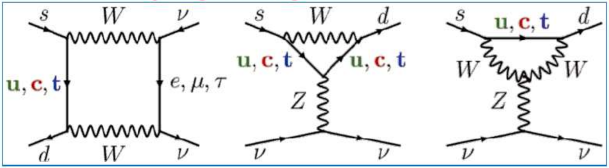


Dominated by $Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$

$$B\left(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)\right) = \kappa_+ (1 + \Delta_{\text{EM}}) \times \left| \frac{V_{ts}^* V_{td} X_t(m_t^2) + \lambda^4 \text{Re} V_{cs}^* V_{cd} \left(P_c(m_c^2) + \delta P_{c,u} \right)}{\lambda^5} \right|^2 .$$

Rare kaon decays: $K \to \pi\nu\bar{\nu}$

SM: box and penguin diagrams



Ultra-rare decays with
the highest CKM suppression:

$$A \sim (m_t/m_W)^2 |V^*_{ts} V_{td}| \sim \lambda^5$$

- SM precision surpasses any other FCNC process involving quarks.
- Measurement of $|V_{td}|$ complementary to those from e.g. B–$\bar{B}$ mixing.
- Main focus of kaon physics: measurement of both $K^+ \to \pi^+\nu\nu$ and $K_L \to \pi^0\nu\nu$ decays.

SM branching ratios
Buras et al., JHEP 1511 (2015) 033

Mode	$BR_{SM} \times 10^{11}$
$K^+ \to \pi^+\nu\bar{\nu}(\gamma)$	8.4±1.0
$K_L \to \pi^0\nu\bar{\nu}$	3.4±0.6

The uncertainties are largely parametric (CKM)

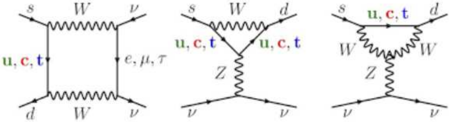
Theoretically clean, almost unexplored, sensitive to new physics.

5

E. Goudzovski / CLFV 2019, Fukuoka, 18 June 2018

Motivations for $K^+ \to \pi^+ \nu\bar{\nu}$

❑ Ultra rare FCNC decay forbidden at tree level



❑ CKM suppressed ($\sim |V_{ts}V_{td}|^2$)

- Dominant short-distance t quark contribution
- Small c quark contribution
- Small long-distance corrections
- Hadronic matrix element extracted from $Br(K^+ \to \pi^+ e\nu)$

} Theoretically very clean

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = (9.11 \pm 0.72) \times 10^{-11}$$

[A.J. Buras et al., JHEP 1511 (2015) 033]

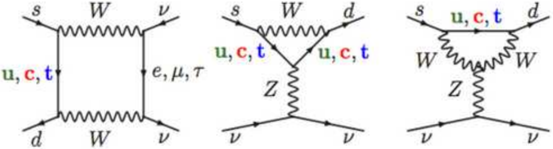
❑ Best measurement based on 7 events (E787/E949)

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{Exp} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$

[E787/E949, Phys.Rev.Lett.101, 191802, 2008]

$K^+ \to \pi^+ \nu\bar{\nu}$ in the Standard Model

- $K \to \pi\nu\bar{\nu}$ are the most precisely predicted FCNC decays involving quarks
- $B_{SM}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = \left(7.8 \pm 0.8\right) \times 10^{-11}$



- A single effective operator: $\left(\bar{s}_L \gamma^\mu d_L\right)\left(\bar{\nu}_L \gamma_\mu \nu_L\right)$
- Dominated by top quark
- Hadronic matrix element shared with $K^+ \to \pi^0 e^+ \nu_e$
- Dominant uncertainty from CKM matrix elements (expect prediction to improve to ~5%)

ETW: Kaon13, Ann Arbor, MI May 1, 2013

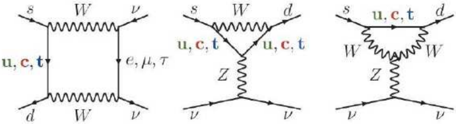
4

$K \to \pi\nu\bar{\nu}$ - The golden modes for kaons

Why $\mathbf{K} \to \pi\nu\bar{\nu}$ **decays** are among the few golden channels to search for New Physics:

1) They are extremely rare !

Loop-induced **FCNC processes**, transition described by **Z-penguin** and **box diagrams**:



2) The SM prediction is exceptionally precise !

$$\mathrm{BR}(K^0 \to \pi^0\nu\bar{\nu}) = \kappa_L \mathrm{Im}(V_{ts}^* V_{td})^2 X(m_t, m_W)/|V_{us}|^5$$

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ (V_{ts}^* V_{td})^2 X(m_t, m_W)/|V_{us}|^5$$ **+ charm contr.**

The hadronic matrix element can be extracted from the precisely measured $K \to \pi e \nu$ decay.

Charm contribitions in charged mode only, precision recently reduced by NNLO calculation.

$$\mathrm{BR}(K^0 \to \pi^0\nu\bar{\nu}) = (2.43 \pm 0.39) \times 10^{-11}$$

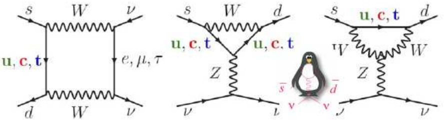
$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = (7.81 \pm 0.80) \times 10^{-11}$$

The SM prediction is tiny !

(Brod et al., PRD83 (2011) 034030)

Uncertainty almost only from knowledge on $|V_{ts}|$!

3) In extensions of the SM, the decay remains similarly predictive !



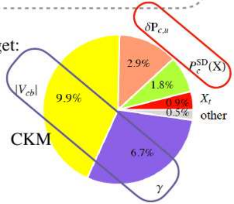
$$A(s \to d\nu\bar{\nu}) \sim$$

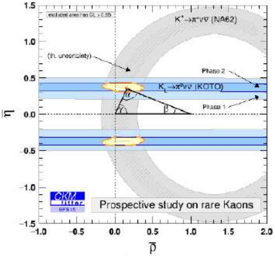
$$\sim \frac{m_t^2}{M_W^2}\lambda_t + \frac{m_c^2}{M_W^2}\ln\frac{M_W}{m_c}\lambda_c + \frac{\Lambda_{QCD}^2}{M_W^2}\lambda_u$$

t(68%) *c(29%)* *u(3%)*

Short distance ***Long distance***

Error budget:





Measuring both $K^+ \to \pi^+ \nu\nu$ and $K_L \to \pi^0 \nu\nu$ provides the CKM unitarity triangle independently from measurements in B mesons sector.

$$BR(K^+ \to \pi^+ \nu\nu^-) = (8.4 \pm 1.0) \times 10^{-11}$$

Buras et al., JHEP11(2015)033

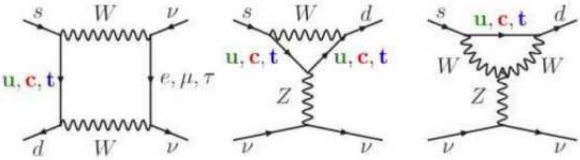
Experimental result before NA62:

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})_{\rm exp} = (17.3^{+11.5}_{-10.5}) \cdot 10^{-11}$$

E949, Phys. Rev. D 77, 052003 (2008)
Phys. Rev D 79, 092004 (2009)]

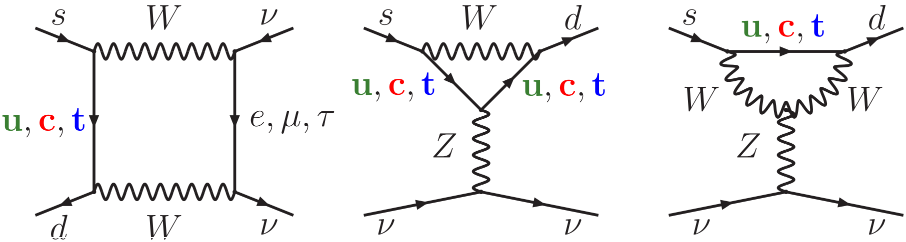
$K \to \pi\nu\bar{\nu}$ in the SM

- FCNC and thus one loop and very small



- The hadronic matrix elements of the form $\langle\pi|\hat{O}|K\rangle$ are obtained using isospin from semileptonic kaon decays
- K_L decay is CP-violating
- K^+ decay is sensitive to the top coupling
- The combination is an excalent probe of the CKM parameters, independent of B physics

The solution of the mystery:



[J. Brod, talk at CKM 2008]

$K \to \pi\nu\bar{\nu}$ – SM

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = 7.73(16)(25)(54) \times 10^{-11}$$

$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) = 2.59(6)(2)(28) \times 10^{-11}$$

- Updated SM prediction: [Brod, Gorbahn, Stamou, 2105.02868]
  - NLO QCD for top [Misiak et al., hep-ph/9901278]
  - NNLO QCD for charm [Buras et al., hep-ph/0603079]
  - NLO electroweak [Brod, Gorbahn, 0805.4119; Brod et al. 1009.0947]
  - N(N)LO ChPT + QED for matrix elements [Mescia, Smith, 0705.2025]
  - $\delta P_{c,u}$ +6% [Isidori et al., hep-ph/0503107]
  - Updated m_t and α_s
  - CKM parameters from global SM fit

$K \to \pi\nu\bar\nu$ – NP Sensitivity

Experiment [NA62 2103.15389, KOTO 1810.09655]:

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar\nu) = (10.6^{+4.0}_{-3.4}|_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}}) \times 10^{-11}$$

$$\mathrm{Br}(K_L \to \pi^0\nu\bar\nu) < 300 \times 10^{-11} \quad @90\%\mathrm{CL}$$

$$\mathcal{H}_{\mathrm{eff}} = \left[\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_w}C^{\mathrm{SM}} + \frac{1(i)}{\Lambda^2}\right](\bar s_L\gamma^\mu d_L)(\bar\nu_L\gamma_\mu\nu_L) + \mathrm{h.c.}$$

90% CL	current	10% ("100 events")	3% ("1000 events")
K^+, Re	$\Lambda_+ \gtrsim 340$ TeV	$\Lambda_+ \gtrsim 735$ TeV	$\Lambda_+ \gtrsim 894$ TeV
K^+, Im	$\Lambda_+ \gtrsim 209$ TeV	$\Lambda_+ \gtrsim 278$ TeV	$\Lambda_+ \gtrsim 292$ TeV
K_L, Re	no bound	no bound	no bound
K_L, Im	$\Lambda_L \gtrsim 139$ TeV	$\Lambda_L \gtrsim 1307$ TeV	$\Lambda_L \gtrsim 1476$ TeV

- Note that the SM corresponds to $\Lambda_+ = 257$ TeV, $\Lambda_L = 487$ TeV.

$$\Lambda \to n\nu\bar{\nu}$$

$\Lambda \to n\nu\bar{\nu}$ – SM

$$\mathrm{Br}(\Lambda \to n\nu\bar{\nu}) = \frac{3\alpha^2 G_F^2 \Delta^5 \lambda^{10} \tau_\Lambda}{240\pi^5 \sin^4\theta_w}\left(1 - \frac{3}{2}\delta\right)\left(f_1^2 + 3g_1^2\right) \times \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t\right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda} P_c + \frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t\right)^2\right]$$

- Updated SM prediction (my numerics and error estimate)
  - NLO QCD for top [Misiak et al., hep-ph/9901278]
  - NNLO QCD for charm [Buras et al., hep-ph/0603079]
  - NLO electroweak [Brod, Gorbahn, 0805.4119; Brod et al. 1009.0947]
  - NLO SU(3) for form factors / matrix elements [Geng, Camalich, Shi, 2112.11979]
  - PDG input; CKM parameters from global SM fit

$$\mathrm{Br}(\Lambda \to n\nu\bar{\nu}) = 5.95(13)(28)(44) \times 10^{-13}$$

$\Lambda \to n\nu\bar{\nu}$ – (axial)vectors

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_w} C^{\text{SM}} (\bar{s}_L \gamma^\mu d_L)(\bar{\nu}_L \gamma_\mu \nu_L)$$

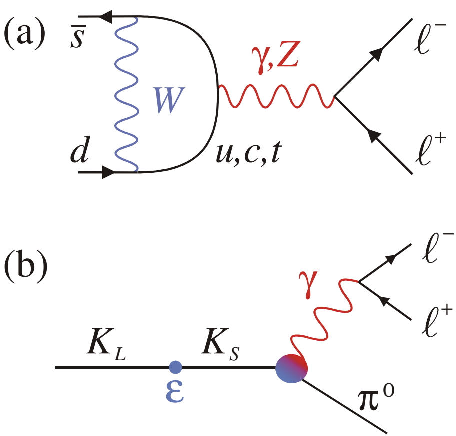
$$+ \frac{1(i)}{\Lambda_V^2} (\bar{s}\gamma^\mu d)(\bar{\nu}_L \gamma_\mu \nu_L) + \frac{1(i)}{\Lambda_A^2} (\bar{s}\gamma^\mu \gamma_5 d)(\bar{\nu}_L \gamma_\mu \nu_L) + \text{h.c.}$$

Sensitivity:

90% CL	30% ("10 events")	10% ("100 events")
Re	$\Lambda_V \gtrsim$ 331 TeV	$\Lambda_V \gtrsim$ 474 TeV
Re	$\Lambda_A \gtrsim$ 170 TeV	$\Lambda_A \gtrsim$ 179 TeV
Im	$\Lambda_V \gtrsim$ 208 TeV	$\Lambda_V \gtrsim$ 242 TeV
Im	$\Lambda_A \gtrsim$ 300 TeV	$\Lambda_A \gtrsim$ 399 TeV

$K \to \pi \ell^+ \ell^-$

$K_L \to \pi^0 \ell^+ \ell^-$



[Mescia, Smith, Trine, hep-ph/0606081]

$K_L \to \pi^0 \ell^+ \ell^-$ – SM

SM prediction [Mescia, Smith, Trine, hep-ph/0606081]:

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = 3.54^{+0.98}_{-0.85} \left[1.56^{+0.62}_{-0.49}\right] \times 10^{-11}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) = 1.41^{+0.28}_{-0.26} \left[0.95^{+0.22}_{-0.21}\right] \times 10^{-11}$$

- Needs: $K_S \to \pi^0 \ell^+ \ell^-$, $K_S \to \pi^0 \gamma\gamma$, ϵ_K
- Sign of DCPV-ICPV interference term unclear (positive preferred?)
  - Can be determined from forward-backward asymmetry

$K_L \to \pi^0 \ell^+ \ell^-$ – NP Sensitivity

Experimental 90% CL bounds [KTeV hep-ex/0309072, hep-ex/0001006]

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 28 \times 10^{-11}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 38 \times 10^{-11}$$

Many operators can contribute:

- Additional contributions to Vector, Axialvector
- Scalar, Pseudoscalar, Tensor, Pseudotensor

Generically, expect helicity suppression factor $m_\ell m_s / M_W^2$ for the latter

$K_L \to \pi^0 \ell^+ \ell^-$: "real" (axial)vectors

For generic interactions in addition to SM:

$$\mathcal{H}_{\text{eff}} = \frac{1}{\Lambda_V^2}(\bar{s}\gamma^\mu d)(\bar{\ell}\gamma^\mu \ell) + \frac{1}{\Lambda_A^2}(\bar{s}\gamma^\mu d)(\bar{\ell}\gamma^\mu \gamma_5 \ell) + \text{h.c.}$$

Sensitivity ("10%" assumes half SM error):

90% CL	current	30% ("10 events")	10% ("100 events")
e^+e^-	$\Lambda_V \gtrsim$ **110** TeV	$\Lambda_V \gtrsim$ **191** TeV	$\Lambda_V \gtrsim$ **221** TeV
e^+e^-	$\Lambda_A \gtrsim$ **129** TeV	$\Lambda_A \gtrsim$ **328** TeV	$\Lambda_A \gtrsim$ **482** TeV
$\mu^+\mu^-$	$\Lambda_V \gtrsim$ **87** TeV	$\Lambda_V \gtrsim$ **205** TeV	$\Lambda_V \gtrsim$ **237** TeV
$\mu^+\mu^-$	$\Lambda_A \gtrsim$ **75** TeV	$\Lambda_A \gtrsim$ **276** TeV	$\Lambda_A \gtrsim$ **410** TeV

- The SM corresponds to $\Lambda_V = 414$ TeV, $\Lambda_A = 400$ TeV.

$K_L \to \pi^0 \ell^+ \ell^-$: "imaginary" (axial)vectors

For generic interactions in addition to SM:

$$\mathcal{H}_{\text{eff}} = \frac{i}{\Lambda_V^2}(\bar{s}\gamma^\mu d)(\bar{\ell}\gamma^\mu \ell) + \frac{i}{\Lambda_A^2}(\bar{s}\gamma^\mu d)(\bar{\ell}\gamma^\mu \gamma_5 \ell) + \text{h.c.}$$

Sensitivity ("10%" assumes half SM error):

90% CL	current	30% ("10 events")	10% ("100 events")
e^+e^-	$\Lambda_V \gtrsim 118$ TeV	$\Lambda_V \gtrsim 251$ TeV	$\Lambda_V \gtrsim 337$ TeV
e^+e^-	$\Lambda_A \gtrsim 101$ TeV	$\Lambda_A \gtrsim 146$ TeV	$\Lambda_A \gtrsim 154$ TeV
$\mu^+\mu^-$	$\Lambda_V \gtrsim 91$ TeV	$\Lambda_V \gtrsim 288$ TeV	$\Lambda_V \gtrsim 401$ TeV
$\mu^+\mu^-$	$\Lambda_A \gtrsim 68$ TeV	$\Lambda_A \gtrsim 141$ TeV	$\Lambda_A \gtrsim 152$ TeV

- The SM corresponds to $\Lambda_V = 414$ TeV, $\Lambda_A = 400$ TeV.

$K_L \to \pi^0 \ell^+ \ell^-$: (pseudo)scalars

For a generic effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{1}{\Lambda_S^2}(\bar{s}d)(\bar{\ell}\ell) + \frac{1}{\Lambda_P^2}(\bar{s}d)(\bar{\ell}\gamma_5\ell) + \text{h.c.}$$

Sensitivity ("10%" assumes half SM error):

90% CL	current	30% ("10 events")	10% ("100 events")
e^+e^-	$\Lambda_S \gtrsim$ **75** TeV	$\Lambda_S \gtrsim$ **145** TeV	$\Lambda_S \gtrsim$ **171** TeV
e^+e^-	$\Lambda_P \gtrsim$ **74** TeV	$\Lambda_P \gtrsim$ **142** TeV	$\Lambda_P \gtrsim$ **177** TeV
$\mu^+\mu^-$	$\Lambda_S \gtrsim$ **48** TeV	$\Lambda_S \gtrsim$ **140** TeV	$\Lambda_S \gtrsim$ **184** TeV
$\mu^+\mu^-$	$\Lambda_P \gtrsim$ **56** TeV	$\Lambda_P \gtrsim$ **135** TeV	$\Lambda_P \gtrsim$ **157** TeV

Compare to constraint from BR$(K_L \to e^+e^-) = 9^{+6}_{-4} \times 10^{-12}$:

90% CL	current	10% ("100 events")
$K_L \to e^+e^-$	$\Lambda_{S,P} \gtrsim$ **760** TeV	$\Lambda_{S,P} \gtrsim$ **820** TeV

$K_L \to \pi^0 \ell^+ \ell^-$: (pseudo)tensors

For a generic effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{1}{\Lambda_T^2}(\bar{s}\sigma^{\mu\nu}d)(\bar{\ell}\sigma_{\mu\nu}\ell) + \frac{1}{\Lambda_{\tilde{T}}^2}(\bar{s}\sigma^{\mu\nu}d)(\bar{\ell}\sigma_{\mu\nu}\gamma_5\ell) + \text{h.c.}$$

Sensitivity ("10%" assumes half SM error):

90% CL	current	30% ("10 events")	10% ("100 events")
e^+e^-	$\Lambda_T \gtrsim$ **28** TeV	$\Lambda_T \gtrsim$ **54** TeV	$\Lambda_T \gtrsim$ **68** TeV
e^+e^-	$\Lambda_{\tilde{T}} \gtrsim$ **28** TeV	$\Lambda_{\tilde{T}} \gtrsim$ **54** TeV	$\Lambda_{\tilde{T}} \gtrsim$ **67** TeV
$\mu^+\mu^-$	$\Lambda_T \gtrsim$ **25** TeV	$\Lambda_T \gtrsim$ **92** TeV	$\Lambda_T \gtrsim$ **137** TeV
$\mu^+\mu^-$	$\Lambda_{\tilde{T}} \gtrsim$ **16** TeV	$\Lambda_{\tilde{T}} \gtrsim$ **43** TeV	$\Lambda_{\tilde{T}} \gtrsim$ **55** TeV

Compare to:

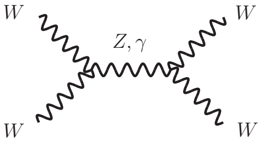
90% CL	current	10% ("100 events")
$K^+ \to \pi^+\nu\bar{\nu}$	$\Lambda_{T,\tilde{T}} \gtrsim$ **45** TeV	$\Lambda_{T,\tilde{T}} \gtrsim$ **69** TeV

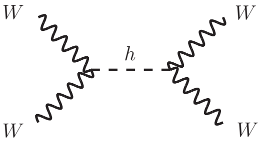
Generic New Physics

Can we generalize the SM loop functions?

- EFT (SMEFT, HEFT, ...)?
  - Generalities: E.g. chirality [Alonso et al. 1407.7044]
  - Nonrenormalizable (intrinsic cutoff)
  - Proliferation of operators
- Models?
  - Renormalizable (if complete)
  - Which parameters are independent?
  - Proliferation of models
- Generic loop functions [Brod, Gorbahn 1903.05116; Bishara et al. 2104.10930]
  - Explicit results – no need to calculate
  - Minimal number of parameters

Perturbative Unitarity



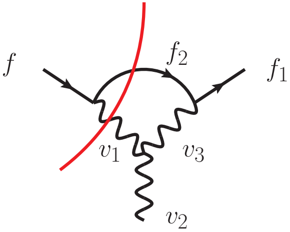


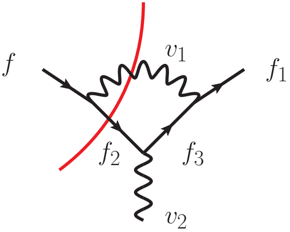
- Classic Example: $\boldsymbol{W}$ scattering [Lee et al., Phys. Rev. D 16 (1977) 1519]
  - $\boldsymbol{Z}$, γ exchange alone leads to unbounded growth $\boldsymbol{\mathcal{A} \sim s + t}$
  - Including Higgs channel leads to bounded high-energy growth
  - Relies on intricate relation between Gauge – Gauge and Gauge – Higgs couplings
- Provided by "Ward identities" for SSB $\rightarrow$ "Slavnov-Taylor identities"
  [Llewellyn Smith Phys. Lett. B 46 (1973) 233;
  Cornwall et al. Phys. Rev. Lett. 30 (1973) 1268]

Example: The Z penguin in the SM

- We need the following couplings:
  - $g^L_{W^+\bar{u}_i d_j} = \frac{e}{\sqrt{2}\sin\theta_w} V_{ij}$, $\qquad g^L_{G^+\bar{u}_i d_j} = \frac{m_{u_i}}{M_W}\frac{e}{\sqrt{2}\sin\theta_w} V_{ij}$
  - $g^L_{Z\bar{u}_i u_j} = \frac{2e}{\sin 2\theta_w}\left(I^3_{u_i} - Q_{u_i}\sin^2\theta_w\right)\delta_{ij}$,
    $g^R_{Z\bar{u}_i u_j} = -\frac{2e}{\sin 2\theta_w} Q_{u_i}\sin^2\theta_w\delta_{ij}$
  - $g_{ZW^+W^-} = \frac{e}{\tan\theta_w}$, $\qquad g_{ZW^+G^-} = -e\tan\theta_w$,
    $g_{ZG^+G^-} = \frac{e}{\tan\theta_w}\left(1 - \frac{1}{2\cos^2\theta_w}\right)$
- "Unphysical" and part of physical couplings fixed by STI
- GIM $\lambda_u = -\lambda_c - \lambda_t$ fixed by STI
- $\cos\theta_w \equiv M_W/M_Z$ fixed by STI
- The result is finite

"Unitarity Sum Rule"





$$\sum_{v_3} g^{L/R}_{v_3 \bar{f}_1 f_2} g_{v_1 v_2 \bar{v}_3} = \sum_{f_3} \left(g^{L/R}_{v_1 \bar{f}_1 f_3} g^{L/R}_{v_2 \bar{f}_3 f_2} - g^{L/R}_{v_2 \bar{f}_1 f_3} g^{L/R}_{v_1 \bar{f}_3 f_2} \right)$$

"Unitarity Sum Rule" – Generalized GIM

- General rule:

$$\sum_{v_3} g^{L/R}_{v_3 \bar{f}_1 f_2} g_{v_1 v_2 \bar{v}_3} = \sum_{f_3} \left(g^{L/R}_{v_1 \bar{f}_1 f_3} g^{L/R}_{v_2 \bar{f}_3 f_2} - g^{L/R}_{v_2 \bar{f}_1 f_3} g^{L/R}_{v_1 \bar{f}_3 f_2} \right)$$

- E.g. SM fermions $f_1 \to s$, $f_2 \to d$; heavy vectors $v_1 \to W_1^+$, $v_2 \to W_2^-$

- Off-diagonal couplings of neutral gauge bosons vanish $\Rightarrow$

$$0 = \sum_{f_3} g^{L/R}_{W_2^- \bar{s} f_3} g^{L/R}_{W_1^+ \bar{f}_3 d}$$

- Unitarity of "generalized CKM matrix" follows from universality and diagonality of neutral gauge boson couplings

- $\Rightarrow$ (generalized) GIM mechanism!

Generalized Penguin-Box Functions

- Write general dimension-four Lagrangian
- Implement sum rules
- Generalize penguin and box functions
  - Gauge independent and finite
  - Minimal number of couplings
- Public `mathematica` code `WellPut`

Summary

Exciting times ahead!