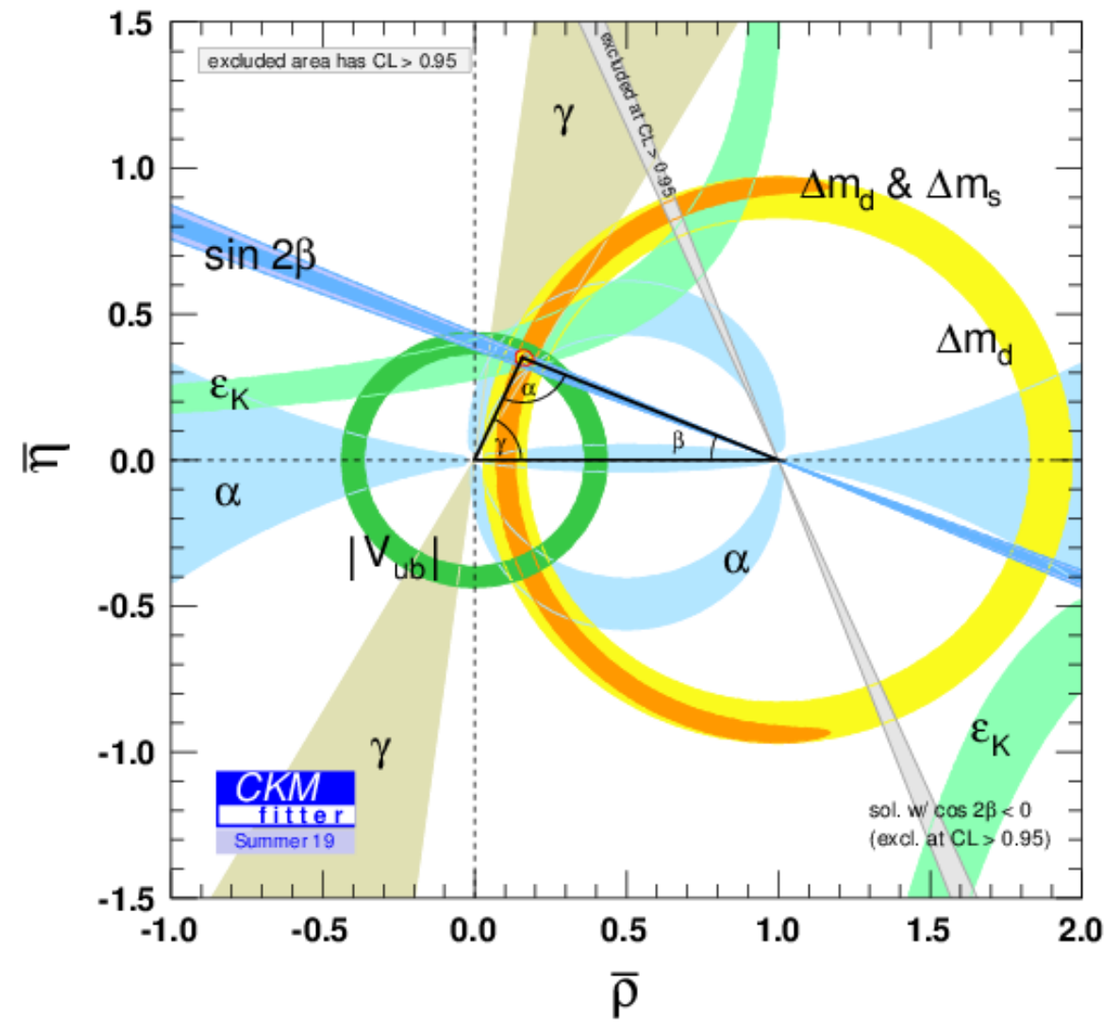
Aiming for New Heights with Kaons workshop, March 31st 2022

$K \to \mu^+\mu^-$ as a clean probe of CKM parameters

Avital Dery



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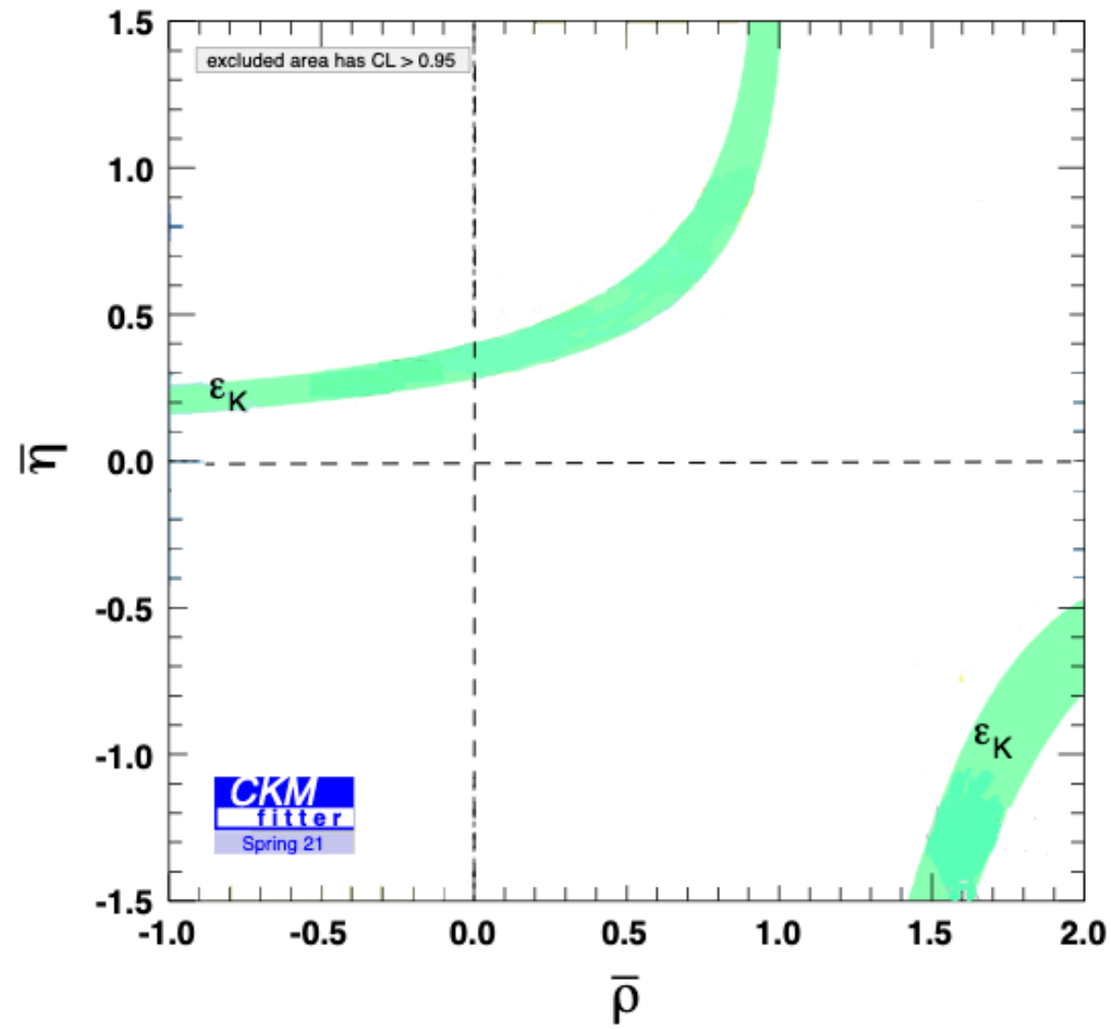

$K \to \mu^+\mu^-$

$$\mathscr{B}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$$

$$\mathscr{B}(K_S \to \mu^+\mu^-) < 2.1 \cdot 10^{-10}$$

LD SD

$$\mathscr{B}(K_L \to \mu^+\mu^-)_{\rm SM} = \begin{cases} (6.85 \pm 0.80 \pm 0.06) \times 10^{-9}\,(+) \\ (8.11 \pm 1.49 \pm 0.13) \times 10^{-9}\,(-) \end{cases}$$

LD SD

$$\mathscr{B}(K_S \to \mu^+\mu^-)_{\rm SM} \approx (4.99 + 0.19) \times 10^{-12} = (5.18 \pm 1.50 \pm 0.02) \times 10^{-12}$$

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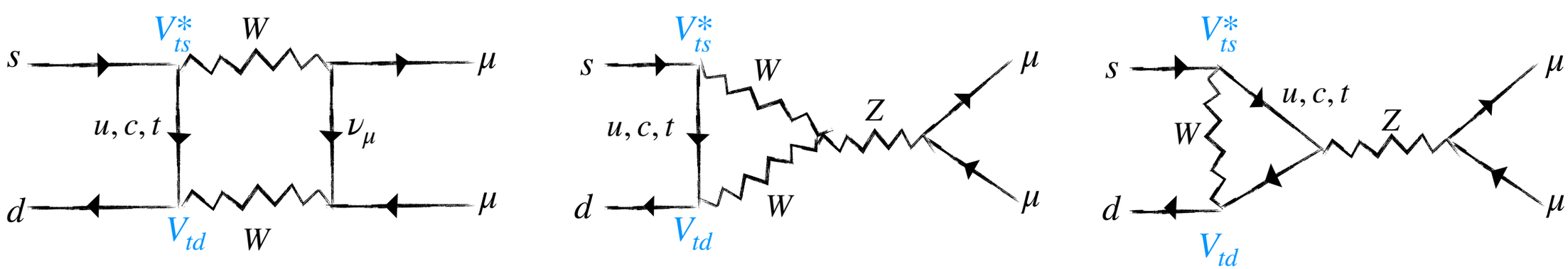
Short-distance diagrams



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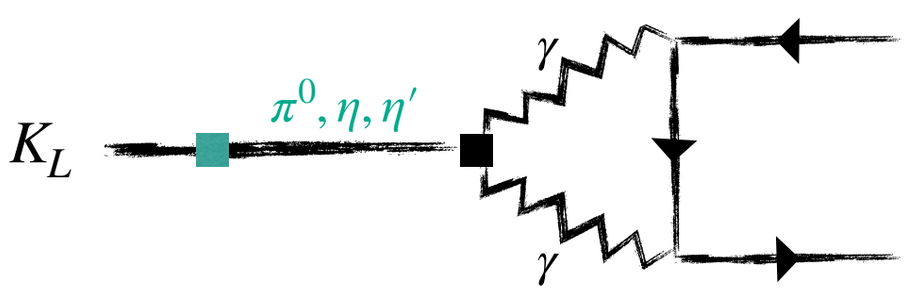
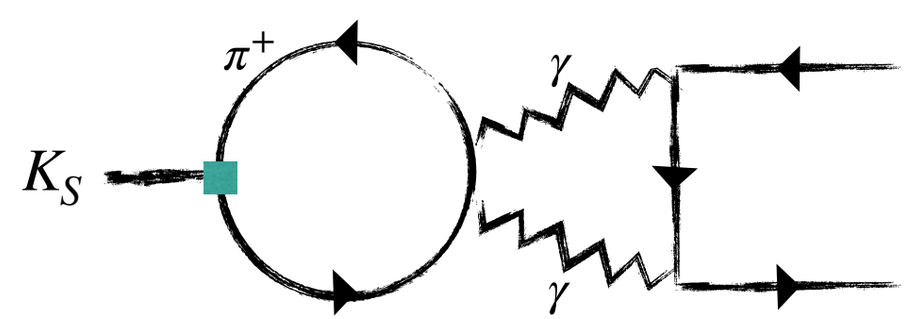
Both rates dominated by long-distance physics.





The key: **CPV comes from the weak (SD) diagrams**

CP Analysis of $K \to \mu^+\mu^-$

Initial state: kaon mass eigenstates are approximately also CP eigenstates,

$$\underset{\textit{CP-odd}}{K_L}, \qquad \underset{\textit{CP-even}}{K_S}$$

Final state: since the kaon has $J = 0$, the dimuon state can have either $S = 0, \ell = 0$ or $S = 1, \ell = 1$

corresponding to final states:

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In practice, we measure the **incoherent sum,**

$$\Gamma(K_{L,S} \to \mu^+\mu^-)_{meas.} = \Gamma(K_{L,S} \to (\mu^+\mu^-)_{\ell=0}) + \Gamma(K_{L,S} \to (\mu^+\mu^-)_{\ell=1})$$

If we could extract the CPV modes, we would have a similar situation (theoretically) to $K_L \to \pi^0\nu\bar{\nu}$

K_S-K_L INTERFERENCE

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G. D'Ambrosio and T. Kitahara, *Direct CP Violation in $K \to \mu^+\mu^-$*, *Phys. Rev. Lett.* **119** (2017) 201802 [arXiv:1707.06999] [INSPIRE].

$$I(t) = \frac{1+D}{2}\left|\langle f|\mathscr{H}_{\text{eff}}^{\Delta S=1}|K^0(t)\rangle\right|^2 + \frac{1-D}{2}\left|\langle f|\mathscr{H}_{\text{eff}}^{\Delta S=1}|\overline{K}^0(t)\rangle\right|^2$$

$$= \frac{1}{2}|A(K_1)|^2 e^{-\Gamma_S t} + |A(K_2)|^2 e^{-\Gamma_L t} + \boxed{2D\,\text{Re}\left[e^{-i\Delta M_K t}\,A(K_1)^* A(K_2)\right]}\, e^{-\frac{\Gamma_S+\Gamma_L}{2}t} + \mathcal{O}(\epsilon)$$

$$D = \frac{K^0 - \overline{K}^0}{K^0 + \overline{K}^0}$$

Non-zero D is required!

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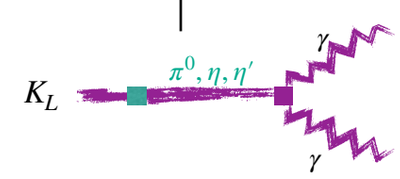
$$\propto \{A^{\mu}_{L\gamma\gamma} - 2\pi \sin^2\theta_W(\mathrm{Re}[\lambda_t]y'_{7A} + \mathrm{Re}[\lambda_c]y_c)\}$$



Total BR measurement of effective "K_S + int." , with $\mathcal{O}(1)$ D, can allow determination of the unknown sign of $A^{\mu}_{L\gamma\gamma}$.

K_S-K_L INTERFERENCE

Time dependent rate:

[AD, M. Ghosh, Y. Grossman, S. Schacht, *JHEP* **07** (2021) 103, [arXiv:2104.06427]]

$$f(t) = C_L\, e^{-\Gamma_L t} + C_S\, e^{-\Gamma_S t} + 2\left[C_{\cos}\, \cos(\Delta m t) + C_{\sin}\, \sin(\Delta m t)\right] e^{-\Gamma t}, \qquad \left(\frac{d\Gamma}{dt}\right) = N_f\, f(t)$$

4 Experimental parameters

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4 Experimental parameters

4 amplitudes, 2 are pure CPC and 2 are pure CPV:

$|A(K_S \to (\mu^+\mu^-)_{\ell=0})|$

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$$\varphi_0 \equiv \arg(A(K_S)_0^* A(K_L)_0)$$

$$\varphi_1 \equiv \arg(A(K_S)_1^* A(K_L)_1)$$

Neglecting $\mathcal{O}(\varepsilon)$ effects,
(i) K_S, K_L are CP eigenstates,
(ii) LD physics is CPC
=> CPV amplitudes are **pure SD**

A priori, 6 theory parameters

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4, 1 of which is pure SD

Neglecting $\mathcal{O}(\varepsilon)$ effects,
(i) K_S, K_L are CP eigenstates,
(ii) LD physics is CPC
=> CPV amplitudes are **pure SD**

The leading-order SM weak operator only contributes to the $\ell = 0$ final state (with corrections only at $\mathcal{O}\left(m_K^2/m_W^2\right)$).

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$$C_L = |A(K_L)_0|^2$$

$$C_S = |A(K_S)_0|^2 + \beta_\mu^2 |A(K_S)_1|^2$$

$$C_{\cos} = D\,|A(K_S)_0 A(K_L)_0|\cos(\varphi_0)$$

$$C_{\sin} = D\,|A(K_S)_0 A(K_L)_0|\sin(\varphi_0)$$

$$\boxed{\frac{1}{D^2}\frac{C_{\sin}^2 + C_{\cos}^2}{C_L} = |A(K_S)_0|^2}$$

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$$\mathscr{B}(K_S \to \mu^+\mu^-)_{\ell=0} = \mathscr{B}(K_L \to \mu^+\mu^-) \times \frac{1}{D^2} \times \frac{\tau_S}{\tau_L} \times \left(\frac{C_{int}}{C_L}\right)^2, \qquad C_{int}^2 = C_{\cos}^2 + C_{\sin}^2.$$

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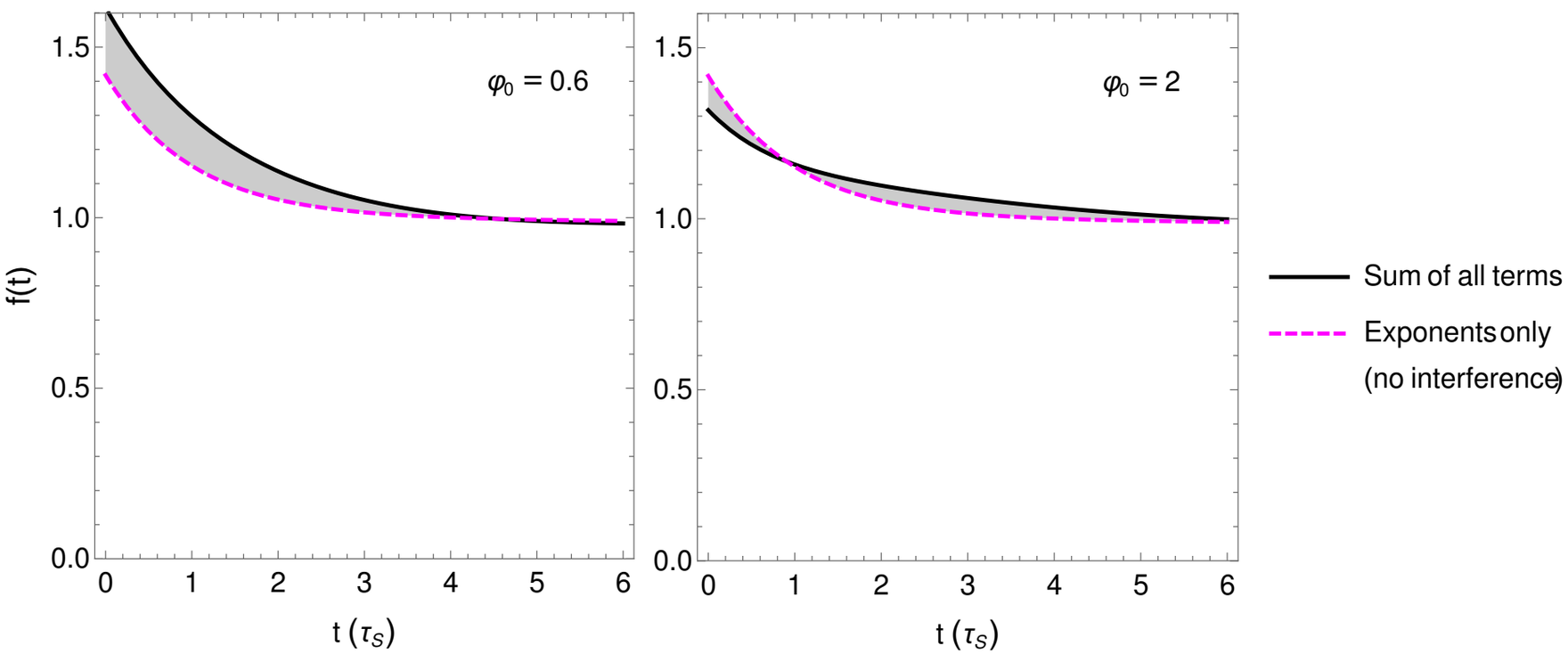

Approximate SM prediction, $D = 1$:

$(C_L)_{\rm SM} \equiv 1$

$(C_S)_{\rm SM} \approx 0.43$

$(C_{Int.})_{\rm SM} = \sqrt{C_{\cos}^2 + C_{\sin}^2} \approx 0.12$

We can measure $|A(K_S)_{\ell=0}|^2$

[AD, M. Ghosh, Y. Grossman, S. Schacht, *JHEP* **07** (2021) 103, [arXiv:2104.06427]]

Only hadronic parameter, $\mathcal{O}(1\%)$ uncertainty from isospin breaking

$$|A(K_S)_0|^2 = \left| \frac{G_F}{2} \frac{2\alpha_{em} m_K m_\mu Y(x_t)}{\pi \sin^2\theta_W} \times f_K \times V_{cs}V_{cd}\,\mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{cs}^* V_{cd}}\right)\right|^2$$

Relevant CKM parameter: $\bar{\eta}_{ds} = \mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{cs}^* V_{cd}}\right) = A^2\lambda^4\bar{\eta} + \mathcal{O}(\lambda^7)$

Current error on $\bar{\eta}_{ds}$ is $\mathcal{O}(5\%)$

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An experimental effort to measure $\mathcal{B}(K_S \to \mu^+\mu^-)_{\ell=0}$ can supplement the current program dedicated to $\mathcal{B}(K_L \to \pi^0\bar{\nu}\nu)$, $\mathcal{B}(K^+ \to \pi^+\bar{\nu}\nu)$.

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Releva

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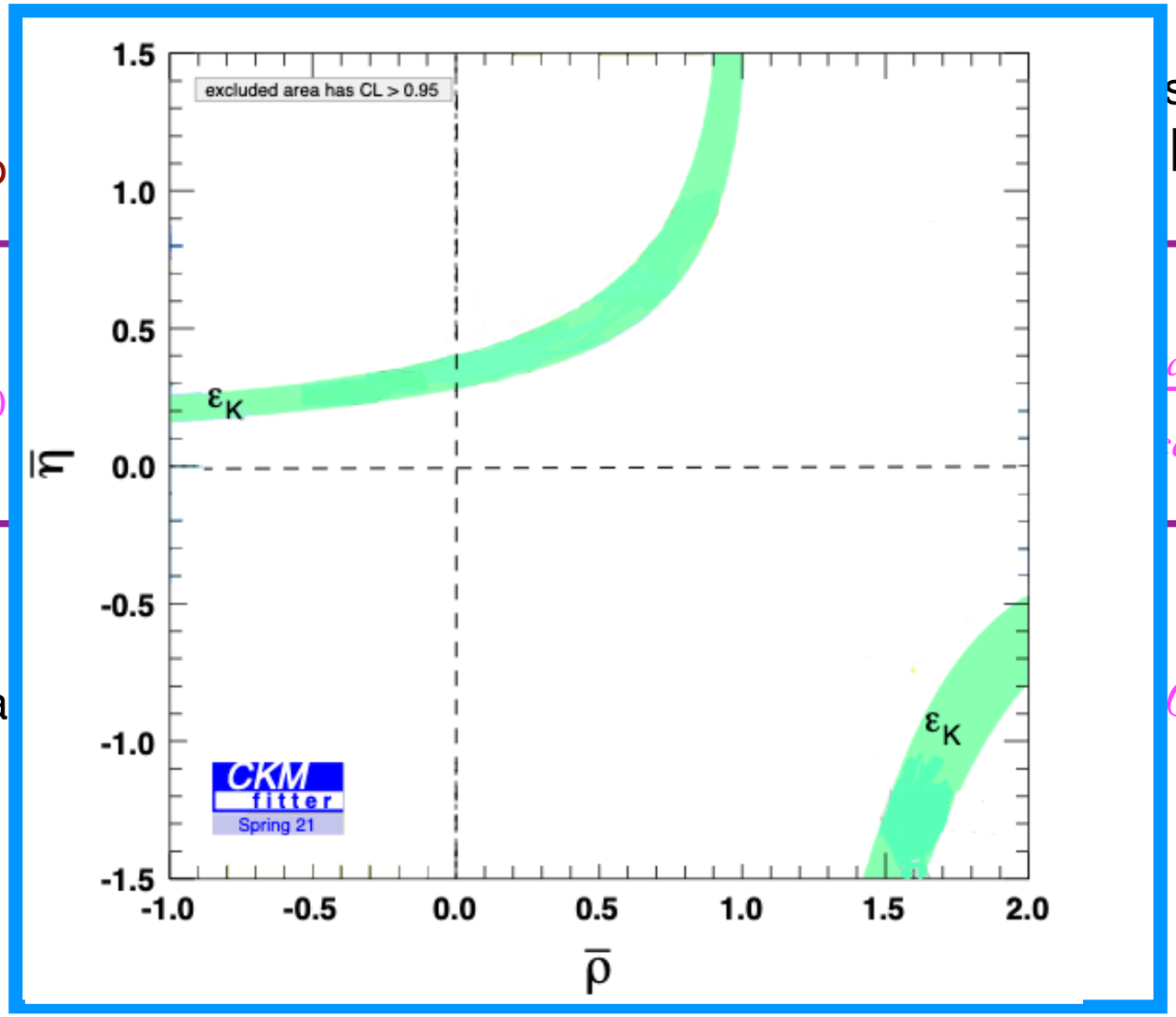

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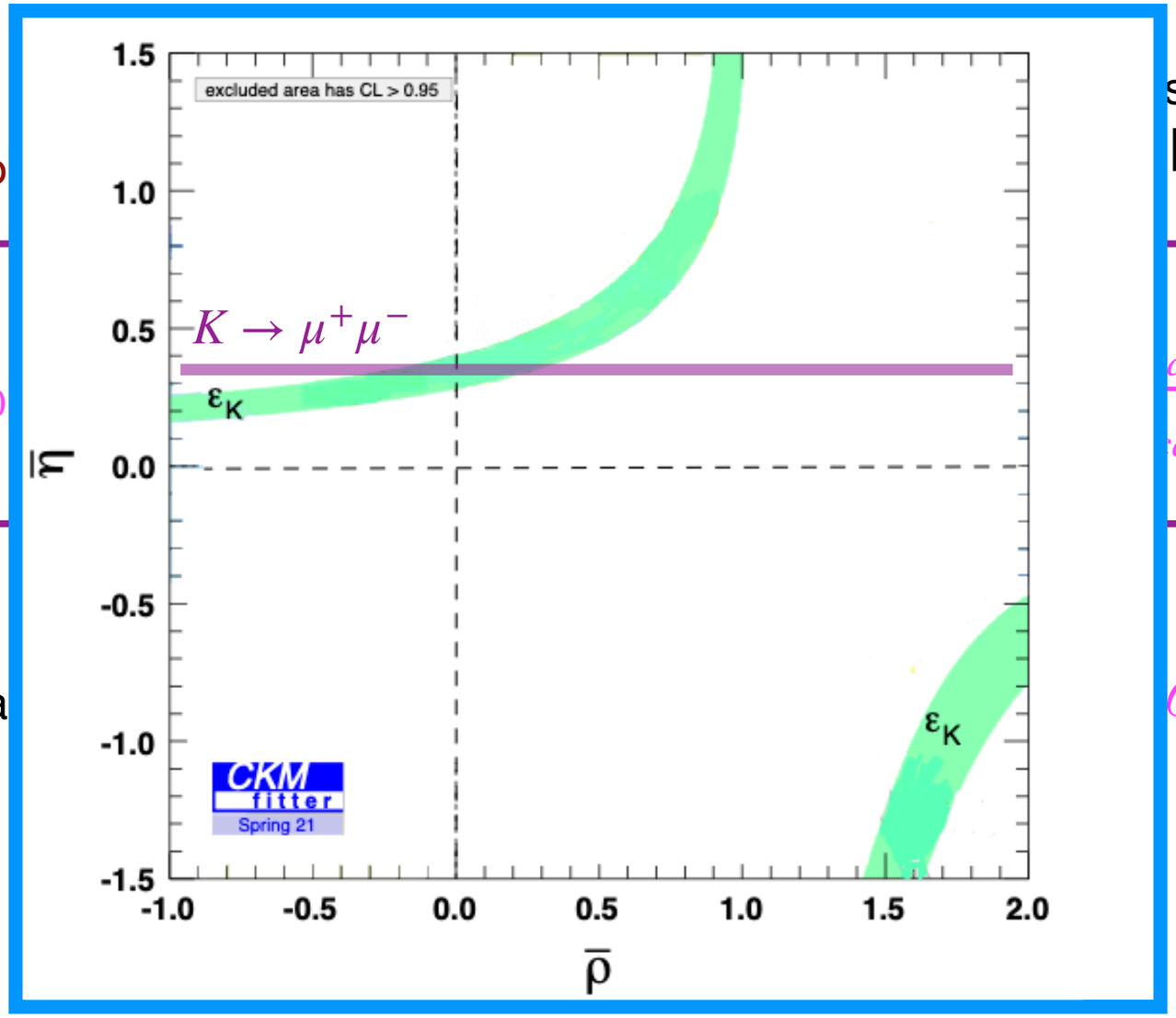

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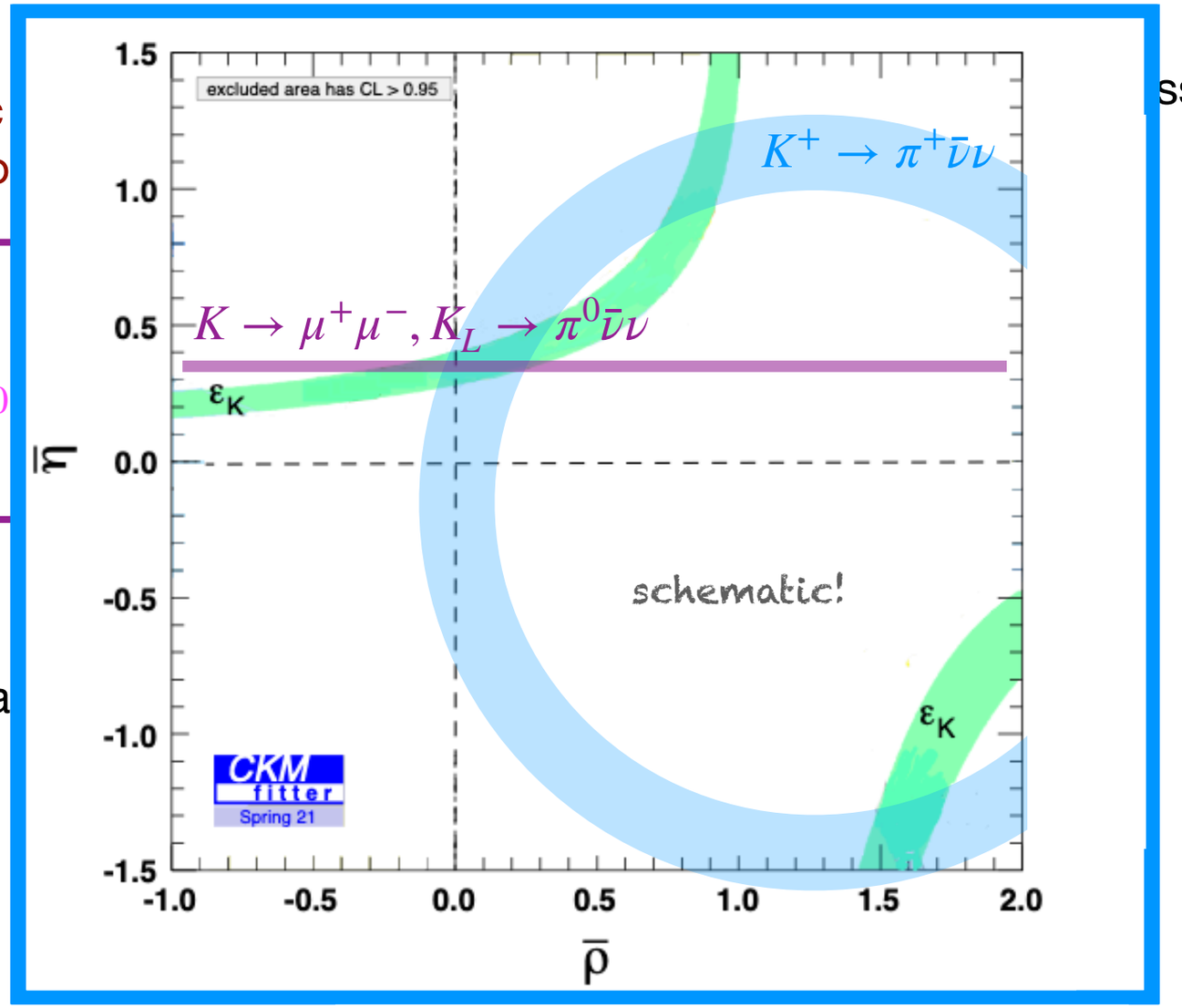

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Eliminating $|V_{cb}|$- related uncertainty

A. J. Buras and E. Venturini,
[arXiv:2109.11032]

Use four basic CKM parameters: $\lambda = |V_{us}|, \quad |V_{cb}|, \quad \beta, \quad \gamma$

Tension between inclusive and exclusive determinations of $|V_{cb}|$:

$$|V_{cb}|_{B\to X_c} = (42.16 \pm 0.50) \cdot 10^{-3}$$
$$|V_{cb}|_{B\to D^{(*)}\ell\nu} = (39.36 \pm 0.68) \cdot 10^{-3}$$

$$R_{\rm SL} = \frac{\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SD}}{\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})} = 1.55 \times 10^{-2} \left[\frac{\lambda}{0.225}\right]^2 \left[\frac{Y(x_t)}{X(x_t)}\right]^2$$

Independent of any SM parameters other than λ, m_t .

$K \to \mu^+\mu^-$ beyond the SM

[AD, M. Ghosh, *JHEP* **03** (2022) 048, [arXiv:2112.05801]]

$$R(K_S \to \mu^+\mu^-)_{\ell=0} \equiv \frac{\mathscr{B}(K_S \to \mu^+\mu^-)_{\ell=0}}{\mathscr{B}(K_S \to \mu^+\mu^-)^{\rm SM}_{\ell=0}} \lesssim 1280$$ [LHCb]

NP operators

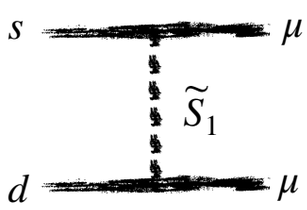
$$O_{VLL} = (\overline{Q}_L\gamma^\mu Q_L)(\overline{L}_L\gamma_\mu L_L)\,; \qquad O_{VLR} = (\overline{Q}_L\gamma^\mu Q_L)(\overline{e}_R\gamma_\mu e_R)\,,$$
$$O_{VRL} = (\overline{d}_R\gamma^\mu d_R)(\overline{L}_L\gamma_\mu L_L)\,; \qquad O_{VRR} = (\overline{d}_R\gamma^\mu d_R)(\overline{e}_R\gamma_\mu e_R)\,,$$

$K_L \to \pi^0\bar{\nu}\nu$

$$O_{SLR} = (\overline{Q}_L d_R)(\overline{e}_R L_L)\,,$$
$$O_{SRL} = (\overline{d}_R Q_L)(\overline{L}_L e_R)\,.$$

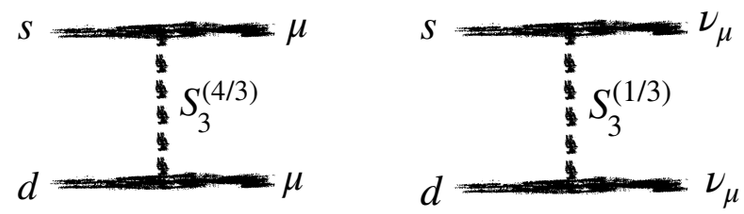
Example models:

A. Scalar Leptoquark $\widetilde{S}_1 \sim (\bar{3},1)_{4/3}$



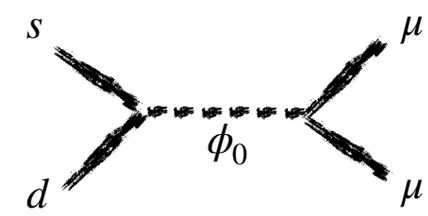
Can saturate the current bound while satisfying all existing constraints

B. Scalar Leptoquark $S_3 \sim (\bar{3},3)_{1/3}$



$R(K_S \to \mu^+\mu^-)_{\ell=0} \lesssim 26$, bounded by the GN bound on $R(K_L \to \pi^0\bar{\nu}\nu)$

C. 2HDM $\Phi \sim (1,2)_{1/2} = \begin{pmatrix}\phi^+ \\ \phi_0\end{pmatrix}$



Can saturate the current bound while satisfying all existing constraints

Summary

- ✦ Time-dependent rate of $K \to \mu^+\mu^-$ can **cleanly probe SD physics,** allowing for an <u>independent determination of CKM parameters from kaon physics</u>.

(Kross-check!)

- ✦ Naive estimate: need $\mathcal{O}(10^{13})$ kaons to get close to SM sensitivity.

- ✦ The theoretical uncertainty is of at most $\mathcal{O}(1\%)$!

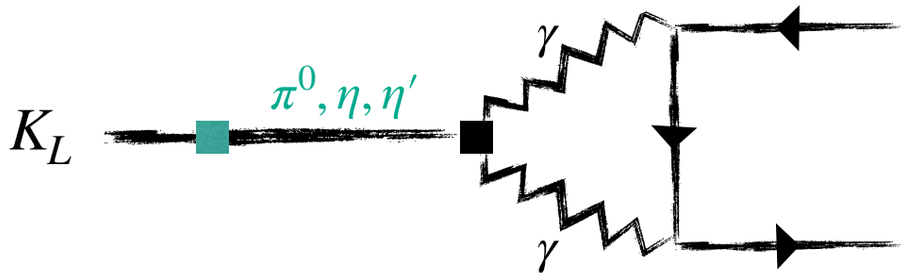
- ✦ The relevant CKM parameter is $\bar{\eta}_{ds} = \mathrm{Im}\left(V_{ts}^* V_{td}/(V_{cs}^* V_{cd})\right) \approx A^2\lambda^4\bar{\eta}$, the same combination that appears in $K_L \to \pi^0\bar{\nu}\nu$.

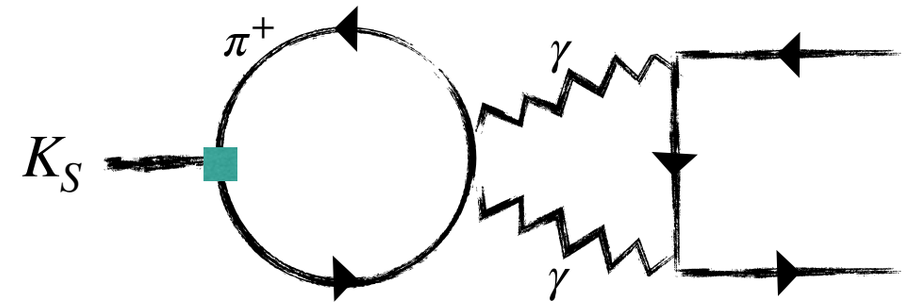
- ✦ The ratio $\mathcal{B}(K_S \to \mu^+\mu^-)_{\ell=0}/\mathcal{B}(K_L \to \pi^0\bar{\nu}\nu)$ is an extremely clean SM observable, dependent <u>only on the parameters λ, m_t .</u>

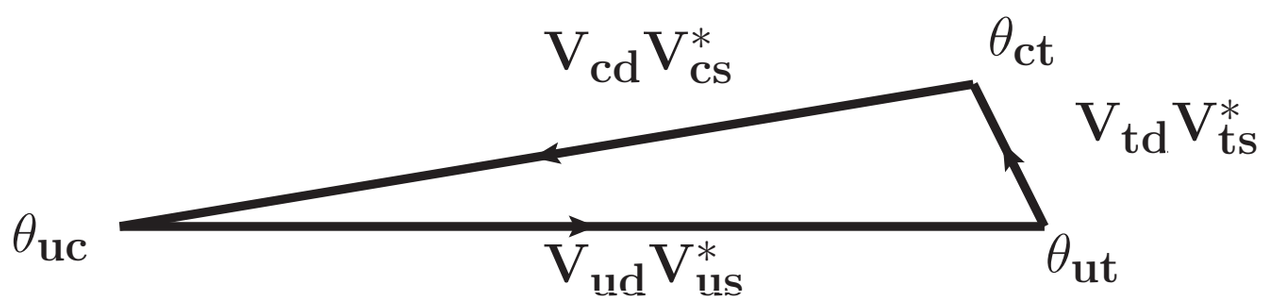
- ✦ $K \to \mu^+\mu^-$ is a sensitive probe of heavy NP, several simple models can saturate the current bound, $R(K_S \to \mu^+\mu^-)_{\ell=0} \sim 10^3$.

Backup

LD contribution is CPC to $\mathcal{O}(\lambda^4)$







$$\theta_{uc} = arg\left(-\frac{V_{cd}V_{cs}^*}{V_{ud}V_{us}^*}\right) \sim \lambda^4$$

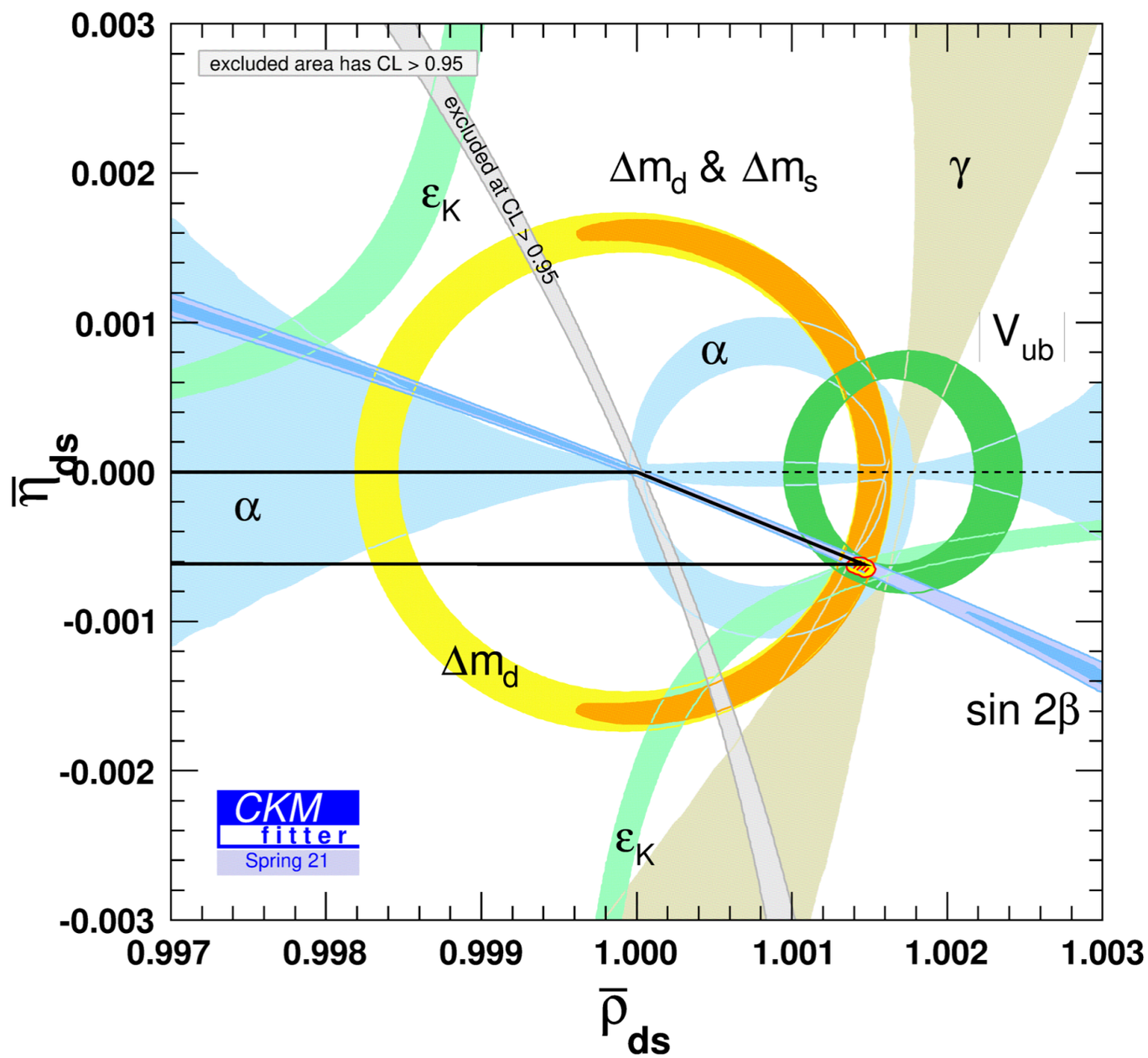
We can measure $|A(K_S)_{\ell=0}|^2$

Only hadr
uncertaint

nan, S. Schacht,
Xiv:2104.06427]

$|A($

$)|^2$



$A^2\lambda^4\bar{\eta} + \mathcal{O}(\lambda^7)$

Current error on
$\bar{\eta}_{ds}$ is $\mathcal{O}(5\%)$

is would be a
rmination of $\bar{\eta}_{ds}$
from a **pure kaon**
s measurement.