Mitigation of performance-limiting mechanisms in Nb3Sn films

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Advantages and disadvantages of Nb3Sn

- Larger SC gaps and $T_c$ results in a lower BCS surface resistance than in Nb

\[ R_s = \frac{\mu_0^2 \lambda^3 \omega^2 \Delta}{\rho_n kT} \ln \left( \frac{9kT}{4\hbar \omega} \right) e^{-\Delta/kT} + R_i \]

- For Nb3Sn, one could get the same low-field $R_s$ at 4.2K as for Nb at 2K.

- Higher critical field can translate into potentially higher accelerating gradients
  - Nb3Sn ($T_c = 18$K) has $H_c = 540$ mT versus 200 mT for Nb ($T_c = 9$K)

- Low $H_c$1 (2-10 times smaller than $H_c$1 = 170-180 mT in Nb)
- Current-blocking grain boundaries (GBs)
- Sensitivity of SC properties to local nonstoicheometry (segregation of Sn on weak-linked GBs)
- Low thermal conductivity $\kappa_{Nb3Sn} \sim 10^{-3} \kappa_{Nb} \sim 10^{-2} \text{ W/mK}$
- RF overheating in Nb3Sn layers thicker than 2-3 microns at 2K and 1-2 GHz
- Nb3Sn is more prone to flux trapping than Nb
Grain boundaries impeding SRF currents

GBs are **fully transparent** to the RF currents

Critical current density $J_c$ of GB is close to $J_d$

No penetration of vortices along GBs

At $J_c = 0.1J_d$, Abrikosov-Josephson vortices penetrate along GBs in Nb$_3$Sn at $H > 20$-$50$ mT
Grain boundaries pin vortices in Nb$_3$Sn

- Flux flow channels along GBs. Anisotropic pinning of vortices caged within grains
- GBs critical current density $J_c$ is much smaller than $J_d = H_c/\lambda$
- Bulk pinning force depends on the grain size $d$: GB partially block supercurrents

*References*


Flux flow channels along GB

PRB 50, 13563 (1994); PRL 88, 097001 (2002); PRB 65, 214531 (2002).
Mitigation of penetration of vortices along GBs

Sn segregation on GBs turns them into Josephson weak links

\[ H > H_p \simeq \lambda J_c \simeq H_c J_c / J_d \]

Higher \( H_c = 540 \text{ mT} \) for \( \text{Nb}_3\text{Sn} \) can result in better high-field SRF performance only if GBs are strongly coupled, \( J_c > 0.2 J_d \)

SIS multilayers: blocking penetration of vortices in the bulk

Big reduction of RF vortex losses localized in the first thin S layer

Mismatch of GB structures in S coating layers and the Nb cavity: GB joints and I layer stop vortices

Penetration of vortices along GBs does not go beyond the first S layer
Effect of overheating on the surface resistance

Thermal feedback for trapped vortices and linear BCS:

\[
\left( R_i(H_a) + R_0 e^{(T-T_0)\Delta/T_0^2} \right) \frac{H_a^2}{2} = (T - T_0)g,
\]

Effective thermal impedance of the cavity wall:

\[
g = \frac{1}{\alpha K} + \frac{d}{\kappa}
\]

Kapitza Heat diffusion

Maximum overheating and thermal breakdown field at \( R_i < R_0 \)

\[
T_b - T_0 = \frac{k_B T_0^2}{\Delta} \left( 1 + \frac{R_i}{e R_0} \right)
\]

\[
H_b = \sqrt{\frac{2\kappa\alpha_K k_B T_0^2}{e\Delta(\kappa + d\alpha_K) R_0} \left( 1 - \frac{R_i}{2e R_0} \right)}
\]

Nb3Sn have larger RF losses caused by trapped vortices

Trapped vortices cause strong overheating which reduces Q(H)

2-3 microns thick Nb3Sn film has the same thermal impedance as a 2-3 mm thick Nb

Trapped vortex driven by RF Meissner current

An elastic vortex is driven by the Lorentz force $\mathbf{f}_L = \phi_0 \mathbf{J} \times \mathbf{z}$ perpendicular to $\mathbf{J}$:

$$J(z, t) = \left(\frac{H_a}{\lambda}\right)e^{-z/\lambda} \sin \omega t$$

The surface Lorentz force is balanced by viscous drag force and bending stress

At $H_a = 100 \text{–} 200 \text{ mT}$, $J(0)$ approaches the depairing limit

$$J_d \simeq \frac{H_c}{\lambda}$$

Typical depinning $J_c = 10 \text{–} 100 \text{ kA/cm}^2$ in Nb are some 4 orders of magnitude lower than $J_d = \frac{H_c}{\lambda} = 500 \text{ MA/cm}^2$

Pinning is too weak to stop the vortex tip at the surface above $H > 0.01H_c = 2 \text{ mT}$
RF Campbell length

Dynamic eq for displacements $u(x,t)$ of a vortex driven a weak RF field $H_a \ll H_c$

$$\eta \dot{u} = \epsilon u'' - \frac{H_a}{\lambda} e^{-x/\lambda} \sin \omega t$$

Elastic RF ripple length – Campbell penetration depth:

$$L_\omega = \sqrt{\frac{\epsilon}{\eta \omega}} = \frac{\xi}{2\lambda} \sqrt{\frac{g \rho_n}{\pi \mu_0 f}}$$

- Campbell length $L_\omega$ can be much greater than $\lambda$.
- $L_\omega$ can be either larger or smaller than the pin distance from the surface.
  If $\ell > L_\omega$ the effect of pinning is weak.

<table>
<thead>
<tr>
<th>Clean Nb</th>
<th>$\lambda \approx \xi$, $\rho_n = 1$ $n\Omega m$, $f = 2$ $GHz$, $L_\omega \approx 180$ $nm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Nb}_3\text{Sn}$</td>
<td>$\lambda/\xi \approx 20$, $\rho_n = 0.2$ $\mu\Omega m$, $f = 2$ $GHz$, $L_\omega \approx 126$ $nm$</td>
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</table>
Low-field RF power of an oscillating vortex

- Low frequencies. The whole vortex segment swings: $L_\omega \gtrsim \ell$

\[ P \simeq \frac{4\pi B_p^2 \ell^3 \omega^2}{3\rho_n \xi^2} \]

Decreases strongly as the pin spacing decreases

- Intermediate $\omega$: $\lambda \lesssim L_\omega \lesssim \ell$

\[ P \simeq \pi \mu_0^{-3/2} B_p^2 \lambda \xi \sqrt{\omega \rho_n} \]

No dependence on the pin spacing

- High $\omega$: $L_\omega \lesssim \lambda$

\[ P = \frac{H^2 n^2}{2} \]

No dependence on the pin spacing

$P \sim 0.13 \mu W$ at $B = 100 \text{ mT}$ and $2 \text{ GHz}$.

Hotspots revealed by thermal maps require regions $\sim$ few mm with $\sim 10^6$ vortices

Gurevich and Ciovati. PRB 87, 054502; (2013)
At $H = H_c$, the superflow velocity of Cooper pairs reaches the critical pairbreaking value $v_c = \Delta/p_F$.

How fast can the vortex tip move at the pairbreaking limit?

$$v \approx \frac{J_d \phi_0}{\eta} \approx \frac{\rho_n \xi}{2 \mu_0 \lambda^2}$$

This rough estimate yields $v = 10$ km/s, which exceeds both the speed of sound (2-4 km/s) and $v_c = \Delta/p_F = 1$ km/s.

How can a supersonic vortex tip remain connected to a subsonic elastic vortex line in the bulk?

SRF cavity is a unique testbed to study the extreme dynamics of a vortex driven by non-dissipative Meissner currents at the pairbreaking limit.
How can a vortex move faster than the current superflow which propels it?

- Vortex core stretches along the direction of motion
- Vortex can move much faster than the drift velocity of supercurrent
- $V$ can exceed the pairbreaking velocity

A sailboat can move much faster than the wind if drag is weak and the sail is nearly perpendicular to the wind blow.
75 nm thick Pb film: imaging of penetrating vortices with a nanoscale SQUID on tip

Velocities can reach \(10–20 \text{ km/s}\) as \(J(x,y)\) at the edge reaches \(J_d\) (\(H = H_s\) for the SRF cavities)

If \(v = 10 \text{ km/s}\), a vortex penetrates by the distance

\[ L \sim v/f \sim 10\mu m \gg \lambda, \quad \text{at } 1 \text{ GHz} \]

Vortices penetrate almost instantaneously through the Meissner RF layer

Hot vortex branching trees. No materials defects can stop such superfast vortices.
What happens to the vortex core at high velocities?

- A nearly round vortex core of radius $\approx \xi$
- Cloud of dissipative quasiparticles is locked onto the moving core
- Bardeen-Stephen vortex drag independent of $v$

**Slow vortex**

- Velocity-dependent LO vortex drag
  \[ \eta(v) = \frac{\eta_0}{1 + (v/v_0)^2} \]

**Fast vortex**

- The core stretches along $v$ as the recovery length of $\Delta(x, t)$ behind the core increases with $v$:
  \[ L_\Delta \approx v \tau_\Delta, \quad v > \xi/\tau_\Delta \]
- A cloud of diffusive nonequilibrium quasiparticles is lagging behind the core
- Vortex drag decreases with $v$:
  \[ \eta(v) \simeq \frac{\phi_0^2}{2\pi \rho_n \xi L_\Delta} \]
Larkin-Ovchinnikov instability

Balance of drag and Lorentz forces for a straight vortex in a thin film:

\[
\frac{\eta_0 v}{1 + (v/v_0)^2} = \phi_0 J
\]

Observations on different materials:

Acceleration of a runaway vortex at \( v > v_0 \), jumps on the V-I curves

The observed \( v_0(T) \) is \( \sim 0.1 - 1 \) km/s near \( T_c \) and decreases as \( T \) decreases.

Can be masked by heating effects
LO instability of a trapped vortex

Since the LO critical velocity $v_0 \sim 0.1 - 1$ km/s is 1-2 orders of magnitude smaller than velocities of a vortex at $H = 10 - 100$ mT, the LO instability can be essential in SRF cavities.

- What happens to the vortex if its fast tip is LO-unstable while the rest of the vortex is LO-stable?
- Can a vortex be shredded into disconnected pieces by strong surface current?
- Dependence of RF losses and the residual surface resistance caused by trapped vortices on the RF field.
- The extreme vortex dynamics in SRF cavities is not masked by strong overheating typical of dc transport measurements at $T \ll T_c$. 
Nonlinear dynamic equations for a vortex

\[ M \ddot{y} + \eta(y) \dot{y} = \epsilon/R - (H_a/\lambda) e^{-x/\lambda} \sin \omega t \]

Dynamic eq. for a dimensionless vertical displacement

\[ u(x,t) = y(x,t)/\lambda, \quad x \rightarrow x/\lambda : \]

\[ \mu \frac{\partial}{\partial t} \left( \frac{\dot{u}}{\sqrt{1+u'^2}} \right) + \frac{\gamma \dot{u} \sqrt{1+u'^2}}{1+u'^2 + \alpha \gamma^2 \dot{u}^2} = \frac{u''}{(1+u'^2)^{3/2}} - \beta e^{-x} \sin(2\pi t) \]

Takes into account vortex inertia, and nonlinearities of the LO vortex drag and bending rigidity

\[ \gamma = f/f_0, \quad f_0 = H_{c1}\rho_n/H_{c2}\lambda^2\mu_0 \]

\[ \alpha = (\lambda f_0/v_0)^2, \quad \beta = H_a/H_{c1} \]

\[ f_0 = 22 \text{ GHz for Nb.} \]
Nonlinear vortex losses and residual resistance

Dissipated power per vortex:
\[ p = \int \langle \eta(v)v^2 \rangle ds \]

Surface resistance \( R_i \) for the mean trapped flux density \( B_0 \) is obtained from
\[ pB_0/\phi_0 = R_i H_a^2/2 \]:

\[ R_i(\beta) = \frac{R_0 \gamma^2}{\beta^2} \int_0^1 dt \int_0^l \frac{(1 + u'^2)^{1/2} \dot{u}^2 dx}{1 + u'^2 + \alpha \gamma^2 \dot{u}^2} \]

\[ R_0 = \frac{2 \rho_n B_0}{\lambda B_{c2}} \]

For Nb at 1-2 GHz, we have \( \gamma \sim 10^{-1} \), and \( \alpha \sim 10^2 - 10^4 \). At small \( f \) and \( H_a \), the LO term in the denominator is negligible and \( R_i \) is independent of \( H_a \)

As \( H_a \) and \( f \) increase, \( \dot{u}^2 \) cancels out and \( R_i \) becomes nearly independent of frequency and decreases with the RF field:

\[ R_i \propto H_a^{-2} \]
LO mechanism of the low-field $Q(H)$ rise

The surface resistance $R_i(H)$ starts decreasing with the field amplitude as the frequency increases.

Calculated for different values of

$$\gamma = f/f_0 \quad @ \quad l = 4\lambda, \quad \alpha = 3 \cdot 10^3$$

Fit to the experimental data of for a 1.47 GHz Nb cavity

Ciovati, JAP 96, 1591 (2004)

$$l = 3\lambda, \quad B_0 = 0.73 \, \mu T,$$

$$v_0(2K) = 30 \, m/s,$$

$$v_0(1.37K) = 35 \, m/s,$$
Effect of frequency on the field dependence of $R_i(H_a)$

transition from quasi-harmonic to relaxation oscillations at the peak in $R_i(H)$. The Campbell length increases with $H_a$:

$$L_\omega(H_a) \sim \sqrt{\epsilon/\omega \eta(\nu)}$$

- $L_\omega(H_a, \omega) < l$ before the peak
- $L_\omega(H_a, \omega) > l$ after the peak
Tuning the LO vortex dynamics by impurities

Calculated for $\alpha_0 = 1.6 \cdot 10^4$, $l = 3\lambda_0$

- Making the surface dirtier and decreasing the impurity m.f.p. shifts the anomalous drop of the vortex surface resistance $R_i(H)$ to lower fields.
- May pertain to the low-field $Q(H)$ drop observed on many Nb cavities.
Random pinning

Numerical modeling of nonlinear dynamics of a curvilinear elastic vortex driven by strong RF current in a film

Mesoscopic effects in RF response for different pinning configurations

LO instability in the presence of random pinning

Effect of overheating

\[
M \frac{\partial^2 \mathbf{R}}{\partial t^2} + \eta \frac{\partial \mathbf{R}}{\partial t} = \epsilon \frac{\partial^2 \mathbf{R}}{\partial X^2} - \nabla U(X, \mathbf{R}) - \hat{\gamma} \phi_0 H e^{-X/\lambda} \sin \omega t,
\]

\[
U(X, \mathbf{R}) = - \sum_{n=1}^{N} \frac{U_n \xi^2}{\xi^2 + (X - X_n)^2 + |\mathbf{R} - \mathbf{R}_n|^2}
\]
Averaging over statistical realizations of the pinning potential often (but not always) yields a linear low-field dependence of $R_s(H)$.

Quasi-static collective pinning:
Overheating effects

Mattis-Bardeen

$h = \frac{R_i}{R_0}$ is proportional to the flux of trapped vortices

Quasi-linear dependence caused by heating

High-field upturn of $R_s(H)$

Larkin-Ovchinnikov

Heating mitigates the negative Q slope
Could strong pinning mitigate SRF vortex losses?

**GOOD**
- Artificial pinning centers (APCs) which take 10% of current-carrying cross-section can produce critical current densities $J_s \approx 0.1J_d$
- For cavities this can only be effective below the depinning field $H < 0.1H_c = 20$ mT = 10% of the SRF breakdown field for Nb.
- Reduction of vortex losses only in a small low-$H$ part of the field operation range

**BAD**
- 10% of metallic APCs produce huge ohmic losses above the proximity effect breakdown field. Incompatible with high $Q$ controlled by the BCS surface resistance
- 10% of dielectric APCs block the current-carrying cross section, greatly increasing the field penetration depth and the BCS surface resistance
- Above the depinning field, high Bean’s hysteretic losses make high-$J_c$ SRF cavities no better than the normal Cu cavities

α-Ti ribbons in a Nb-Ti alloy (*D. Larbalestier & P. Lee*)
Conclusions

Taking advantage of the high-field potential of Nb3Sn requires mitigating the issues which have been taken for granted in Nb:

- Small lower critical field
- Current-blocking grain boundaries
- Sensitivity of SC properties to local nonstoicheometry (segregation of Sn on weak-linked GBs)
- Much lower thermal conductivity
- RF overheating in Nb3Sn layers thicker than 2-3 microns at 2K and 1-2 GHz
- Nb3Sn is more prone to flux trapping than Nb
- Pinning may only be effective at very low RF fields. Dense APC structures greatly increase BCS and eddy current losses.

Opportunities to probe the extreme nonlinear dynamics of trapped vortices in SRF cavities at low T. Vortex mechanism of the negative Q(H) slope.