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Higher Order Mode and Cavity Studies at the University of Rostock since HOMSC16

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- *meanwhile at other places*

ICFA Mini Workshop on Higher Order Modes in Superconducting Cavities (HOMSC2018)

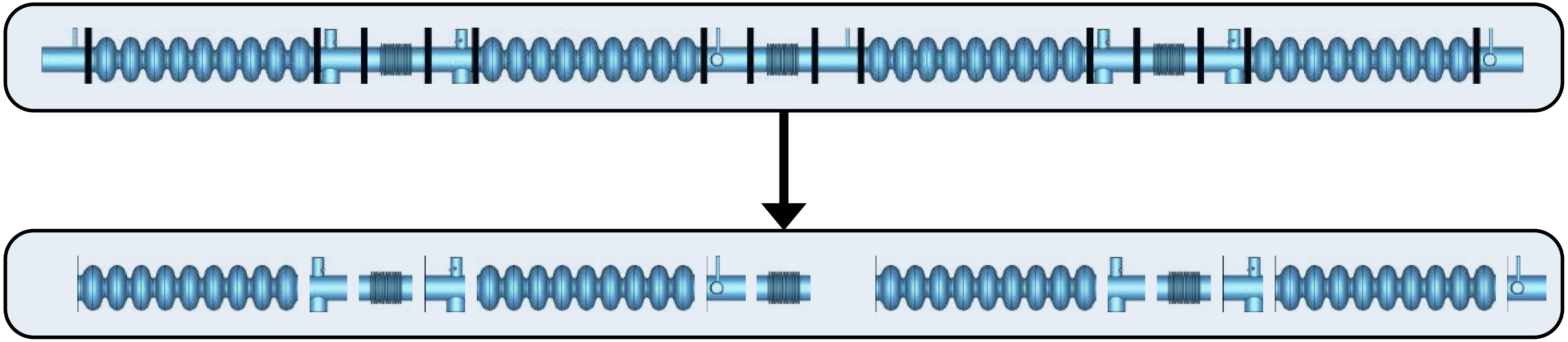
Cornell, Ithaca, USA

Overview

- Motivation for Domain Decomposition Approach
- State-Space Concatenations (SSC)
  - Application to FLASH
  - Application for BESSY VSR
  - Computation of External Quality Factors
- Uncertainty Quantification Techniques and Stochastic Models for SRF Cavities
- Future Circular Collider (FCC) studies – see Shahnam’s presentation this morning
- Deflecting Cavity for Beam Separation at ELBE

Domain Decomposition Methods for Large, Complex SRF Accelerators

- SRF cavities are often too complex to solve Maxwell's equations using standard numerical methods on standard workstations (i.e. computation takes impractically long)
- Usage of domain decomposition methods
  - Decompose the domain at regions of constant cross-section into several subdomains /segments
  - Solve Maxwell's equations for each subdomain and couple the results
- Allows for computationally efficient investigation of multi-cavity modes

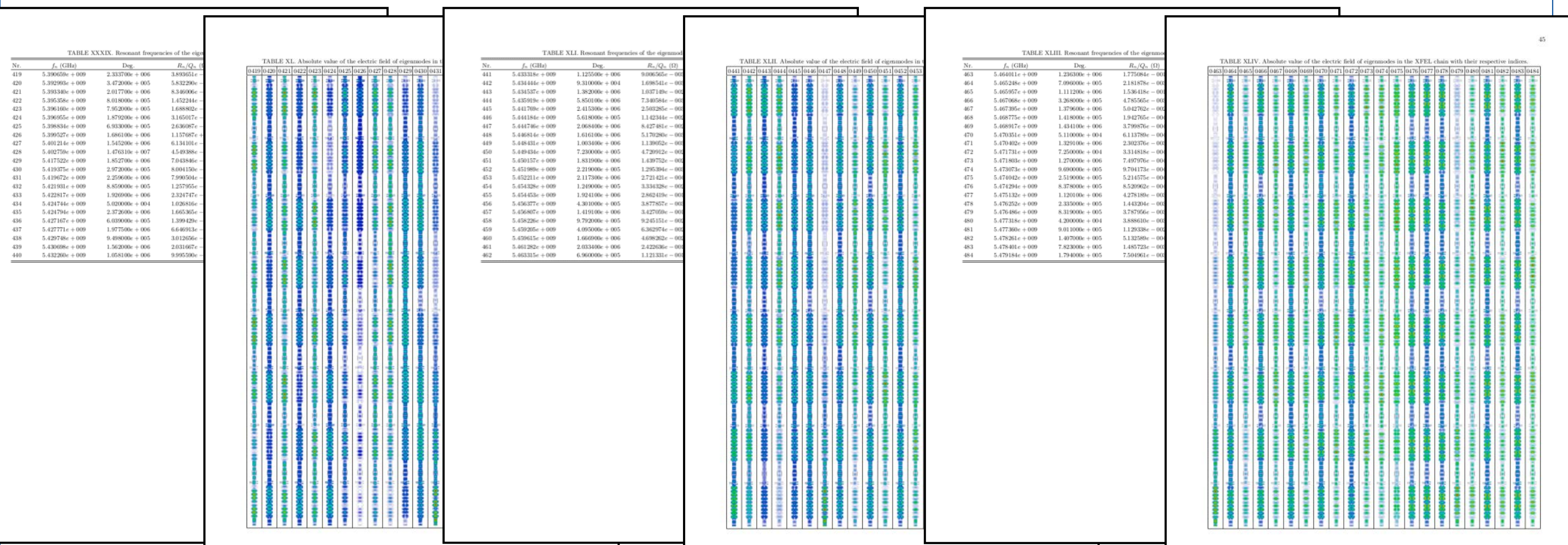


State-Space Concatenations (SSC)

- Consideration of segments as blocks with terminals
  - Modal voltages $v_{r,p,m}(t)$ correspond to tangential electric fields of 2D port modes
  - Modal currents $i_{r,p,m}(t)$ correspond to tangential magnetic fields of 2D port modes
- Generation of second-order state-space equations for segments
  - Wave equation (PDE) → state-space system (ODE)
- Model-order reduction for state-space system via orthogonal projection
  - Original state vector of dimension ~ 10^6 → reduced state vector of dimension < 10^2
- Concatenation of reduced-order state-space system
  - Coupling constraints according to Kirchhoff‘s laws
  - Arbitrary topologies and number of 2D port modes supported
- Computation of RF properties by means of the reduced-order model
  - Impedance- and scattering parameters; frequencies and field distributions of eigenmodes; 3D field distribution due to port excitations; external quality factors

T. Flisgen,. Dissertation, Universität Rostock, 2015, http://purl.uni-rostock.de/rosdok/id00001811

Compendium of Eigenmodes for FLASH 3.9 GHz Module



T Flisgen, J Heller, T Galek, L Shi, N Joshi, N Baboi, RM Jones, U van Rienen. “Eigenmode compendium of the third harmonic module of the European X-ray Free Electron Laser”, Physical Review Accelerators and Beams, 20(4), 2017, 042002

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Compendium of Eigenmodes for FLASH 3.9 GHz Module

- Computed 1,479 eigenmodes in the interval 1 GHz to 8 GHz of the chain by means of a standard workstation computer
- Compendium contains resonant frequencies, external quality factors, beam coupling impedances, and electric field distributions
- In total, the compendium consists of 137 pages
- $T_{comp} \approx$ 8 h on an Intel Xeon E5-2687W v2 @ 3.40 GHz plus several hours for exporting matrices and plotting the field distributions

T Flisgen, J Heller, T Galek, L Shi, N Joshi, N Baboi, RM Jones, U van Rienen. “Eigenmode compendium of the third harmonic module of the European X-ray Free Electron Laser”, Physical Review Accelerators and Beams, 20(4), 2017, 042002

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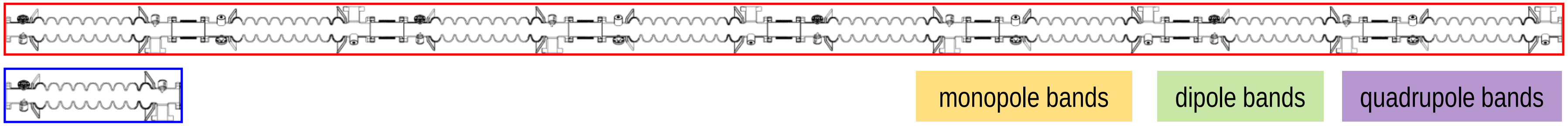
Q Factors in Single Cavity and in Chain of Cavities - Pure Bellow Modes at 5.1524 GHz



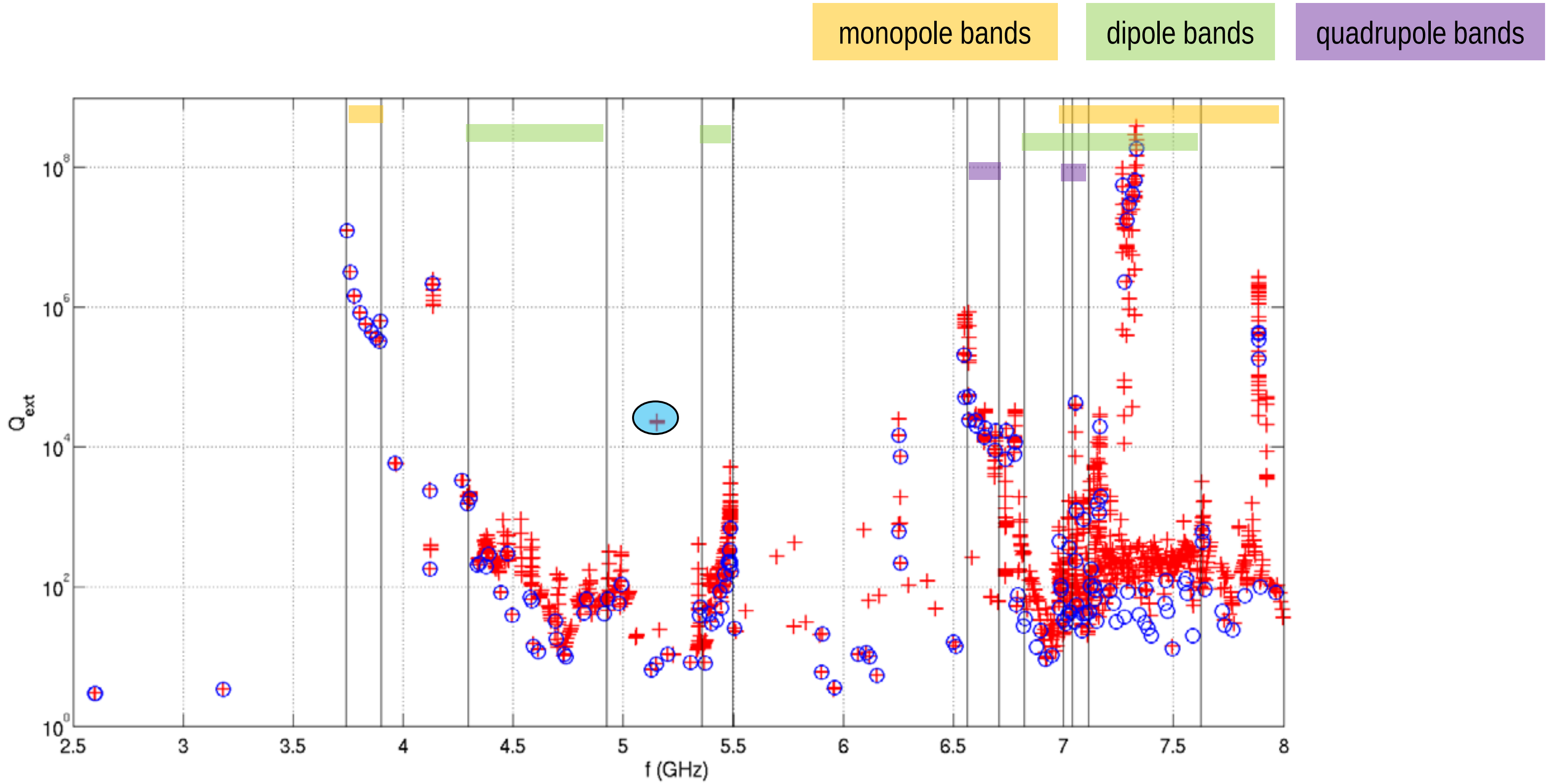
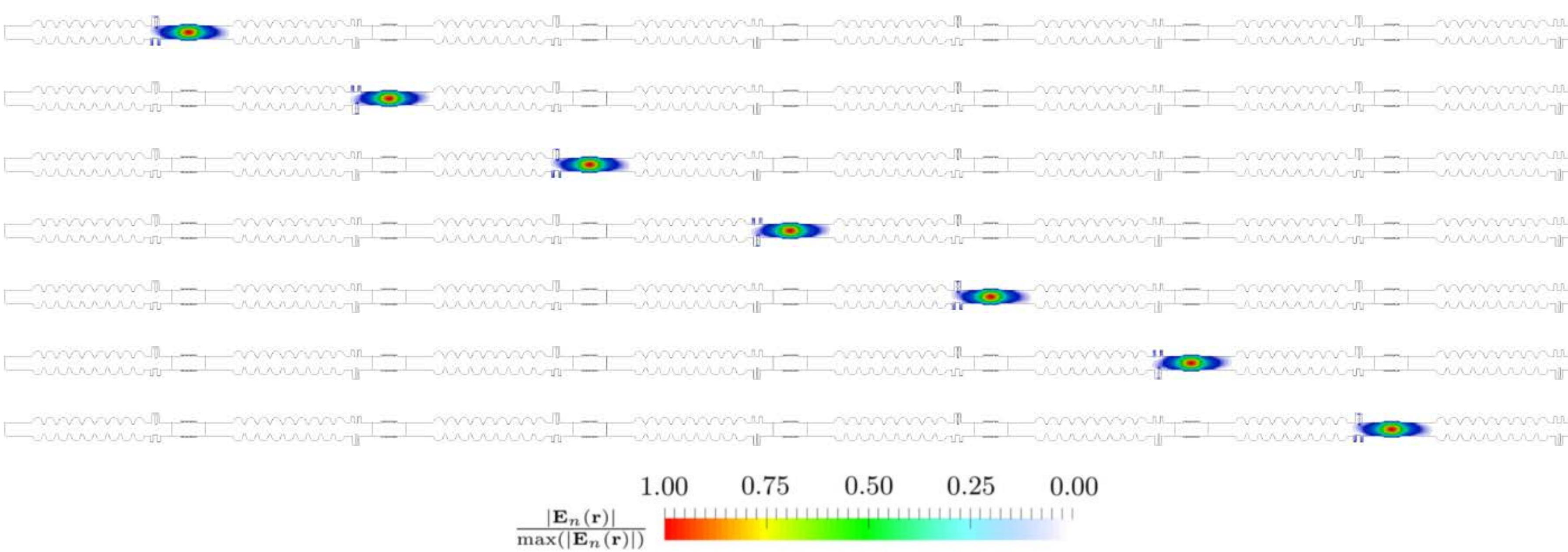


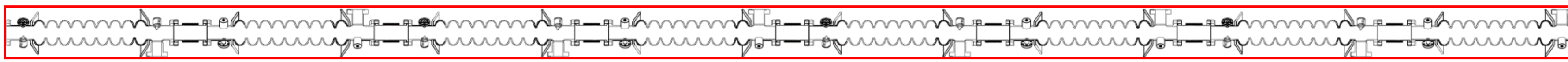
Figure of cavity chain courtesy of E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008

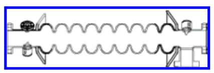
Pure Bellow Modes at 5.1524 GHz

- The seven bellows act as a cavity
- Beam could excite these fields and thus interact with following bunches



Q Factors in Single Cavity and in Chain of Cavities – Multi-Cavity Modes in the Vicinity of 5.5 GHz





monopole bands | dipole bands | quadrupole bands

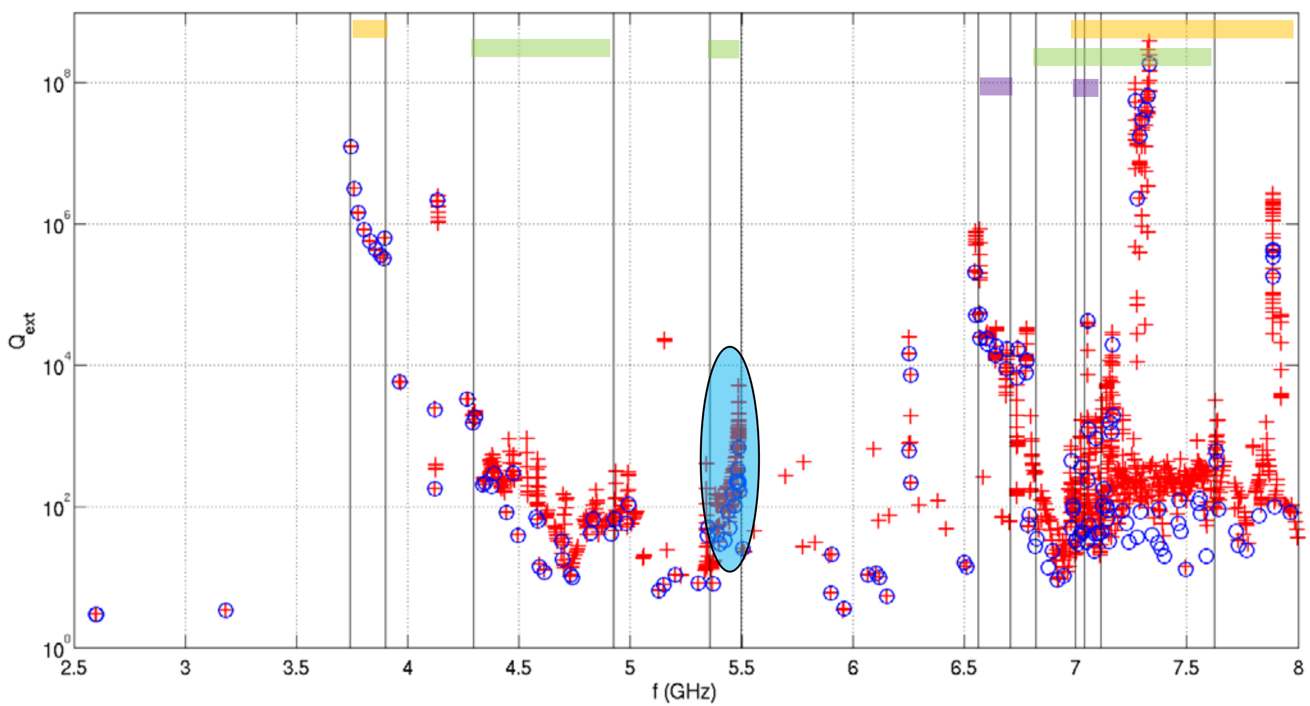
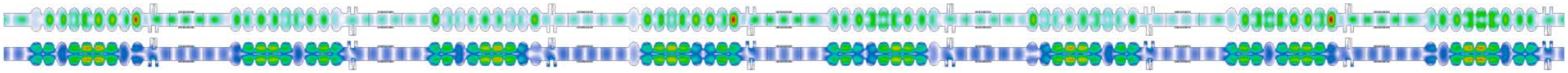
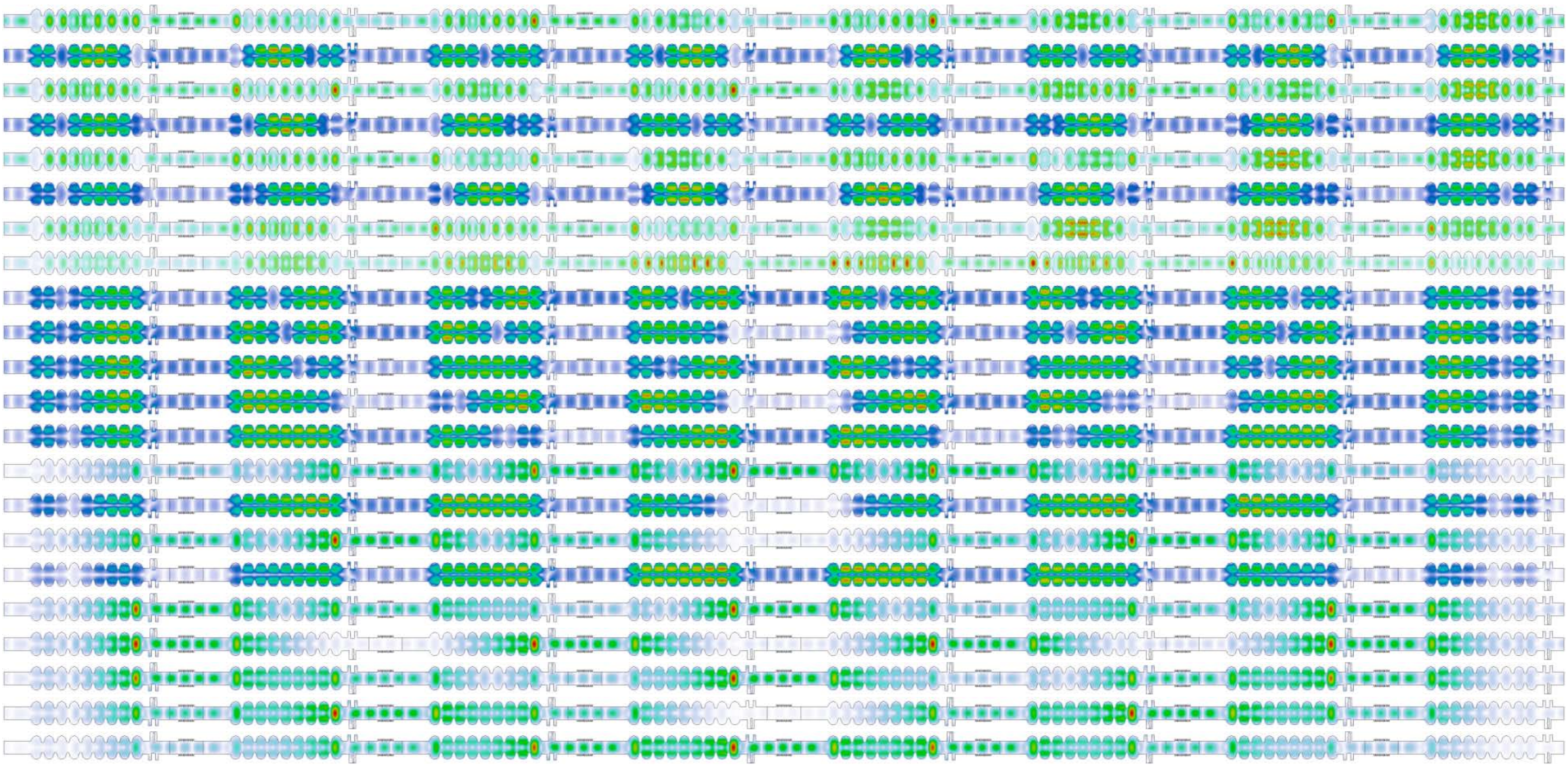
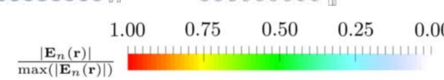


Figure of cavity chain courtesy of E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008



Multi-Cavity Modes in the Vicinity of 5.5 GHz

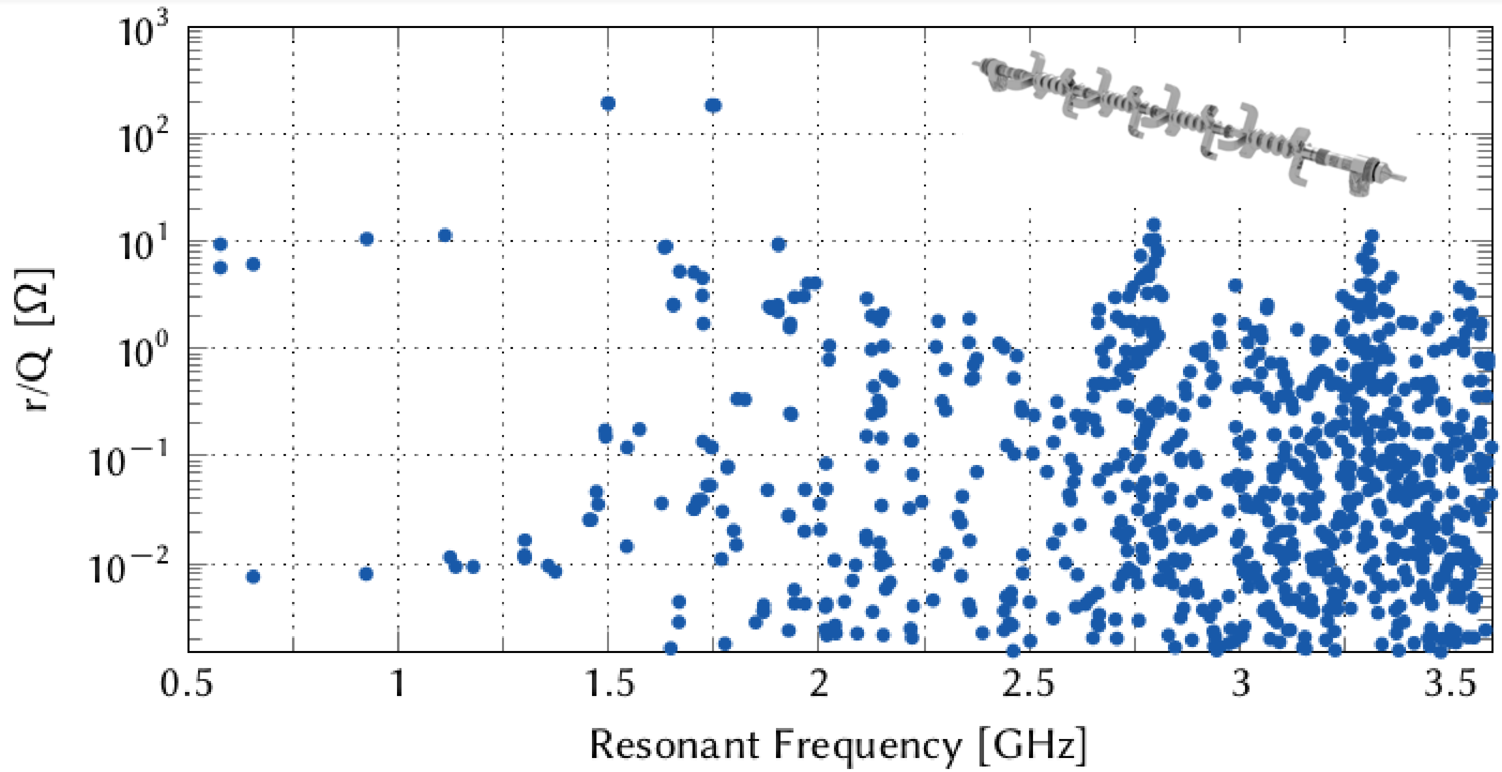




The plots show the normalized absolute value of the electric field.

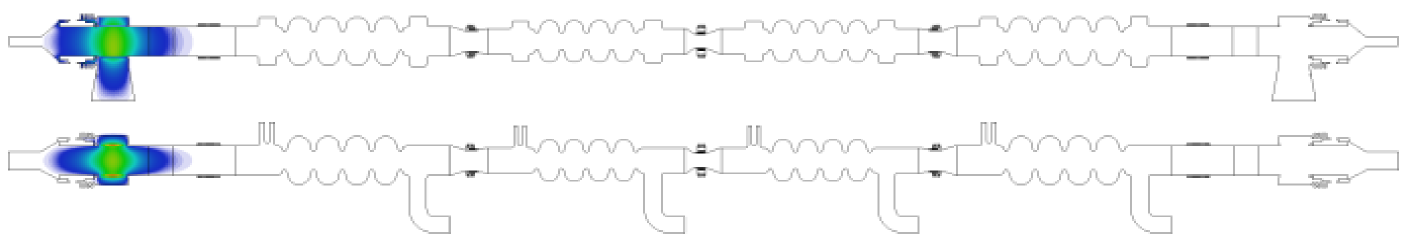
Selected Results for BESSY VSR

Comparison of multiple designs with regard to R/Q and Q_{ext} – one example:

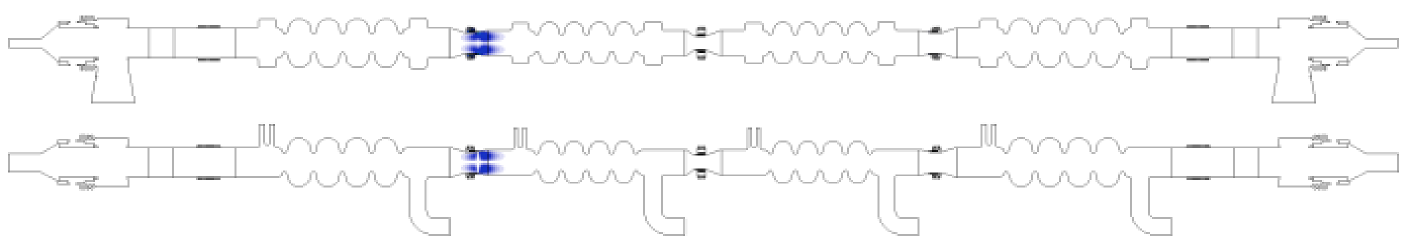


Selected Results for BESSY VSR

Many modes are trapped in the taper and show external quality factors above the intrinsic quality factors of the cavity → further investigation if relevant



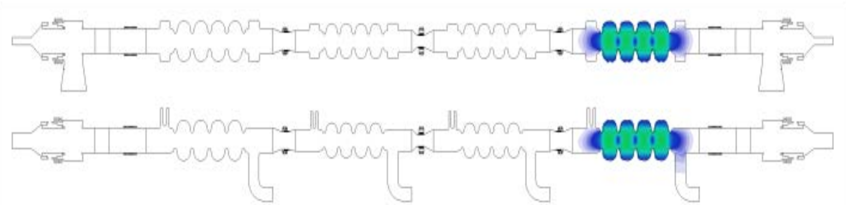
Taper-mode resonating at 1.225 GHz with Q_{ext} *of* $\approx 10^{13}$.



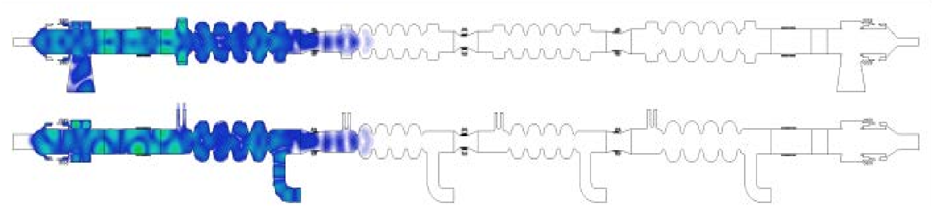
Taper-mode resonating at 2.299 GHz with Q_{ext} *of* $\approx 10^{8}$.

Selected Results for BESSY VSR

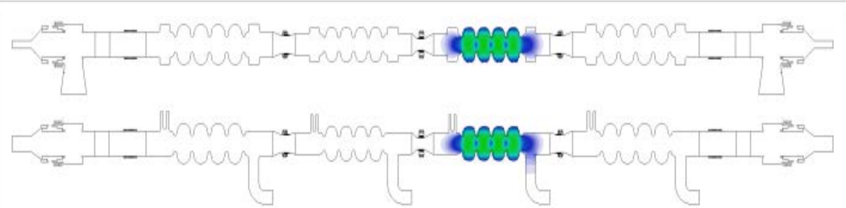
- Two π-modes at 1.5 and 1.75 GHz, each - very important that they can't couple to any other modes
- Several interesting modes that can have a potentially dangerous R/Q



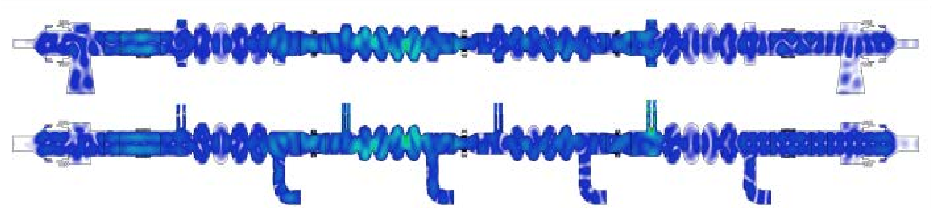
π-mode resonating at 1.5 GHz.



Mode resonating at 2.797 GHz and r/Q of 14.27 Ω.



π-mode resonating at 1.75 GHz.



Mode resonating at 3.315 GHz and r/Q of 11.18 Ω.

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TECHNISCHE UNIVERSITÄT DARMSTADT

DESY

DFG

Uncertainty Quantification Techniques and Stochastic Models for SRF Cavities

- Networking activities funded by the German Research Foundation DFG (Deutsche Forschungsgemeinschaft)
- Participating partners from DESY, TU Braunschweig, TU Darmstadt, University of Rostock
- Scientific network dealing with modelling and determination of uncertainties in the stochastic parameters and outputs of SRF resonators
- Goal: Further develop and exchange models between the network partners to describe the input uncertainties (e.g., geometry parameters) as well as methods for determining the uncertainties in the outputs (e.g., eigenmodes)
- Long-term goal: Integrate the stochastic processes into the optimization process for future resonator designs to determine robust designs

Uncertainty Quantification

Probability Sampling Methods:

- Standard approach: Monte Carlo simulations (MCS)
  - Compute model solution for N randomly chosen probability samples of uncertain model parameters
  - Mean value precision improves by $1/\sqrt{N}$ (Rule of thumb: 1 digit accuracy $\rightarrow$ 100 x more samples)
  - Requires large number N of model realizations
- Surrogate model using generalized polynomial chaos technique
  - Approximate desired quantity by polynomial expansion
  - Compute expansion coefficients by executing deterministic code
  - Perform MCS on polynomial expansion

$\Rightarrow$ Non-intrusive: Deterministic code remains unchanged

Uncertainty Quantification Techniques and Stochastic Models for SRF Cavities

- Currently, joint journal publication in preparation:

Global Sensitivity Analysis of Manufacturing on the Fundamental Mode Spectrum of the European XFEL Cavities

J. Corno,[1] N. Georg,[1,2] S. Gorgi Zadeh,[3] J. Heller,[3] V. Gubarev,[4] T. Roggen,[5]
U. Römer,[2] C. Schmidt,[3] S. Schöps,[1] A. Sulimov,[4] and U. van Rienen[3]

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[2] *Technische Universität Braunschweig, Braunschweig, Germany*
[3] *Universität Rostock, Rostock, Germany*
[4] *DESY, Hamburg, Germany*
[5] *CERN, Geneva, Switzerland*

- There's also a contribution to ICAP 2018 (end of October)

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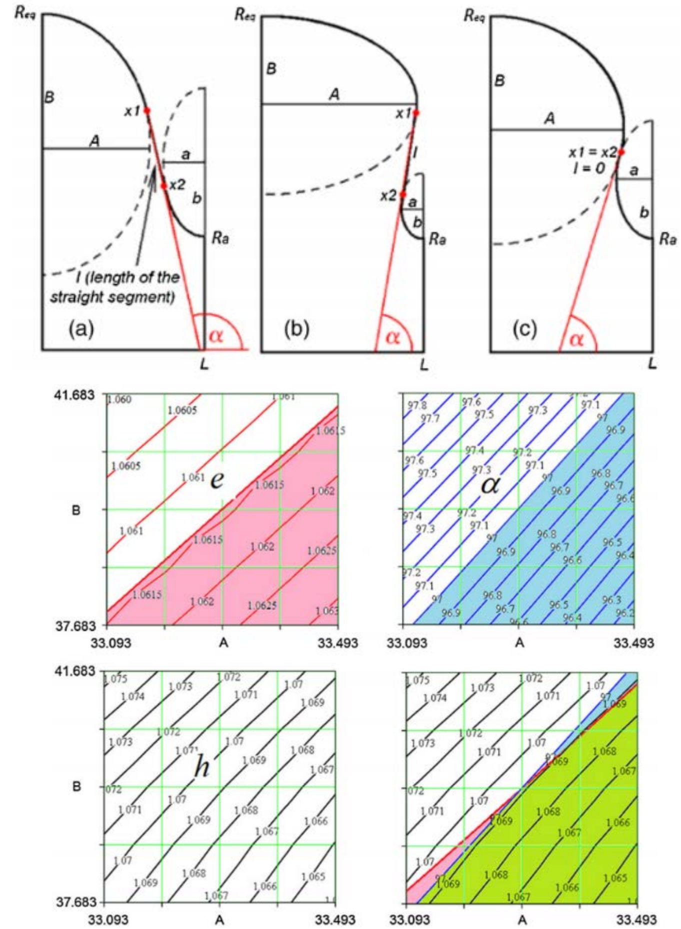
Cavity Optimization

- Optimized cavity shape could deviate from the optimal point without significantly deteriorating E_{pk}/E_{acc}, H_{pk}/E_{acc} and wall angle but changing significantly the values of A and B
- These changes give us freedom to avoid multipactor or/and to tune the most dangerous HOMs to a non-dangerous position
- Optimization technique could be expanded to take also geometrical uncertainties into account



e: normalized surface electric field
h: normalized surface magnetic field
α: wall angle
Colored region represent "forbidden" geometries and white region represents feasible geometrical shapes

V. Shemelin, S. Gorgi Zadeh, J. Heller, U. van Rienen. "Systematical study on superconducting radio frequency elliptic cavity shapes applicable to future high energy accelerators and energy recovery linacs". Physical Review Accelerators and Beams, 2016.

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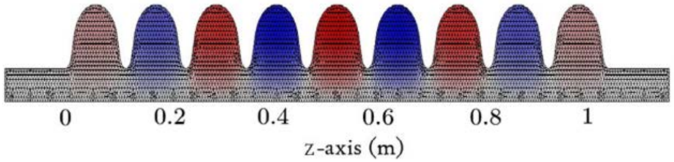
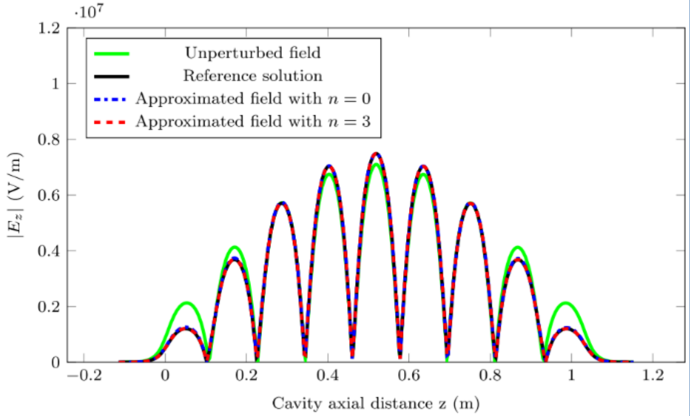
Eigenmode Computation of Cavities with Perturbed Geometry



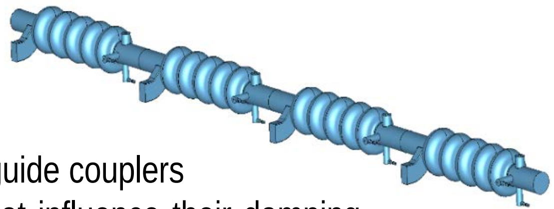
- Discretization of Helmholtz equation gives rise to generalized eigenvalue problem $\mathbf{Kx} = \lambda\mathbf{Mx}$
  - Eigenvectors ($\mathbf{x}$) contain modal field distribution
  - Eigenvalues (λ) contain the frequencies of the resonances
- Geometrical perturbations of the structure lead to new eigenvalue problems with different system matrices $\widetilde{\mathbf{K}}\tilde{\mathbf{x}} = \tilde{\lambda}\widetilde{\mathbf{M}}\tilde{\mathbf{x}}$
  - Aim: Approximate the eigenvalues and eigenvectors of a slightly modified system from the known eigenvalues and eigenvectors of the unperturbed system without solving the eigenvalue problem for the perturbed geometry i.e. to approximate $\tilde{\mathbf{x}}$ and $\tilde{\lambda}$ from $\mathbf{K}$, $\mathbf{x}$, λ, $\mathbf{M}$, $\mathbf{x}$, $\widetilde{\mathbf{K}}$, $\widetilde{\mathbf{M}}$



Approximation of the electric field profile on z-axis when the geometrical parameter A of the end-cells are perturbed by 2mm

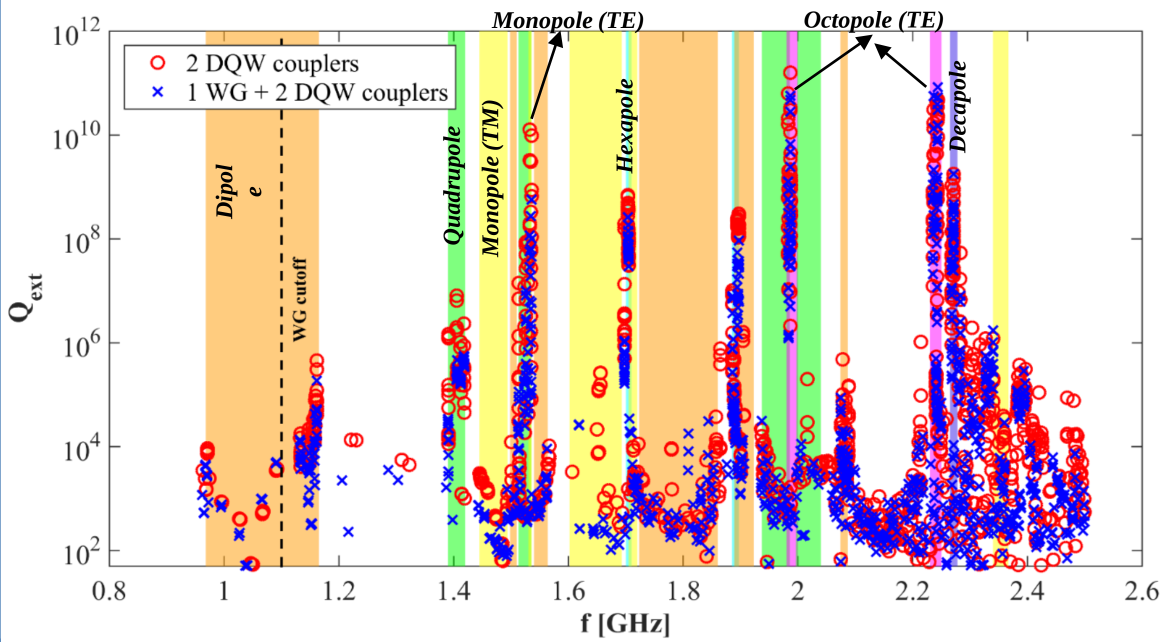
S. Gorgi Zadeh, T. Flisgen, U. van Rienen. “Eigenmode computation of cavities with perturbed geometry using matrix perturbation methods applied on generalized eigenvalue problems”. Journal of Computational Physics, 2018.

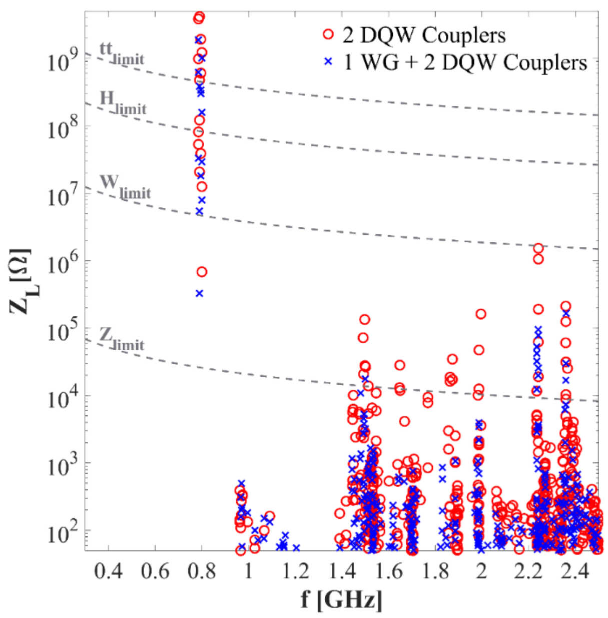
Future Circular Collider (FCC) studies



see Shahnam's presentation this morning

- Damping of monopole, dipole and quadrupole bands is improved by adding waveguide couplers
- Octopole and decapole modes are trapped, thus adding another coupler does not influence their damping - these modes however are not excited by on-axis beams





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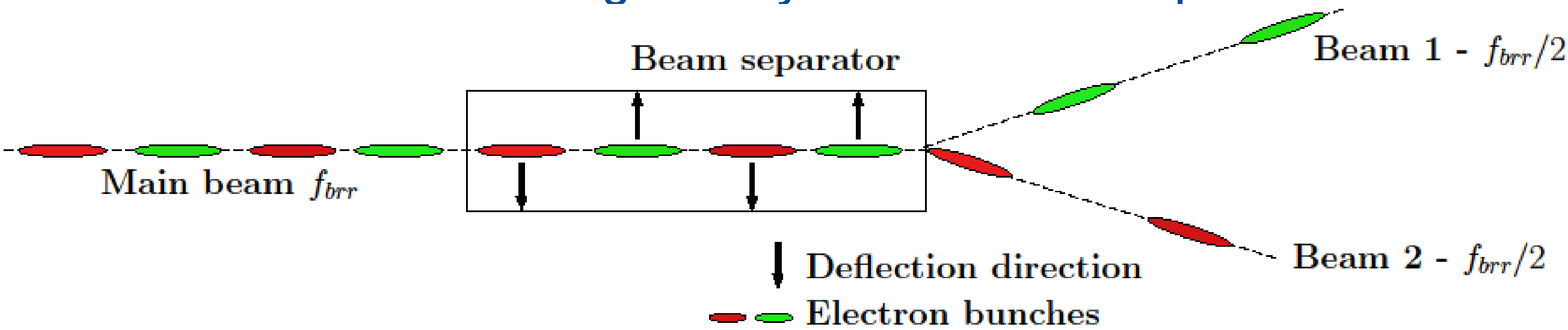
HZDR HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

Transverse deflecting cavity as a beam separator



- Currently, the ELBE e^- linac at HZDR has single beam line → single beam user at any given time
- Different users have distinct beam settings
- Beam capacity is under-utilized
- Using beam separator → multiple beam lines → multiple users → maximal utilization of the beam

Beam and cavity parameters		
Beam Energy	100	MeV
Bunch repetition rate	1	MHz
Beam size (σ_x/σ_y)	2	mm
Deflection angle	3	mrad
RF frequency	260.5	MHz
Transverse kick voltage	300	kV

G Hallilingaiah, A Arnold, U Lehnert, P Michel, U. van Rienen. "Numerical Studies of Normal Conducting Deflecting Cavity Designs for the ELBE Accelerator", 9th Int. Particle Accelerator Conf.(IPAC'18), Vancouver, THPAL074, 2018

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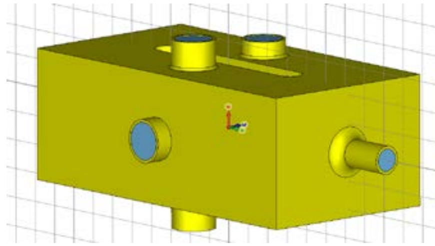

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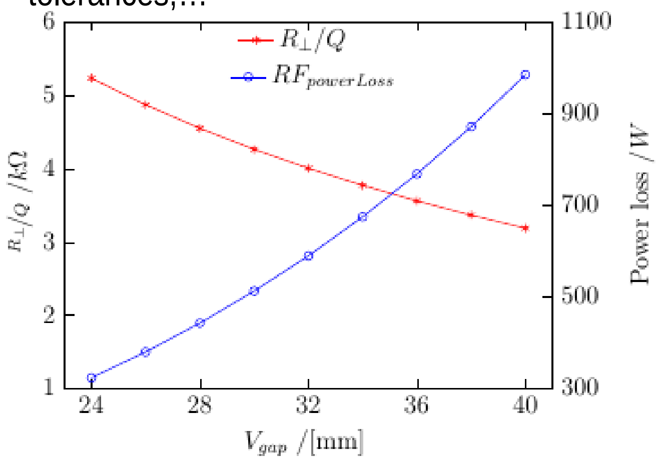
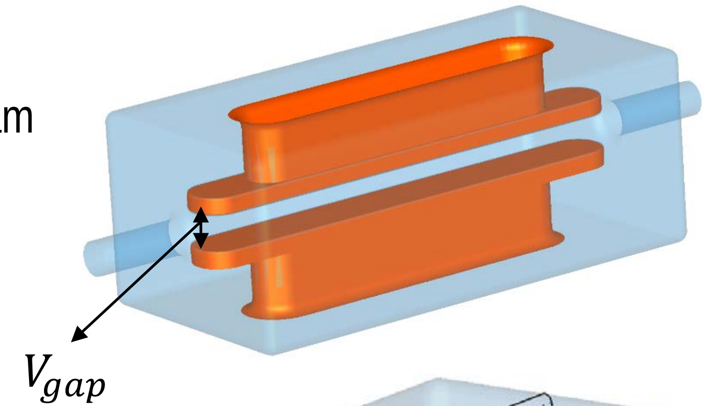
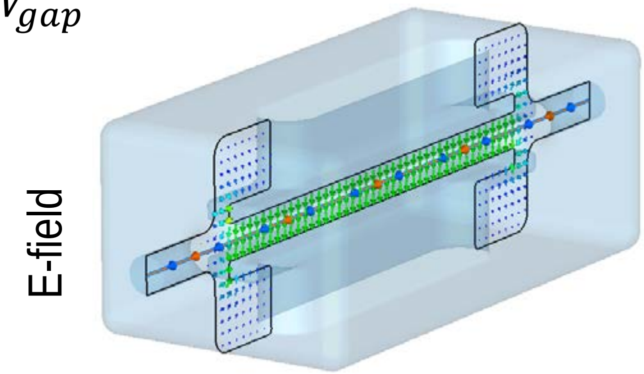
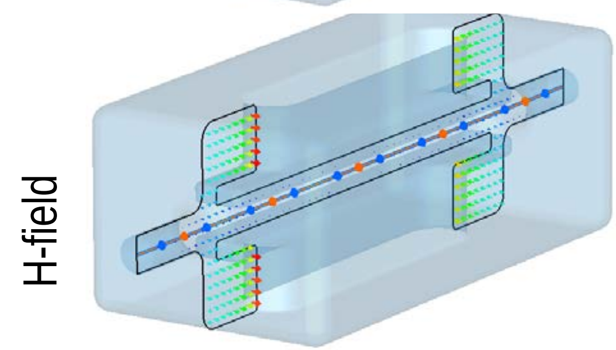
Transverse deflecting cavity

- Normal conducting cavity design was proposed as a beam separator for ELBE[*]
- Net deflection is along the E-field (vertical)
- RF power loss of $\approx 1\text{kW}$ for V_{gap} of 40 mm
- Vertical gap ↓ → E-field ↑→ RF power requirement ↓
- What is the optimum vertical gap ?
  - Multipacting, beam dynamics, **Wakefield analysis**, manufacturing tolerances,…









[*] G Hallilingaiah et al. “Numerical Studies of Normal Conducting Deflecting Cavity Designs for the ELBE Accelerator”, IPAC'18, Vancouver, THPAL074, 2018

Resonant HOM excitation

- Multiple bunches pass through the cavity → resonant HOM excitation may occur
- For resonant case, power loss due to longitudinal HOM[*] is

$$P_{max} = \frac{R_{||}/Q\omega_0 q^2}{4t_b}\left(\frac{e^{t_b/\tau}+1}{e^{t_b/\tau}-1}\right)$$

t_b is the bunch spacing, $\tau = 2Q/\omega_0$, q is the bunch charge, ω_0 is the eigenmode frequency

- Similarly, steady state resonant amplitude of HOM transverse voltage for off-axis beam is

$$|V_{\perp}|_{max,r=a} = \frac{R_{\perp}/Q\omega_0 qak}{4}\left(\frac{e^{t_b/\tau}+1}{e^{t_b/\tau}-1}\right)$$

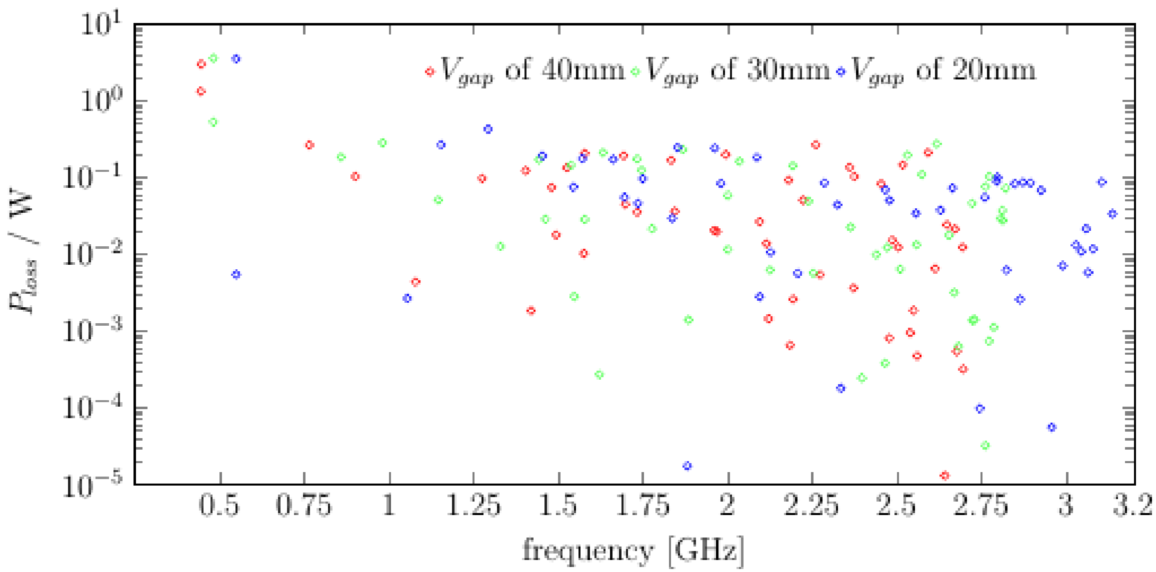
where a is the offset distance (5mm), k is the wavenumber

- q of 1nC and $f_{brr} = 1/t_b = 1\ \mathrm{MHz}$
- P_{max} and $|V_{\perp}|_{max}$ are calculated for different vertical gap width

[*] A. Lunin, T. Khabiboulline, N. Solyak, A. Sukhanov, and V. Yakovlev, “Resonant excitation of high order modes in the 3.9 GHz cavity of the Linac Coherent Light Source,” Phys. Rev. Accel. Beams, vol. 21, no. 2, p. 22001, Feb. 2018.

Resonant HOM excitation

Average power loss due to resonant excitation

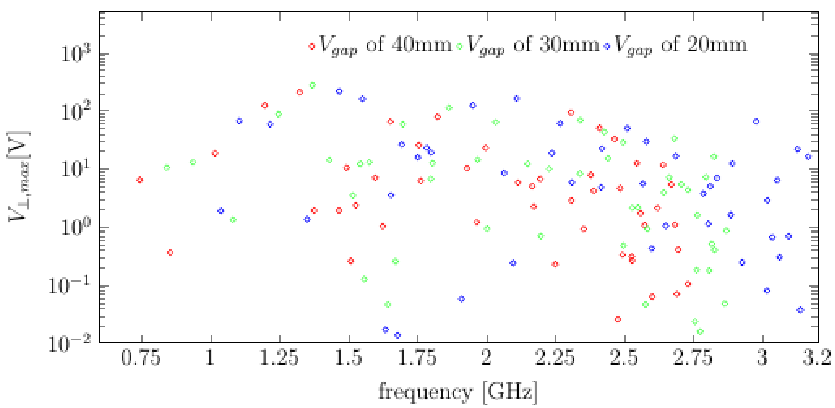


Vertical gap	$\sum P_{HOM}$
40 mm	5.17 W
30 mm	7.19 W
20 mm	8.24 W

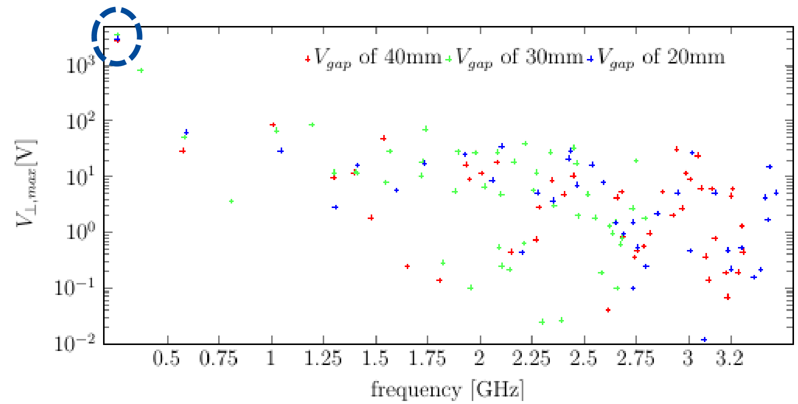
- Total RF power loss for 3 mrad kick is $\approx 1\ \mathrm{kW}$
- Average power loss due to resonant excitation is much less compared to RF power loss
- No significant load on the cooling
- Resonant excitation power loss is significant for higher beam current

Resonant HOM excitation

$V_{\perp,max}$ due to horizontal HOM



$V_{\perp,max}$ due to vertical HOM



- $V_{\perp,max}$ are calculated for beam offset of 5 mm
- Horizontal kick voltage is not significant compared to vertical kick voltage
- For the RF kick voltage of 300 kV, kick voltage from beam is less than 1.5%
- Resonant $V_{\perp}$ due to transverse HOM is less significant

Vertical gap	$\sum V_{Hor}$	$\sum V_{ver}$	$V_{ver,260.5MHz}$
40 mm	838 V	3092 V	2669 V
30 mm	967V	3500 V	3127 V
20 mm	1242 V	4939 V	3489 V

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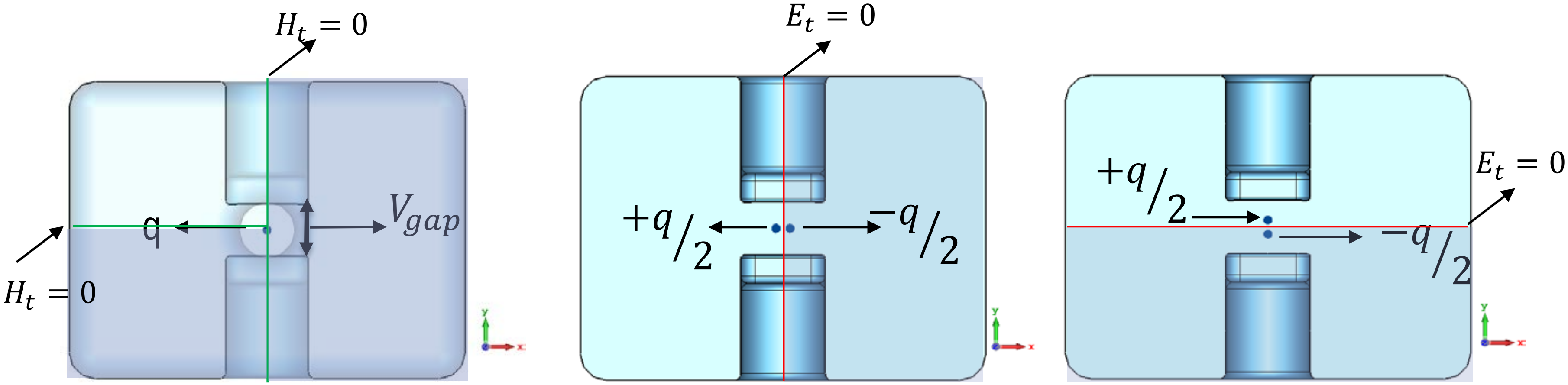
Wakefield analysis



Monopole modes Z_{long} | Dipole modes (Horizontal) Z_{horz} | Dipole modes (Vertical) Z_{vert}

- Impedance spectra for different vertical gap size of a bare cavity are calculated
- Rotationally asymmetric RF structure – symmetry plane is considered [*]
- Gaussian bunch of $\sigma_z = 2$ cm, q = 1 nC, and horizontal offset = vertical offset = 5 mm

[*] F. Marhauser, R. Rimmer, and H. Wang, "Narrowband and Broadband Impedance Budget of the 1497 MHz HOM-Damped Five-Cell High Current Cavity," JLAB-TN-08-002, 2008.

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Wakefield analysis

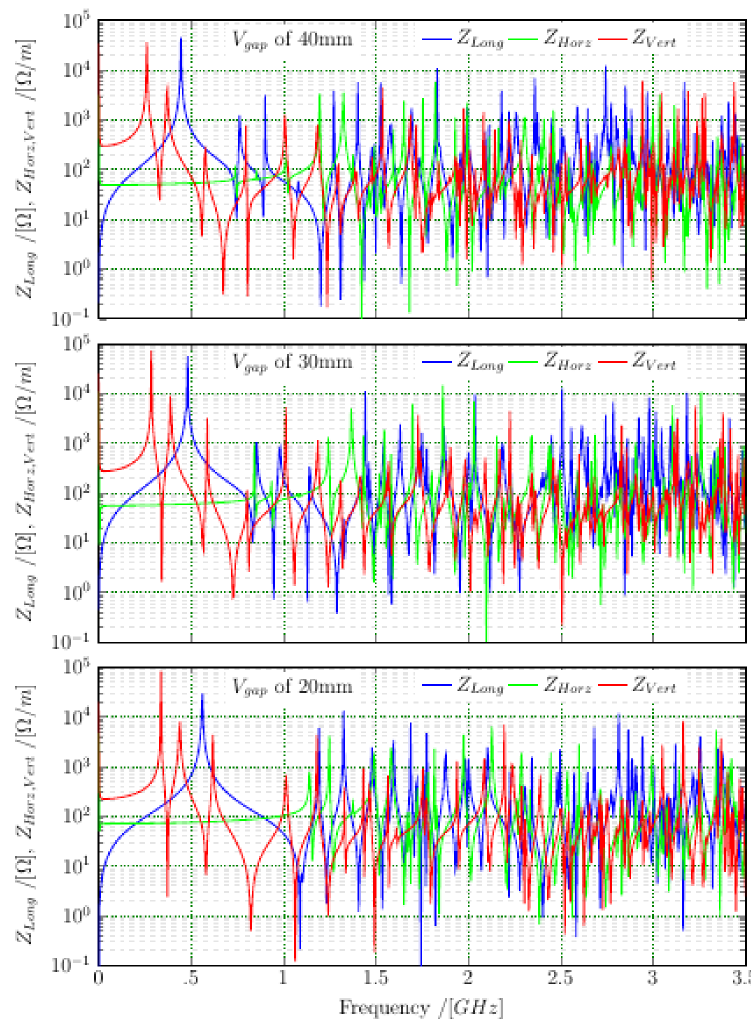
- Gaussian bunch of $\sigma_z = 2\ cm$, q=1 nC, and beam offset ($du_{vert,horz}$= 5 mm)
- Longitudinal and transverse impedance are given as,

$$Z_{long}(\omega) = -\int_{-\infty}^{\infty} W_{||}(s)e^{-(j\omega s)}\,ds$$

$$Z_{horz,vert}(\omega) = \frac{j}{du_{horz,vert}}\int_{-\infty}^{\infty} W_{\perp}(s)e^{-(j\omega s)}\,ds$$

where $W_{||}(s)$ and $W_{\perp}(s)$ are the longitudinal and transverse wake functions

- Smaller vertical gap
  - longitudinal modes are pushed further away from the deflecting mode
  - easy to extract HOM
  - optimum: 30 mm → 600 W power requirement



Wakefield analysis

- Longitudinal loss and transverse kick factor are given by,

$$k_{long} = -\int_{-\infty}^{\infty} W_{||}(s)\lambda(s)\, ds$$

$$k_{horz,vert} = \frac{1}{du}\int_{-\infty}^{\infty} W_{\perp}(s)\lambda(s)\, ds$$

where $\lambda(s)$ is the normalized charge density and du is the beam offset

	Loss factor	Kick factor	
Vertical gap	k_{long} $[v/nC]$	k_{horz} $[v/nCm]$	k_{vert} $[v/nCm]$
40 mm	0.365	18.174	23.321
30 mm	0.385	21.113	23.521
20 mm	0.438	27.406	23.981

- Smaller vertical gap
  - Marginal increase in longitudinal loss factor and vertical kick factor
  - Horizontal kick factor increase by 50% → tail of the bunch gets a stronger horizontal transverse kick from the head of the bunch
- Vertical gap of 30 mm is reasonable to avoid kick from the offset beam

  (Further investigation is required with respect to multipacting, beam dynamics, fabrication tolerance,…)

Conclusion

- Domain decomposition methods combined with model order reduction methods as implemented in SSC allow to compute modal atlases for long, complex SRF chains
- These may also be combined with efficient 2D optimization approaches
- This can then be combined with non-intrusive methods for uncertainty quantification
- In preparation: Inclusion of Surface Losses via Perturbation Approaches into SSC

- Further specific studies:
  - Design studies for FCC cavities and HOM damping
  - Transverse deflecting cavity as beam separator

Acknowledgements

We gratefully acknowledge

- Contributions by Tomasz Galek
- Cooperation with our partners from CERN, DESY, HZB, HZDR, TU Darmstadt, TU Dortmund
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  - DFG under contract SCHM 3127/2-1
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  - HZDR

Federal Ministry of Education and Research

DFG

EuCARD²

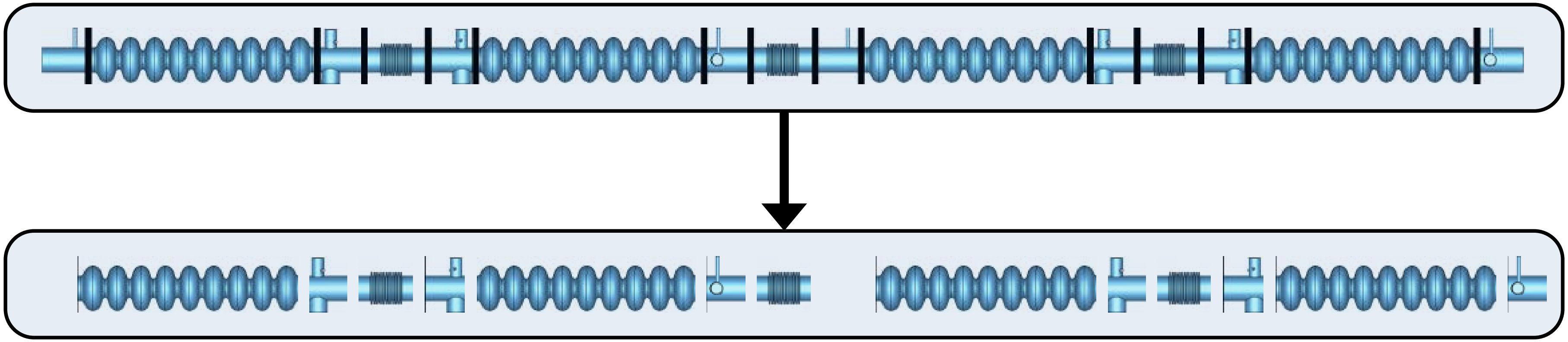
CAPACITIES

CERN

HZDR HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

State-Space Concatenations (SSC)

1. Decomposition of the Structure at Regions of Constant Cross Section

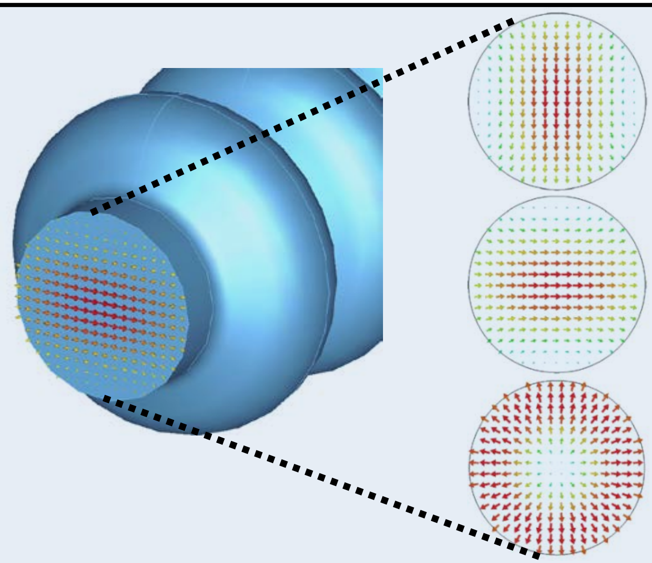


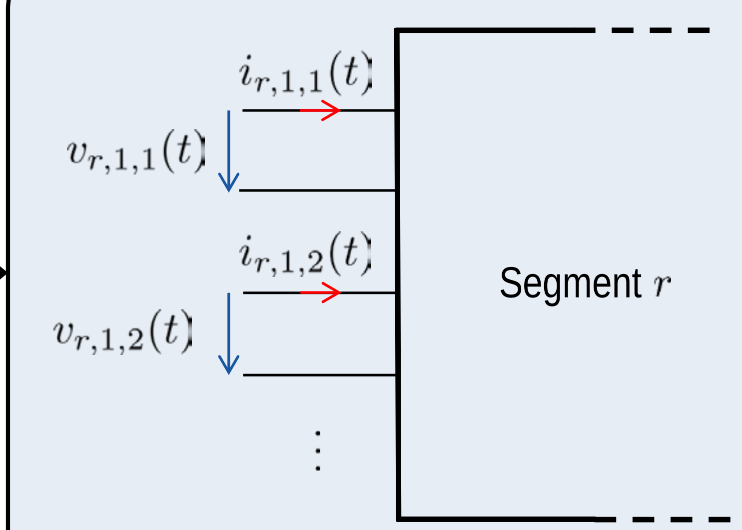
Important properties:

- (numerical) treatment of segments is computationally less demanding
- single treatment of identical segments
- segments with simple geometry can be treated semi-analytically, which is very fast
- employment of symmetry of segments is feasible

State-Space Concatenations (SSC)

2. Consideration of Segments as Blocks with Terminals

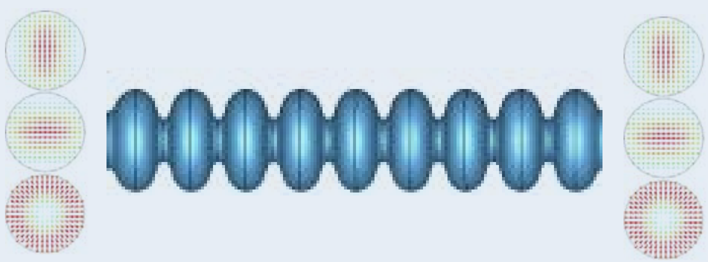

E-field of first three 2D port modes



- Modal voltages $v_{r,p,m}(t)$ correspond to tangential electric fields of 2D port modes
- Modal currents $i_{r,p,m}(t)$ correspond to tangential magnetic fields of 2D port modes

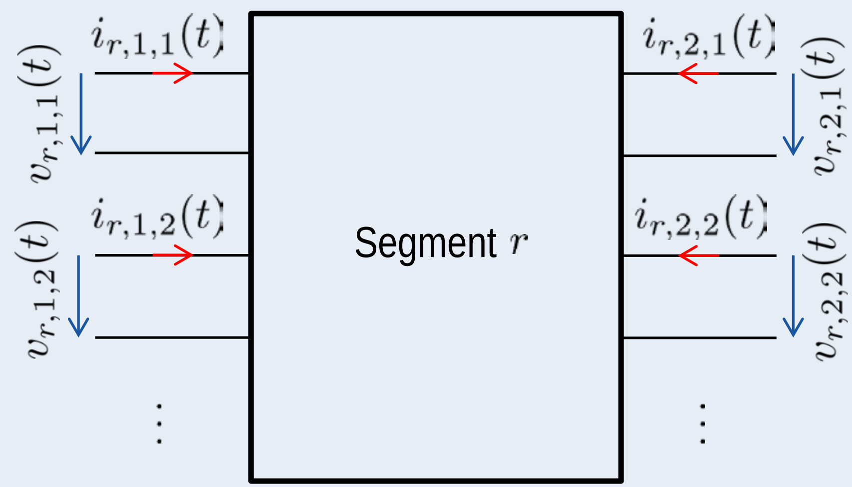
State-Space Concatenations (SSC)

3. Generation of Second-Order State-Space Equations for Segments



wave equation (PDE):

$$\Delta\mathbf{E}(\mathbf{r},t) = \varepsilon\mu\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) + \mu\frac{\partial}{\partial t}\mathbf{J}(\mathbf{r},t)$$



state-space system (ODE):

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_r(t) = \mathbf{A}_r\mathbf{x}_r(t) + \mathbf{B}_r\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_r(t)$$

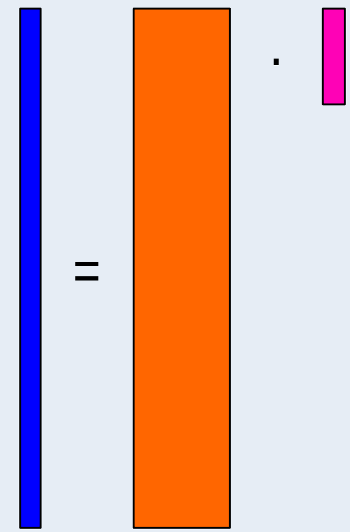
$$\mathbf{v}_r(t) = \mathbf{B}_r^{\mathrm{T}}\mathbf{x}_r(t)$$

State-Space Concatenations (SSC)

4. Model-Order Reduction for State-Space Systems

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_r(t) = \mathbf{A}_r\mathbf{x}_r(t) + \mathbf{B}_r\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_r(t)$$

$$\mathbf{v}_r(t) = \mathbf{B}_r^{\mathrm{T}}\mathbf{x}_r(t)$$



$$\mathbf{x}_r(t) = \mathbf{W}_r \cdot \mathbf{x}_{\mathrm{rd},r}(t)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_{\mathrm{rd},r}(t) = \underbrace{\mathbf{W}_r^{\mathrm{T}}\mathbf{A}_r\mathbf{W}_r}_{\mathbf{A}_{\mathrm{rd},r}}\mathbf{x}_{\mathrm{rd},r}(t) + \underbrace{\mathbf{W}_r^{\mathrm{T}}\mathbf{B}_r}_{\mathbf{B}_{\mathrm{rd},r}}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_r(t)$$

$$\mathbf{v}_r(t) = \underbrace{\mathbf{B}_r^{\mathrm{T}}\mathbf{W}_r}_{\mathbf{B}_{\mathrm{rd},r}^{\mathrm{T}}}\mathbf{x}_{\mathrm{rd},r}(t)$$

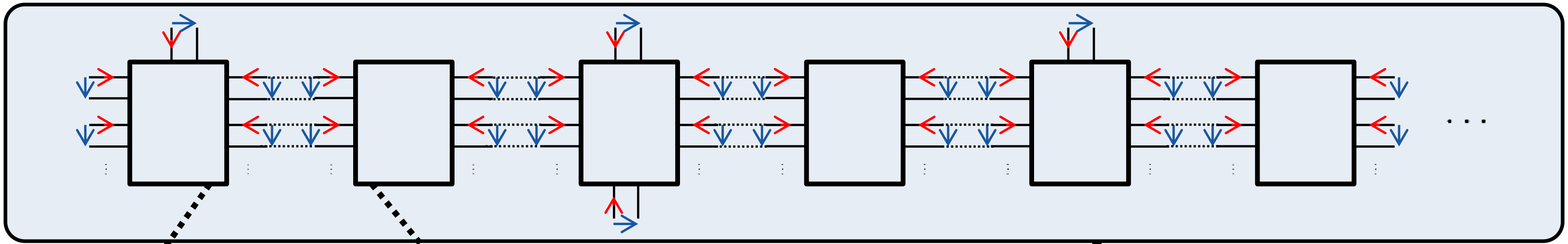
state vector: $\mathbf{x}_r(t) \in \mathbb{R}^{N_\mathrm{s}},\ N_\mathrm{s} \approx 10^5$

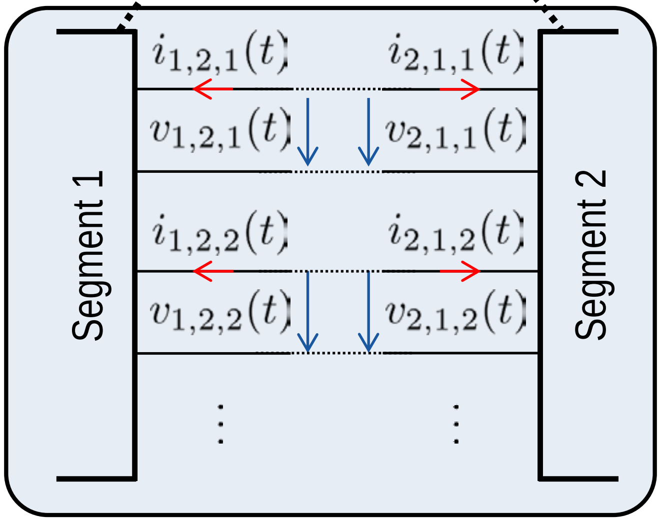
orthogonal projection matrix: $\mathbf{W}_r \in \mathbb{R}^{N_\mathrm{s} \times N_\mathrm{srd}},\ \mathbf{W}_r^{\mathrm{T}}\mathbf{W}_r = \mathbf{I}$

reduced state vector: $\mathbf{x}_{\mathrm{rd},r}(t) \in \mathbb{R}^{N_\mathrm{srd}},\ N_\mathrm{srd} < 10^2$

State-Space Concatenations (SSC)

5. Concatenation of Reduced-Order State-Space System



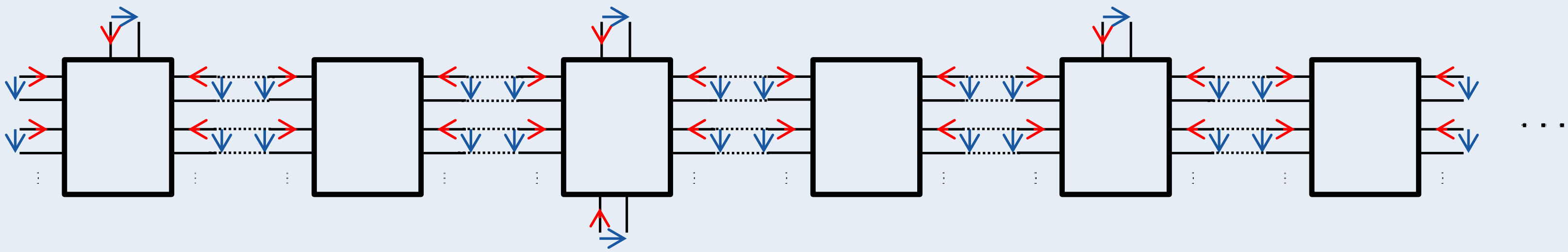


$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_{\mathrm{rdc}}(t) = \mathbf{A}_{\mathrm{rdc}}\mathbf{x}_{\mathrm{rdc}}(t) + \mathbf{B}_{\mathrm{rdc}}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}(t)$$
$$\mathbf{v}(t) = \mathbf{B}_{\mathrm{rdc}}^{\mathrm{T}}\mathbf{x}_{\mathrm{rdc}}(t)$$

- Coupling constraints according to Kirchhoff's laws
- Arbitrary topologies and number of 2D port modes supported

State-Space Concatenations (SSC)

6. Computation of RF Properties by Means of the Reduced-Order Model

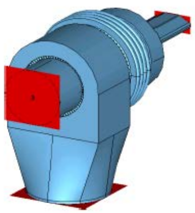


$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{x}_{\mathrm{rdc}}(t) = \mathbf{A}_{\mathrm{rdc}}\mathbf{x}_{\mathrm{rdc}}(t) + \mathbf{B}_{\mathrm{rdc}}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}(t)$$

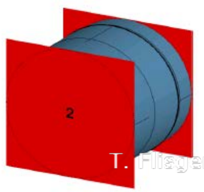
$$\mathbf{v}(t) = \mathbf{B}_{\mathrm{rdc}}^{\mathrm{T}}\mathbf{x}_{\mathrm{rdc}}(t)$$

- impedance- and scattering parameters
- 3D field distribution due to port excitations
- frequencies and field distributions of eigenmodes
- external quality factors

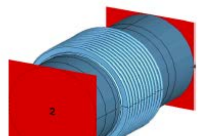
Decomposition of BESSY VSR Chain into Segments



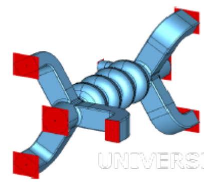
(1) Warm Taper Up



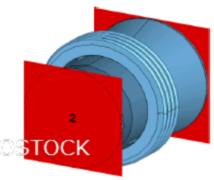
(2) Valve Up



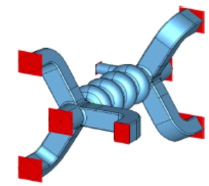
(3) Bellow 1



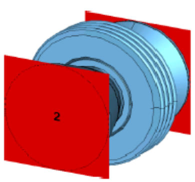
(4) Cavity 1 with $f_\pi = 1.5\,\mathrm{GHz}$



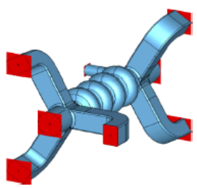
(5) Bellow 2



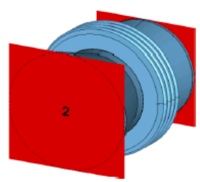
(6) Cavity 2 with $f_\pi = 1.75\,\mathrm{GHz}$



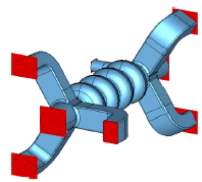
(7) Bellow 3



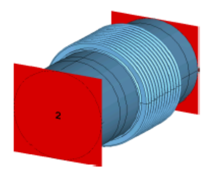
(8) Cavity 3 with $f_\pi = 1.75\,\mathrm{GHz}$



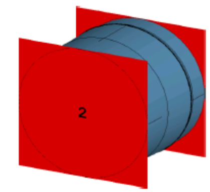
(9) Bellow 4



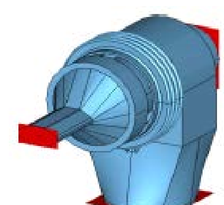
(10) Cavity 4 with $f_\pi = 1.5\,\mathrm{GHz}$



(11) Bellow 5



(12) Valve Down



(13) Warm Taper Down

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Modal Atlas for BESSY VSR Cold String