The Matrix Element Method at NLO

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Outline

- MEM: a brief Introduction
- MEM at LO
- MEM at NLO
- Example : DY
- Example : $H \longrightarrow ZZ$

The Matrix Element Method (MEM)

- All measurements of SM parameters and searches for new physics rely on matrix elements at some level.

- The Matrix element contains the maximal amount of theoretical information available (for the hard scattering process).

- The goal of the MEM is to perform a measurement using the matrix element to create a probability distribution function.

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega} \int dx_1 dx_2 \, d\Phi(\mathbf{y}) \frac{f(x_1)f(x_2)}{x_1 x_2 s} |\mathcal{M}_\Omega(\mathbf{y})|^2 W(\mathbf{x},\mathbf{y}) \ .$$

- Then this can be used to obtain a Likelihood for the model under investigation.

$$\mathcal{L}(\mathbf{x}|\Omega) = f(N) \prod_{i=1,N} \mathcal{P}(\mathbf{x_i}|\Omega).$$

Pros and cons of the method.

- Clean separation between theory and experimental inputs
- Utilizes full ME.
- Many potential applications.
- Ripe for parallelisation

- Computationally expensive
- Need for simplifications:
  - Transfer function form
  - LO ME elements

This talk

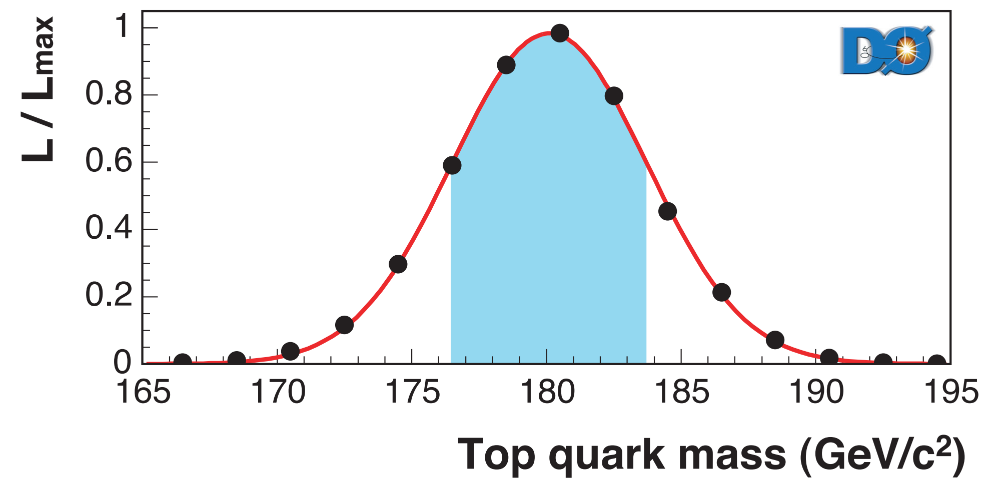
$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega} \int dx_1 dx_2 \, d\Phi(\mathbf{y}) \frac{f(x_1) f(x_2)}{x_1 x_2 s} |\mathcal{M}_\Omega(\mathbf{y})|^2 W(\mathbf{x}, \mathbf{y}) \ .$$

Theory input

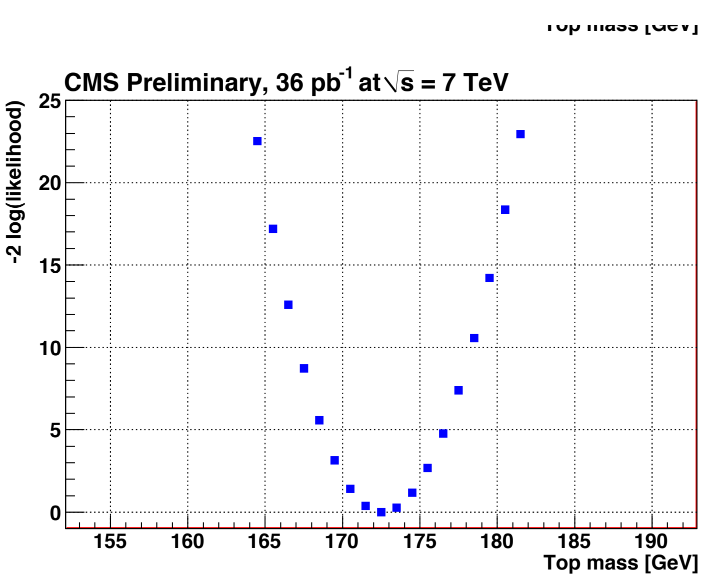
Experimental input

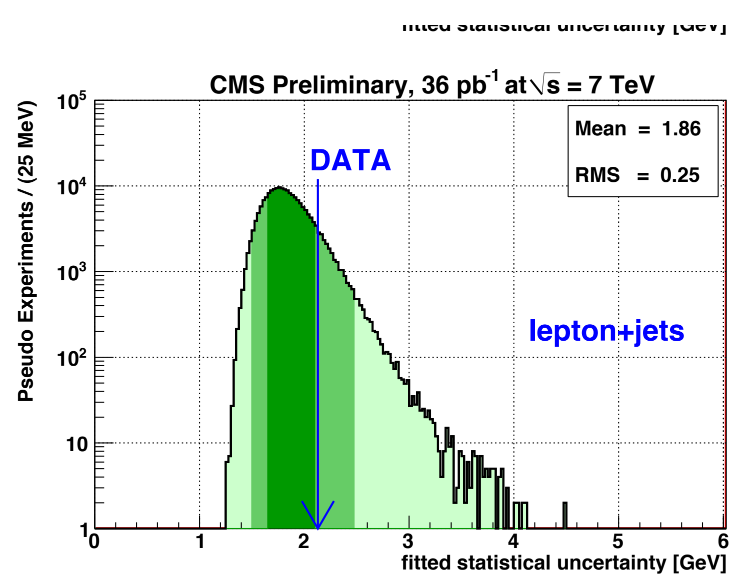
An example of the MEM in HEP.

- Most famous application: Top mass measurement!
- With a large enough data set the Log likelihood is quadratic function.



D0: Nature **429**, 638-642





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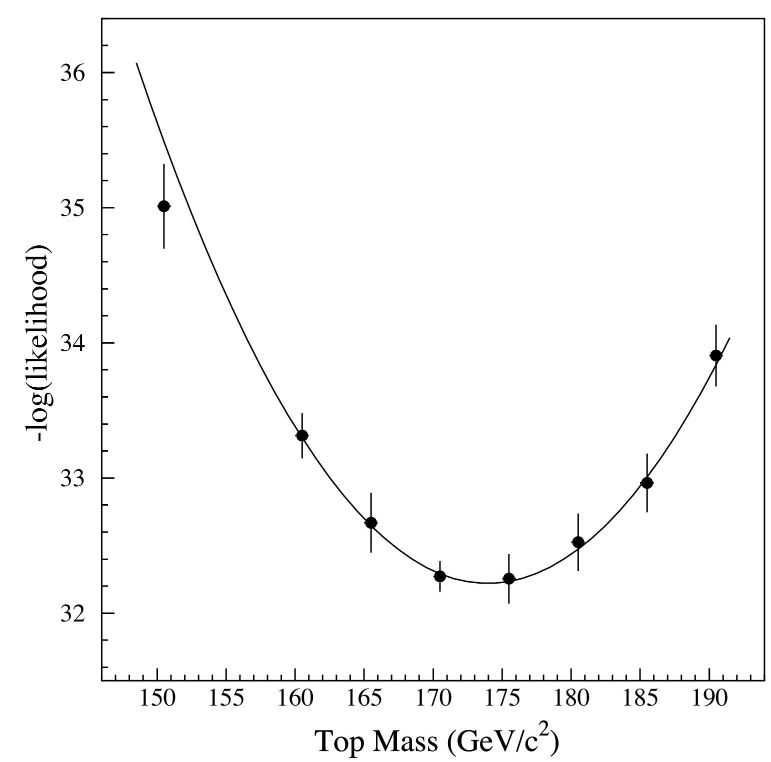


CDF: Phys. Rev. D**50**, 2966 (1994).

Another example of the MEM

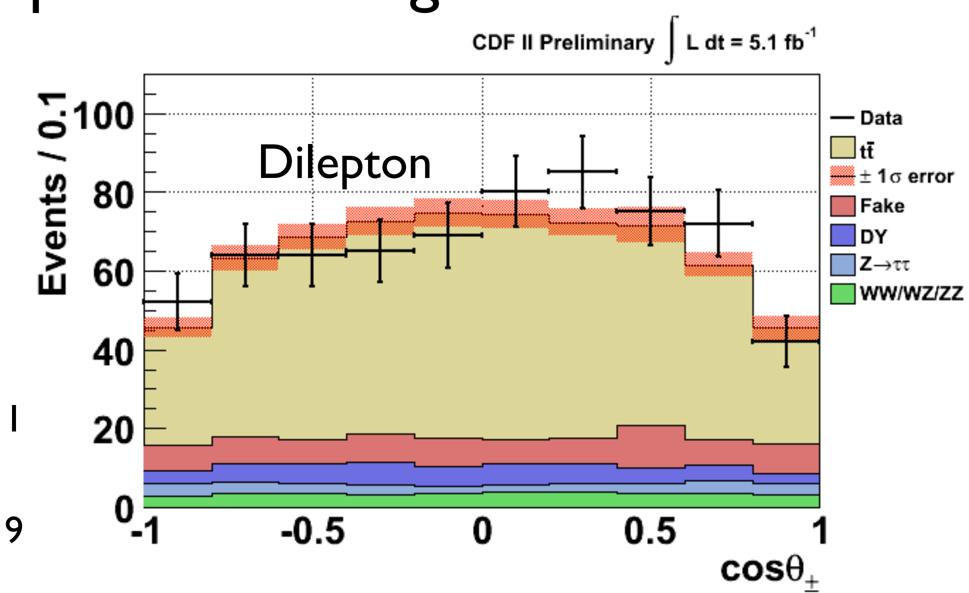
- Slide from David Mietlicki's Moriond talk:

Measuring the Spin Correlation

- Results shown here assume spin quantized along beam axis
  - CDF:
    - Template fits based on decay product angular distributions

$$\kappa^{CDF}_{Lep+Jet} = 0.72 \pm 0.69$$ CDF Conf. Note 10211

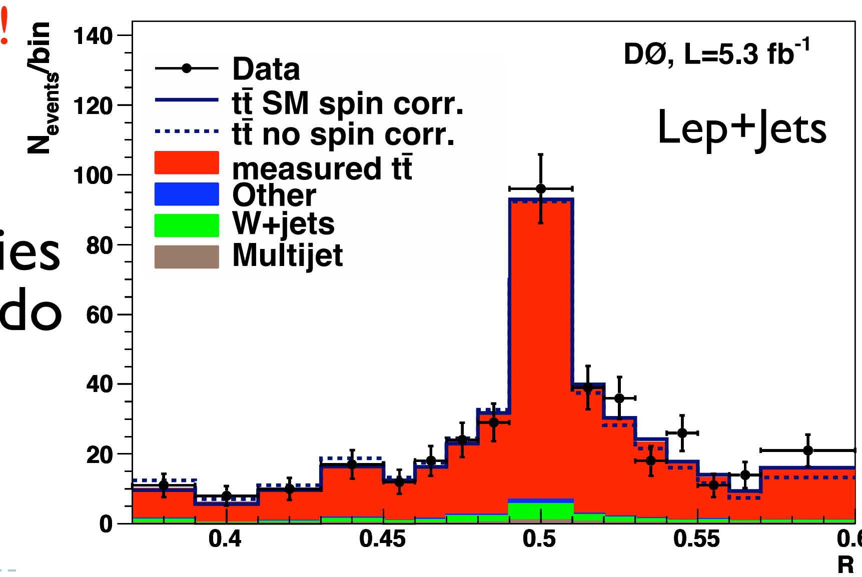
$$\kappa^{CDF}_{Dilepton} = 0.042 \pm 0.563$$ CDF Conf. Note 10719



- NB SM prediction: 0.78(4)

- D0: 3 σ Evidence For Spin Correlations!
  - New matrix element approach
    - Significantly increased sensitivity
  - Likelihood fit based on probabilities that events are signal events and do (or do not) contain SM spin correlation

$$\kappa^{D0}_{Combo(Dil,Lep+Jet)} = 0.66 \pm 0.23$$



 PRL **108**, 032004 (2012) D. Mietlicki Moriond 2012

Theoretical MEM tools.

- Experimentalists have multiple in-house MEM codes (for top mass etc.) using various LO MEs.

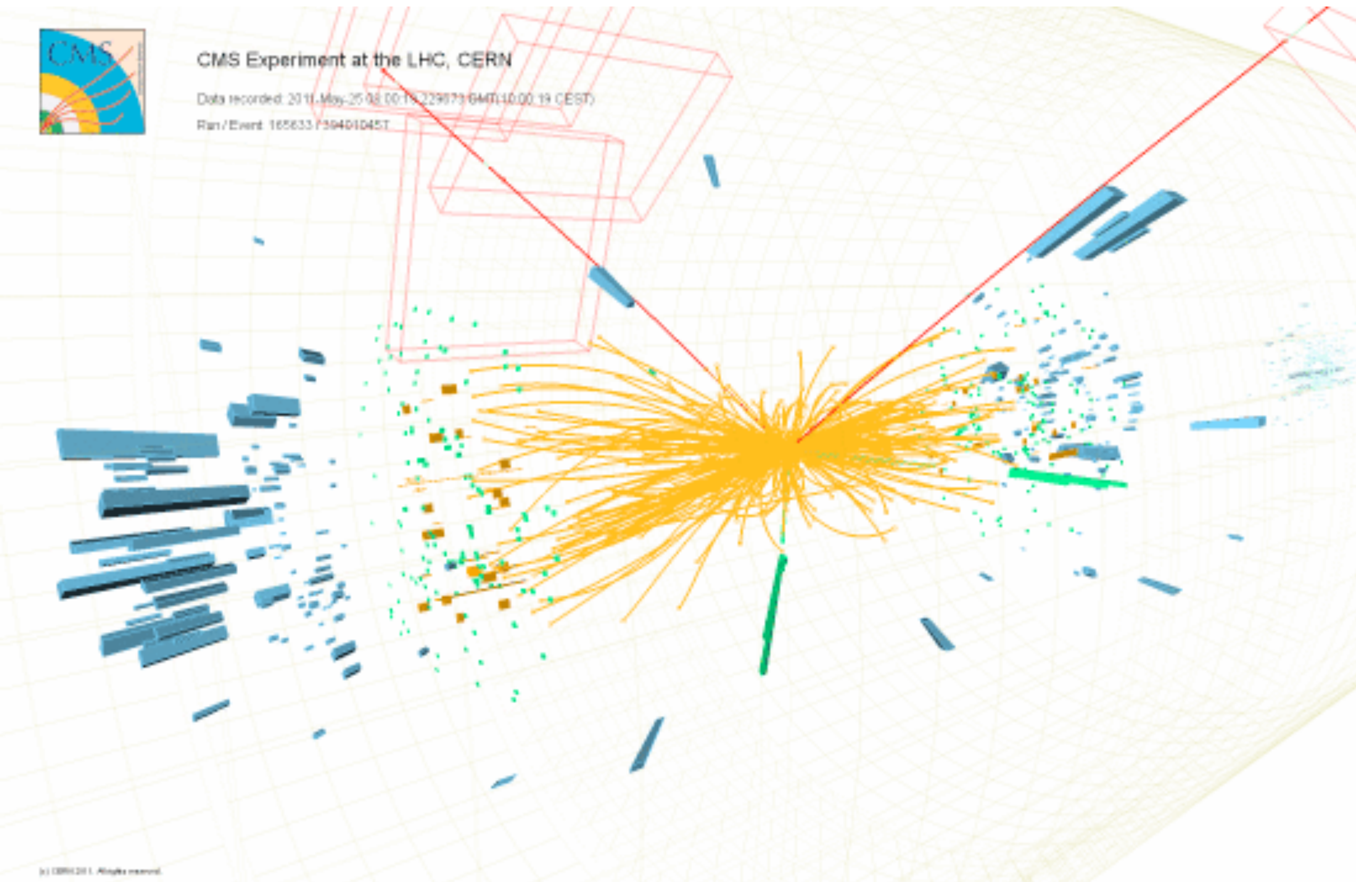
- A nice implementation of the MEM for general BSM scenarios has been provided at LO in the Madgraph framework. (Artoisenet, Lemaitre, Maltoni, Mattelaer 1007.3300).

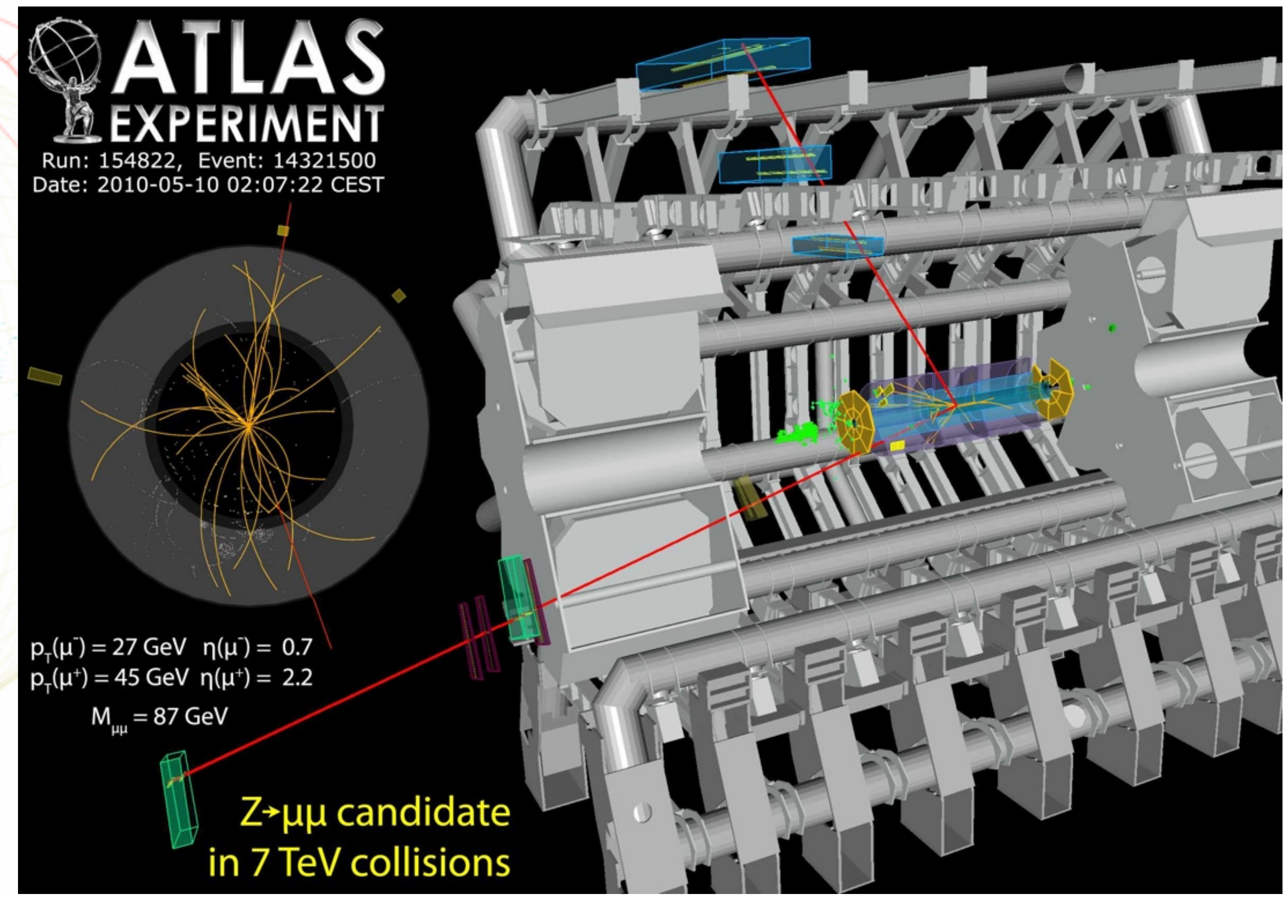
- This has also been extended to include some ISR modeling. (Alwall, Freitas, Mattelaer 1010.2263).

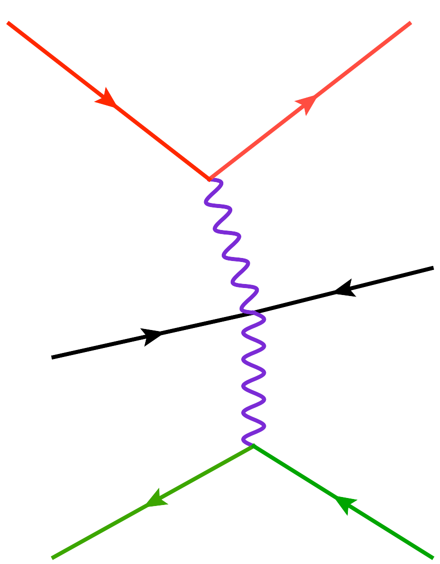
- Would be nice to have the situation where we can have NLO background + LO BSM signal

- Providing the NLO background is the goal of this work!

Experimental events versus fixed order weights.

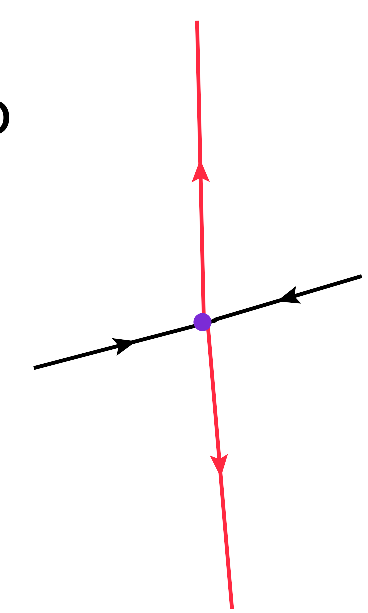




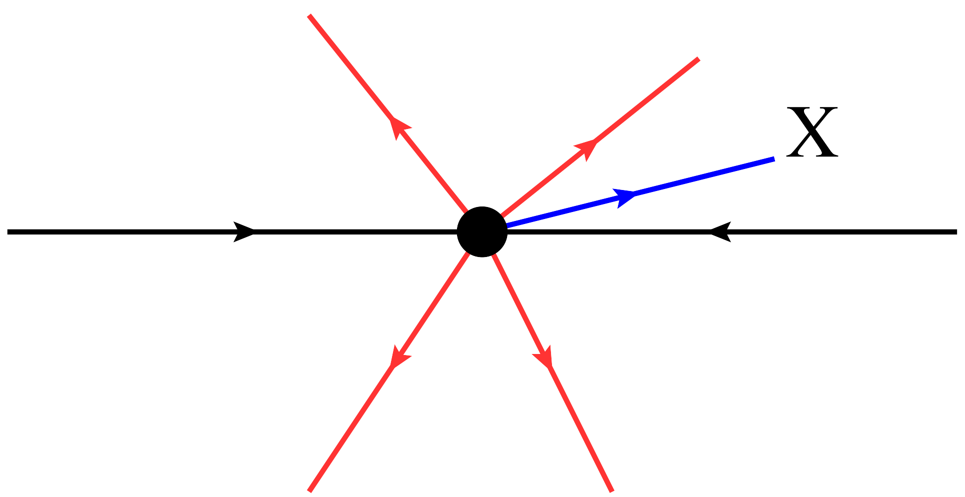


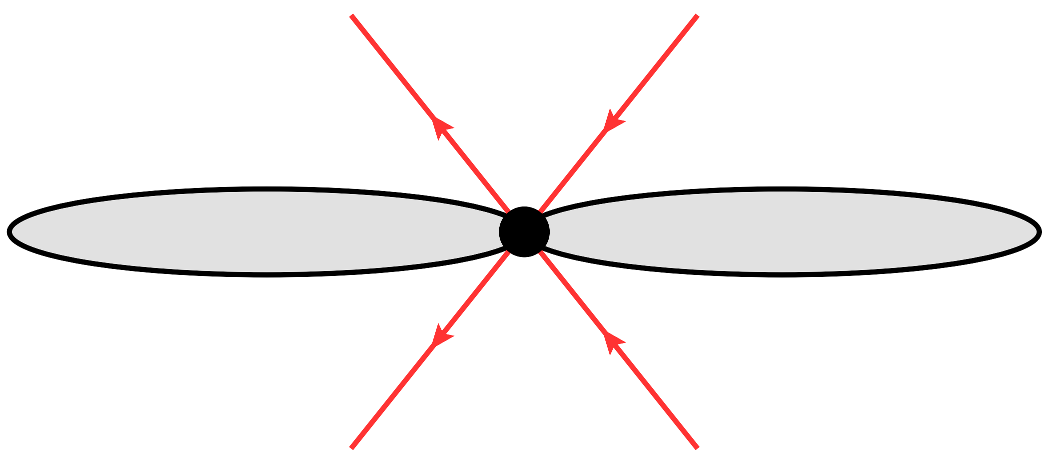
How can we relate an experimental event to our theoretical model?

Experimental events are unbalanced (for the LO final state), LO events have exact balance.



Going to the MEM frame





- We can cluster all of the event which is not in the Born final state into one vector X.

- One can perform an Lorentz boost to the frame in which X is at rest in the transverse plane.

- This frame is the MEM frame.

- Now we have a Born final state, and X is in the longitudinal direction.

Welcome to the MEM frame.

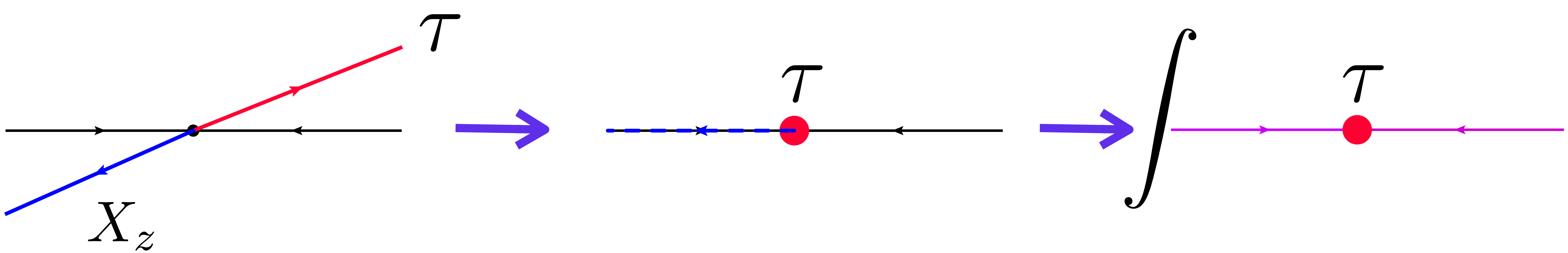
- This boost CANNOT be avoided, if we give the LO Matrix element an unbalanced phase space point, we can produce any number we like!

- However, since the boost is not unique, we had better integrate over allowed boosts, the resulting weight is made up of two pieces,

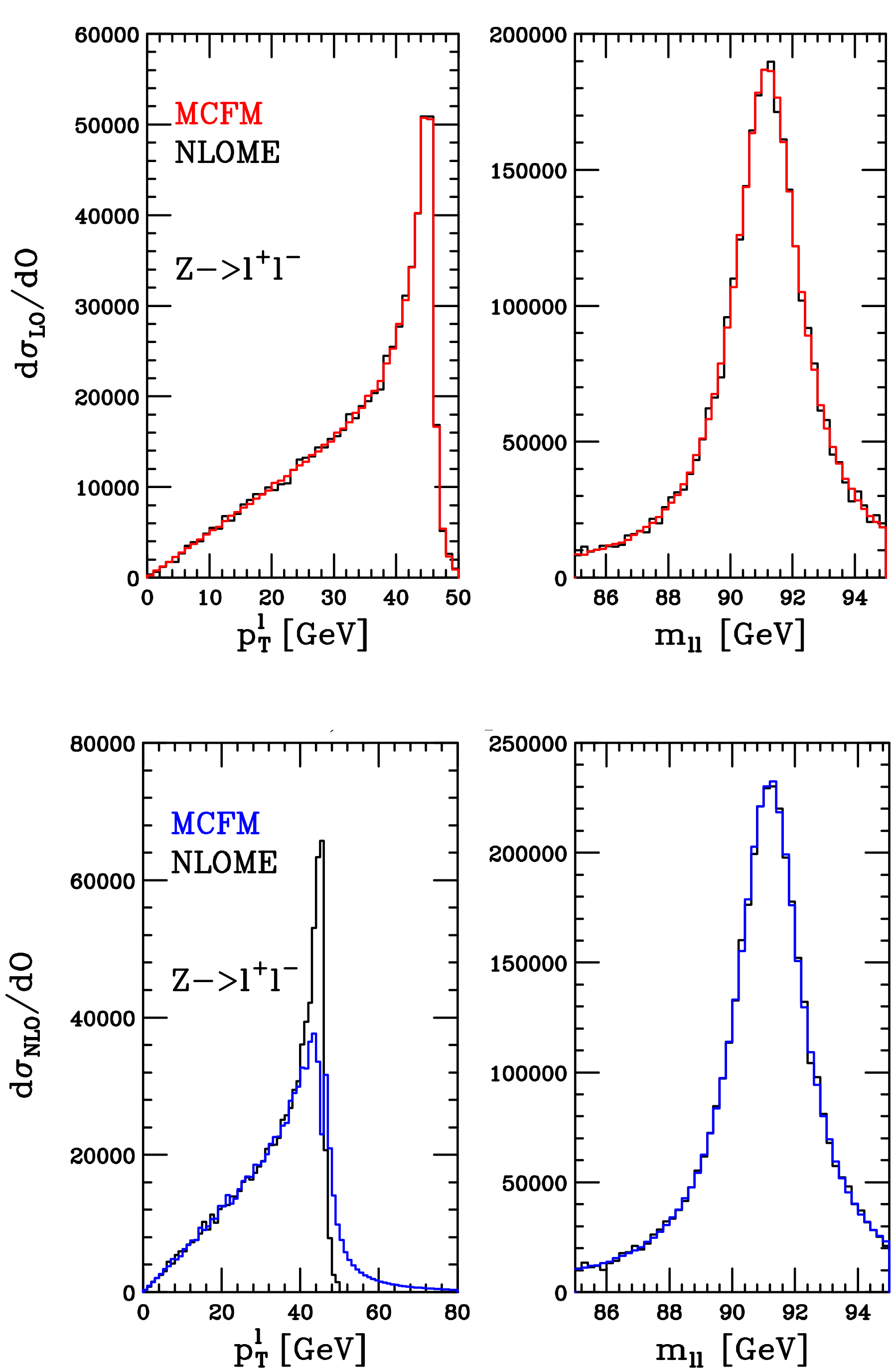
$$\mathcal{P}^{(n)}(\{p_i\}|\Omega) = \frac{1}{\sigma_\Omega}\mathcal{L}(\tau) \times \left|\mathcal{M}_\Omega^{(n)}(\tau, \{p_i\})\right|^2 ,$$

- Of these, only the "luminosity function" depends on the boost,

$$\mathcal{L}(\tau) = \int dx_1 dx_2 \frac{f(x_1)f(x_2)}{x_1 x_2 s}\delta(x_1 x_2 - \tau)$$



Physics in the MEM frame.



- Any Observable which is Lorentz invariant is identical in the MEM and lab frame.
- On the contrary, frame dependent quantities like transverse momentum change in the MEM frame compared to the Lab frame.
- We can use Lorentz invariant “p_T”

$$\left(p_T^{(i)}\right)^2 = 2\frac{(p_a\cdot p_i)(p_i\cdot p_b)}{(p_a\cdot p_b)};$$

- Rapidity is defined in the lab frame

$$\eta_i = \frac{1}{2}\log\left(\frac{x_a}{x_b}\frac{(p_b\cdot p_i)}{(p_a\cdot p_i)}\right)$$

- All cuts are performed in the Lab frame

MEM at LO: Summary

- Using our approach one can produce a LO weight for an experimental event of the form,

$$p_1 + p_2 \to Q + X$$

- Where Q is the desired final state. We map to the born by boosting away X.

- We then integrate over all boosts with the limits on the integration set by lab frame rapidity cuts.

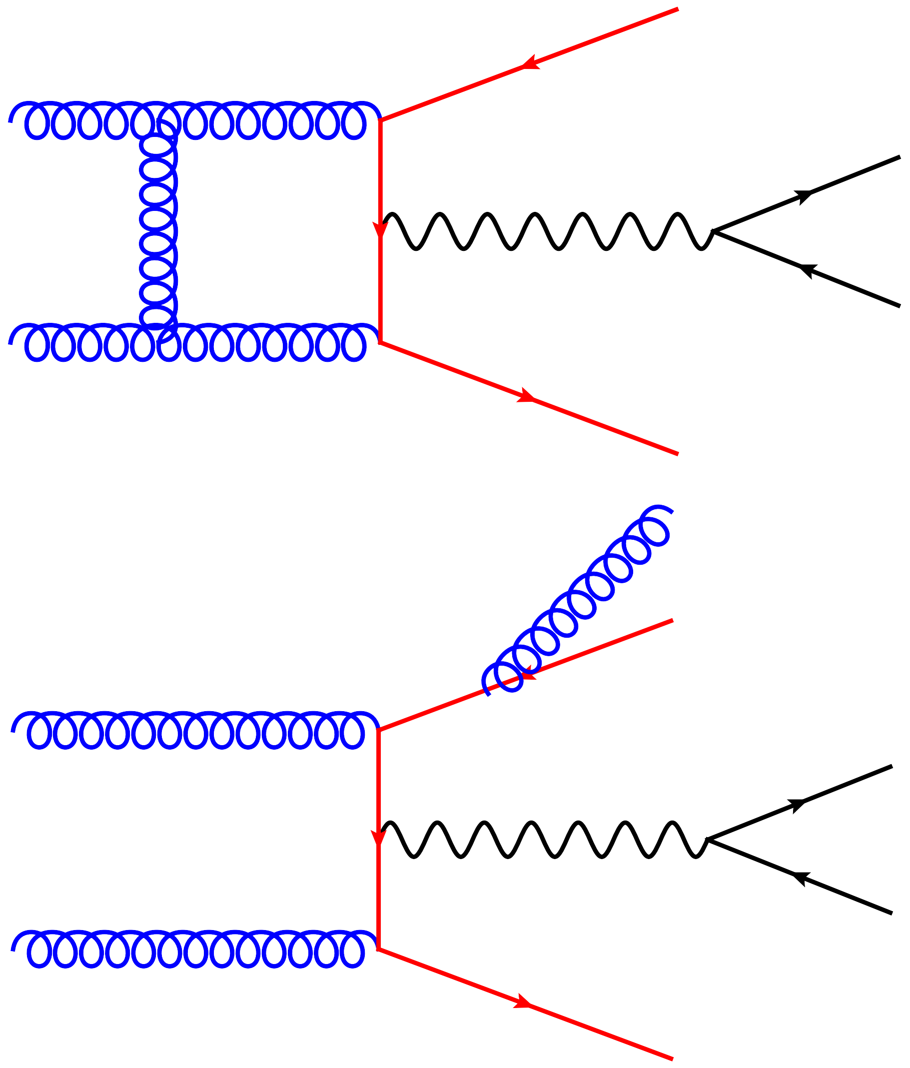
- One expects this to work well provided that X doesn't have a big impact on the observable or the observable isn't strongly dependent on the local pdf shape

- Normalisation is fixed to the cross section,

$$\int \mathcal{P}(\{p_i\}|\Omega) dp_i = 1$$

NLO parton level

- At NLO in perturbation theory one has to deal with divergences
- Virtual diagrams contain UV and IR divergences which typically manifest themselves as poles an analytic.
- Real diagrams contain an emission of an additional parton. Although 4 dimensional they develop singular regions in phase space when the extra parton is unresolved.



$$\sigma_{NLO} = \int_m d\sigma_{LO} + \int_m d\sigma_V + \int_{m+1} d\sigma_R$$

Subtraction schemes.

- To make a NLO Monte Carlo program we need to tackle these divergences, this is done using a subtraction scheme. (Catani, Seymour; Frixione Kunszt, Signer)

- Introduce a counter term which lives in the m+1 phase space. This counter term will cancel a real singularity point by point in the singular region.

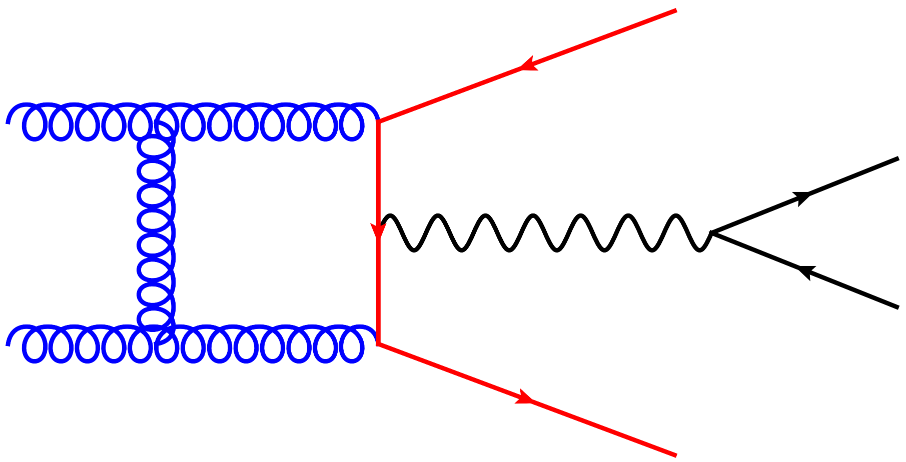
- Next, integrate out one of the counter term particles analytically, arriving at a m-dimensional integral. The singular regions will now manifest as analytic poles.

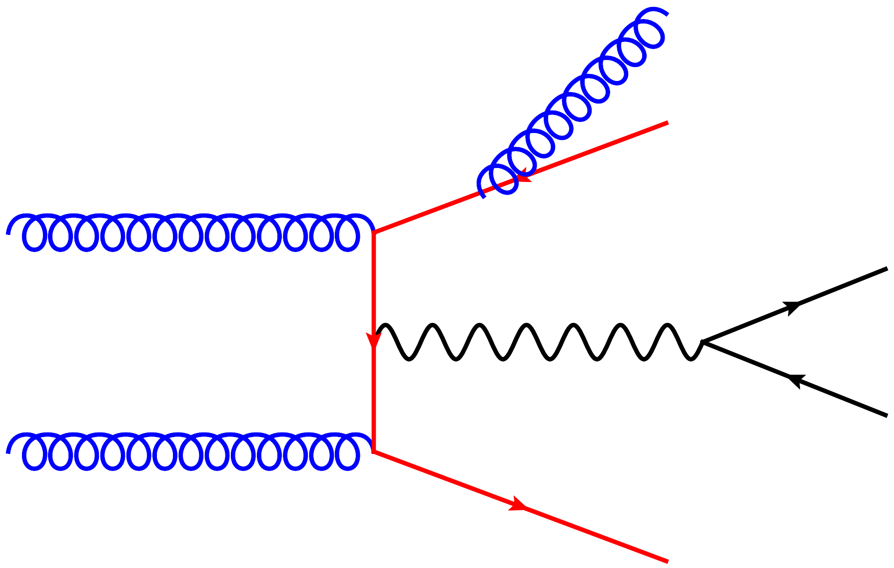
- Cancel these poles against the virtual terms and live happily ever after...

$$\sigma_{NLO} = \int_m d\sigma_{LO} + \int_m [d\sigma_V + \int_1 d\sigma_A]_{\epsilon=0} + \int_{m+1} [(d\sigma_R)_{\epsilon=0} - (d\sigma_A)_{\epsilon=0}]$$

MEM at NLO

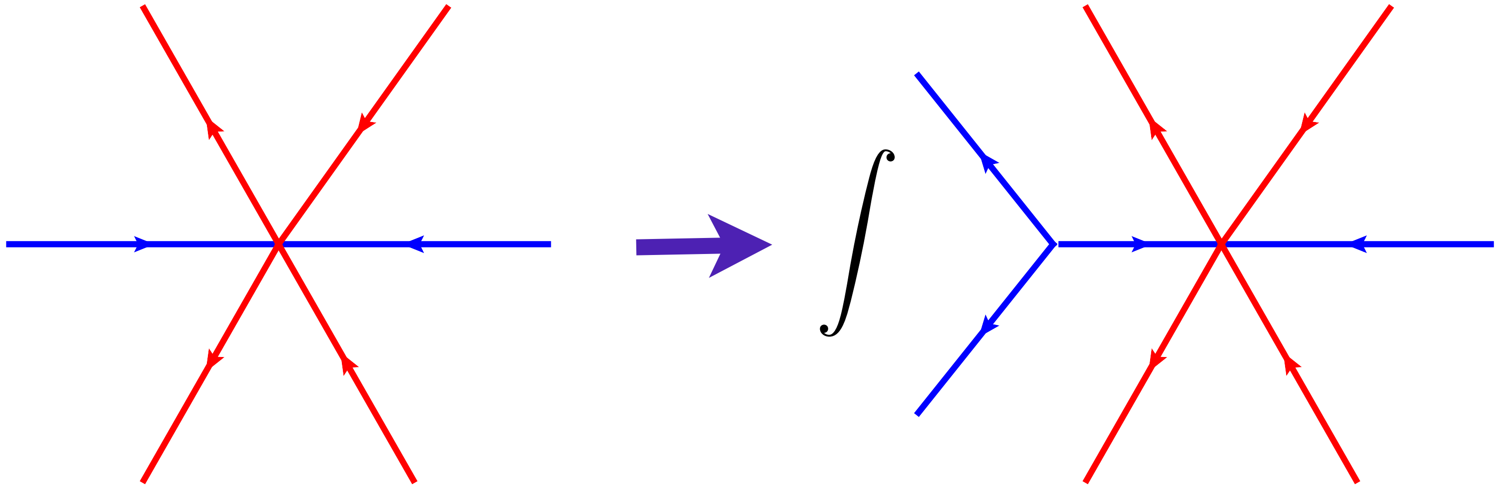
- Naively one might expect the MEM to be impossible at NLO.
- This is because NLO calculations include two sorts of contributions which live in different phase spaces.
- The virtual (loop) diagrams can easily be incorporated into the method, since they share the same phase space as the Born.
- The issue lies in the generation of the real phase space, which contain one extra parton. We need to define a map for these to a Born topology.





The Forward Branching phase space

- To overcome this difficulty, we use the Forward Branching Phase space proposed in : 1106.5045 (Giele, Stavenga, Winter).
- This generation technique allows one to create emitted radiation without changing the final state particles, in this way the final state particles are fixed.
- We integrate over all emissions and as a result cover all of phase space.



- We have implemented this into a new program NLOME based upon MCFM (Campbell, Ellis, CW).

How does the Forward brancher work?

- We wish to factorise the phase space as follows:

$$\frac{1}{2s_{ab}} d\,\Phi_1^{[D]}(p_a + p_b \to Q + p_r) = \frac{1}{2s_{ab}} d\,\Phi_{\mathrm{FBPS}}^{[D]} \times \left[\frac{2\pi}{2\widehat{s}_{ab}} d\,\Phi_1^{[D]}(\widehat{p}_a + \widehat{p}_b \to Q) \right] \ .$$

- To branch particle b, we boost a, such that momentum conservation requires b to have a virtuality.

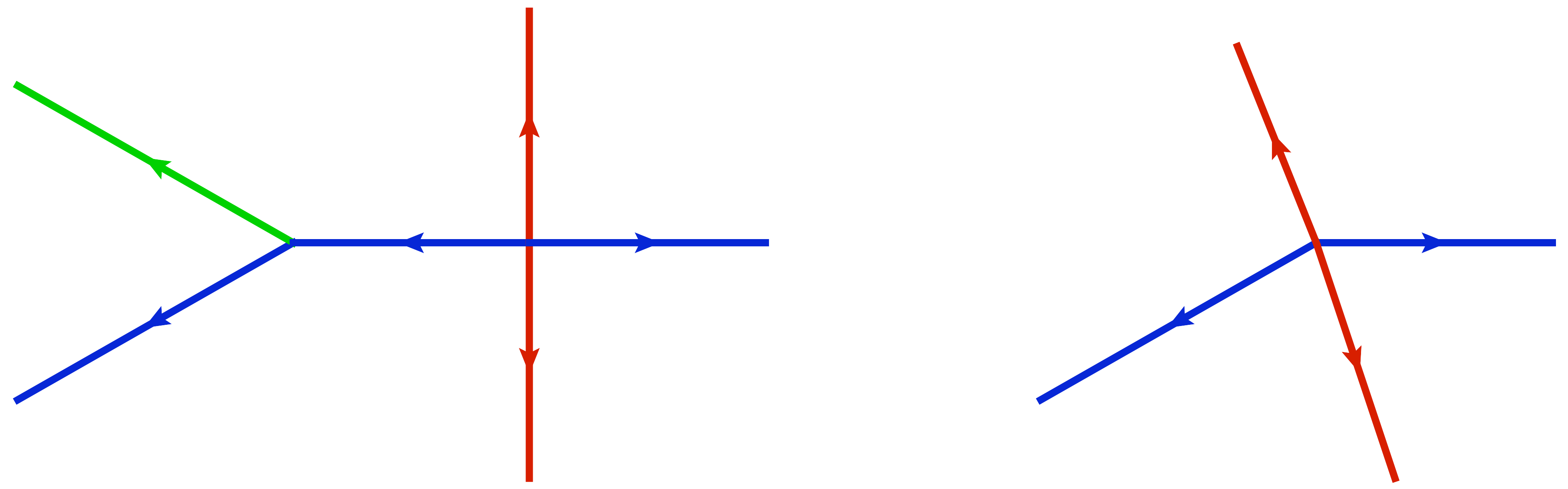
$$\begin{cases} \tilde{p}_a = (1+\beta)\,\widehat{p}_a \\ \tilde{p}_b = \widehat{p}_b - \beta\,\widehat{p}_a \end{cases} ,$$

- Now one can decay this particle into pb and pr. The resulting phase space then factorizes as follows:

$$d\,\Phi_{\mathrm{FBPS}} = \frac{1}{(2\pi)^3} \left(\frac{\widehat{s}_{ab}}{s_{ab}} \right) d\,t_{ar} d\,t_{rb} d\,\phi \ ,$$

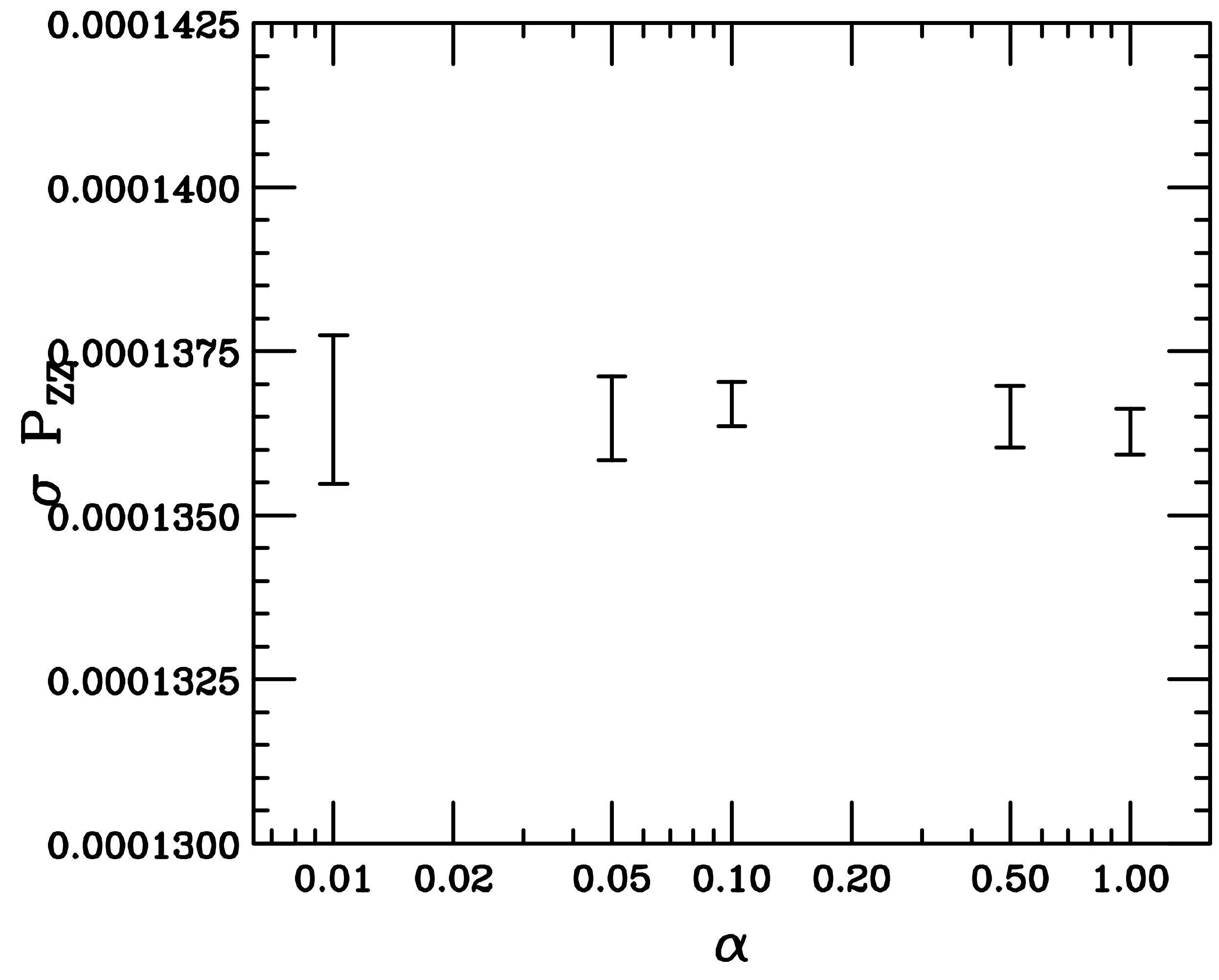
Technical details: Dipole subtractions

- Normal implementation of Catani Seymour dipoles for initial-initial singularities requires a Lorentz transformation on the final state whilst keeping the initial state fixed (up to a rescaling).
- This is bad for us, since this maps different born points to each other.
- Modify dipole phase space such that the map to the original born phase space point is preserved.

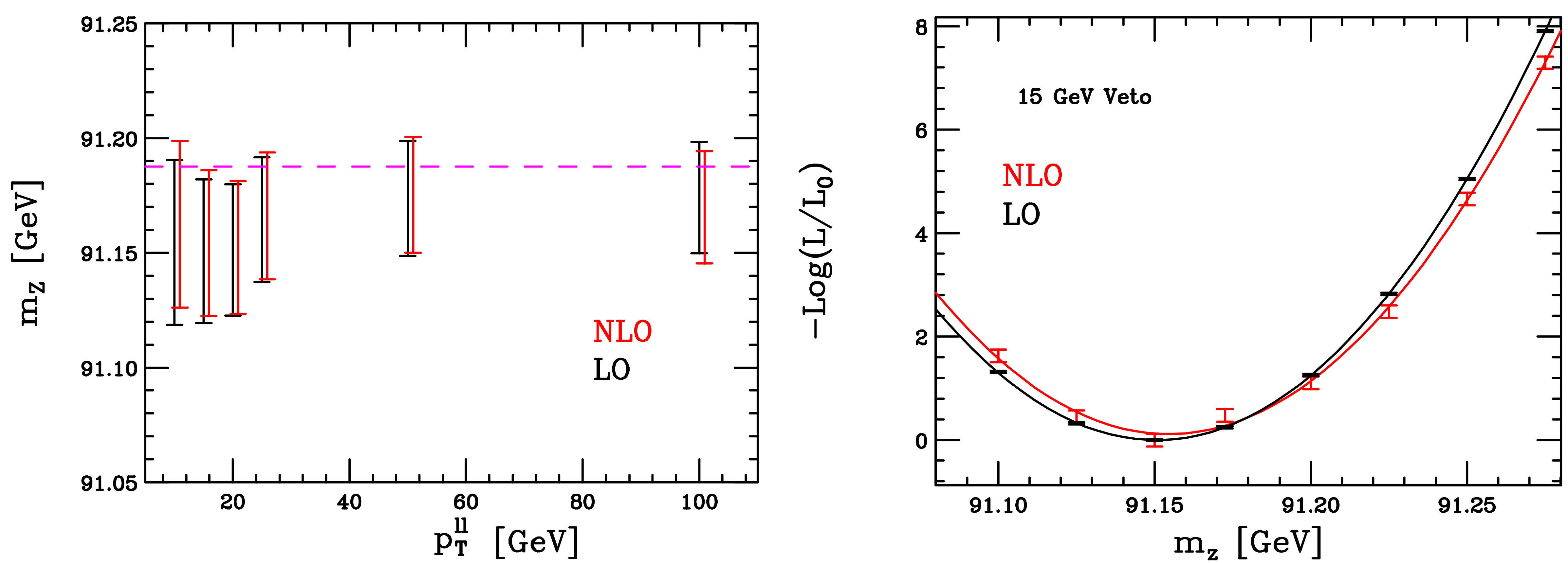


Validating the new dipoles

- Ultimately alpha independence tells us that the code is independent of the dipole subtractions.
- Plot here shows alpha (in)dependence for a single phase space point.



Validation: Measuring the mass of the Z



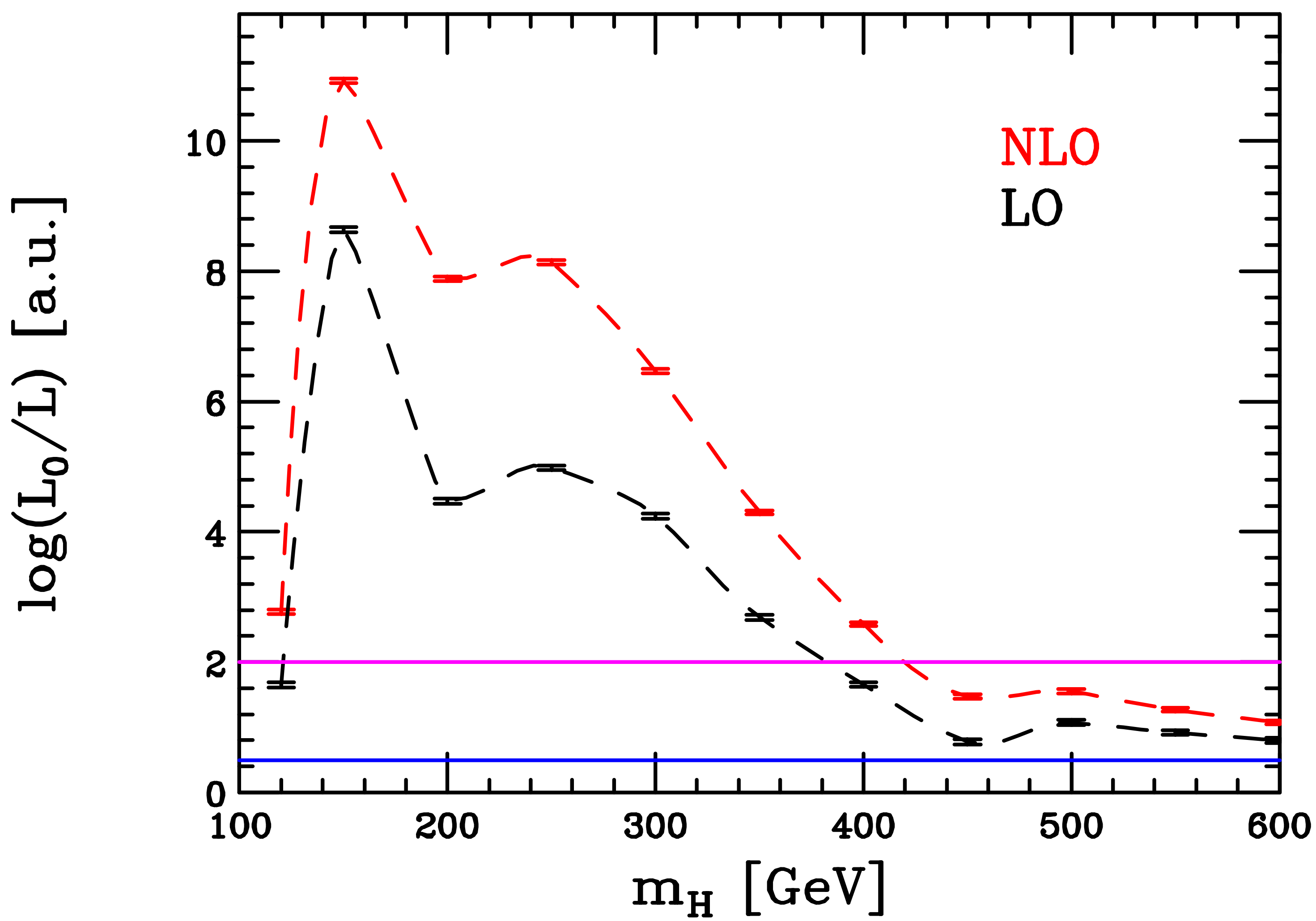
We generated ~5000 (~0.1 fb-1) events using Pythia, applying simple lab frame cuts.

Tests boosts and stability of method, independent code used to generate events

NLO effects are small, which is expected given the physics under investigation.

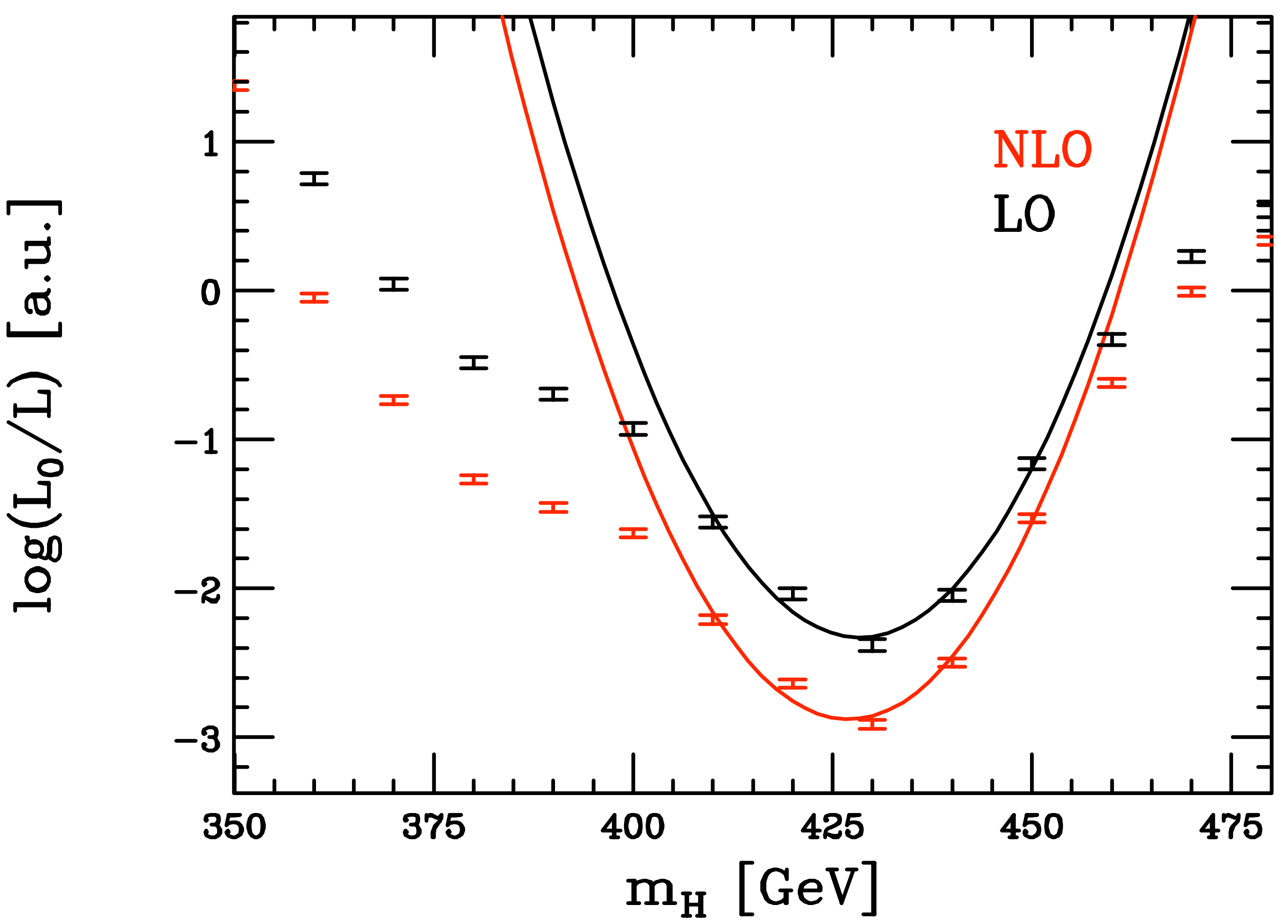
$$\log \mathcal{L}(n\sigma) = \log \mathcal{L}_{max} - n^2/2.$$

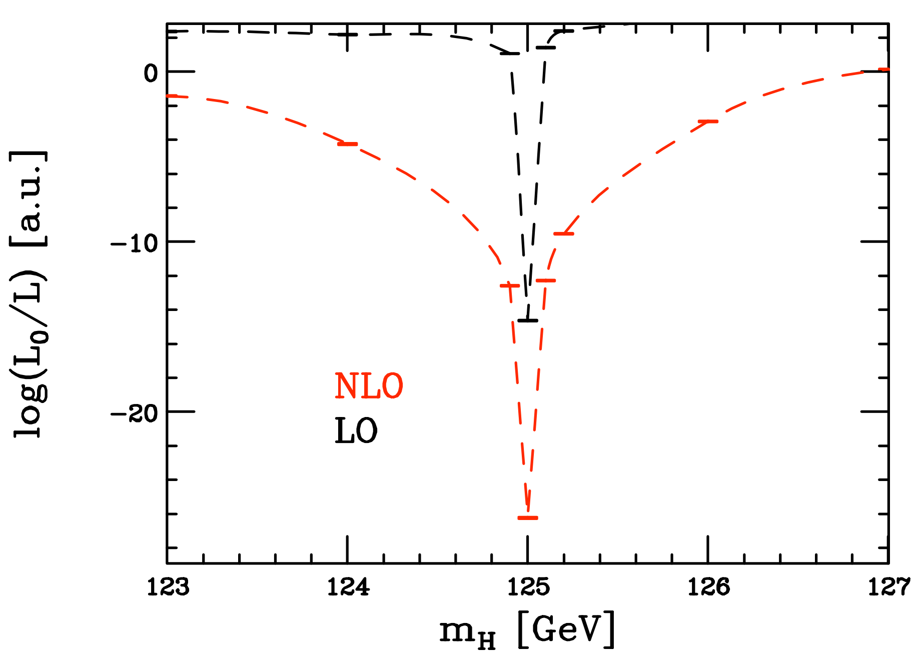
Example: Setting Limits on the Higgs



- A more interesting example is to use the MEM to constrain the Higgs
- We generate ~ 250 (~7fb-1) ZZ->4l unweighted NLO events.
- The background spectrum contains everything in MCFM (e.g. gg->ZZ, singly resonant Z..).
- NLO is able to set better limits since it is a better fit to the background.

Example: Higgs mass measurements.





- The MEM can also be used to measure the mass of the Higgs.
- For a light Higgs the MEM is dominated by the transfer functions.
- For a heavy Higgs the MEM is able to provide a best fit mass (427+/- 14 NLO), (428 +/- 14 LO) with a handful of events.
- Both NLO and LO get the correct mass (minimum of the fit) but NLO deviates more from the background only hypothesis (0)

Conclusions.

- We have illustrated how the MEM can be theoretically well defined at all orders, presented simple examples at NLO of H->4l and Z->ll

- In order to define a fixed order weight for an experimental event one must boost to a frame in which the final state is balanced.

- Since a given boost is not unique, we must integrate over all equivalent boosts, the Matrix Element doesn’t care but the PDFs do.

- Our approach does not change the experimental input (transfer functions).

Future study

We are keen to extend the method to other measurements, in particular....

- Measurement of the top mass at the LHC and Tevatron (flagship application of the MEM).
- Higgs in other channels, associated production, two photons etc. Confirming SM properties, BR, spin etc.
- Measurement of/Limits on triple anomalous gauge couplings.
- ....

We gladly welcome experimental input! Beta code of NLOME is available, first release expected in April/May. Thank you to experimentalists who have helped so far!