TRIUMF

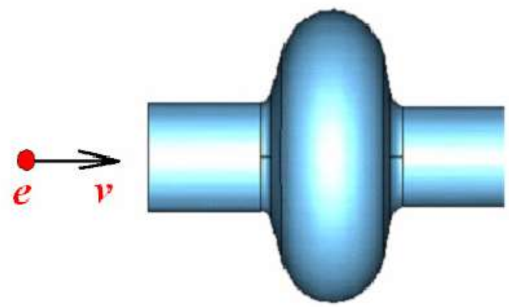
Iterative learning controller for beam loading compensation

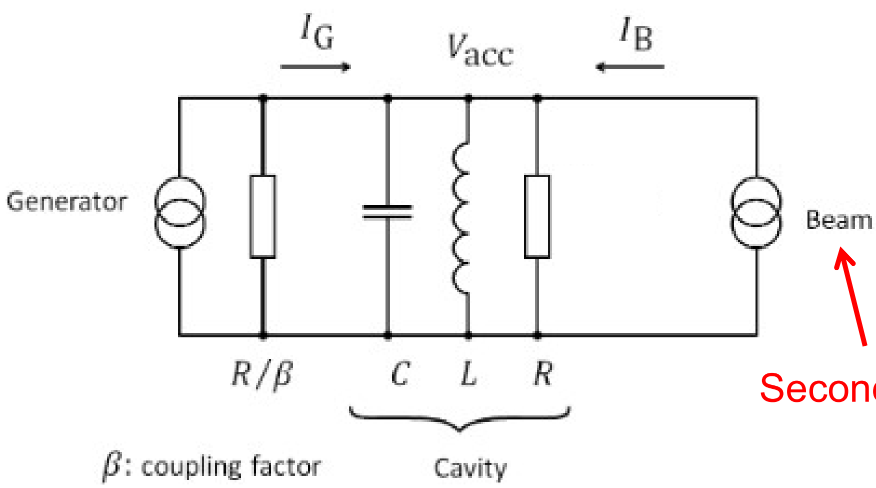
Ken Fong, Ramona Leewe, Yanyun Ma

Outline

- Brief introduction into beam loading and general control strategies
- Concept of manual feed forward leading to adaptive feed forward
- ILC block diagram and equations
- ILC in frequency domain
- Evaluation of different window (learning) functions
- Experimental measurements in TRIUMF's ARIEL facility
- Summary and conclusions

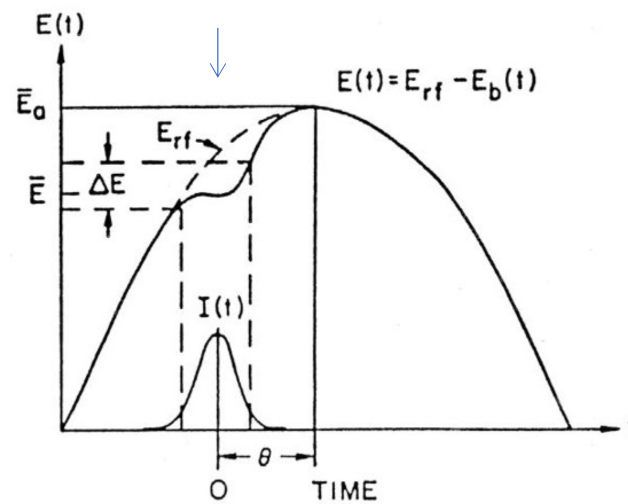
What is beam loading?





Second source

Field is in
opposite direction

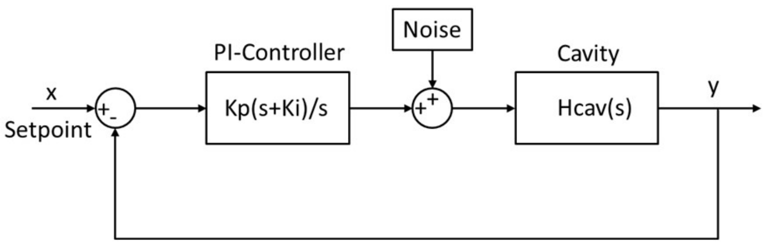


- Beam loading results from the interaction of the beam with the cavity field and causes additional field instabilities

What to do to mitigate the field instabilities

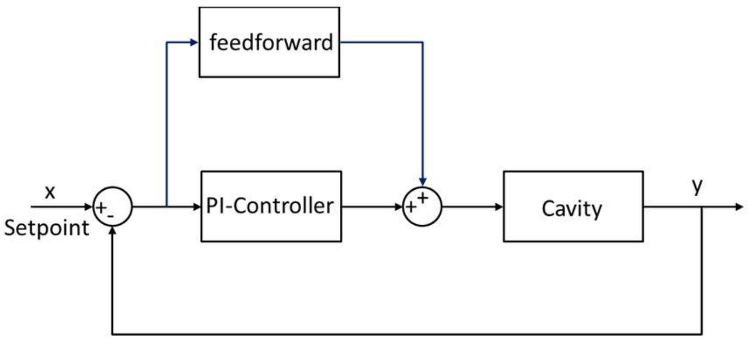
Feedback system

- A feedback systems looks at the output of a system and tries to correct the input to achieve a desired output
- Closed loop system
- Generally used for random slow variation



Feedforward system

- Applies a correctional signal to the input of the plant before an expected disturbance occurs
- Open loop system
- Generally used for very fast (where feedback systems are too slow) and repetitive disturbances



What to do to mitigate the field instabilities

Too slow, e-Linac design requirement for the voltage is 0.1% within 10 us during pulse mode

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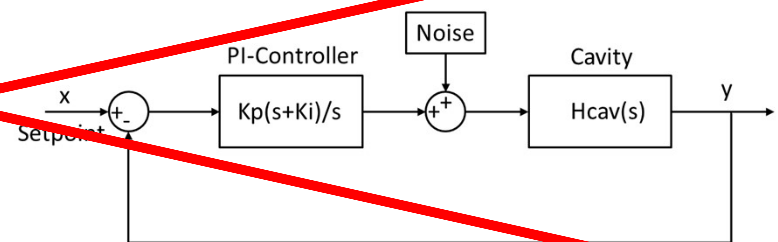

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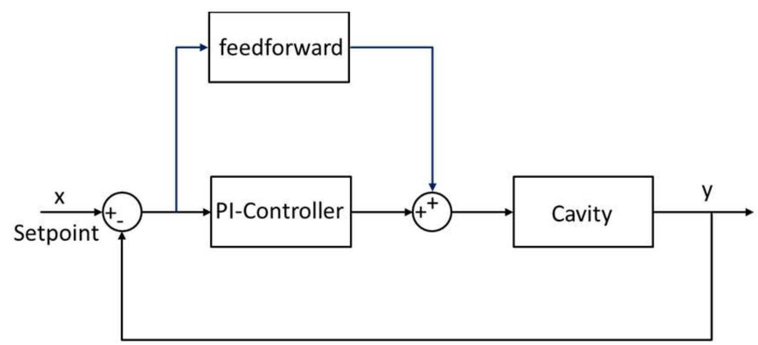
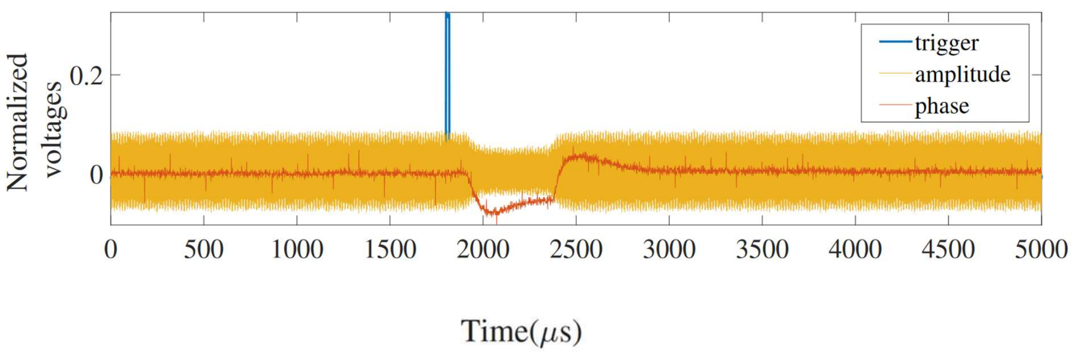


The effect of beam loading on the RF field and the concept of feedforward

Beam loading measurement on a room temperature quarter wave resonator:

- Normalized field amplitude, phase and trigger signal



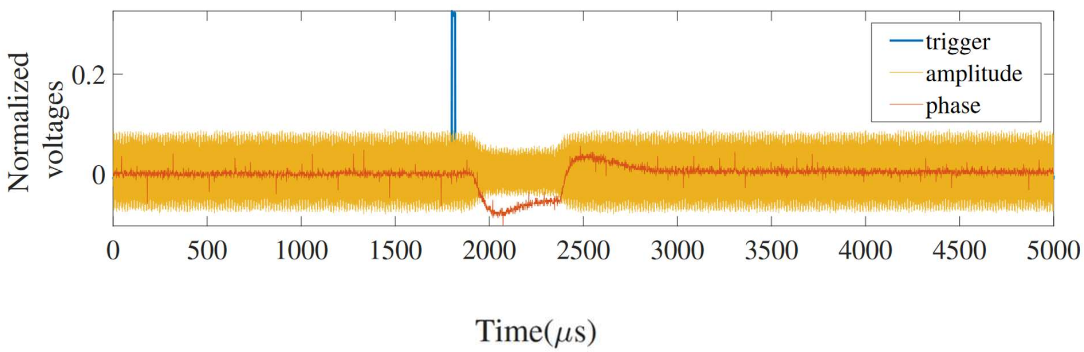
Requirements for manual feed forward

- Knowing the amplitude and phase error
- Trigger/timing signal, (added)

The effect of beam loading on the RF field and the concept of manual feedforward

Beam loading measurement on TRIUMF's inceptor cryomodule

- Normalized field amplitude, phase and trigger signal



Requirements for manual feed forward

- Knowing the amplitude and phase error

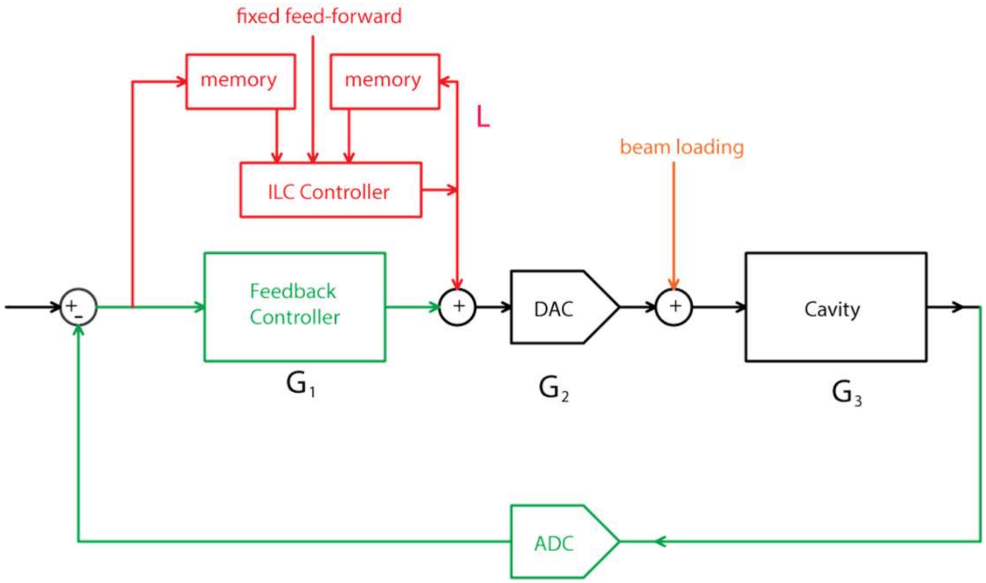
Varying beam current!

- Trigger/timing signal

Adaptive feed forward

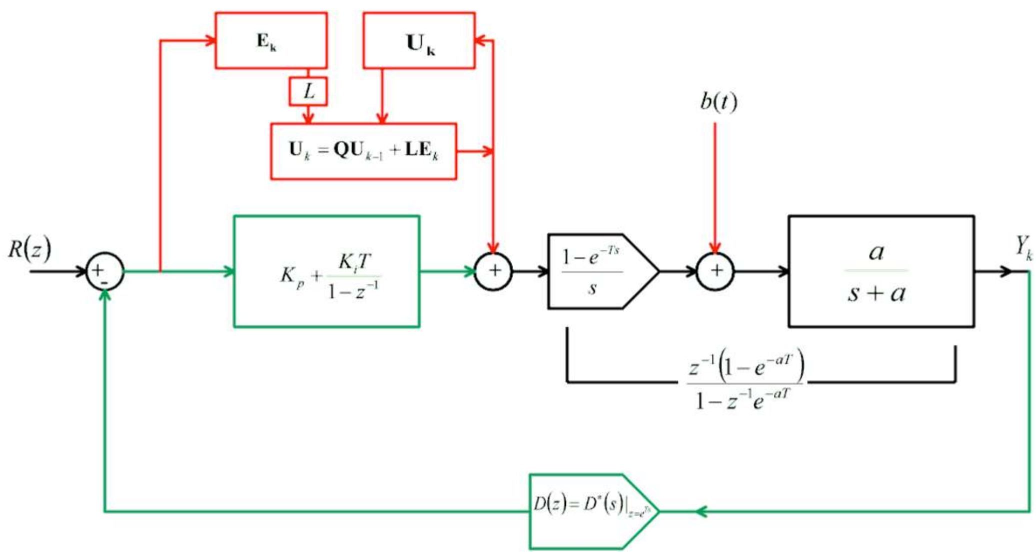
Adaptive feed forward

Block diagram of a typical ILC controller



- ILC (red blocks) generates a feedforward signal that tracks repeating disturbance
- Error voltage and the output drive are stored in the memories and applied towards the correction for the next macro pulse (iteration)
- To be implemented in a digital system, sampling rate determines its performance
- Because of its discrete time nature, most of the analysis is performed in the z-transform

Block diagram and transfer function in terms of Z-transform



Learning function combines the error and correction from previous iteration and constructs a new correction to be applied to the present iteration

- The drive calculated by the ILC for the k^{th} iteration for an ILC:

$$U_k = QU_{k-1} + LE_k$$

- Q, filter often set to 1 (or Q<1), E is the error and L the learning function (window function)
- The cavity output is then

$$Y_k(z) = P(z)\, U_k(z) + Y_0(z)$$

- Where P(z) is the closed loop transfer function

$$P(z) = \frac{\dfrac{\left(1-e^{-aT}\right)}{z-e^{-aT}}}{1 + \dfrac{\left(1-e^{-aT}\right)}{z-e^{-aT}}\left(K_p + \dfrac{zK_iT}{z-1}\right)}$$

ILC in frequency domain

ILC open loop response in time domain:

$$y_n^k = Q y_n^{k-1} + \sum_{m=0}^{N} L_{n-m} e_m^{k-1}$$

- Inputs: e_n^{k-1} outputs: y_n^k
- Learning function L_m is:
  - Causal for $n \geq m \geq 0$
  - Non-causal for $0 > m \geq n - N$
  - Implies that L_m is finite impulse response

- Defining the error input to be a sinus signal at a single discrete frequency f_0:

$$e_n = \cos\left(\frac{2\pi f_0 n}{N}\right) \qquad f_0 \in \left\{0, 1, 2, \ldots, \frac{N}{2} - 1\right\}$$

ILC open loop response in frequency domain:

$$y_n = \sum_{m=0}^{N} L_{n-m} \cos\left(\frac{2\pi f_0 m}{N}\right)$$

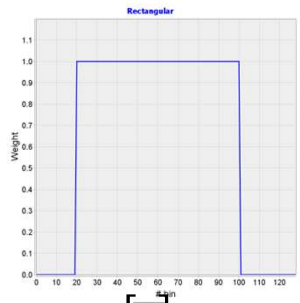
DFT: $$\mathrm{Y}(f) = \sum_{n=0}^{N-1} \sum_{m=0}^{N} L_{n-m} \cos\left(\frac{2\pi f_0 m}{N}\right) e^{-\frac{i2\pi f n}{N}}$$

- For the kth iteration: $\mathrm{Y}^k = \frac{1-Q^k}{1-Q}\mathrm{Y}$
- Convergence of y_k requires $Q < 1$

K. Fong, “Analysis of Iterative Learning Controller in frequency domain”, TRI-DN-17-27, TRIUMF, 2017

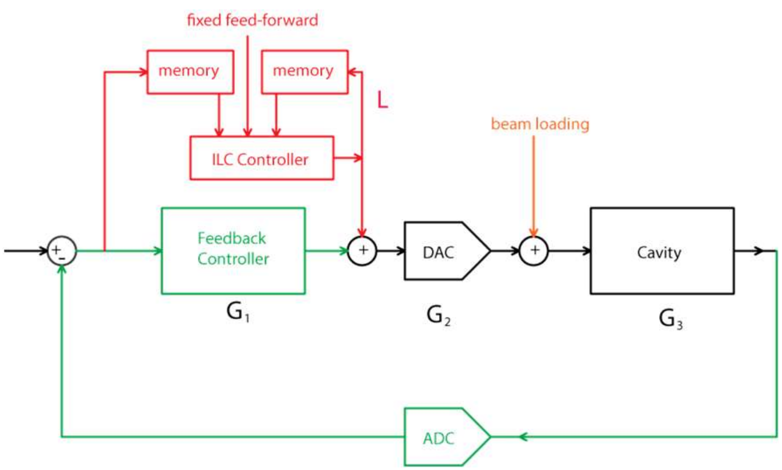
Simulation of ILC in frequency domain

Assuming 3-sample non-causal Simple Moving Average learning function

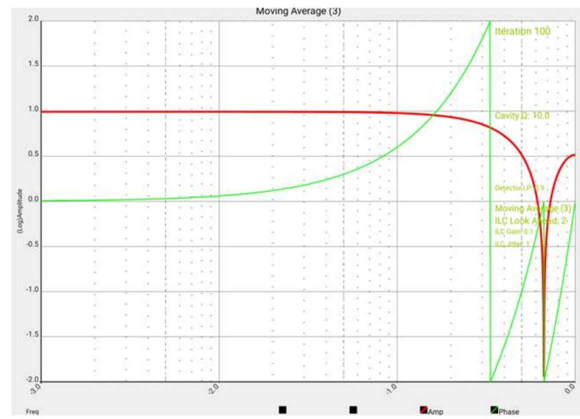


$$w[n] = 1$$

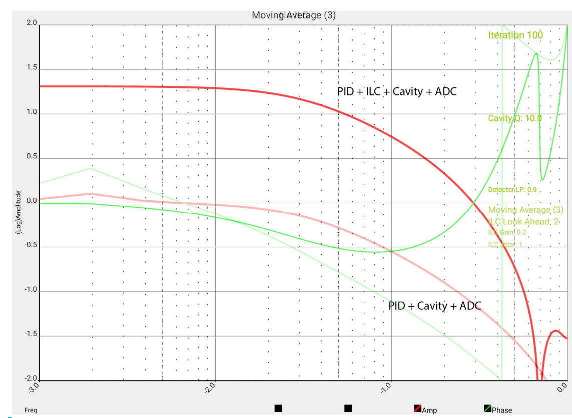
$$\left\{0, \ldots, L_n = 0, L_{n+1} = \frac{1}{3}, L_{n+2} = \frac{1}{3}, L_{n+3} = \frac{1}{3}, 0 \ldots\right\}$$



Bode plot of the window function:



Open loop transfer function including ILC:



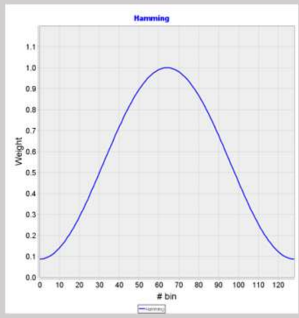
Look ahead =2 (non-causal)

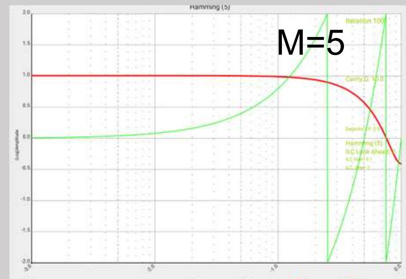
Comparison of different learning functions/ window function

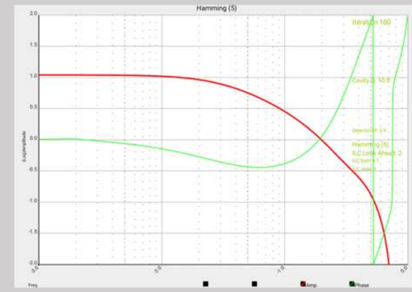
- Hamming Window

$$w[n] = \alpha + (1-\alpha)\cos\left(\frac{2\pi n}{M}\right)$$

$\alpha \ = \ 0.53836$



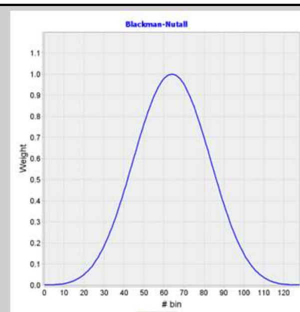


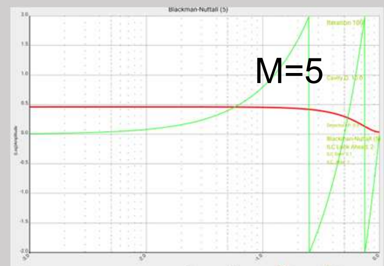


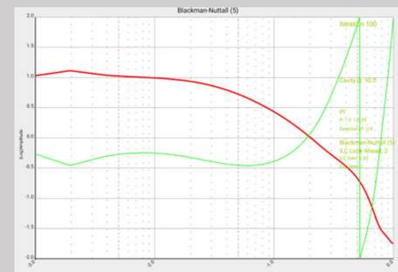
- Blackman-Nutall Window

$$w[n] = a_0 + a_1\cos\left(\frac{2\pi n}{M}\right) + a_2\cos\left(\frac{4\pi n}{M}\right) + a_3\cos\left(\frac{6\pi n}{M}\right)$$

$a_0 \ = 0.364$
$a_1 \ = 0.489$
$a_2 \ = 0.137$
$a_3 \ = 0.011$

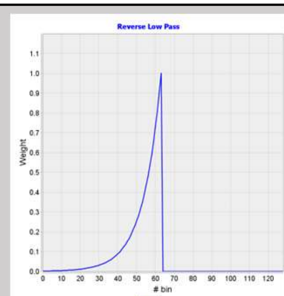


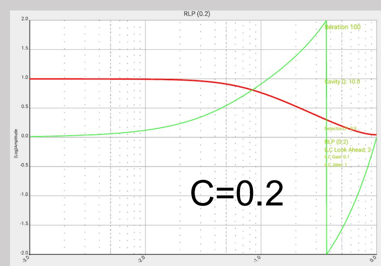


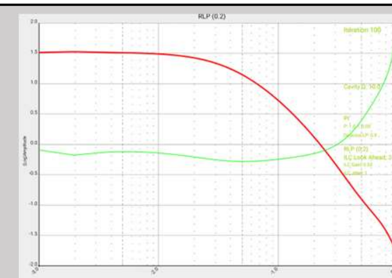


- Reverse Low Pass filter

$$y_n = (1-a)x_n + ax_{n+1} + a^2x_{n+2} + a^3x_{n+3} + \cdots$$







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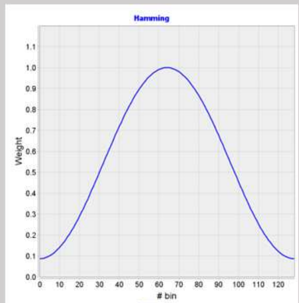

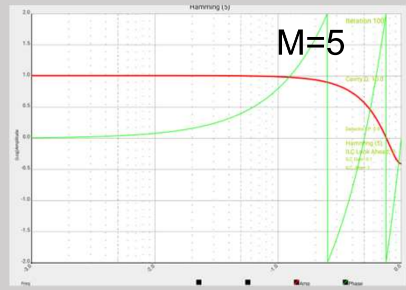

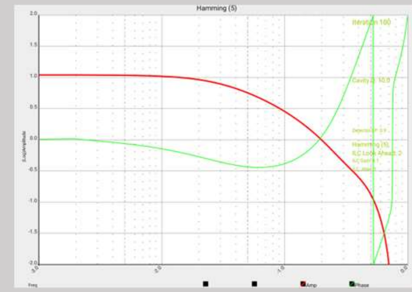

	Number of Samples	Number of MAC operations per sample
Rectangular	3	3
Reverse Low Pass	-	>8
Blackman-Nutall	5	3

- Blackman-Nutall Window

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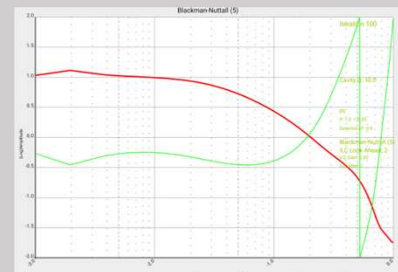

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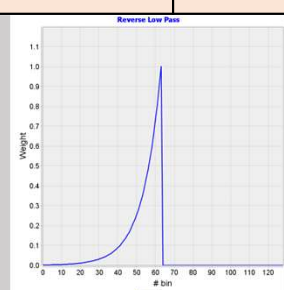
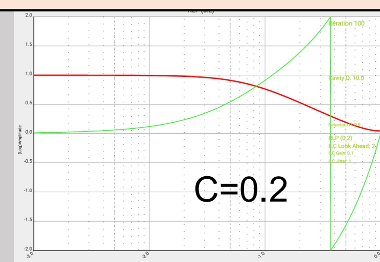
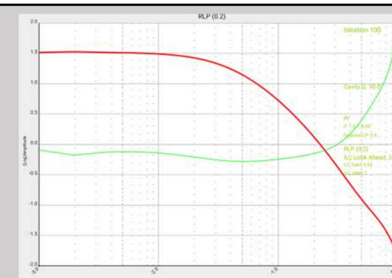






Conclusions from the frequency domain simulations

$$y_n = \sum_{m=0}^{N} L_{n-m} \cos\left(\frac{2\pi f_0 m}{N}\right)$$

- The phase response of the open loop gain of an ILC is independent of the number of iterations. (k is the iteration variable)

- The amount of phase lead is related to the amount of non-causality.

$$\mathrm{Y}^k = \frac{1-Q^k}{1-Q}\mathrm{Y}$$

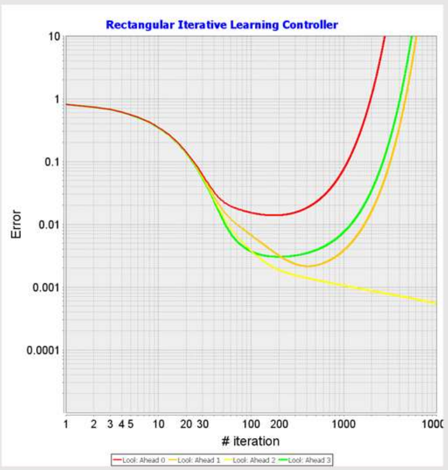
- The amplitude gain of the ILC increases per iteration.

- These mean that the phase response is the determining factor in stability. In general, the total phase lag should not exceed 90^o at all frequencies.

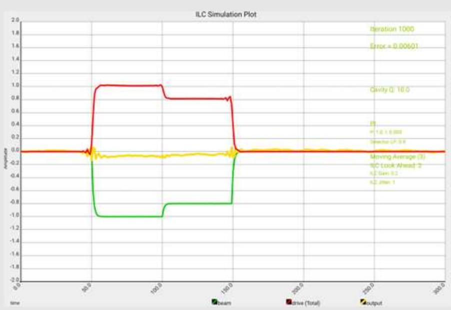
- ILC suppress repetitive transient by:

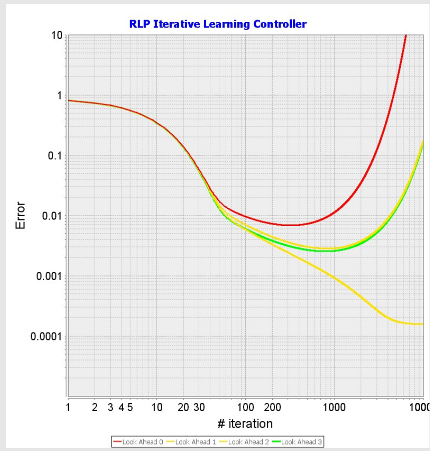
1. Increase mid and high frequency gains, where the feedback gain has rolled off.
2. Additional phase lead to stabilize the system at these frequencies.

Rate of convergence for different windows and different look ahead



Look ahead = 2





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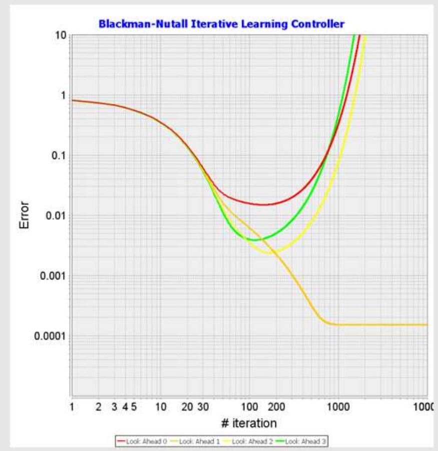
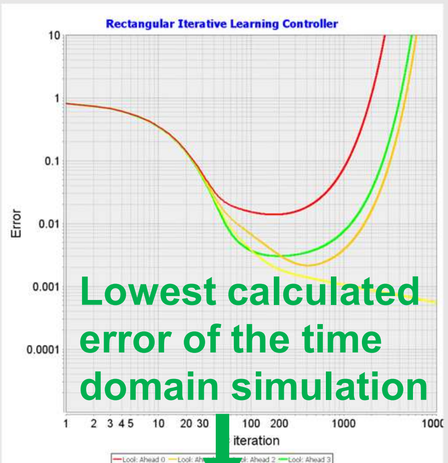


Look ahead = 2

The initial rates of convergence are similar between different windows.
The rates are governed by ILC gain.
Causal window is unstable. (look ahead=0)

- Look-ahead too small – insufficient phase lead at high frequency – reduced phase margin.
- Look-ahead too large – phase wrap around at mid frequency – reduced gain margin.

Rate of convergence for different windows and different look ahead



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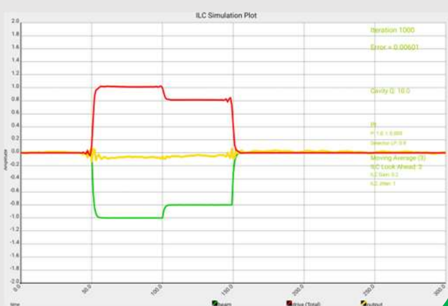

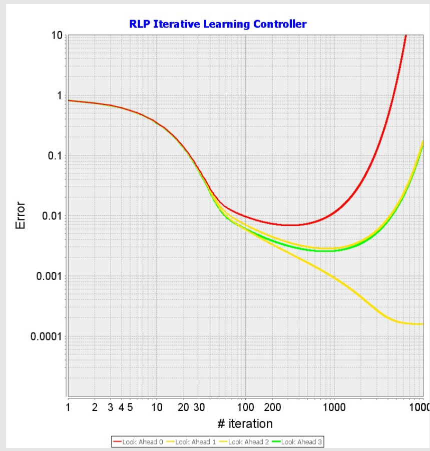

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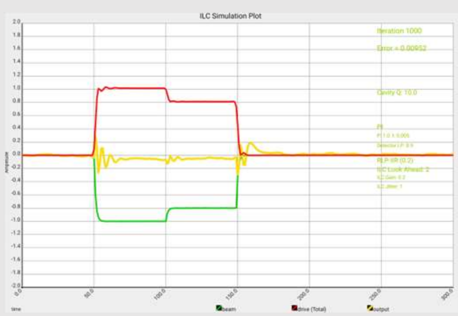

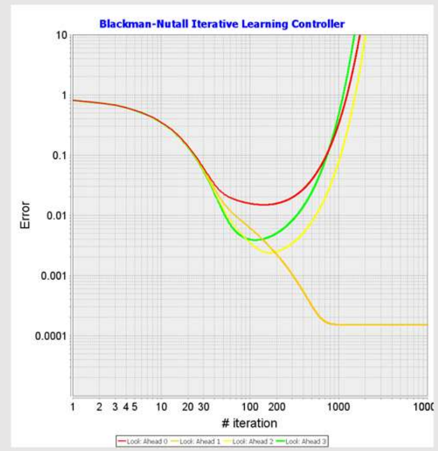

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Summary of time and frequency domain analysis and simulation study

Conclusion:

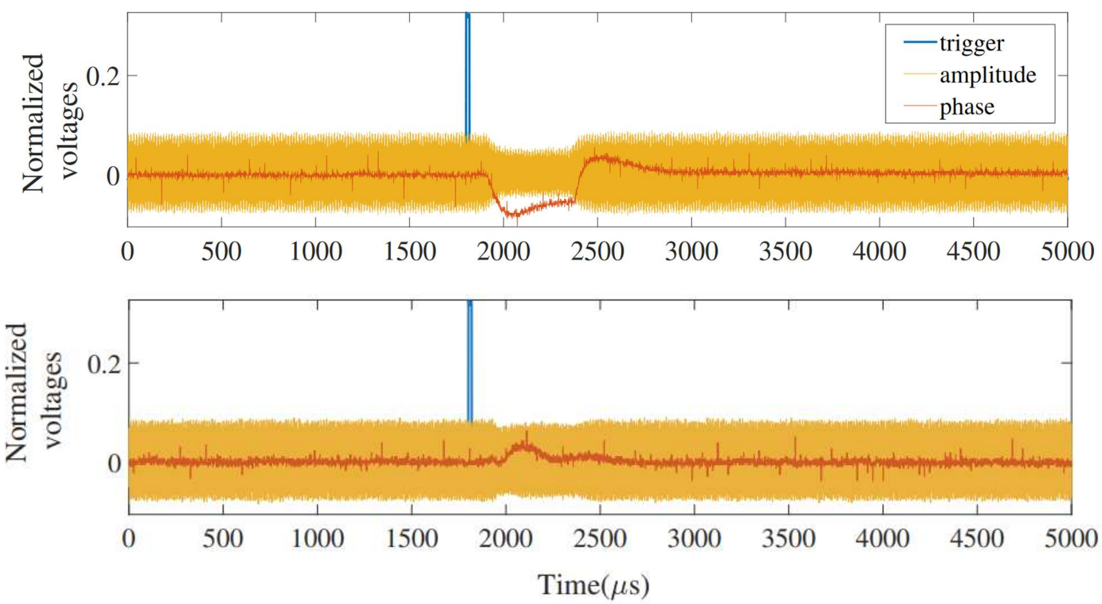
- Frequency domain analysis of Iterative Learning Controller provides visual indication of stability using classical method (phase and amplitude margins in Bode plot)
  - Allows simply analysis of high frequency response
- Stability requires a total phase lag to be less than 90^{o}.
- Phase response is very sensitive to amount of look ahead provided by the learning function

How to choose the right window and look ahead

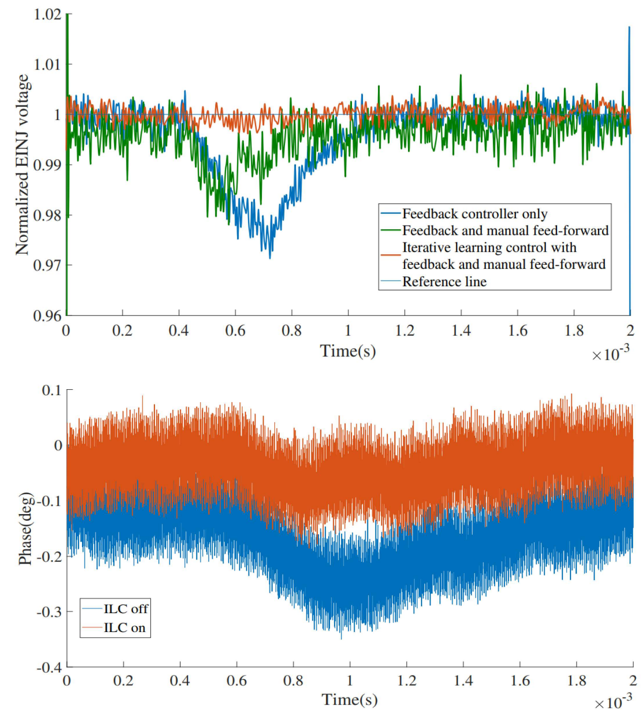
- Frequency domain behavior
- Time domain behavior
- Rate of convergence
- Our choice:
  - **Rectangular window with a look ahead of 2**

RF Measurements on TRIUMF’s e-Linac

Amplitude and phase of a room temperature cavity with and without ILC:



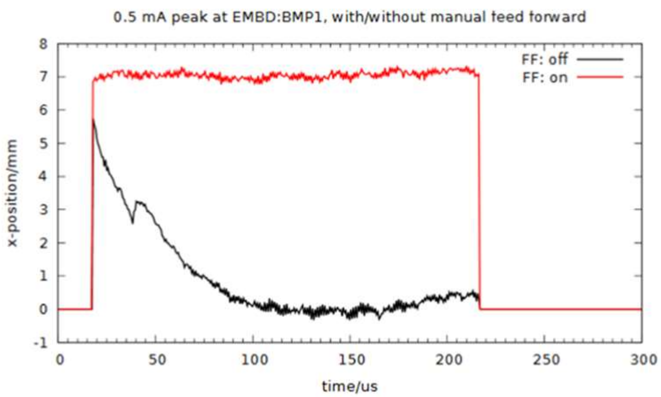
Amplitude and phase of TRIUMF’s injector cryomodule with ILC and manual feed forward



Ramona Leewe, Ken Fong, Yanyun Ma

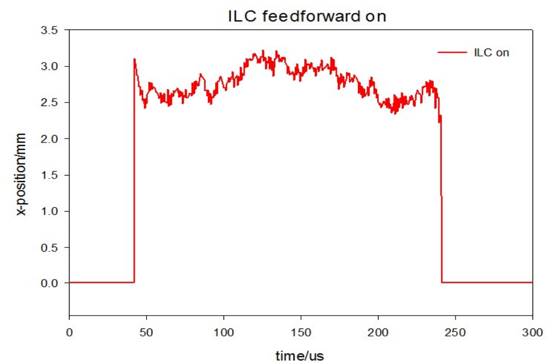
BPM measurement, beam loading effect on the beam profile

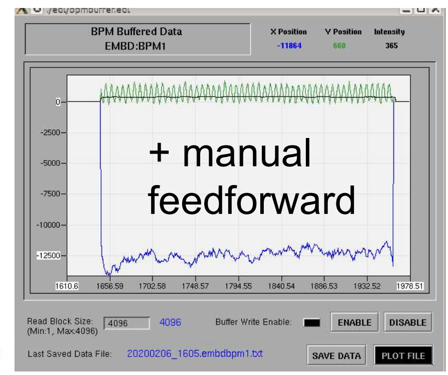
Manual feedforward



- “Perfect” compensation using fixed amplitude feed forward
- Does not work for variable beam energy

ILC (iterative learning controller)

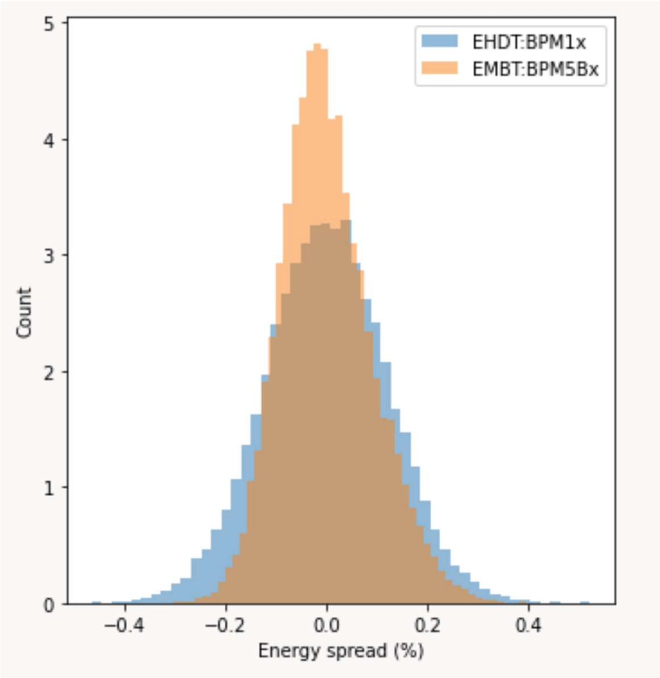




- Good compensation using ILC
- Beam current drifts are likely to affect the performance
- Works for variable beam energy
- Best results combining manual feedforward and ILC

Histogram of beam stability

Last weeks beam stability histogram for 2 different BPM’s with ILC



- Orange signal BPM1 is located after the injector cryomodule
- blue signal BPM5 is located after the acceleration cryomodule
- Energy spread of 0.2%, design specification for beam stability is 0.1% of the energy spread

Summary and outlook

Summary

- Analyzed an Iterative learning controller in time and frequency convergence
  - DFT and frequency analysis adds much more information than z-domain
- Evaluation of different learning functions, behavior and rate of convergence
- Measurements indicate a stability of 0.2%

Outlook

- To achieve ARIEL's design specification of 0.1% beam stability the E-gun will be evaluated
  - Adding a tuning system for the E-gun to stabilize the beam current at the source

Thank you!