Higher Order Mode and Cavity Studies at the University of Rostock since HOMSC16

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• meanwhile at other places

ICFA Mini Workshop on Higher Order Modes in Superconducting Cavities (HOMSC2018)
Cornell, Ithaca, USA
Overview

- Motivation for Domain Decomposition Approach
- State-Space Concatenations (SSC)
  - Application to FLASH
  - Application for BESSY VSR
  - Computation of External Quality Factors
- Uncertainty Quantification Techniques and Stochastic Models for SRF Cavities
- Future Circular Collider (FCC) studies – see Shahnam’s presentation this morning
- Deflecting Cavity for Beam Separation at ELBE
Domain Decomposition Methods for Large, Complex SRF Accelerators

- SRF cavities are often too complex to solve Maxwell’s equations using standard numerical methods on standard workstations (i.e. computation takes impractically long)
- Usage of domain decomposition methods
  - Decompose the domain at regions of constant cross-section into several subdomains /segments
  - Solve Maxwell’s equations for each subdomain and couple the results
- Allows for computationally efficient investigation of multi-cavity modes
State-Space Concatenations (SSC)

- Consideration of segments as blocks with terminals
  - Modal voltages $v_{r,p,m}(t)$ correspond to tangential electric fields of 2D port modes
  - Modal currents $i_{r,p,m}(t)$ correspond to tangential magnetic fields of 2D port modes

- Generation of second-order state-space equations for segments
  - Wave equation (PDE) $\rightarrow$ state-space system (ODE)

- Model-order reduction for state-space system via orthogonal projection
  - Original state vector of dimension $\sim 10^6 \rightarrow$ reduced state vector of dimension $< 10^2$

- Concatenation of reduced-order state-space system
  - Coupling constraints according to Kirchhoff's laws
  - Arbitrary topologies and number of 2D port modes supported

- Computation of RF properties by means of the reduced-order model
  - Impedance- and scattering parameters; frequencies and field distributions of eigenmodes; 3D field distribution due to port excitations; external quality factors

Compendium of Eigenmodes for FLASH 3.9 GHz Module

Compendium of Eigenmodes for FLASH 3.9 GHz Module

- Computed 1,479 eigenmodes in the interval 1 GHz to 8 GHz of the chain by means of a standard workstation computer
- Compendium contains resonant frequencies, external quality factors, beam coupling impedances, and electric field distributions
- In total, the compendium consists of 137 pages
- $T_{\text{comp}} \approx 8$ h on an Intel Xeon E5-2687W v2 @ 3.40 GHz plus several hours for exporting matrices and plotting the field distributions

Q Factors in Single Cavity and in Chain of Cavities -
Pure Bellow Modes at 5.1524 GHz

Figure of cavity chain courtesy of E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008
Pure Bellow Modes at 5.1524 GHz

- The seven bellows act as a cavity
- Beam could excite these fields and thus interact with following bunches
Q Factors in Single Cavity and in Chain of Cavities – Multi-Cavity Modes in the Vicinity of 5.5 GHz

Figure of cavity chain courtesy of E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008
Multi-Cavity Modes in the Vicinity of 5.5 GHz

The plots show the normalized absolute value of the electric field.
Selected Results for BESSY VSR

Comparison of multiple designs with regard to $R/Q$ and $Q_{ext}$ – one example:
Selected Results for BESSY VSR

Many modes are trapped in the taper and show external quality factors above the intrinsic quality factors of the cavity \( \rightarrow \) further investigation if relevant

Taper-mode resonating at 1.225 GHz with \( Q_{\text{ext}} \) of \( \approx 10^{13} \).

Taper-mode resonating at 2.299 GHz with \( Q_{\text{ext}} \) of \( \approx 10^{8} \).
Selected Results for BESSY VSR

- Two $\pi$-modes at 1.5 and 1.75 GHz, each - very important that they can’t couple to any other modes
- Several interesting modes that can have a potentially dangerous R/Q

$\pi$-mode resonating at 1.5 GHz.

Mode resonating at 2.797 GHz and r/Q of 14.27 $\Omega$.

$\pi$-mode resonating at 1.75 GHz.

Mode resonating at 3.315 GHz and r/Q of 11.18 $\Omega$. 
Uncertainty Quantification Techniques and Stochastic Models for SRF Cavities

• Networking activities funded by the German Research Foundation DFG (Deutsche Forschungsgemeinschaft)
• Participating partners from DESY, TU Braunschweig, TU Darmstadt, University of Rostock
• Scientific network dealing with modelling and determination of uncertainties in the stochastic parameters and outputs of SRF resonators
• Goal: Further develop and exchange models between the network partners to describe the input uncertainties (e.g., geometry parameters) as well as methods for determining the uncertainties in the outputs (e.g., eigenmodes)
• Long-term goal: Integrate the stochastic processes into the optimization process for future resonator designs to determine robust designs
Uncertainty Quantification

Probability Sampling Methods:

- Standard approach: Monte Carlo simulations (MCS)
  - Compute model solution for $N$ randomly chosen probability samples of uncertain model parameters
  - Mean value precision improves by $1/\sqrt{N}$ (Rule of thumb: 1 digit accuracy $\rightarrow$ 100 x more samples)
  - Requires large number $N$ of model realizations
- Surrogate model using generalized polynomial chaos technique
  - Approximate desired quantity by polynomial expansion
  - Compute expansion coefficients by executing deterministic code
  - Perform MCS on polynomial expansion
  $\Rightarrow$ Non-intrusive: Deterministic code remains unchanged
Uncertainty Quantification Techniques and Stochastic Models for SRF Cavities

• Currently, joint journal publication in preparation:

Global Sensitivity Analysis of Manufacturing on the Fundamental Mode Spectrum of the European XFEL Cavities

J. Corno,1 N. Georg,1,2 S. Gorgi Zadeh,3 J. Heller,3 V. Gubarev,4 T. Roggen,5 U. Römer,2 C. Schmidt,3 S. Schöps,1 A. Sulimov,4 and U. van Rienen3

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3Universität Rostock, Rostock, Germany
4DESY, Hamburg, Germany
5CERN, Geneva, Switzerland

• There’s also a contribution to ICAP 2018 (end of October)
Cavity Optimization

- Optimized cavity shape could deviate from the optimal point without significantly deteriorating $E_{pk}/E_{acc}$, $H_{pk}/E_{acc}$ and wall angle but changing significantly the values of $A$ and $B$
- These changes give us freedom to avoid multipactor or/and to tune the most dangerous HOMs to a non-dangerous position
- Optimization technique could be expanded to take also geometrical uncertainties into account

Eigenmode Computation of Cavities with Perturbed Geometry

- Discretization of Helmholtz equation gives rise to generalized eigenvalue problem $Kx = \lambda Mx$
  - Eigenvectors ($x$) contain modal field distribution
  - Eigenvalues ($\lambda$) contain the frequencies of the resonances
- Geometrical perturbations of the structure lead to new eigenvalue problems with different system matrices $\tilde{K}\tilde{x} = \tilde{\lambda}\tilde{M}\tilde{x}$
  - Aim: Approximate the eigenvalues and eigenvectors of a slightly modified system from the known eigenvalues and eigenvectors of the unperturbed system without solving the eigenvalue problem for the perturbed geometry i.e. to approximate $\tilde{x}$ and $\tilde{\lambda}$ from $K$, $x$, $\lambda$, $M$, $x$, $\tilde{K}$, $\tilde{M}$

Future Circular Collider (FCC) studies

see Shahnam’s presentation this morning

- Damping of monopole, dipole and quadrupole bands is improved by adding waveguide couplers
- Octopole and decapole modes are trapped, thus adding another coupler does not influence their damping - these modes however are not excited by on-axis beams
Current, the ELBE $e^-$ linac at HZDR has single beam line $\rightarrow$ single beam user at any given time

Different users have distinct beam settings

Beam capacity is under-utilized

Using beam separator $\rightarrow$ multiple beam lines $\rightarrow$ multiple users $\rightarrow$ maximal utilization of the beam

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### Beam and cavity parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Energy</td>
<td>100 MeV</td>
</tr>
<tr>
<td>Bunch repetition rate</td>
<td>1 MHz</td>
</tr>
<tr>
<td>Beam size $(\sigma_x/\sigma_y)$</td>
<td>2 mm</td>
</tr>
<tr>
<td>Deflection angle</td>
<td>3 mrad</td>
</tr>
<tr>
<td>RF frequency</td>
<td>260.5 MHz</td>
</tr>
<tr>
<td>Transverse kick voltage</td>
<td>300 kV</td>
</tr>
</tbody>
</table>
Transverse deflecting cavity

- Normal conducting cavity design was proposed as a beam separator for ELBE[*]
- Net deflection is along the E-field (vertical)
- RF power loss of $\approx 1$ kW for $V_{\text{gap}}$ of 40 mm
- Vertical gap $\downarrow \rightarrow$ E-field $\uparrow \rightarrow$ RF power requirement $\downarrow$
- What is the optimum vertical gap?
  - Multipacting, beam dynamics, Wakefield analysis, manufacturing tolerances,…

[*] G Hallilingaiah et al. “Numerical Studies of Normal Conducting Deflecting Cavity Designs for the ELBE Accelerator”, IPAC’18, Vancouver, THPAL074, 2018
Resonant HOM excitation

- Multiple bunches pass through the cavity → resonant HOM excitation may occur
- For resonant case, power loss due to longitudinal HOM\(^*\) is
  \[
  P_{\text{max}} = \frac{R_{||}/Q\omega_0 q^2}{4t_b} \left(\frac{e^{tb/\tau+1}}{e^{tb/\tau-1}}\right)
  \]
  \(t_b\) is the bunch spacing, \(\tau = 2Q/\omega_0\), \(q\) is the bunch charge, \(\omega_0\) is the eigenmode frequency
- Similarly, steady state resonant amplitude of HOM transverse voltage for off-axis beam is
  \[
  |V_\perp|_{\text{max},r=a} = \frac{R_{\perp}/Q\omega_0 q a k}{4} \left(\frac{e^{tb/\tau+1}}{e^{tb/\tau-1}}\right)
  \]
  where \(a\) is the offset distance (5mm), \(k\) is the wavenumber
- \(q\) of 1nC and \(f_{brr} = 1/t_b = 1\) MHz
- \(P_{\text{max}}\) and \(|V_\perp|_{\text{max}}\) are calculated for different vertical gap width

Resonant HOM excitation

Average power loss due to resonant excitation

- Total RF power loss for 3 mrad kick is ≈ 1 kW
- Average power loss due to resonant excitation is much less compared to RF power loss
- No significant load on the cooling
- Resonant excitation power loss is significant for higher beam current

<table>
<thead>
<tr>
<th>Vertical gap</th>
<th>$\sum P_{\text{HOM}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 mm</td>
<td>5.17 W</td>
</tr>
<tr>
<td>30 mm</td>
<td>7.19 W</td>
</tr>
<tr>
<td>20 mm</td>
<td>8.24 W</td>
</tr>
</tbody>
</table>
Resonant HOM excitation

\( V_{\perp,\text{max}} \) due to horizontal HOM

\[ V_{\perp,\text{max}} \text{ due to vertical HOM} \]

\[ V_{\perp,\text{max}} \text{ due to vertical HOM} \]

- \( V_{\perp,\text{max}} \) are calculated for beam offset of 5 mm
- Horizontal kick voltage is not significant compared to vertical kick voltage
- For the RF kick voltage of 300 kV, kick voltage from beam is less than 1.5%
- Resonant \( V_{\perp} \) due to transverse HOM is less significant

<table>
<thead>
<tr>
<th>Vertical gap</th>
<th>( \sum V_{\text{Hor}} )</th>
<th>( \sum V_{\text{ver}} )</th>
<th>( V_{\text{ver},260.5 MHz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 mm</td>
<td>838 V</td>
<td>3092 V</td>
<td>2669 V</td>
</tr>
<tr>
<td>30 mm</td>
<td>967 V</td>
<td>3500 V</td>
<td>3127 V</td>
</tr>
<tr>
<td>20 mm</td>
<td>1242 V</td>
<td>4939 V</td>
<td>3489 V</td>
</tr>
</tbody>
</table>
Wakefield analysis

- Impedance spectra for different vertical gap size of a bare cavity are calculated
- Rotationally asymmetric RF structure – symmetry plane is considered [*]
- Gaussian bunch of $\sigma_z = 2 \text{ cm}$, $q = 1 \text{ nC}$, and horizontal offset = vertical offset = 5 mm

Wakefield analysis

- Gaussian bunch of $\sigma_z = 2 \, cm$, q=1 nC, and beam offset ($d_{\text{vert, horz}} = 5 \, mm$)
- Longitudinal and transverse impedance are given as,

$$Z_{\text{long}}(\omega) = - \int_{-\infty}^{\infty} W_{\parallel}(s)e^{-(j\omega s)} \, ds$$

$$Z_{\text{horz, vert}}(\omega) = \frac{j}{d_{\text{horz, vert}}} \int_{-\infty}^{\infty} W_{\perp}(s)e^{-(j\omega s)} \, ds$$

where $W_{\parallel}(s)$ and $W_{\perp}(s)$ are the longitudinal and transverse wake functions

- Smaller vertical gap
  - longitudinal modes are pushed further away from the deflecting mode
  - easy to extract HOM
  - optimum: 30 mm → 600 W power requirement
Wakefield analysis

- Longitudinal loss and transverse kick factor are given by,

\[ k_{long} = - \int_{-\infty}^{\infty} W_{||}(s) \lambda(s) \, ds \]

\[ k_{horz,vert} = \frac{1}{du} \int_{-\infty}^{\infty} W_{\perp}(s) \lambda(s) \, ds \]

where \( \lambda(s) \) is the normalized charge density and \( du \) is the beam offset.

- Smaller vertical gap
  - Marginal increase in longitudinal loss factor and vertical kick factor
  - Horizontal kick factor increase by 50% → tail of the bunch gets a stronger horizontal transverse kick from the head of the bunch

- Vertical gap of 30 mm is reasonable to avoid kick from the offset beam
  (Further investigation is required with respect to multipacting, beam dynamics, fabrication tolerance,…)
Conclusion

- Domain decomposition methods combined with model order reduction methods as implemented in SSC allow to compute modal atlases for long, complex SRF chains
- These may also be combined with efficient 2D optimization approaches
- This can then be combined with non-intrusive methods for uncertainty quantification
- In preparation: Inclusion of Surface Losses via Perturbation Approaches into SSC

- Further specific studies:
  - Design studies for FCC cavities and HOM damping
  - Transverse deflecting cavity as beam separator
We gratefully acknowledge

• Contributions by Tomasz Galek
• Cooperation with our partners from CERN, DESY, HZB, HZDR, TU Darmstadt, TU Dortmund
• Financial support by
  • BMBF under contract 05K13HR1
  • DFG under contract SCHM 3127/2-1
  • EU within EuCard-2
  • CERN
  • HZDR
State-Space Concatenations (SSC)

1. Decomposition of the Structure at Regions of Constant Cross Section

Important properties:
- (numerical) treatment of segments is computationally less demanding
- single treatment of identical segments
- segments with simple geometry can be treated semi-analytically, which is very fast
- employment of symmetry of segments is feasible
State-Space Concatenations (SSC)

2. Consideration of Segments as Blocks with Terminals

- Modal voltages $v_{r,p,m}(t)$ correspond to tangential electric fields of 2D port modes
- Modal currents $i_{r,p,m}(t)$ correspond to tangential magnetic fields of 2D port modes
State-Space Concatenations (SSC)


wave equation (PDE):
\[ \Delta E(r, t) = \varepsilon \mu \frac{\partial^2}{\partial t^2} E(r, t) + \mu \frac{\partial}{\partial t} J(r, t) \]

state-space system (ODE):
\[ \frac{d^2}{dt^2} x_r(t) = A_r x_r(t) + B_r \frac{d}{dt} i_r(t) \]
\[ v_r(t) = B_r^T x_r(t) \]
State-Space Concatenations (SSC)

4. Model-Order Reduction for State-Space Systems

\[
\begin{align*}
\frac{d^2}{dt^2} x_r(t) &= A_r x_r(t) + B_r \frac{d}{dt} i_r(t) \\
v_r(t) &= B_r^T x_r(t)
\end{align*}
\]

\[
\begin{align*}
\frac{d^2}{dt^2} x_{rd,r}(t) &= W_r^T A_r W_r x_{rd,r}(t) + W_r^T B_r \frac{d}{dt} i_r(t) \\
v_r(t) &= B_r^T W_r x_{rd,r}(t)
\end{align*}
\]

state vector: \( x_r(t) \in \mathbb{R}^{N_s}, N_s \approx 10^5 \)

orthogonal projection matrix: \( W_r \in \mathbb{R}^{N_s \times N_{srd}}, W_r^T W_r = I \)

reduced state vector: \( x_{rd,r}(t) \in \mathbb{R}^{N_{srd}}, N_{srd} < 10^2 \)
5. Concatenation of Reduced-Order State-Space System

- Coupling constraints according to Kirchhoff’s laws
- Arbitrary topologies and number of 2D port modes supported
State-Space Concatenations (SSC)

6. Computation of RF Properties by Means of the Reduced-Order Model

\[
\frac{d^2 x_{rdc}(t)}{dt^2} = A_{rdc} x_{rdc}(t) + B_{rdc} \frac{d}{dt} i(t)
\]

\[
v(t) = B_{rdc}^T x_{rdc}(t)
\]

- Impedance- and scattering parameters
- Frequencies and field distributions of eigenmodes
- 3D field distribution due to port excitations
- External quality factors
Decomposition of BESSY VSR Chain into Segments

1. Warm Taper Up
2. Valve Up
3. Bellow 1
4. Cavity 1 with $f_\pi = 1.5 \text{ GHz}$
5. Bellow 2
6. Cavity 2 with $f_\pi = 1.75 \text{ GHz}$
7. Bellow 3
8. Cavity 3 with $f_\pi = 1.75 \text{ GHz}$
9. Bellow 4
10. Cavity 4 with $f_\pi = 1.5 \text{ GHz}$
11. Bellow 5
12. Valve Down
13. Warm Taper Down
Modal Atlas for BESSY VSR Cold String