TOWARDS A FLOQUET THEORY OF PERIODICALLY DRIVEN SUPERCONDUCTORS

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SUPERCONDUCTIVITY UNDER EXTREME CONDITIONS

• High pressure (H-rich compounds).
• High-speed vortex (Abrikosov-Josephson).
• High magnetic field (magnets).
• High frequency vs. high field (Superconducting cavities).
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SUPERCONDUCTIVITY UNDER EXTREME CONDITIONS

Grasselino et al. SUST 2013

Positive

Accelerating field

Quality factor
SUPERCONDUCTIVITY UNDER EXTREME CONDITIONS

HOW TO USE FLOQUET THEORY TO REDERIVE THE RESPONSE OF SUPERCONDUCTORS TO ELECTROMAGNETIC FIELDS, (AND HOPEFULLY DESCRIBE NEW PHYSICS…)
OUTLINE

- Cooper problem: Binding energy of two electrons in a filled Fermi sea
- Superconductor gap and ground-state energy using Floquet/BCS theory
- Dissipation: Paradigm shift?
- Final Considerations
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COOPER PROBLEM: BINDING ENERGY OF TWO ELECTRONS IN A FILLED FERMI SEA

Thin slab in an AC field

ELECTRIC FIELD

MAGNETIC FIELD

Ignore boundaries in y and z
**COOPER PROBLEM:** BINDING ENERGY OF TWO ELECTRONS IN A FILLED FERMI SEA

\[
H = \frac{1}{2m} \left( p_1 - eA_1 \right)^2 + \frac{1}{2m} \left( p_2 - eA_2 \right)^2 - eA_0(r_1) - eA_0(r_2) + V(r_1 - r_2)
\]

**TWO INTERACTING ELECTRONS**

The other electrons prevent occupation below the Fermi level
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\]

\[
A_{0i} = 0
\]

\[
A_i = \{0, B(t)x, 0\}
\]

\[
B(t) = B_0 \sin(\omega t)
\]
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FLOQUET THEORY

\[ U(t, t + T) = e^{iH_F T/h} \]
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\[ U(t, t + T) = e^{iH_FT/\hbar} \]

• High frequency limit
• Magnus expansion, BCH, …
• Effective Hamiltonian
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\[ H_F = \sum_{i=1}^{2} \left\{ \frac{p_{i,x}^2}{2m} + \frac{p_{i,y}^2}{2m'} + \frac{p_{i,z}^2}{2m} \right\} + V(r_1 - r_2) \]
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EFFECTIVE MASS

Cyclotron frequency at maximum field

\[ m' = m \left[ 1 - \frac{1}{2} \left( \frac{\omega_c}{\omega} \right)^2 \right] \]
COOPER PROBLEM: BINDING ENERGY OF TWO ELECTRONS IN A FILLED FERMI SEA

THE COOPER PROBLEM

- Change of coordinates and Fourier expansion
- Bethe-Goldstone equation for two interacting electrons
- The Cooper model
- Perturbation theory and binding energy

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\[ V_{k,k'} = \begin{cases} -\frac{V}{L^2 D}, & \text{if } \left| \frac{\hbar^2 k^2}{2m} - \epsilon_F \right| \text{ and } \left| \frac{\hbar^2 k'^2}{2m} - \epsilon_F \right| < \hbar \omega_D \\ 0, & \text{otherwise.} \end{cases} \]
THE COOPER PROBLEM

- Change of coordinates and Fourier expansion
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\[-\epsilon \approx 2\hbar \omega_D e^{-\frac{2}{V\mathcal{N}(0)}} \left[ 1 - \frac{1}{6} \left( \frac{2}{V\mathcal{N}(0)} + \frac{\epsilon_F}{\epsilon_D} e^{\frac{2}{V\mathcal{N}(0)}} \right) \left( \frac{\omega_c}{\omega} \right)^2 \right]\]
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SUPERCONDUCTOR GAP AND GROUND-STATE ENERGY USING FLOQUET/BCS THEORY
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SUPERCONDUCTOR GAP

\[
\frac{\Delta}{\hbar \omega_D} = \left[ \sinh \left( \frac{1}{VN(0) \left( 1 - \frac{1}{6} \left( \frac{\omega_c}{\omega} \right)^2 \right)} \right) \right]^{-1}
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GROUND-STATE ENERGY

\[
\langle \psi_{BCS} | H'_R | \psi_{BCS} \rangle - E_0 = -\frac{1}{2} N(0) \Delta^2 \left[ 1 - \frac{1}{2} \left( \frac{\omega_c}{\omega} \right)^2 \right]
\]
DISSIPATION
THE BCS PICTURE

\[ E + 2\hbar\omega \]

\[ E + \hbar\omega \]

• Absorb a photon: transition to state with energy \( E + \hbar\omega \)
• Emit a photon: transition to state with energy \( E \)
• Conductivity proportional to net transition rate

Semiconductor model: Tinkham, *Introduction to Superconductivity*
DISSIPATION

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\[ E \]
DISSIPATION

THE BCS PICTURE

\[ E + 2\hbar\omega \]

\[ E + \hbar\omega \]

\[ E \]

THE FLOQUET PICTURE

FLOQUET STATE

NO PHOTON EMISSION OR ABSORPTION:
COHERENT SUPERPOSITION OF STATES
WITH ENERGY SEPARATED BY \( \hbar \omega \)

Floquet zone
DISSIPATION: CHANGE OF PARADIGM?

THE BCS PICTURE

\[ E + \hbar \omega \]

\[ E + \hbar \omega \]

\[ E \]

THE FLOQUET PICTURE

FLOQUET STATE

\[ E + 2\hbar \omega \]

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\[ E \]

Floquet zone

DISSIPATION CAN ONLY HAPPEN WHEN THE COHERENT SUM OVER FLOQUET STATES IS BROKEN BY COLLISIONS
DISSIPATION: CHANGE OF PARADIGM?

THE BCS PICTURE

\[ E + 2\hbar \omega \]

\[ E + \hbar \omega \]

\[ E \]

BCS

FLOQUET

THE FLOQUET PICTURE

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\[ E + \hbar \omega \]

\[ E \]

Floquet zone

q

k

k - q
DISSIPATION: CHANGE OF PARADIGM?

THE BCS PICTURE

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THE FLOQUET PICTURE

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\[ E + \hbar \omega \]

\[ E \]

\[ q \]

\[ k \]

\[ k - q \]
In progress...

- Towards a Floquet theory of periodically-driven superconductors.
- Cooper problem in the high-frequency limit.
- Preliminary results of a BCS theory using Floquet states.
- Paradigm shift? Dissipation and the breaking of the coherent sum over Floquet states.
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THANK YOU